Optical Cavity Inference Techniques for Low Noise Interferometry

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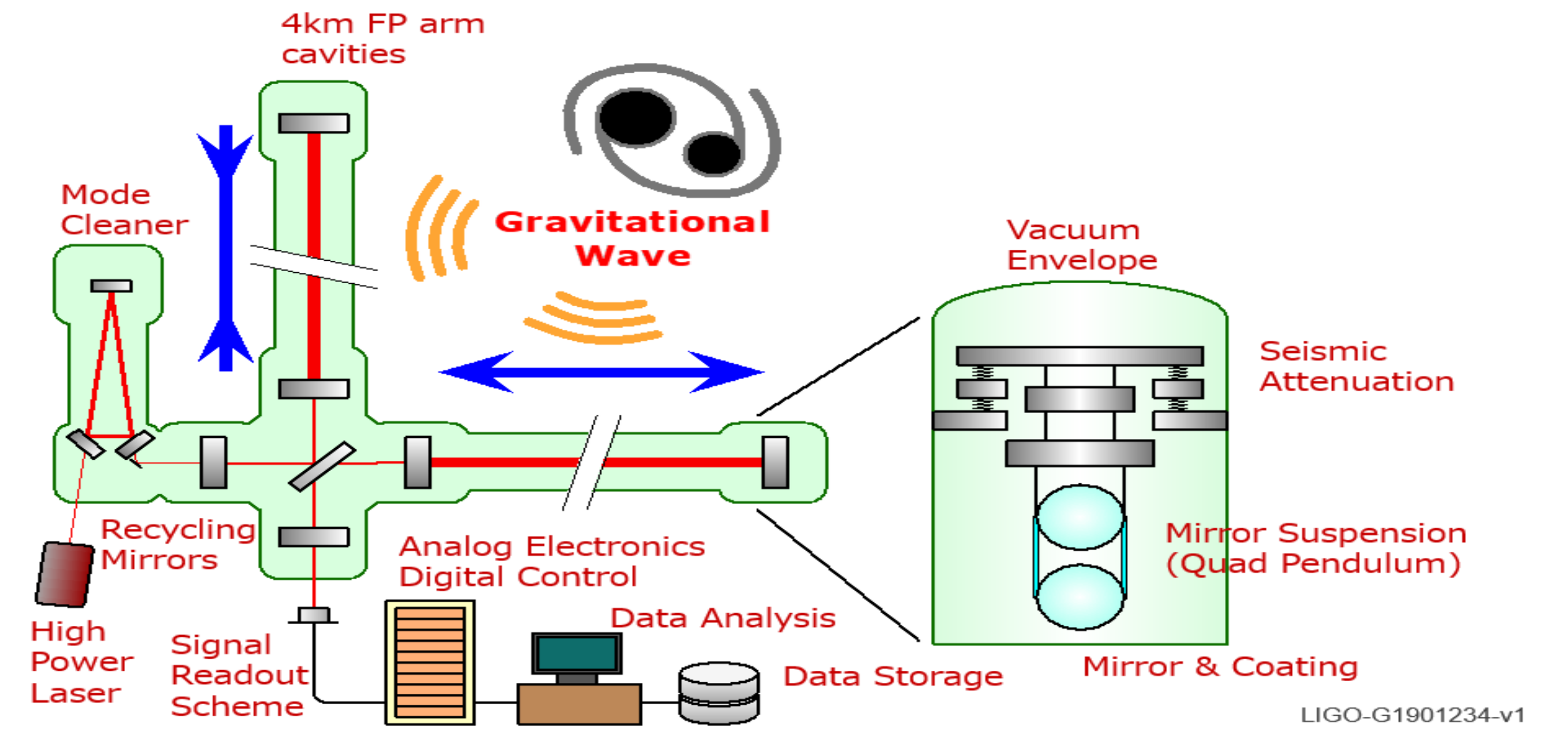
The **LIGO Scientific Collaboration (LSC)** is a global research collaboration dedicated to the detection and analysis of gravitational wave events observed using laser interferometers combined with powerful data analysis tools and simulation software. LIGO currently has 3 interferometers (with more planned) to make their observations into the universe. Each detector is equipped with 4km vacuum chambers with lasers and optical components that allow them to detect ripples in spacetime from gravitational waves inducing a strain as small as $\frac{\Delta L}{L} = 10^{-21}$!

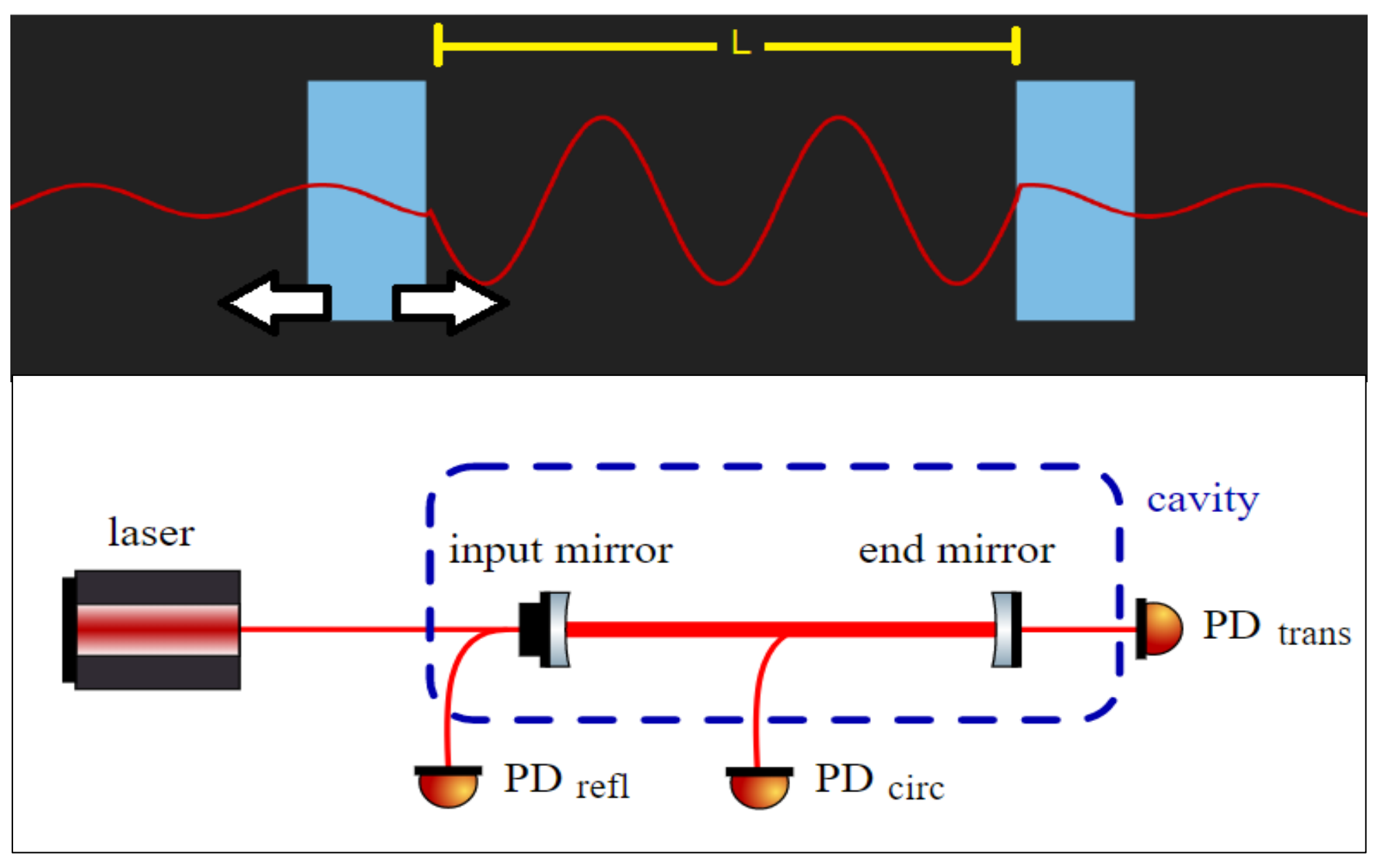


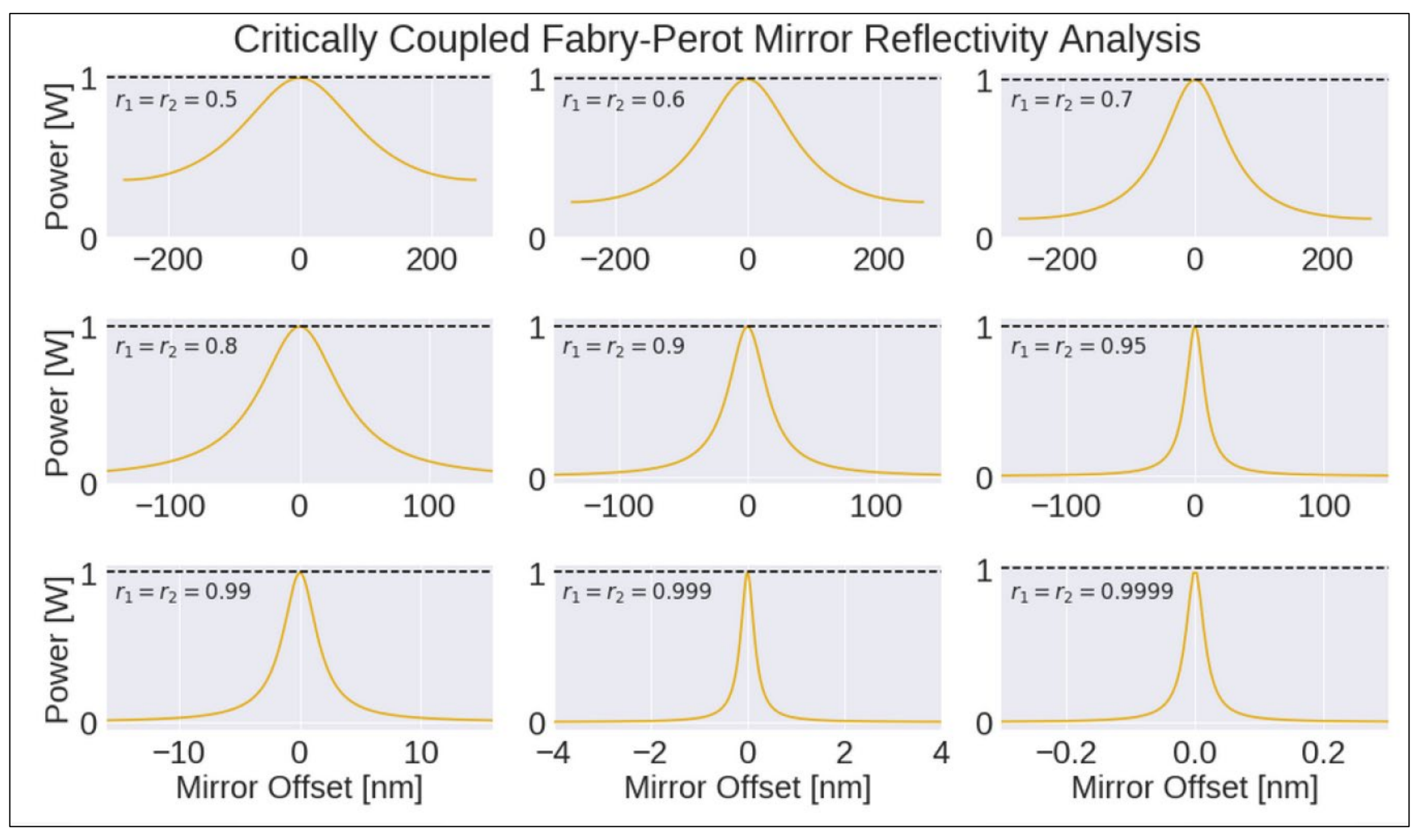
A diagram detailing the various components of the Advanced LIGO interferometer. A gravitational wave will cause one interferometer arm to shrink and expand opposite to the other, causing a phase shift in the beam circulating in the cavities.

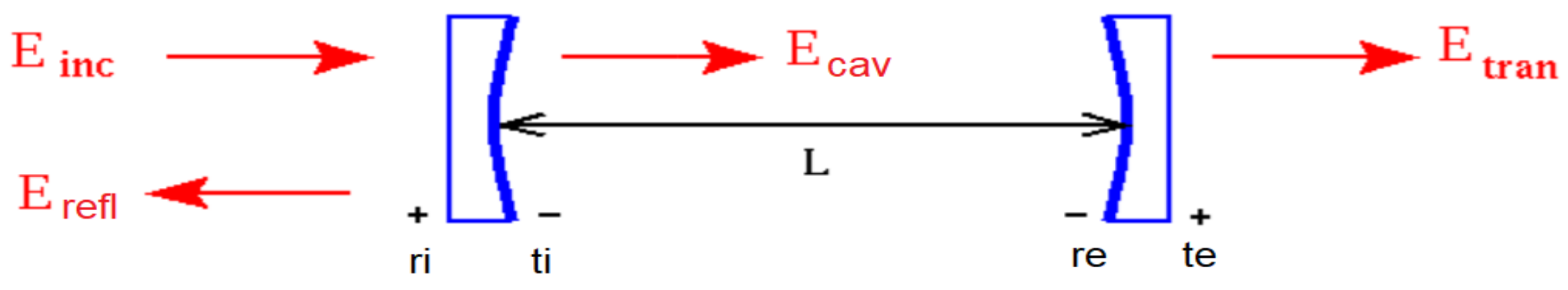
Two aerial photos of the two largest LIGO detectors, each sending a laser down a 4km vacuum chamber with mirrors at each end. The mirrors multiply the path length of the laser, transforming the 4km cavity to one that is effectively 1120km.

What is a Fabry-Perot Cavity?

A Fabry-Perot cavity is a special configuration of mirrors that maximizes the response of an optical system to a change in frequency **OR** wavelength of an input laser. This extreme sensitivity is due to the resonance condition of the cavity. The electric fields will transmit through the cavity and build up inside only if the mirror separation **L** is exactly an integer number of half wavelengths (i.e. $L = \frac{n\lambda}{2}$).







$$E_{tran} = E_{inc}\frac{t_i t_e e^{2i\phi}}{1 - r_i r_e e^{2i\phi}}$$

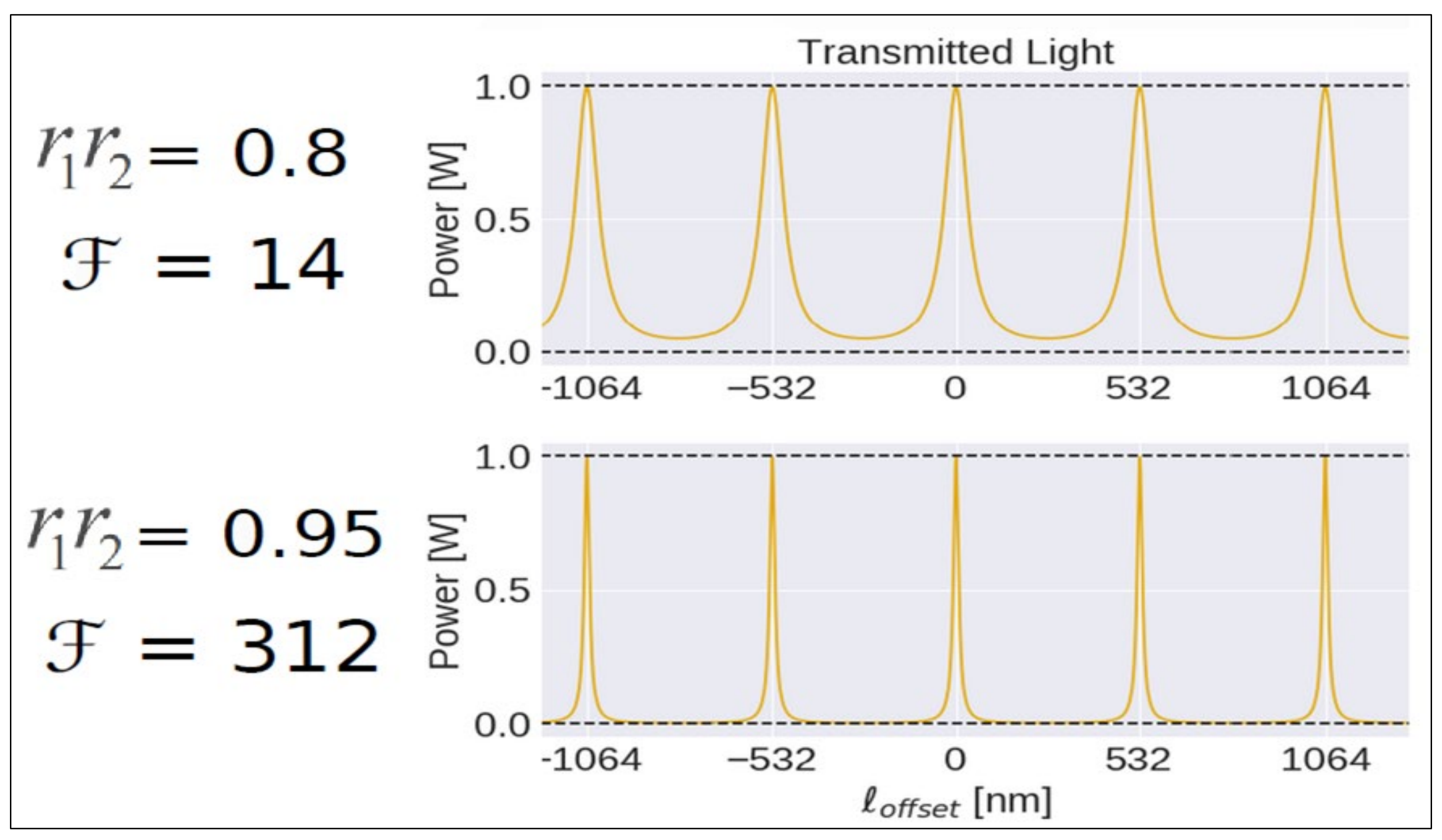
$$E_{cav} = E_{inc}\frac{t_i}{1 - r_i r_e e^{2i\phi}}$$

$$E_{refl} = E_{inc}\left[r_i - \frac{t_i^2 r_e e^{2i\phi}}{1 - r_i r_e e^{2i\phi}}\right]$$

$$\mathfrak{F} = \frac{\pi\sqrt{r_1 r_2}}{1 - r_1 r_2}$$

Why use a Fabry-Perot at LIGO?

A Fabry-Perot cavity will only allow a certain range of wavelengths to pass through. A cavity with high finesse will have very narrow windows of "acceptable" wavelength, reflecting all those that fall outside that window. Offsets in the wavelength of a laser from an incident GW event that are less than a tenth of a nm is enough to have a difference in the two interference patterns of both arm's Fabry-Perot cavities. LIGO then analyzes the difference in patterns as their data signal.

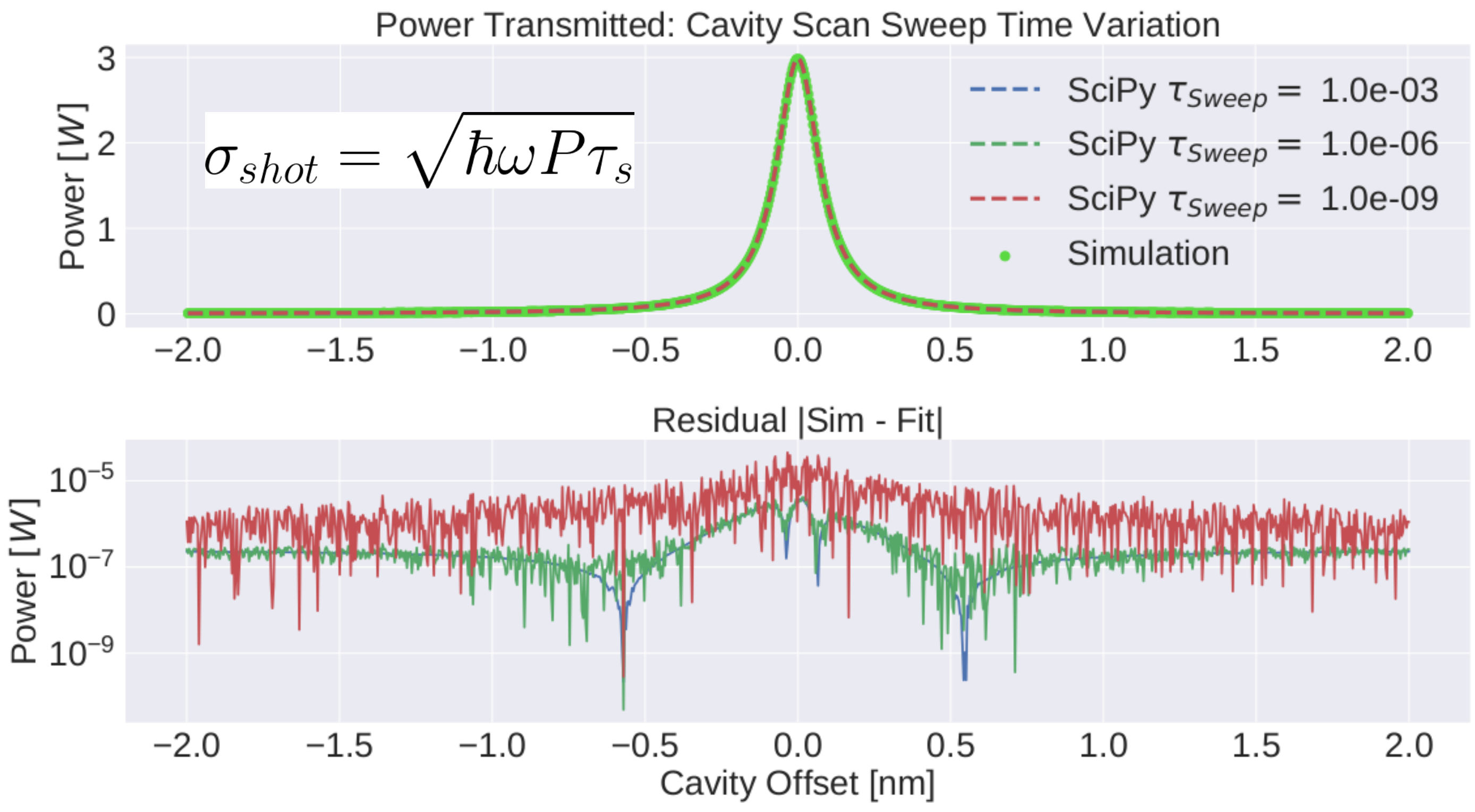


What limits the sensitivity of our measurements?

Utilizing the Fabry-Perot cavity at LIGO to take advantage of its extreme sensitivity to length changes means we must isolate the cavity itself from outside noises that can affect the length.

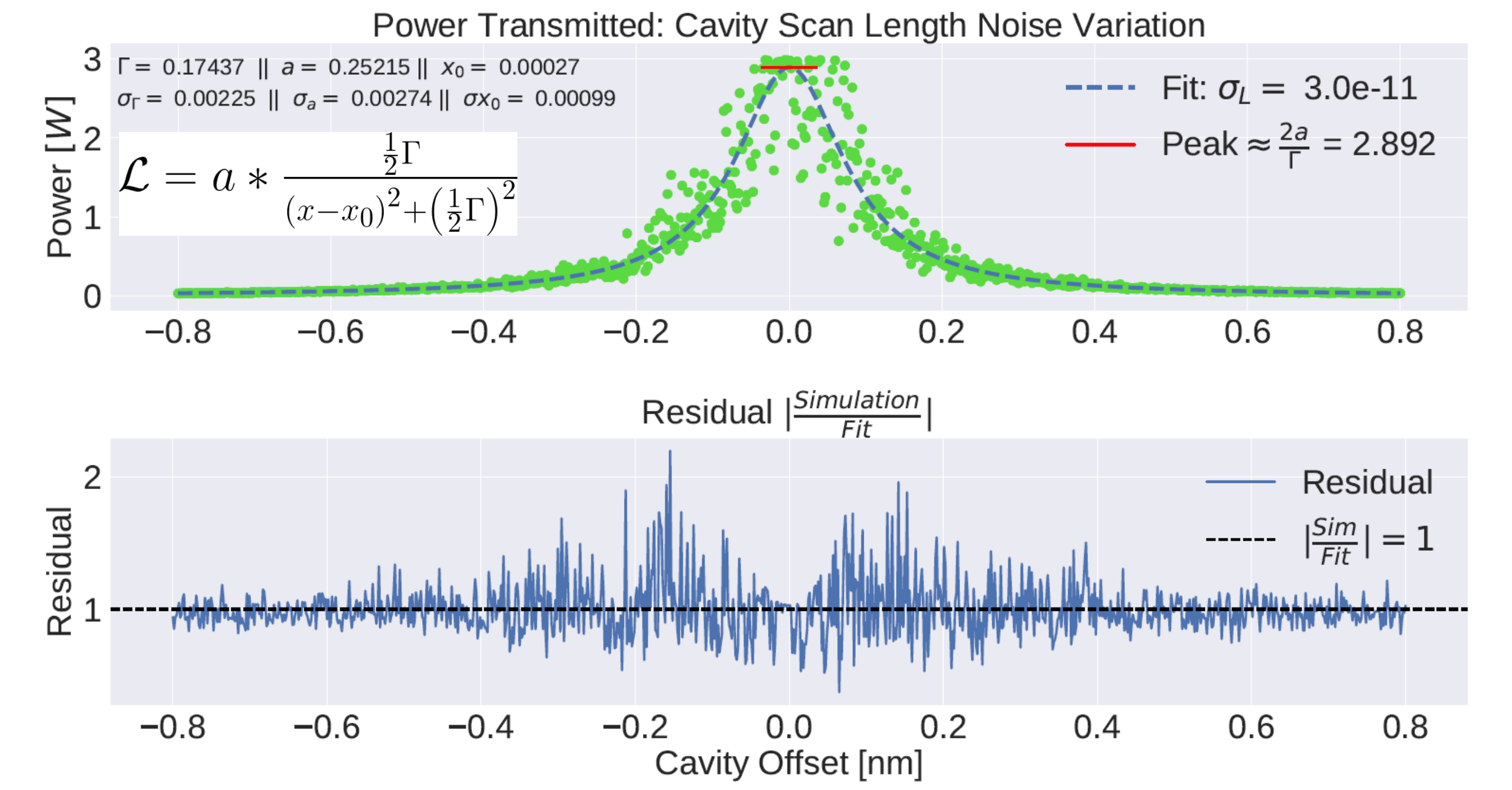
- Seismic Noise – vibrations in the ground supporting the detector
- Thermal Noise – thermal wiggling of the matter composing the mirrors
- Laser Noise – "dirty" laser beams whose purported frequency can wander
- Shot Noise – poissonic nature of counting photons leading to discrepancies
- Length Noise – "catch-all" term for unwanted Gaussian noise in movement of mirrors from imperfect servo motion while scanning cavity frequencies.

Shot Noise Simulation



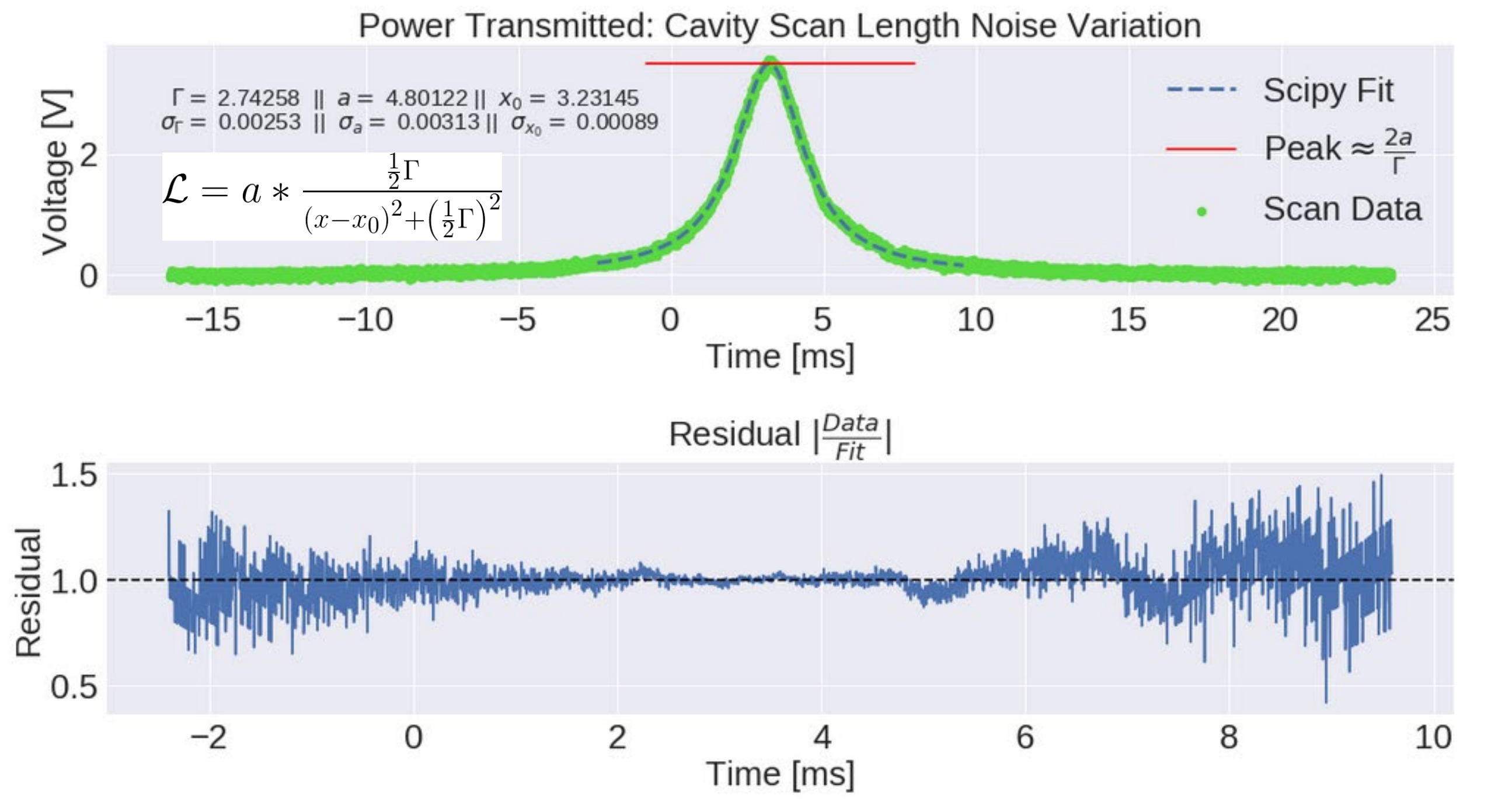
Three shot noise simulations overlaid each with their own scanning speed to show the effect of shot noise compared to a noiseless simulation.

Length Noise Simulation



A simulation with length noise added to each data point of about $\frac{3}{100}\,nm$ which is much higher than expected in a laboratory setting.

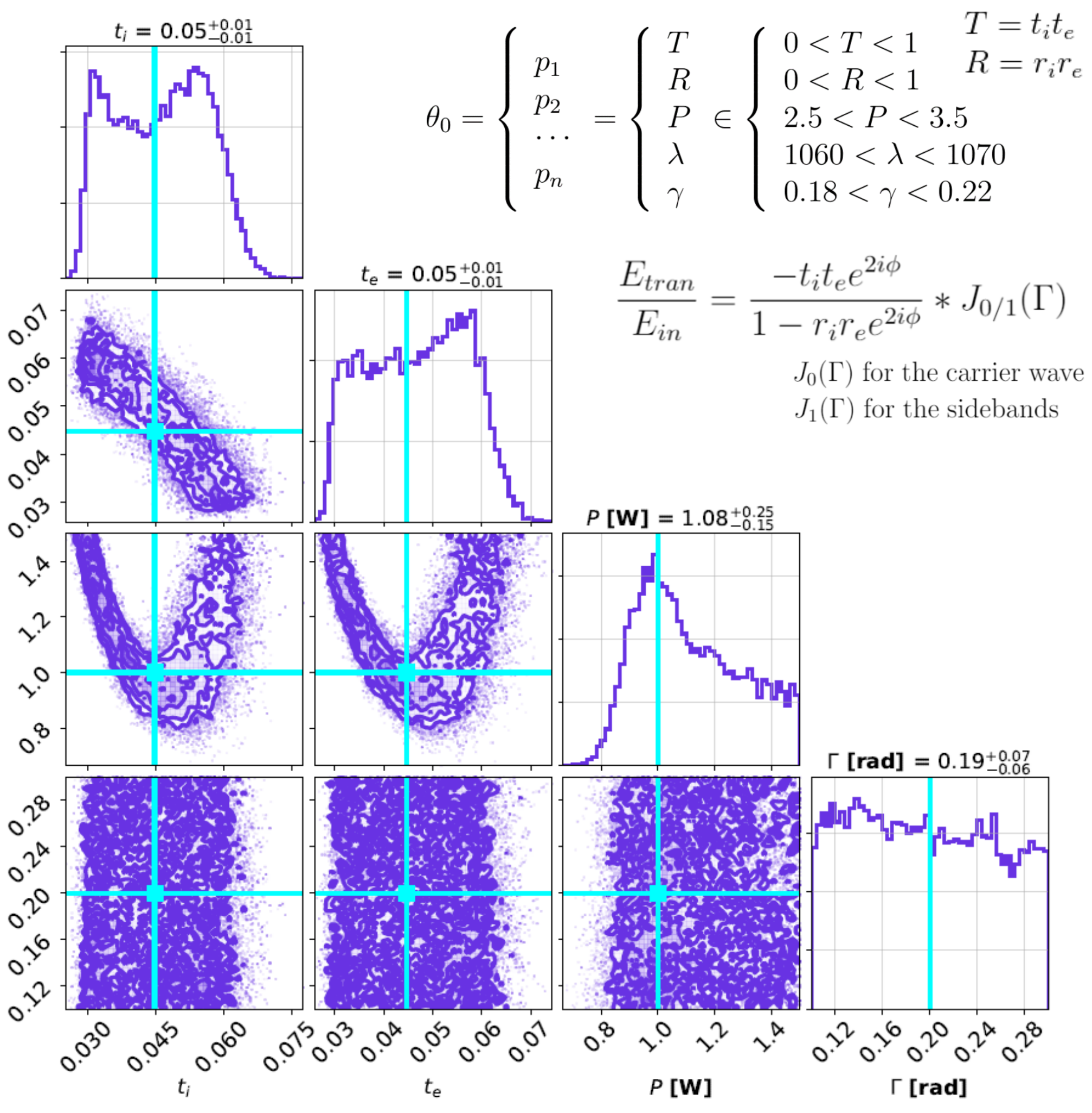
Real Cavity Scan



An actual cavity frequency scan run on May 8th, 2019, with the relatively low uncertainty around the peak that then increases as you move away from resonance.

Monte Carlo Markov Chain Fitting

Python curve fitting routines often overfit or underestimate uncertainties, so there was a desire to utilize MCMC techniques to create fits to real data and compare them. We used the MCMC technique to fit simulated transmitted electric fields by introducing a parameter space, added bounds for parameters, and assigned likelihood functions based off the physical equations of the Fabry-Perot cavity. We then let a thousand "walkers" take random steps in this parameter space until they converged to a fit with a high likelihood.



$$\theta_0 = \begin{cases} p_1 \\ p_2 \\ \dots \\ p_n \end{cases} = \begin{cases} T \\ R \\ P \\ \lambda \\ \gamma \end{cases} \in \begin{cases} 0 < T < 1 \\ 0 < R < 1 \\ 2.5 < P < 3.5 \\ 1060 < \lambda < 1070 \\ 0.18 < \gamma < 0.22 \end{cases} \quad \begin{matrix} T = t_i t_e \\ R = r_i r_e \end{matrix}$$

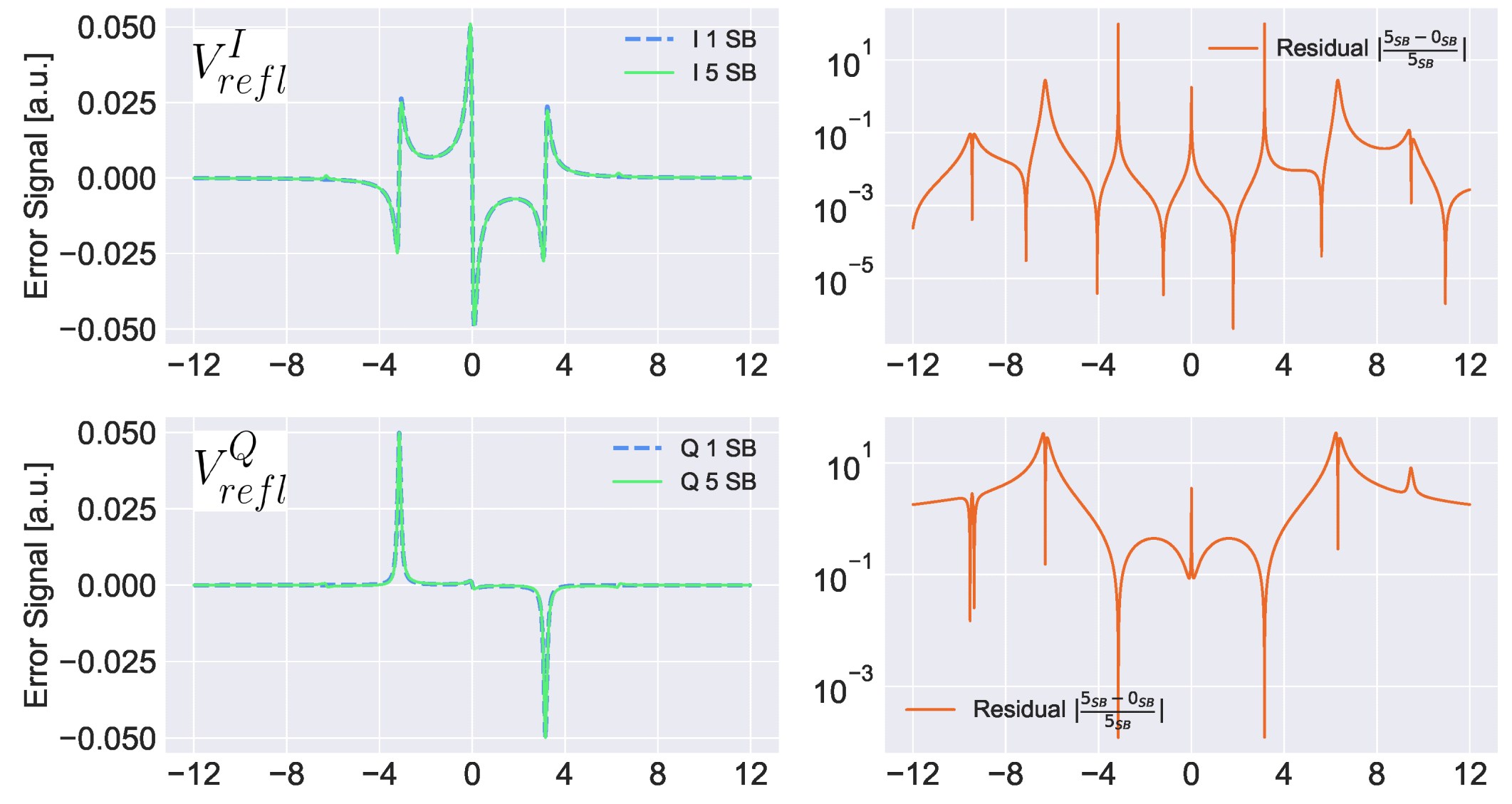
$$\frac{E_{tran}}{E_{in}} = \frac{-t_i t_e e^{2i\phi}}{1 - r_i r_e e^{2i\phi}} * J_{0/1}(\Gamma)$$

$J_0(\Gamma)$ for the carrier wave
$J_1(\Gamma)$ for the sidebands

How does LIGO deal with laser frequency noise?

In order to lock a cavity, the laser beam must be set exactly to the resonant frequency of the cavity such that it is ready to receive any distortions from a gravitational wave event. However, since a frequency shift is indistinguishable from a shift in wavelength, it is critical stabilize the laser frequency. Many layers of control systems exist, the most notable being the Pound-Drever-Hall. This is a technique where the incident laser is modulated and then these newly formed sidebands have frequencies that are not on resonance with the cavity, are reflected, and then measured and corrected.

Error Signal: PDH w/ and w/o Sidebands



A numerical simulation showing an example of a PDH error signal.

$$V^I_{\text{refl}} \simeq J_0(\Gamma)J_1(\Gamma)\,\text{Re}(r^*_{\text{cav}}(\omega)r_{\text{cav}}(\omega+\Omega) + r^*_{\text{cav}}(\omega)r_{\text{cav}}(\omega-\Omega))$$

$$V^Q_{\text{refl}} \simeq J_0(\Gamma)J_1(\Gamma)\,\text{Im}(r^*_{\text{cav}}(\omega)r_{\text{cav}}(\omega+\Omega) - r^*_{\text{cav}}(\omega)r_{\text{cav}}(\omega-\Omega))$$

$\Omega =$ Modulation Frequency
$\Gamma =$ Modulation Depth

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