LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY
- LIGO -
CALIFORNIA INSTITUTE OF TECHNOLOGY
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Technical Note	**LIGO-T1900279-v2**	2020/08/03

Filter Cavity Optics Substrates

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1 Specifications

1.1 Curvatures

Input Optic (FC1) Surface 1 (HR) Flat, ROC: ∞ ($\pm$250nm sagitta)

Input Optic (FC1) Surface 2 (AR) Curved, ROC: +1m ($\pm$5%)

End Optic (FC2) Surface 1 (HR) Curved, ROC: -534m ($\pm$2%)

End Optic (FC2) Surface 2 (AR) Curved, ROC: +1m ($\pm$5%)

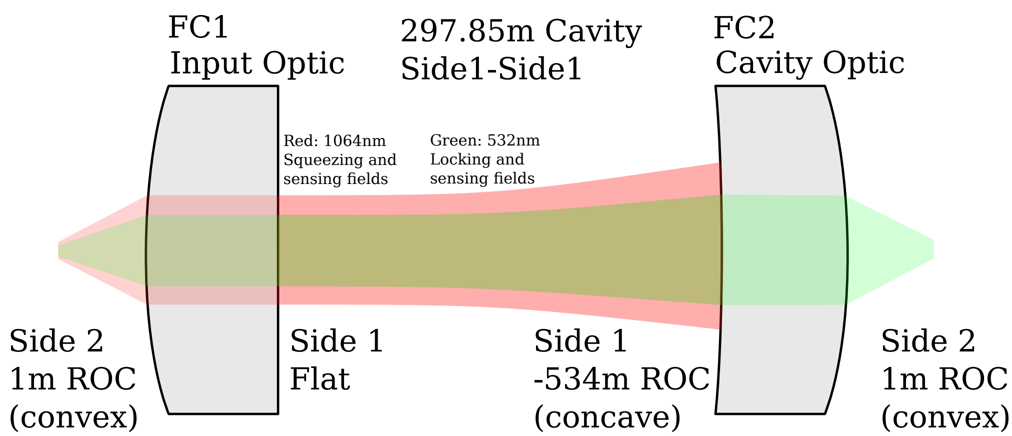

Figure 1: Cavity geometry and mirror specification. The strong side-2 AR-surface curvatures are to reduce the beam size for relay optics.

1.2 Optic Size

The substrate dimensions are for mounting in the HSTS suspension. The specification is 150mm diameter, 75mm thickness as measured from the thickest point when a wedge is included. Related optic drawings are the signal and power recycling mirrors, D0901174 or D0901175 which are also mounted in HSTS suspensions (note that the filter cavity optics will have different surface curvatures than either of these drawings). The following sections indicate that, unlike the SRM and PRM optics, the FC optics do not need to include wedges. Furthermore, the strong AR lens requires that the optic thickness be adjusted to account for the volume of the curved face.

1.3 Wedge

Due to the inclusion of strong lenses on the side-2 surfaces, these optics will not include wedges.

The filter cavity has substantially lower incident light levels on it to cause scatter issues of similar (SRM, PRM) optics. Any direct AR reflection from the convex lensing surface will rapidly diverge.

There are two reasons to avoid a wedge. The first reason is that the 1m ROC indicates that the beam will experience a strong wedge if it is displaced slightly so a 1deg wedge may be implemented by displacing the beam by 17mm. This is approximately two beam radii as of FC1.

Secondly, both 532nm light and 1064nm light are incident on the filter cavity. The index of the glass is 1% different at the two wavelengths, and so the two beams will be deferentially tilted by 82uRad. This is large compared to the beam divergence of 67uRad. Because this ratio is order-1, when the beams are 90deg Gouy phase from the FC1 optic, the 532 and 1064 beams will be displaced by approximately 1 beam radius, causing a potential for clipping and difficulty co-aligning the beams at the two wavelengths.

1.4 Thickness

The edge thickness is adjusted for the optics so that they are suspended the same within the HSTS as the wedged optics. The following calculations give the adjustment to the thickness from the optic curvature.

```
[radius of optic]
R = 0.075m
[ROC of S2]
r = 1m
[saggitta]
s = r - sqrt(r^2 - R^2)
[volume within curved region]
V = int_0^s pi (2 * r * x - x^2) dx = pi (r s^2 - s^3 / 3)
[edge adjustment depth]
d
[volume of edge adjustment]
V_std = pi * R^2 * d
[condition of matching adjustment to curved volume]
V = V_std

[narrower optic as measured from edge]
d = V / (pi * R^2) = s/2 = 1.408mm
V = 24.897 cc
s = 2.816mm

[narrower optic for unwedged case]
d' = d + 0.075 * 1/180 * pi = 2.716mm
```

This gives the edge width to be 72.3mm for the FC1 and FC2 optics, compared to the 75mm edges of other flatter HSTS optics with 1 degree wedges.

2 Substrates

These optics will not carry high power in either 1064 or 532 and so do not need low substrate absorption. The principle design requirement is on uncorrectable mode-matching losses from substrate index homogeneity of the input mirror (FC1).

The target substrate for quick procurement is Corning 7979 Class-0AA or Class-0A glass.

2.1 Homogeneity

Corning 7979 Class-0AA glass is specified for an inhomogeneity $\Delta n \leq 0.5 \cdot 10^{-6}$. This specification is the maximum deviation from the mean. Taking the specification at face value and assuming it would correspond directly to an RMS inhomogeneity, this would cause a wavefront mismatch loss after two passes through the substrate as large as

$$L_{\mathrm{mm}} \leq \left(0.075\mathrm{m}\Delta n \frac{4\pi}{n\lambda}\right)^2 \approx (4\pi \cdot 0.025 \text{ wave})^2 \approx (0.3)^2 \approx 10\%\text{loss} \quad (1)$$

However the spatial scales of the inhomogeneity are large compared to the beamsize, and as is typical for phase-map specifications. Furthermore, the RMS fluctuation is typically smaller than the peak-valley which is dominated at the edges.

Transmission interferometry of 100mm thick Corning 0A glass was performed on the initial LIGO ETMs, documented in T080085. These measurements indicate that in general, the RMS fluctuations in transmission phase maps are generally 3x smaller than the peak-valley. Furthermore, the Zernike polynomials of order ≤ 3 removed (uncorrectable phase deviations) are another factor ≈ 3 lower in RMS, achieving typically $\leq$ 0.03-Wave RMS deviation, corresponding to a wavefront matching loss of:

$$L_{\mathrm{mm}} \leq (4\pi \cdot 0.003\text{wave})^2 \approx 0.14\% \text{ loss} \quad (2)$$

This indicates that Corning IR-Grade 7979 Class-0AA or Class-0A glass should be sufficient to meet index homogeneity requirements of the input optic. The output optic may be a lower grade, as it will only be directed to photodectors and sensors, and will not be required to overlap any other defined mode.

Furthermore, alternative substrate materials such as Heraeus Suprasil do not specify index homogeneity below the $0.5 \cdot 10^{-6}$ level ("Material Specs" from L1300216). Given that no vendor specifies better homogeneity, whichever may deliver the fastest is prefered. If the specification cannot or is not met, the alternative option is for surface polish compensation of the wavefront as per sec. 4.2.2.4.2 (Transmission OPD errors) of T000127. Given that the FC1 optic is flat on side-1, Such a polishing technique may be feasible, but the metrology may not be feasible for an optic with a 1m radius of curvature.

2.2 ROC/Sagitta Tolerancing

As shown in the following section, the tolerances for sagitta error of the flat FC1 side1 flat surface and the relative error in the ROC of the FC2 side1, -534m is not strict. Not only are those errors correctable through mode-matching, as much as 500nm sagitta error on FC1-side1 or 5% ROC relative error on FC2 side1 will cause less than 0.25% mode overlap loss against the designed cavity mode. The choice of 2% tolerancing above was to keep the Gouy phase and ISC frequencies well-determined, but could be relaxed if needed. Below, the plots of the HOM frequencies shows the allowable tolerances. In practice, vendors can hit a specific curvature measurement rather precisely, but are limited by the calibration error in their Fizeau interferometer.

3 Choice of Mirror Parameters

The choice of mirror parameter is driven by the requirements of T1800447. This section provides a short summary of how the chosen parameters meet the following requirements:

- Beam sizes must be appropriate to minimize loss on the mirrors.
- Nearest relay optic feeding the squeezed beam is 44in from FC1 Mirror.
- Gouy phase must give acceptable alignment sensing/cavity stability.
- Gouy phase must not alias high-order modes with the fundamental.
- Gouy phase must not alias high-order modes on useful ISC frequencies, primarily the CLF frequency used for the squeezer, but also the 40MHz used for green sensing.

The beam size requirements as a function of the possible curvatures for a 297.85m cavity are provided by fig 2. The vertical dashed magenta line is the chosen 534m ROC. The Gouy phase of the cavity and its placement of high order modes is shown in fig. 3 and its zoomed version fig. 4. The aliasing study (second panel in those figures) takes the fractional phase of the HOM, multiples by the mode order, and then takes that number modulo 1.0. This is done for all modes up to a given order and the smallest remainder term is plotted. This shows which mode approaches the closest to the HG00 mode and could alias in frequency. The aliasing distance should be large compared to the inverse of the Finesse in each mode. The finesse of the HG00 mode is high (≈ 5000), but clipping loss will be dominant for modes order ≥ 5, and so they will have reduced finesse.

3.1 Cavity Length

The specific cavity length of 297.85m scales the mirror ROC's, and is only weakly driven by the ROC tolerances. It is primarily constrained by the existing 3.125MHz CLF control frequency to not potentially alias on the HG01 frequency, which, while unlikely, could cause considerable ISC issues. To determine the chosen length, a number of lengths and ROC's were considered to simultaneously meet all criteria. The plots of Fig 5, using frequencies of table 1

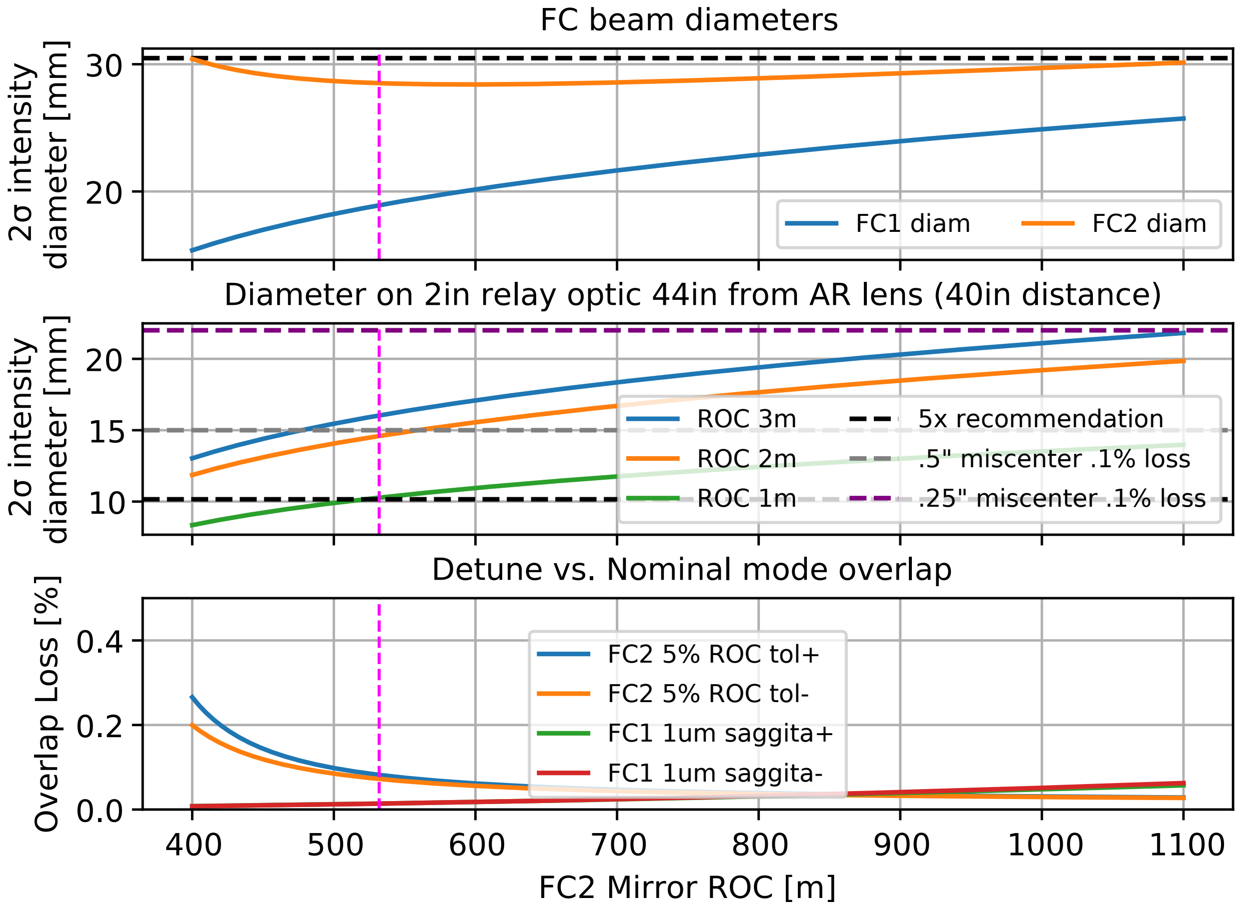

Figure 2: Beam sizes and tolerances for a 297.85m filter cavity. Top: Beam sizes for the 1064 Squeezed beam on each FC Mirror as a function of the FC2 radius of curvature for a 297.85m cavity. Mid: The beamsizes on the nearest input relay optic as a function of the FC1-side1 lens curvature. Limits for this size (dashed) are parameterized by the allowable miscentering on the optic for loss. Bottom: tolerancing for the ROC and sagitta of the FC mirrors from effective uncertainty in the cavity mode (correctable).

RF9	9.099471MHz
RF45	45.497355 MHz
RF40	40 MHz
RF80	80 MHz
CLF	3.125 MHz
FCLF(A)	3.019564 MHz
FCLF(B)	3.522825 MHz

Table 1: Table of used ISC frequencies for fig 5

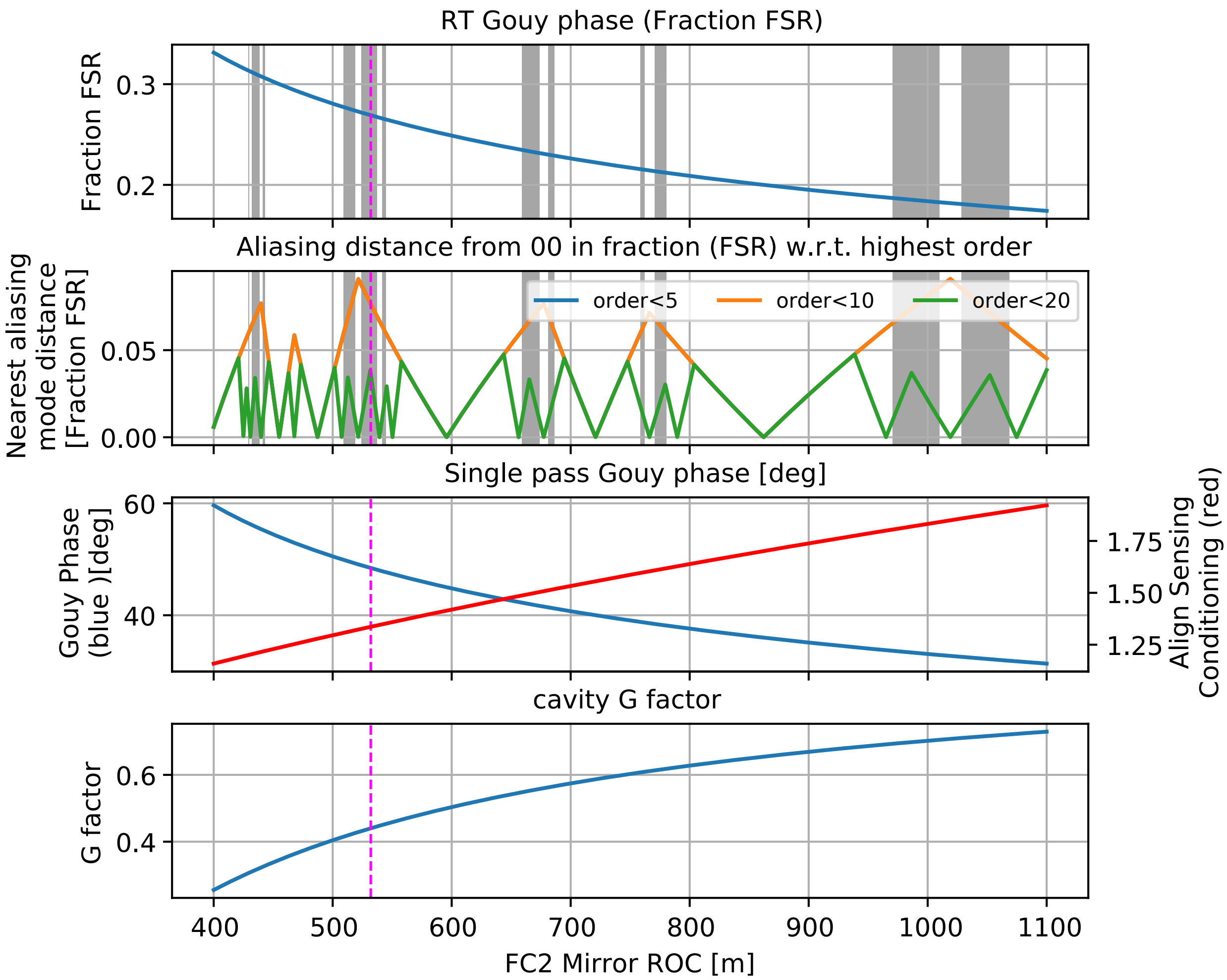

Figure 3: Gouy phase features of 297.85m filter cavity. Top: The Gouy phase as a fraction of the FSR. Second: the aliasing distance of high-order modes to the HG00 in fractions fSR. Third: The single pass Gouy phase useful for alignment sensing, as well as the conditioning of alignment sensing matrix inversion indicating the noise enhancement factor for the worst-sensed DOF. Bottom: The G-factor of the cavity.

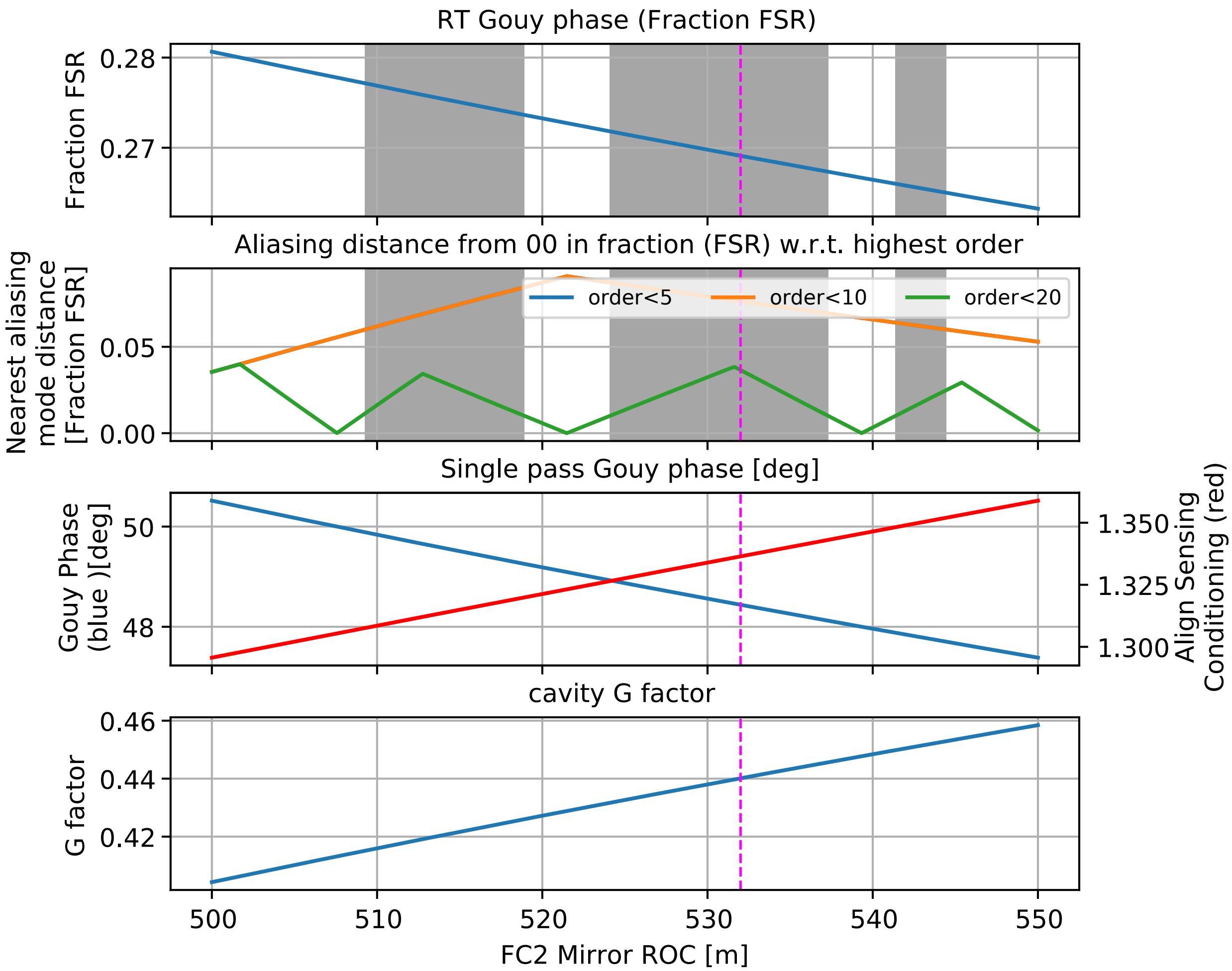

Figure 4: A zoom of fig 3, to show local detail of the aliasing distance. The nearest aliasing mode up to order 10 is of order-7.

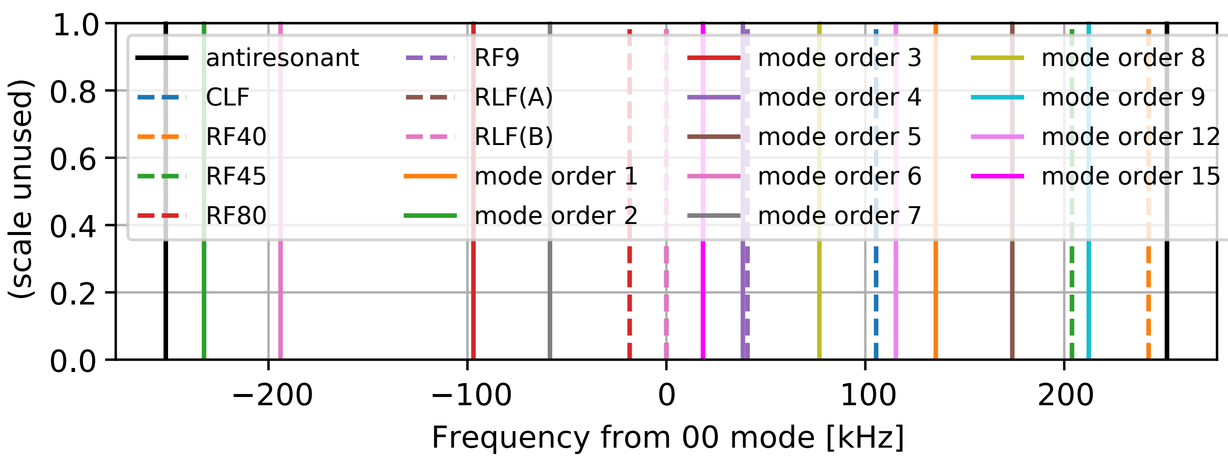

Figure 5: ISC Frequencies and aliased HOM frequencies to show separation and usability of all convenient frequencies. In particular the existing 3.125MHz line used for the squeezer is well separated from all low modes.

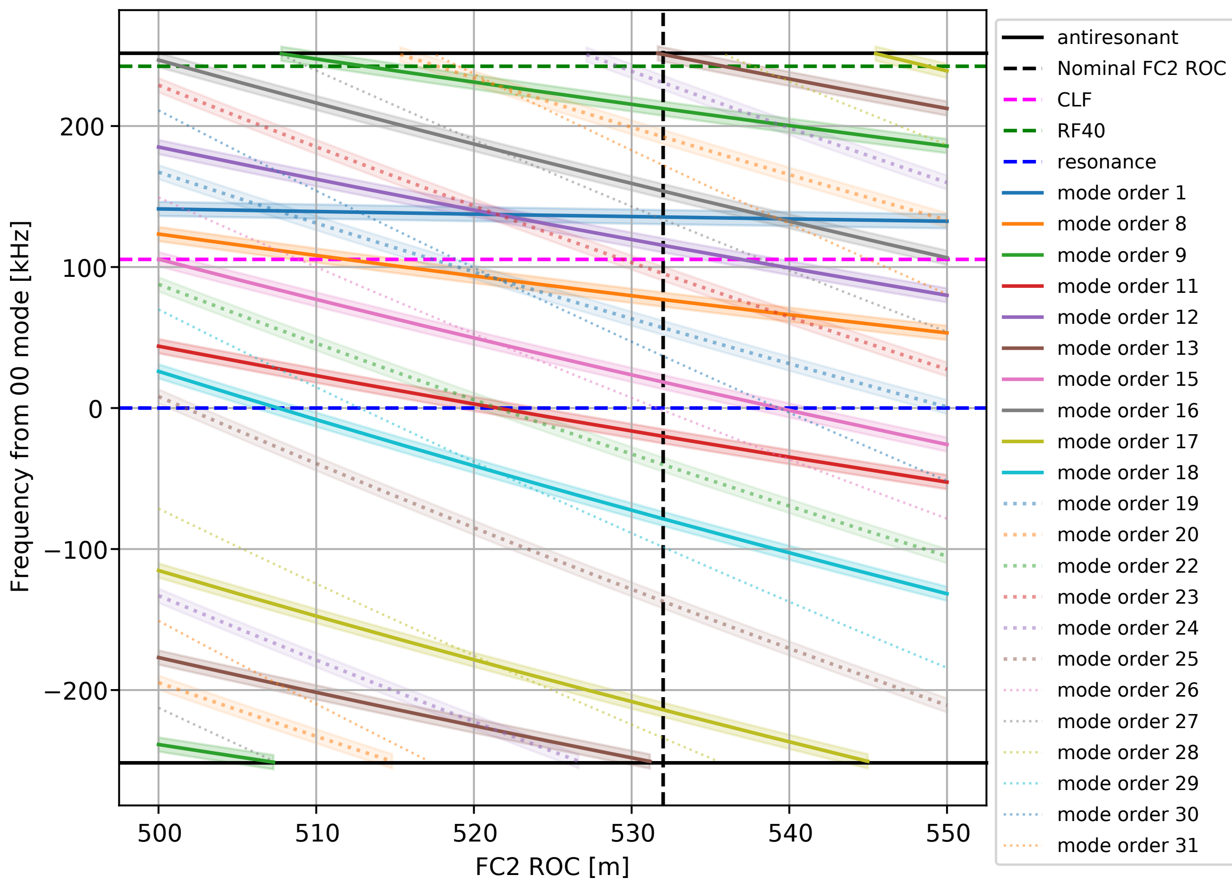

Figure 6: HOM Scan diagram with fundamental, CLF, and 40MHz frequencies, as well as HOM's that cross them within the given X-axis. Modes not crossing either frequency are suppressed. The errorbars reflect a round-trip phase shift of $1\% \cdot \lambda/2 \approx 5$nm w.r.t the fundamental mode, which is due to mirror surface fluctuations that affect HOMs differently than the fundamental. For a nominal FC2 ROC in the region 532m-536m, the nearest high order modes have a Gouy phase shift of 15x the fundamental, corresponding to HG modes summing to $n + m = 15$. The second nearest HOMS are order 26 and 30.

3.2 Aperture Masking

The 16m prototype cavity has observed glitches in its squeezing and control fields from aliasing high-order modes. This occurs because the cavity is high finesse and the mirrors are substantially larger beam size, so even very high order modes have low clipping losses. As the beam and mirrors move, high-order-mode resonance frequencies are perturbed and drift, causing them to cross the fundamental. For A+, the glitches should be reduced from improved alignment control, but if still present would still create a strong noise coupling mechanism.

For the A+ cavity, the relative size of the optic face to the beam is about a factor of 5. This is less than the 16m prototype, but still a large ratio. The mode-scan of figure 6 suggests that modes of order 13-15 should be suppressed to remove any possibility of aliasing. The design challenge is to add an aperture mask to the coating which sufficiently attenuates the high order modes without creating losses or noise couplings in the fundamental, even assuming some miscentering of the fundamental mode.

Figures 7 and 8 plot the cumulative intensity of transverse and radial transverse modes, as well has the masking locations for a radial aperture. The HG modes are calculated from:

$$u_{\mathrm{HG}}(x;n) = \left(\frac{2}{\pi}\right)^{1/4}\sqrt{\frac{1}{2^n n! w}}H\left(\frac{\sqrt{2}x}{w};n\right)e^{-\frac{x^2}{w^2}} \tag{3}$$

$$\mathrm{CDF}_{\mathrm{HG}}(x;n) = \int_{-\infty}^{x}|u_{\mathrm{HG}}(x';n)|^2\mathrm{d}x' \tag{4}$$

which only represents the mode-number of a single axis, not the total number that contributes to the Gouy phase. The order 15 mode that is co-resonant would be composed of the x and y-axis modes summing to that order, for instance a 7th and 8th order distributions.

The LG modes are calculated from:

$$u_{\mathrm{HG}}(r;p,l) = \left(\frac{\sqrt{2}r}{w}\right)^{l}\sqrt{\frac{2p!}{\pi(p+l)!}}L\left(\frac{\sqrt{2}r^2}{w^2};p,l\right)e^{-\frac{r^2}{w^2}} \tag{5}$$

$$\mathrm{CDF}_{\mathrm{LG}}(r;p,l) = 2\pi\int_{r}^{\infty}|u_{\mathrm{LG}}(r';p,l)|^2 r'\mathrm{d}r' \tag{6}$$

Where p, l are the radial and azimuthal orders, and the mode number is $n = 2p + l$.

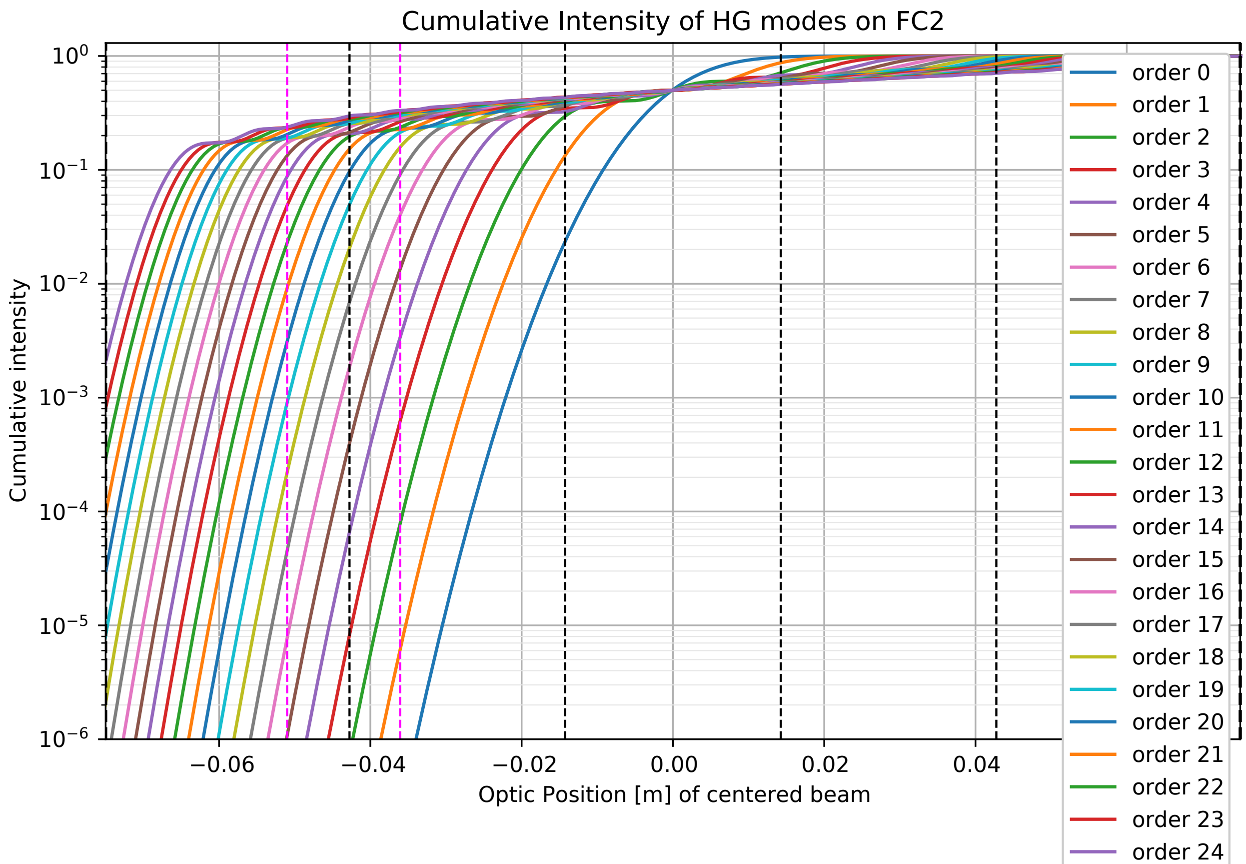

Figure 7: The Cumulative intensity of Hermite-Gaussian modes integrated over 1 transverse axis on the FC2 mirror. E.g. The order 15 mode of figure 6 could be composed of an order 8 and order 7 mode here as an HG78 or HG87. The cumulative intensity indicates the clipping loss of these modes for a given aperture. The black dashed bars are multiples of the beam radius from the center of the optic. The dashed magenta lines indicate the chosen aperture of 51mm (2in) and a factor of square-root 2 lower. The square root factor indicates the effective mask clipping level for grid-modes.

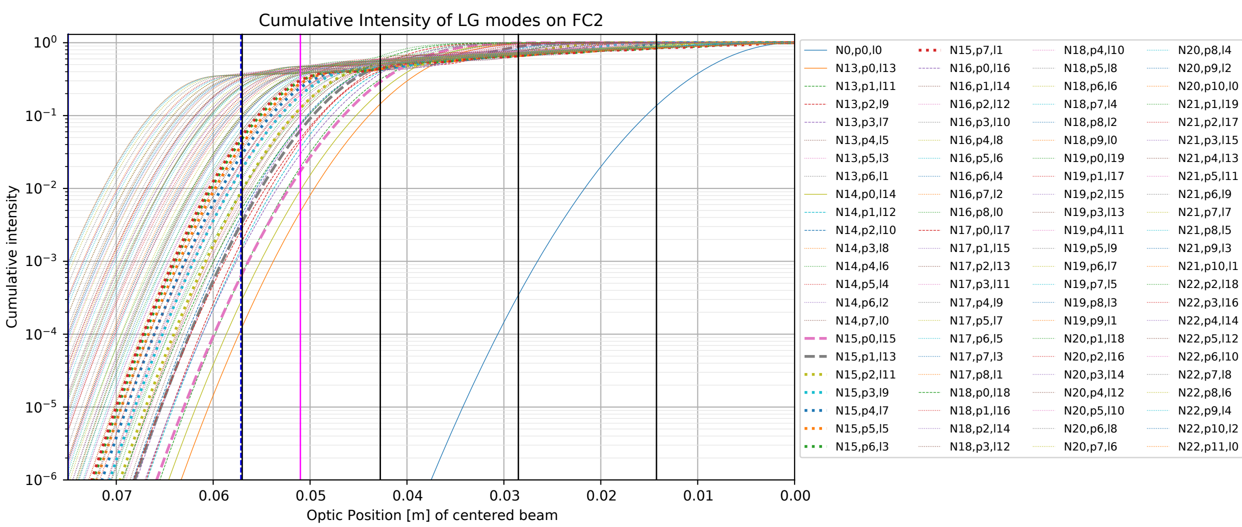

Figure 8: The Cumulative intensity of Laguerre-Gaussian modes integrated radially on the FC2 mirror. These modes are more useful to determine the clipping for a radial mask, although usually small cavity astigmatism makes the HG basis the more correct for sustained modes in the cavity. Only modes higher than 13 are shown and are indexed in the legend by the Gouy phase multiple, n, then radial node number, p, and axial node number, l. The cumulative intensity indicates the clipping loss of these modes for a given aperture. The black dashed bars are multiples of the beam radius from the center of the optic. The dashed magenta line indicates the chosen aperture of 51mm (2in). The order 15 Laguerre modes are clipped to be 2%-10%.

3.3 HOM Enhancement model

When a HOM resonance becomes degenerate with the fundamental, the HOM and fundamental hybridize to create an avoided crossing. The hybridization is particularly bad as it causes the cavity to act on the symmetric and antisymmetric combinations of the degenerate modes, rather than on the fundamental, preventing the cavity from functioning properly for it goal of frequency dependent squeezing. This may be explained with the following model of the filter cavity reflectivity, using a reduced 2-mode system representing the HG00/LG00 mode on-resonance, and some other HOM which has some clipping loss.

$$\mathbf{U}_{\mathrm{FC}} = \begin{bmatrix} \sqrt{1-C} & \sqrt{C} \\ \sqrt{C} & \sqrt{1-C} \end{bmatrix} \qquad \boldsymbol{\eta}_{\mathrm{FC}} = \begin{bmatrix} 1 & 0 \\ 0 & \sqrt{1-L_{\mathrm{HOM}}} \end{bmatrix} \tag{7}$$

$$T = 1000\mathrm{ppm} \qquad r = \sqrt{1-T} \tag{8}$$

$$\eta_{\mathrm{SQZ}}(\Omega) = \left| \begin{bmatrix} 1 \\ 0 \end{bmatrix}^{T} \mathbf{R}(\Omega) \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right|^{2} \qquad \mathbf{R}(\Omega) = r\mathbf{1} - \frac{T e^{i\Omega L/c} \boldsymbol{\eta}_{\mathrm{FC}} \mathbf{U}_{\mathrm{FC}}}{\mathbf{1} - r e^{i2\Omega L/c} \boldsymbol{\eta}_{\mathrm{FC}} \mathbf{U}_{\mathrm{FC}}} \tag{9}$$

In this model, the $\mathbf{U}_{\mathrm{FC}}$ matrix represents the scattering loss or coupling, C, between the fundamental mode and the high-order mode. This is the scattering loss without cavity effects, and the square roots convert from, C being measured as a loss of power from one mode to another into the matrix which acts on fields. The cross term of this matrix could additionally contain a phase for the field coupling, but this phase does not affect the model. The round-trip efficiency is described by $\boldsymbol{\eta}_{\mathrm{FC}}$. This matrix represents the loss that is applied to the high-order mode. It could also contain the scattering and absorption losses of the fundamental, but the model is qualitatively accurate to have negligible losses in the fundamental. The power transmissivity is given by T and field reflectivity by r. The final two equations give the cavity reflectivity for the two transverse models. The η_{SQZ} factor is the reflection efficiency (in power) for the fundamental mode at the chosen sideband frequency, whereas $\mathbf{R}$ is the matrix field reflectivity between the two modes.

Figures 9 and 10 show the mode-splitting and enhancement of losses that occurs when high-order-modes become degenerate. The two figures show two regimes, one where the HOM is low loss and has the same finesse as the fundamental and one where the HOM experiences strong losses and a finesse a factor of 10 lower than the fundamental.

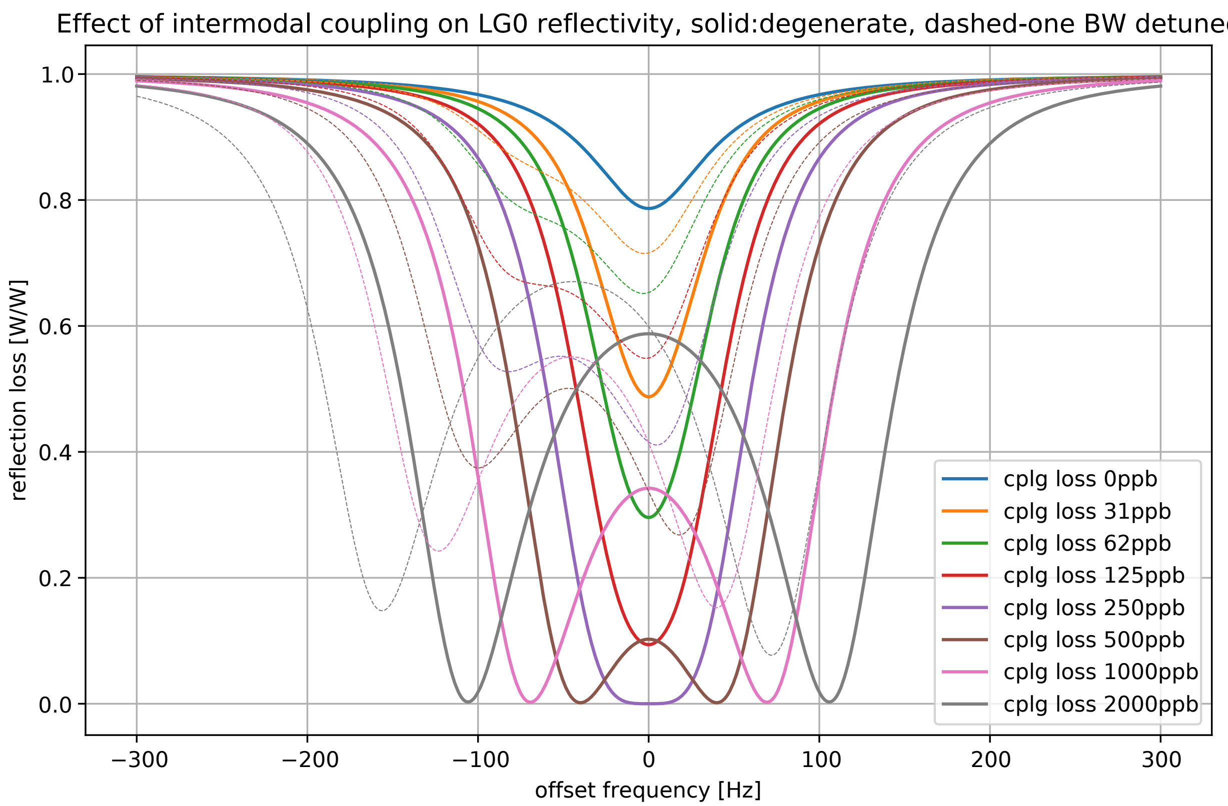

Figure 9: This figure represents a cavity with a single high-order that is degenerate with the fundamental (solids) or a high-order mode 1-bandwidth away (thin dashed). The Y-axis is the (power) reflectivity of the fundamental from the cavity as a function of frequency (X-axis). The cplg loss represents the power scattering between the fundamental and the high-order mode. Without the cavity, this represents a power loss of the fundamental, but with the cavity, this loss becomes enhanced by both the fundamental and HOM resonance. In this case, both the fundamental and HOM have the same round-trip losses and corresponding finesse. For large cross-couplings, the fundamental and HOM hybridizes and a frequency splitting occurs.

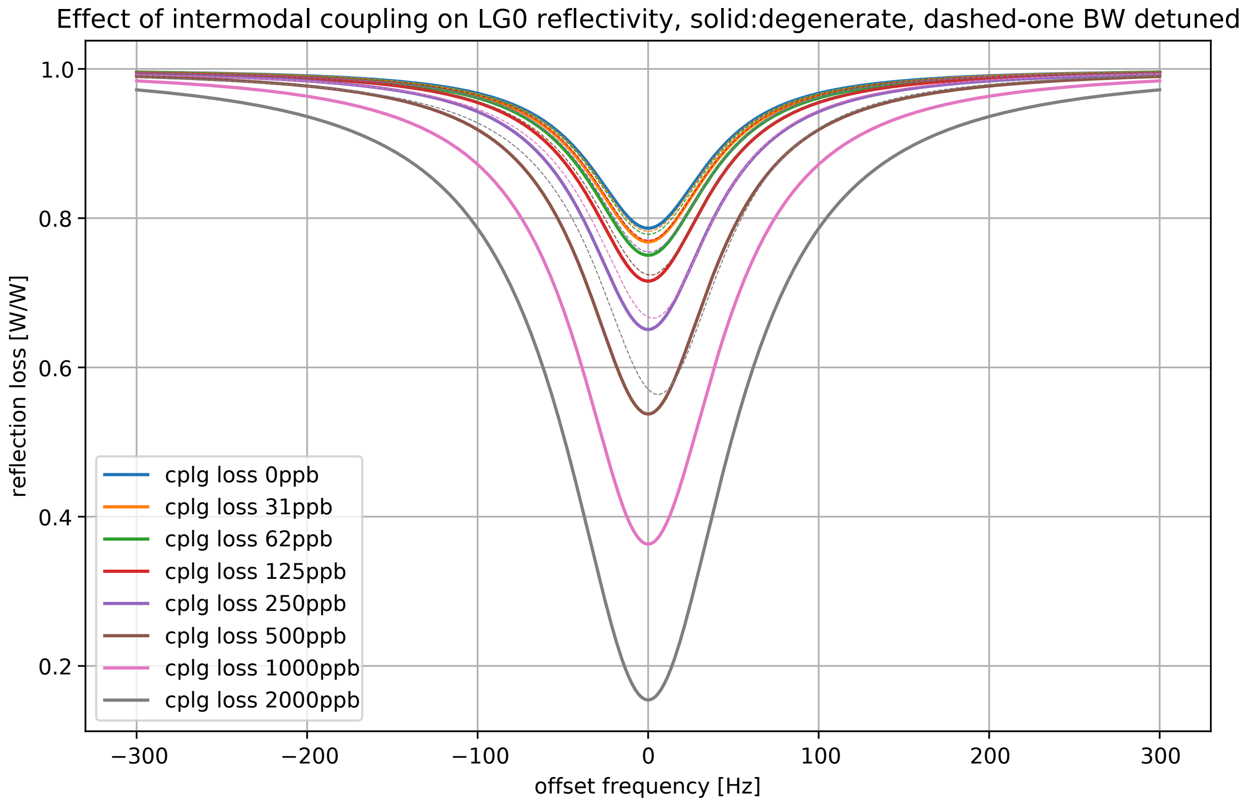

Figure 10: This figure represents a cavity with a single high-order that is degenerate with the fundamental (solids) or a high-order mode 1-bandwidth away (thin dashed). The Y-axis is the (power) reflectivity of the fundamental from the cavity as a function of frequency (X-axis). The cplg loss represents the power scattering between the fundamental and the high-order mode. Without the cavity, this represents a power loss of the fundamental, but with the cavity, this loss becomes enhanced by both the fundamental and HOM resonance. For this plot, the HOM experiences a loss of 2% due to clipping. In this case the sensitivity to coupling is strongly reduced and mode-splitting does not occur.