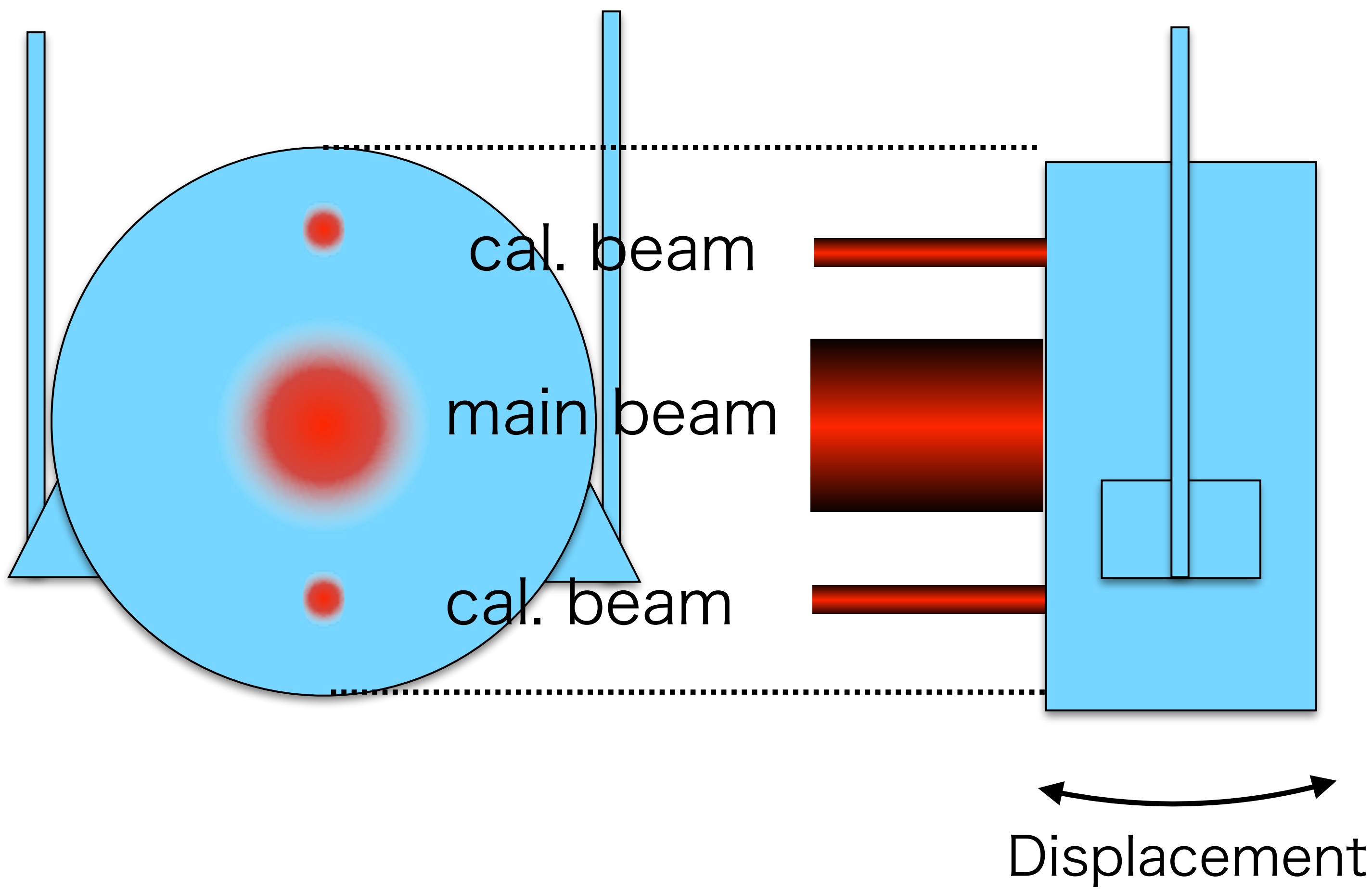
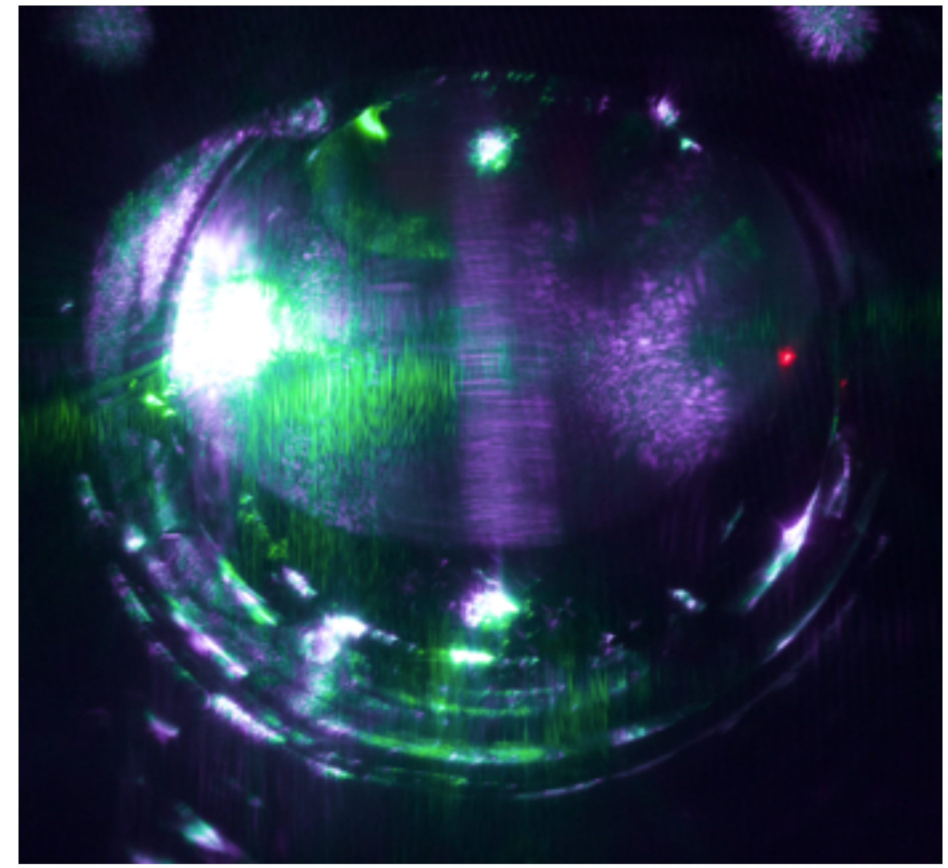
KAGRA Gravity field Calibrator

National Central University
Yuki Inoue
on behalf of KAGRA calibration group

Introduction





$$dx = \frac{2P\cos\theta}{c}\, s(f) \left(1 + \frac{M}{I}\vec{a}\cdot\vec{b}\right)$$

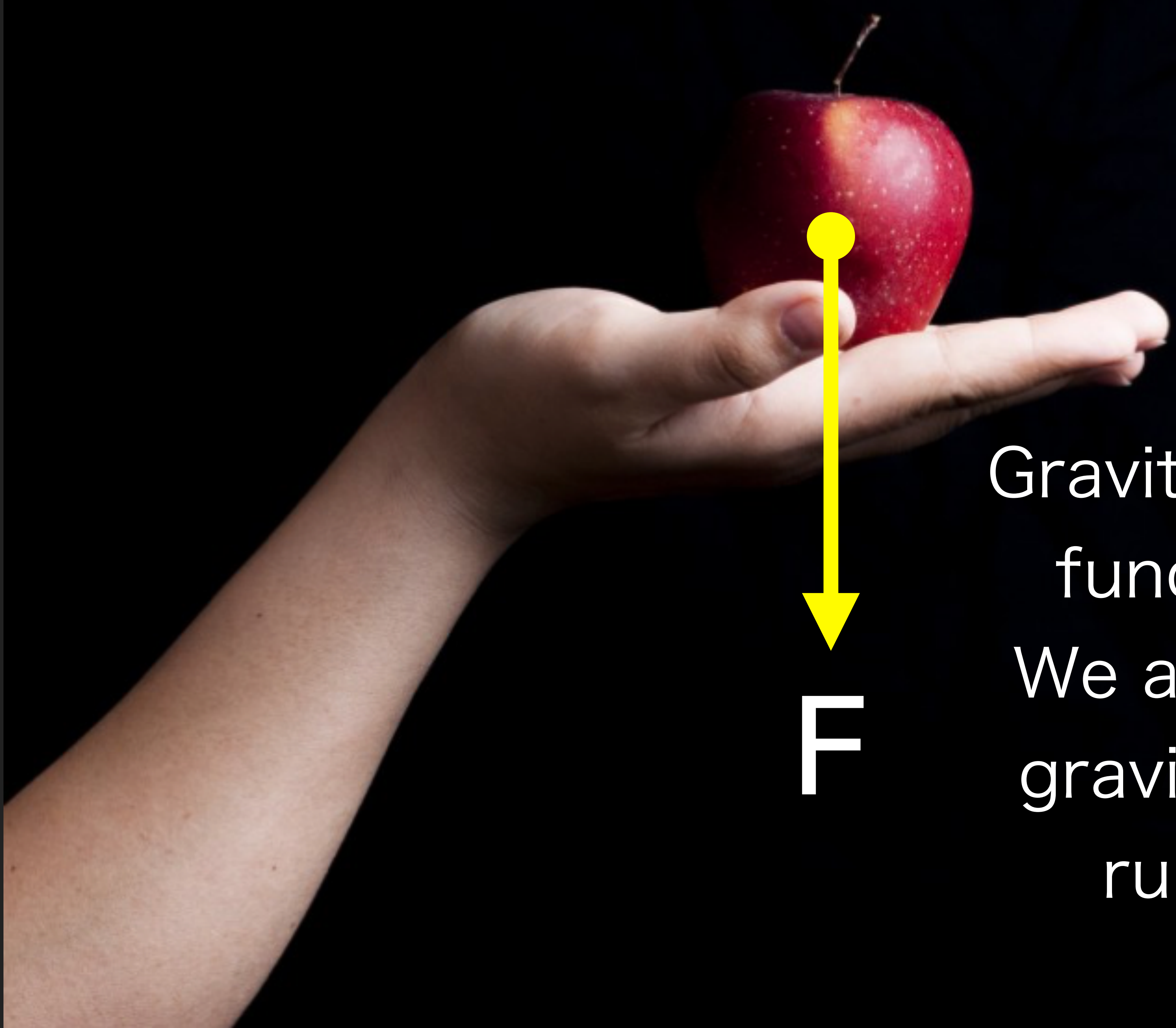
Force(N) | Transfer function(m/N) | Geometrical factor

Photon Calibrator is one of the powerful tools to calibrate the displace meant of the mirror.

Motivation: How can we measure and make sure the absolute laser power?

GRAVITY

$$F=GmM/r^2$$

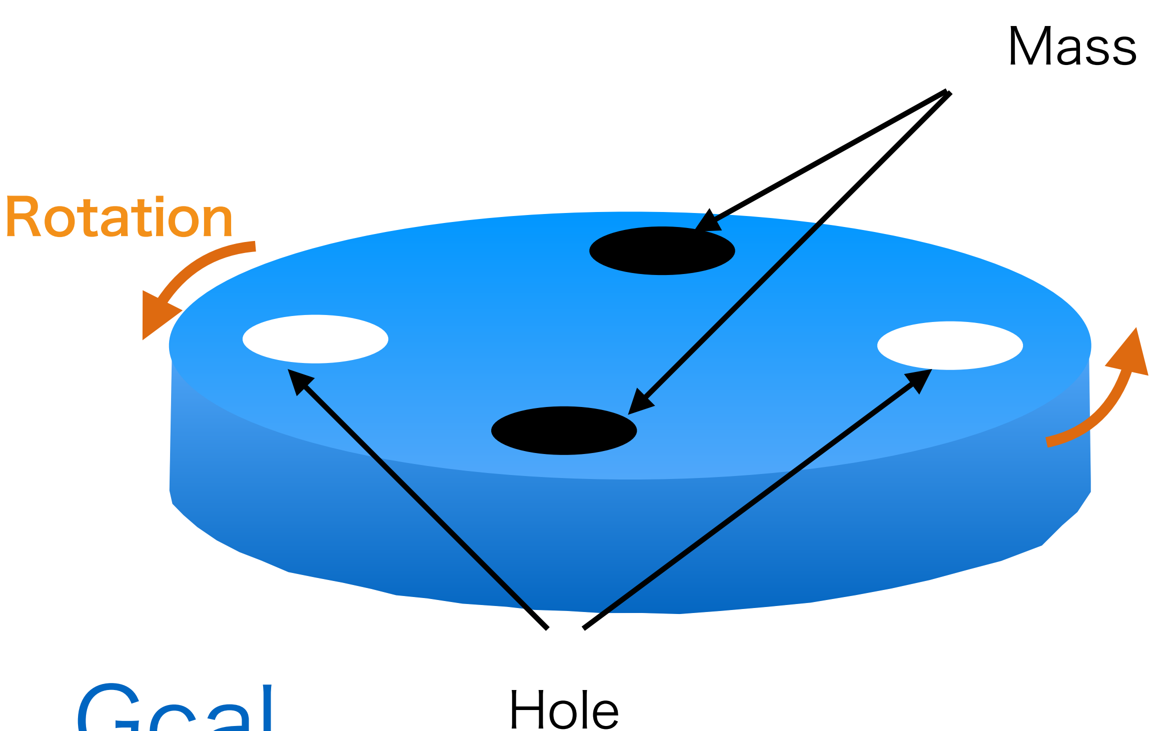


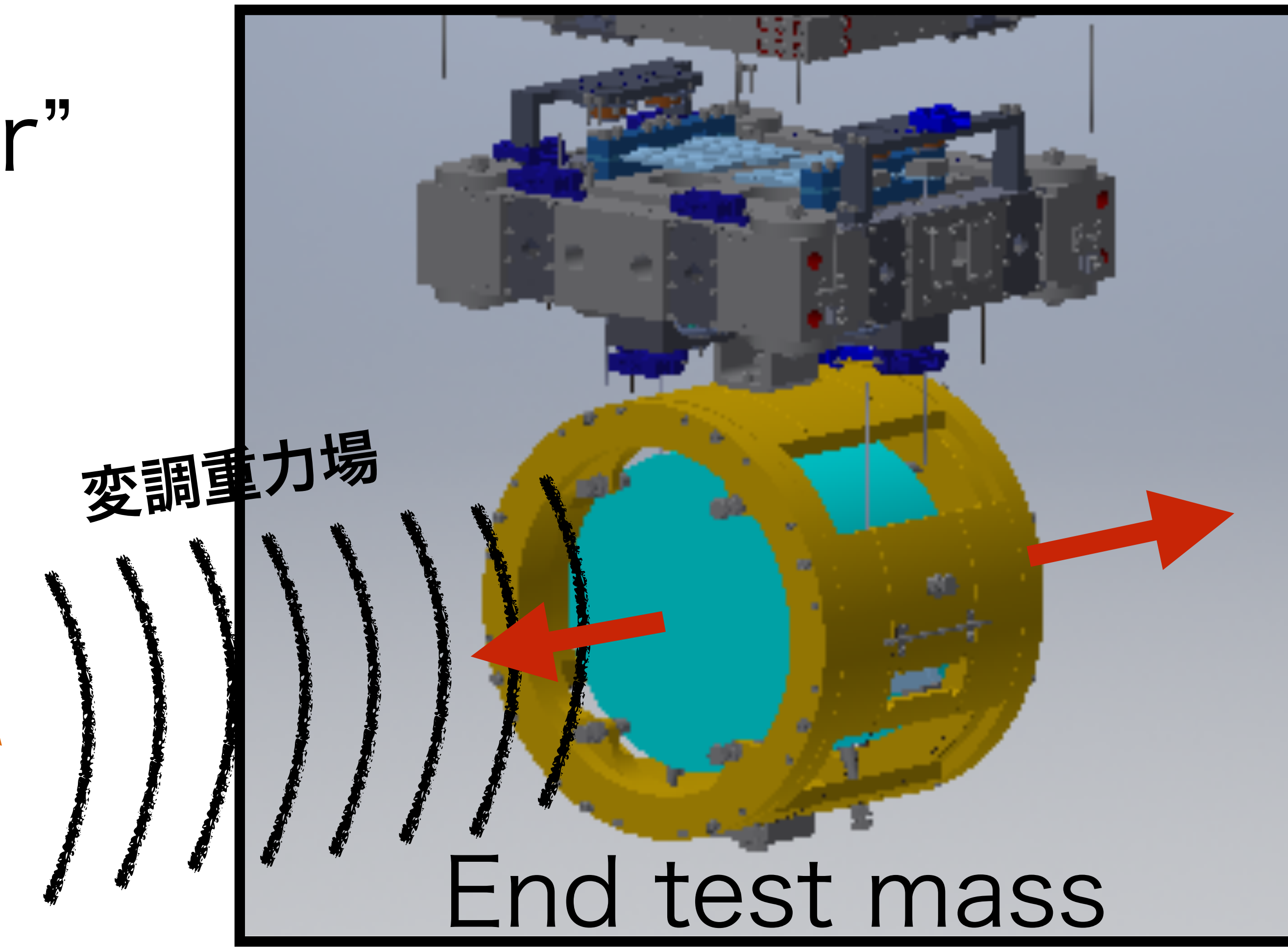
Gravity is one of the most fundamental Sources.
We are trying to use the gravity field as a kind of ruler for the power calibration.

Gravity field calibrator

"Gravity field calibrator"

=Gcal





Gcal

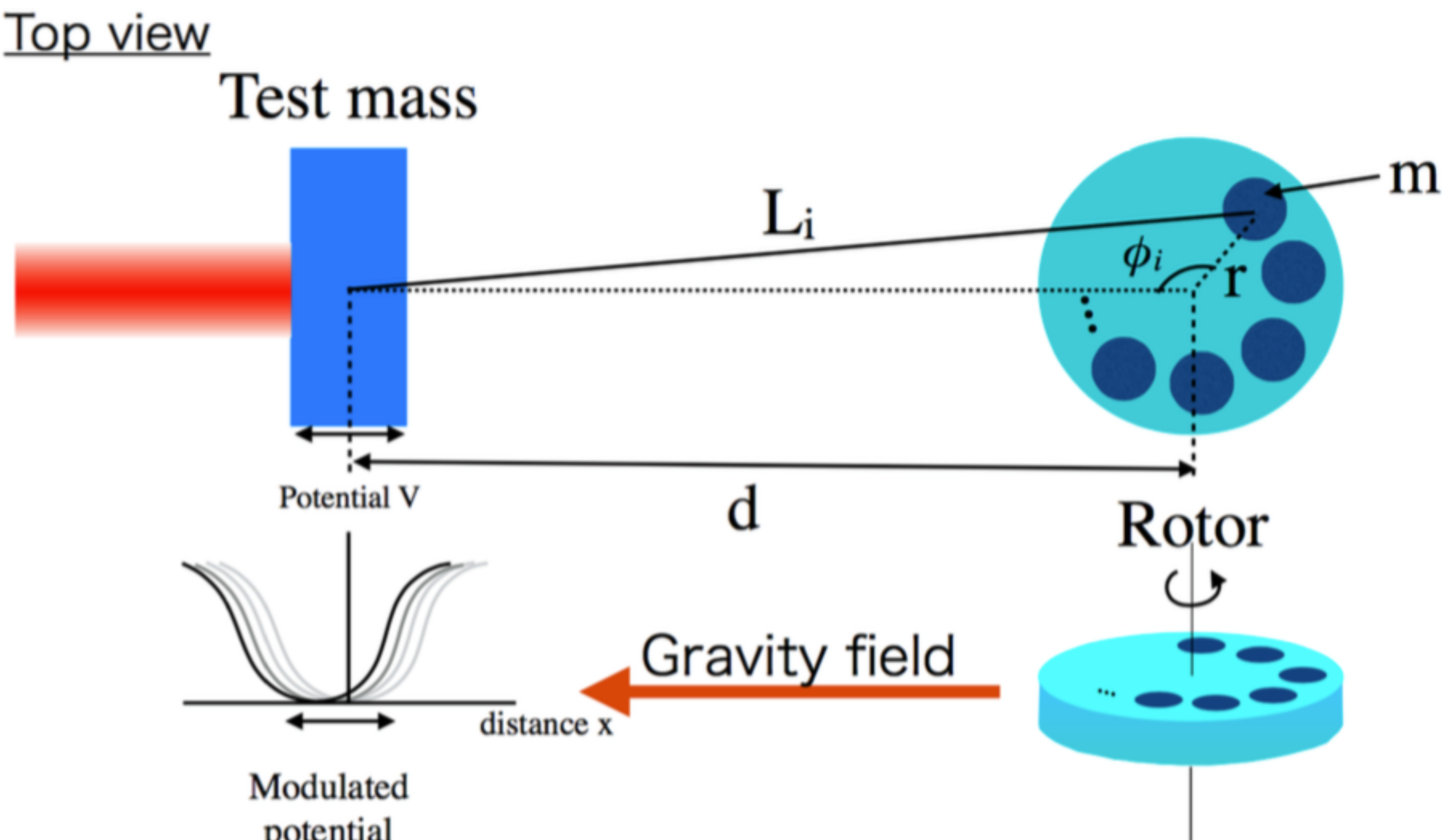
$$x_{2\mathrm{f}} = 9\frac{GMm_{\mathrm{q}}r_{\mathrm{q}}^2}{d^4}s(\omega).$$

PCal

$$dx = \frac{2P\cos\theta}{c}s(f)\left(1+\frac{M}{I}\vec{a}\cdot\vec{b}\right)$$

J. Sinsky and J. Weber. New Source for Dynamical Gravitational Fields. Physical Review Letters, 18:795–797, May (1967).
Matone, L. et al., "Benefits of artificially generated gravity gradients for interferometric gravitational-wave detectors," Classical and Quantum Gravity 24(9), 2217 (2007).

Multipole expansion



$$x_{2f} = 9\frac{GMm_q r_q^2}{d^4}s(\omega).$$

History

"In 1967, Forward and Miller [1] developed a gravity field generator that allowed them to calibrate an orbiter sensor capable of measuring the lunar mass distribution."

"A similar technique was used by Weber *et al* [2, 3] to calibrate a gravitational-wave bar detector,"

"At the University of Tokyo, in the 1980s, a series of experiments were conducted to test the law of gravitation up to a distance of 10 m [4–8]."

"In the 1990s, the gravitational-wave group at the University of Rome developed and carried out experiments [9, 10] "

US
[1] Forward R L and Miller L R 1967 *J. Appl. Phys.* **38** 512
[2] Sinsky J and Weber J 1967 *Phys. Rev. Lett.* **18** 795–7
[3] Sinsky J A 1968 *Phys. Rev.* **167** 1145

Japan
[4] Hirakawa H, Tsubono K and Oide K 1980 *Nature* **283** 184
[5] Oide K, Tsubono K and Hirakawa H 1980 *Japan. J. Appl. Phys.* **19** L123
[6] Suzuki T, Tsubono K and Kuroda K 1981 *Japan. J. Appl. Phys.* **20** L498
[7] Ogawa Y, Tsubono K and Hirakawa H 1982 *Phys. Rev.* D **26** 729
[8] Kuroda K and Hirakawa H 1985 *Phys. Rev.* D **32** 342

U. of Rome
[9] Astone P *et al* 1991 *Z. Phys.* C **50** 21
[10] Astone P *et al* 1998 *Eur. Phys. J.* C **5** 651

Citation: Malone. et. al.

2007. Apr.

Benefits of artificially generated gravity gradients for interferometric gravitational-wave detectors

L Matone[1], P Raffai[2], S Márka[1], R Grossman[1], P Kalmus[1], Z Márka[1], J Rollins[1] and V Sannibale[3]

→ Basic idea of Gcal/ Ncal for interferometer

→ Uncertainty of distance

2018. Sep.

Improving the absolute accuracy of the gravitational wave detectors by combining the photon pressure and gravity field calibrators

Yuki Inoue,[1,2] Sadakazu Haino,[1,2] Nobuyuki Kanda,[3] Yujiro Ogawa,[2,4] Toshikazu Suzuki,[2,5,6] Takayuki Tomaru,[2,4,5,6] Takahiro Yamanmoto,[7] and Takaaki Yokozawa[7]

→ Systematic errors

→ Quadrupole/Hexapole method

→ Combination with Photon calibrator (Mentioned EURAMET comparison)

2018. Nov.

First Tests of a Newtonian Calibrator on an Interferometric Gravitational Wave Detector

D. Estevez, B. Lieunard, F. Marion, B. Mours, L. Rolland, D.Verkindt

→ First demonstration with interferometer

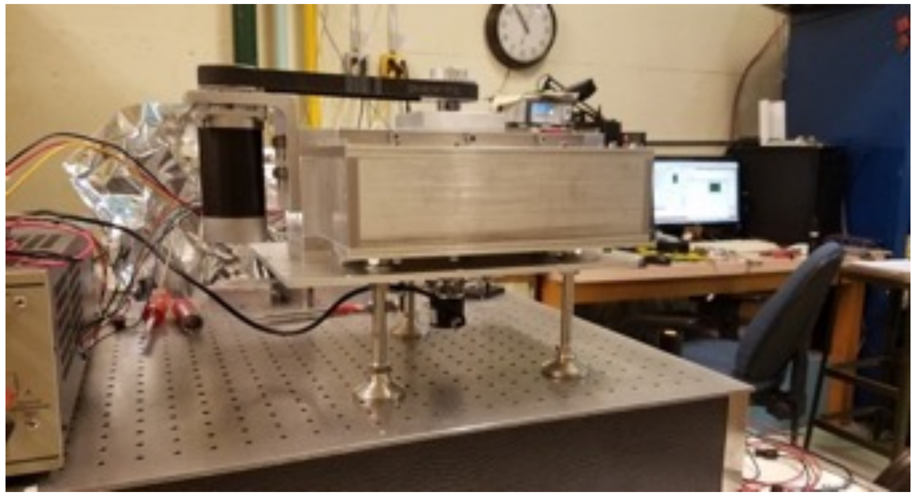
→ Consistency check with other method

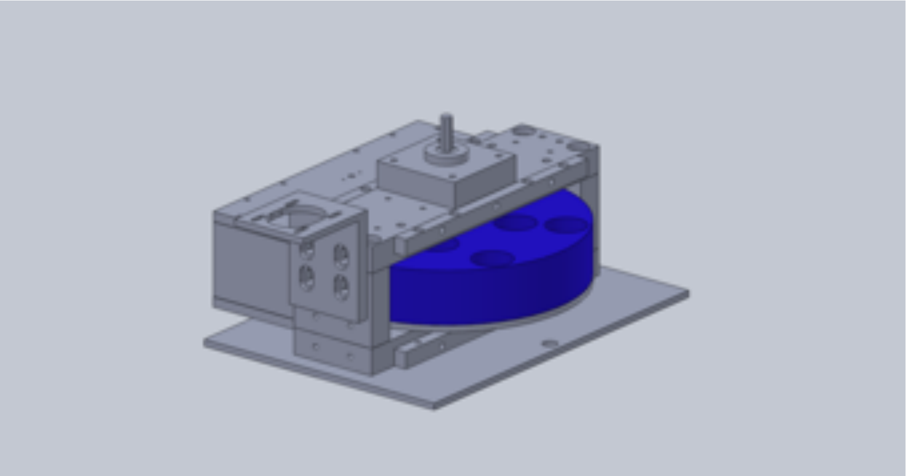
→ Suspended design

Gravity field calibrator and Newtonian Calibrator

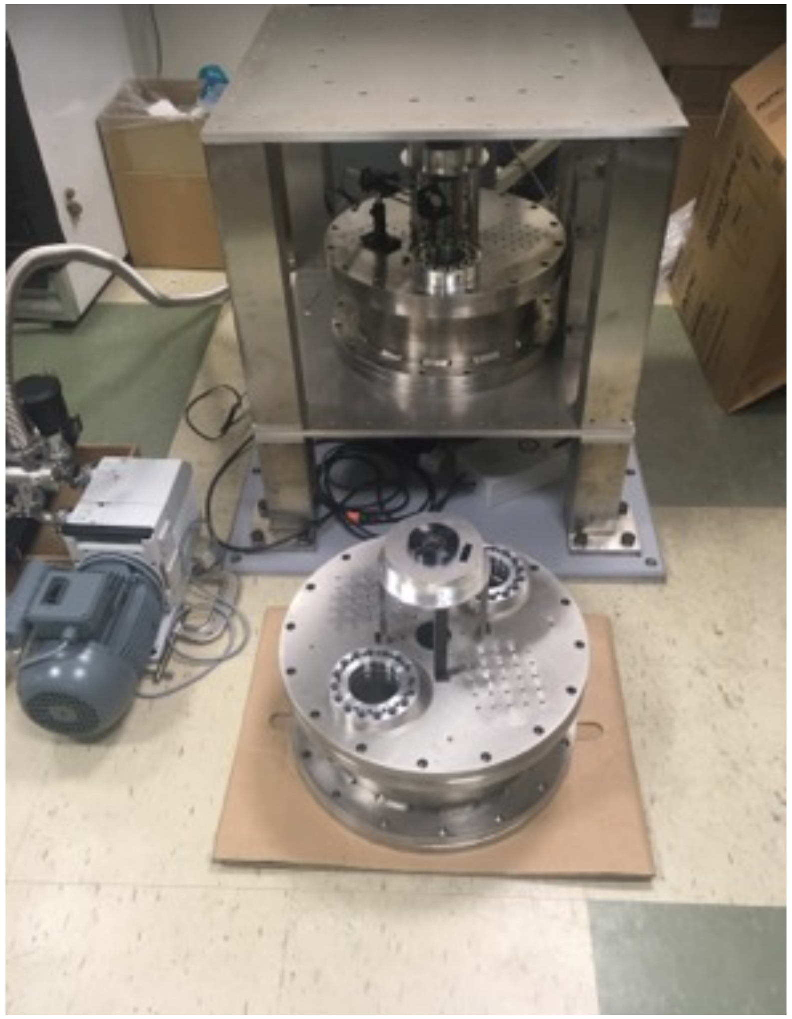
Virgo

LIGO



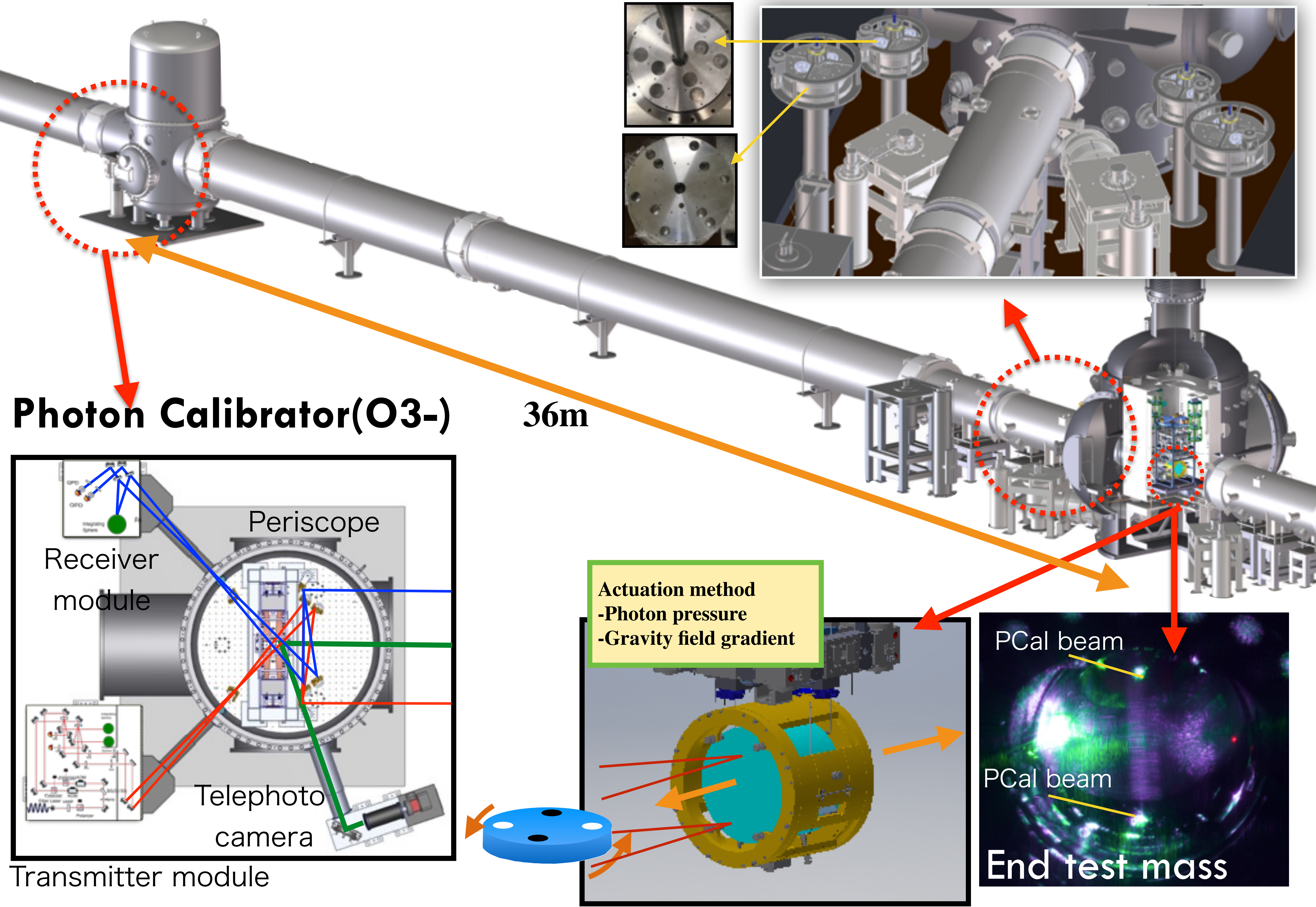


KAGRA



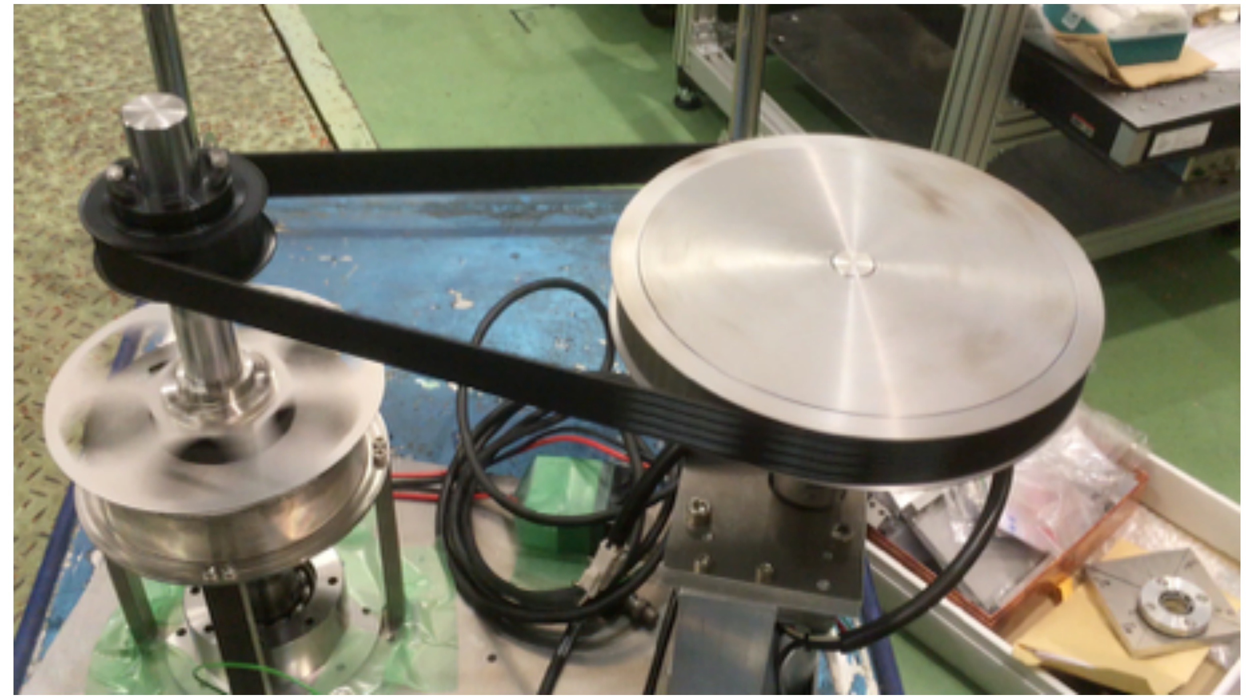
KAGRACalibration instruments

Gravity Field Calibrator(O4-)

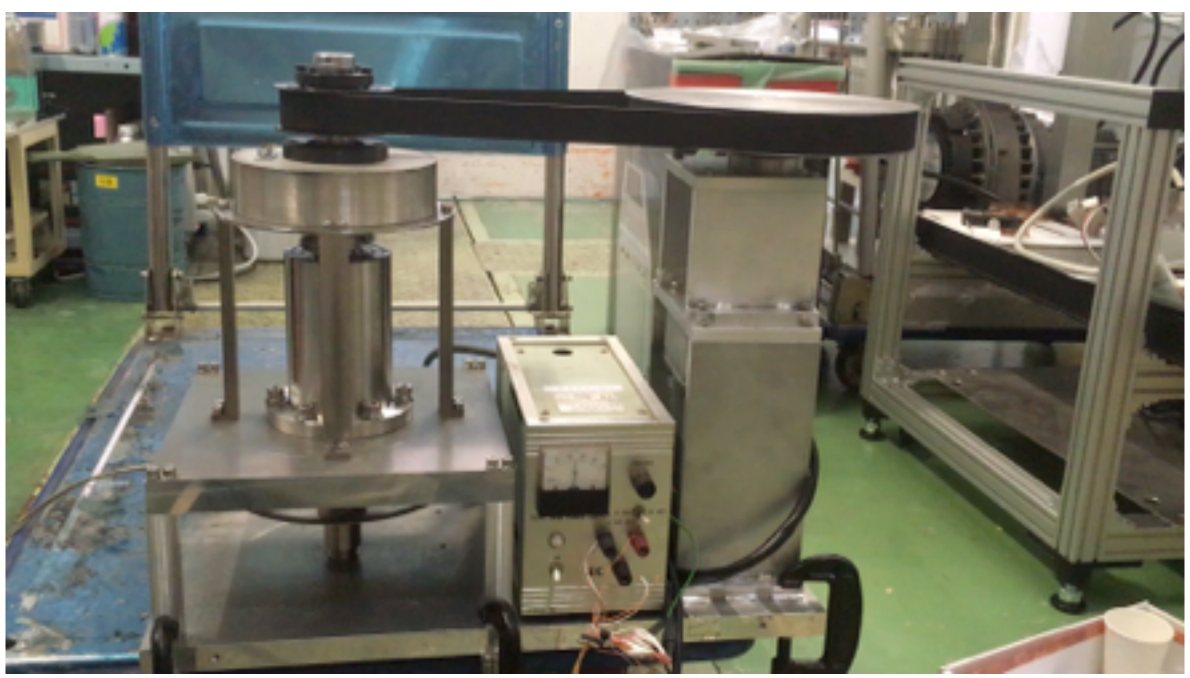


History of KAGRA Gravity field calibrator

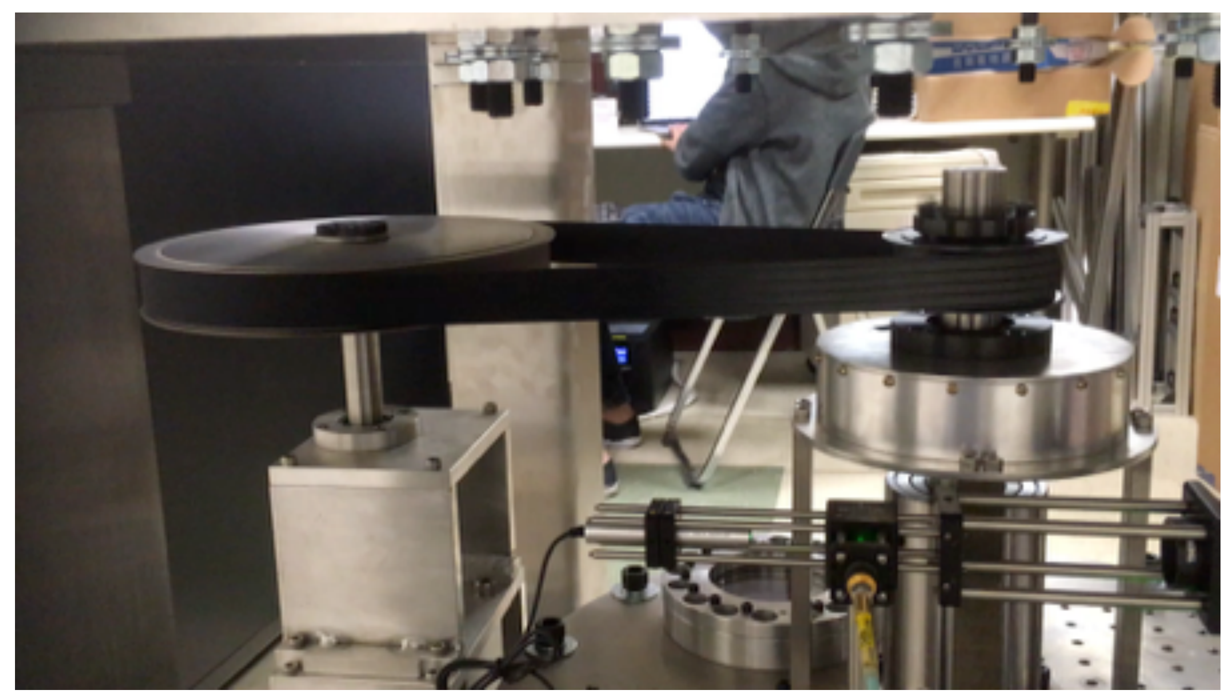
1980 Oide et.al.



2016 Prototype rotation test

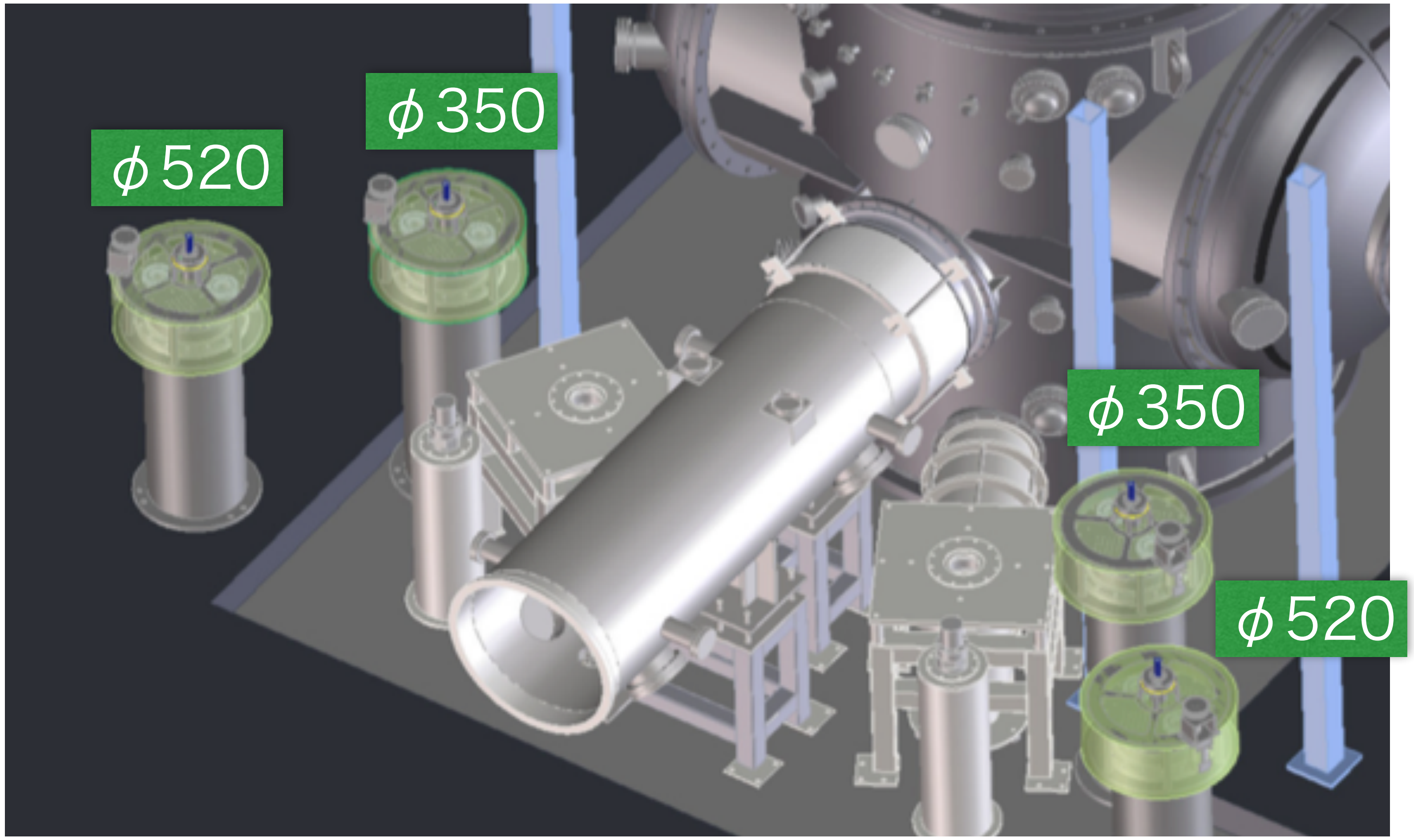


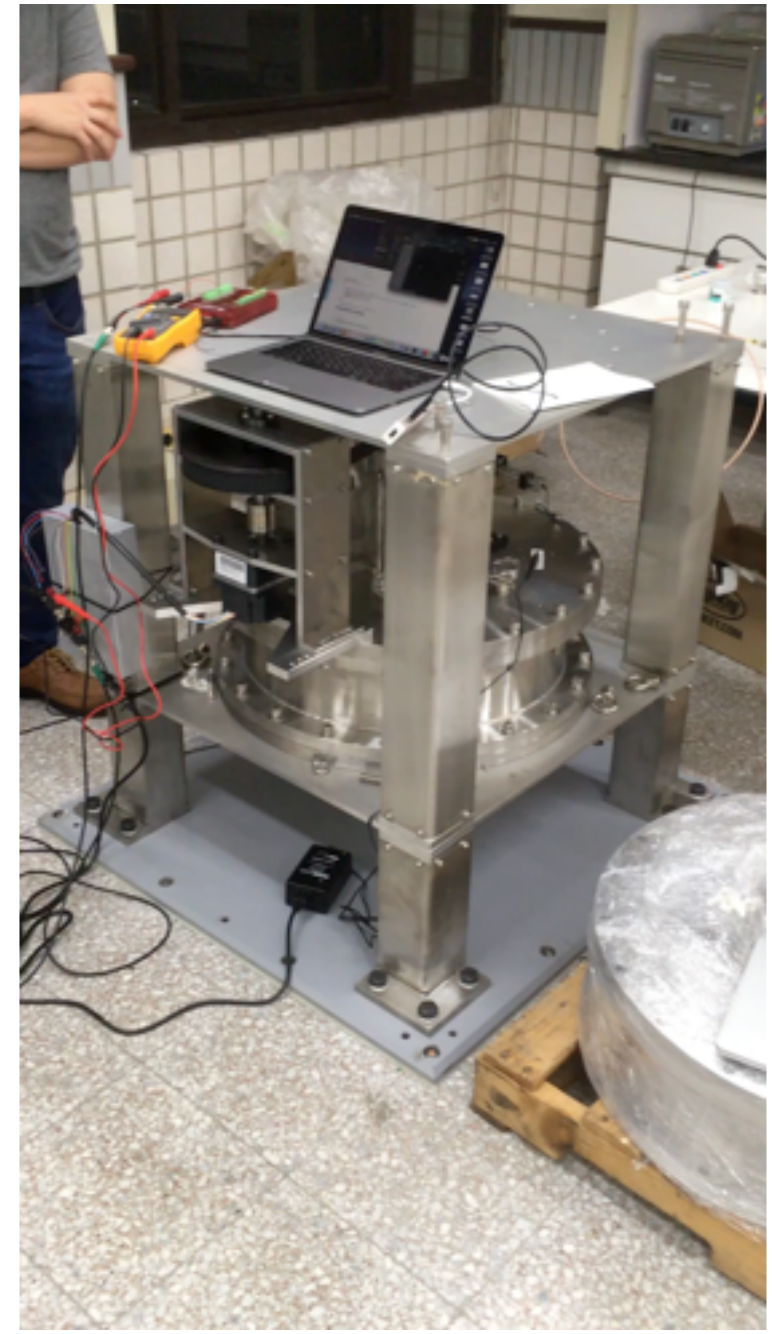
2016 Prototype rotation test



2017 350mm Gcal Test

KAGRA Gravity field Calibrator for O4



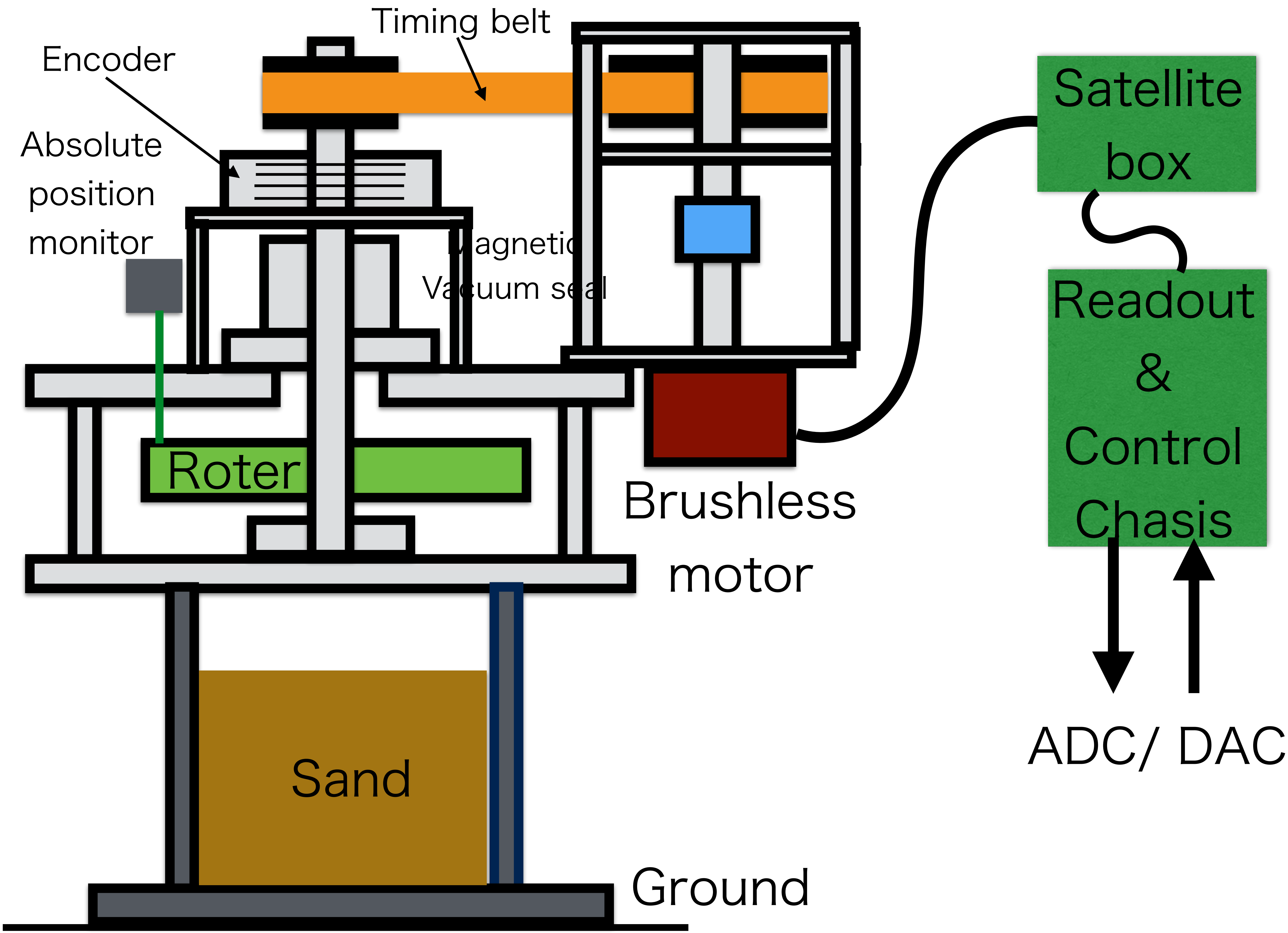


2018 520mm Gcal Test

Remote control test

We have succeeded to the remote control test!

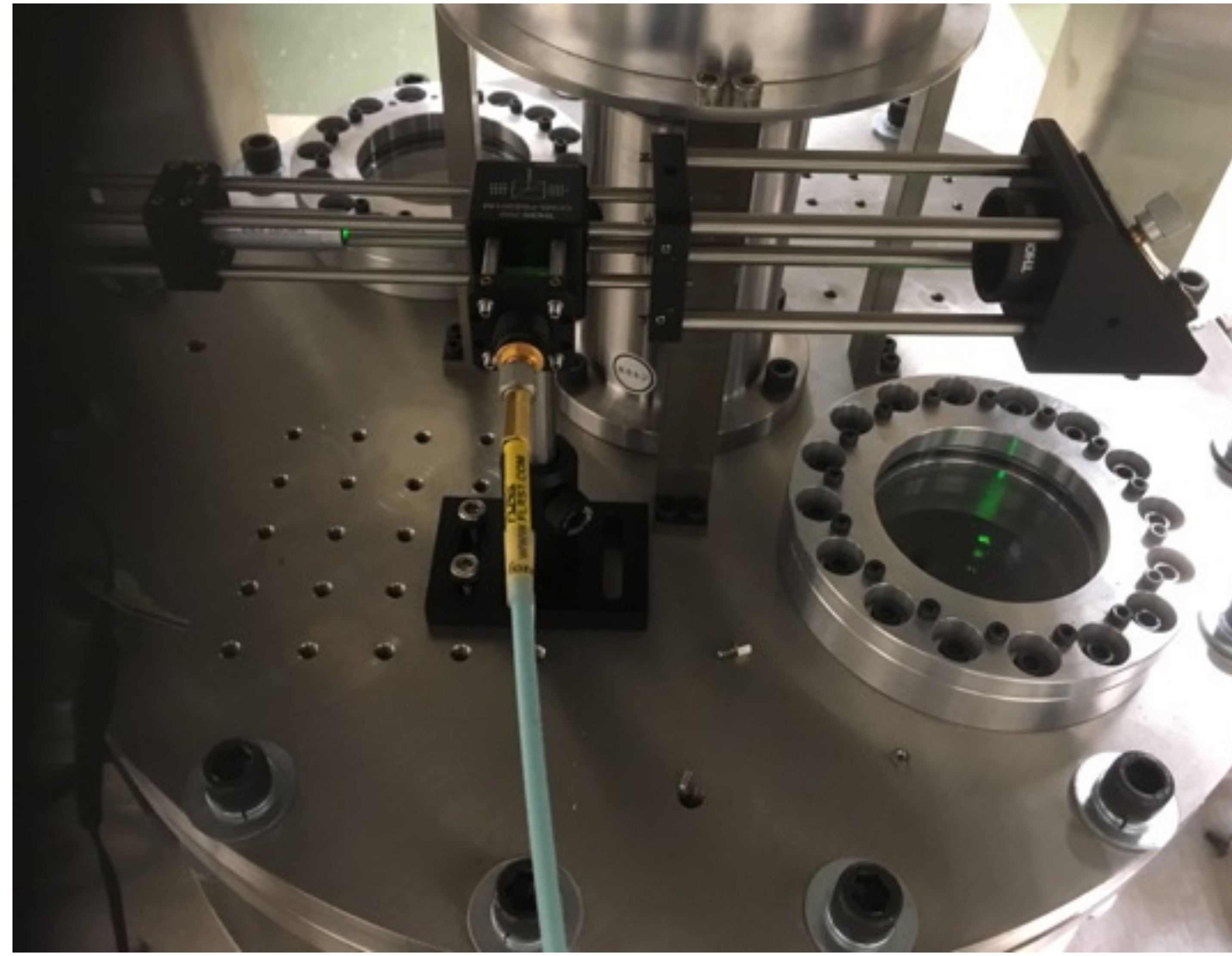
Instruments

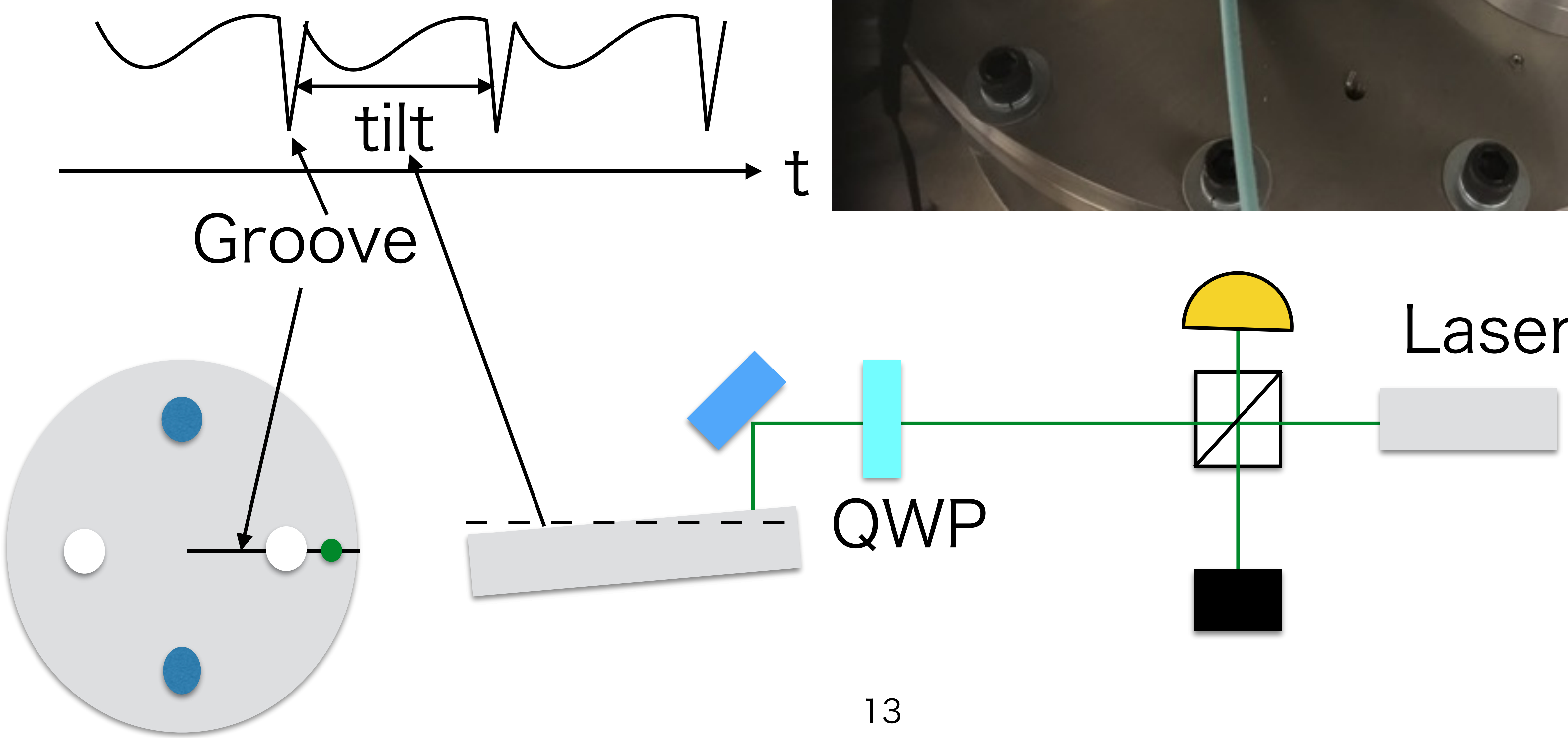


Sensors

1.Absolute position

ϕ , θ

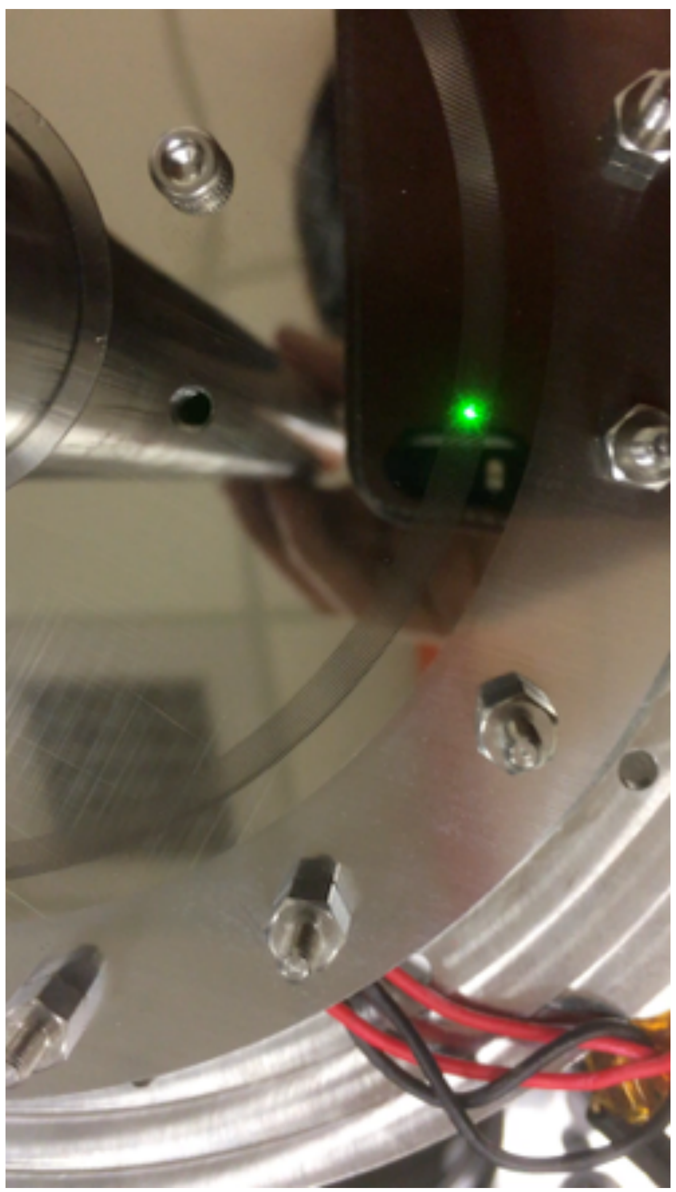


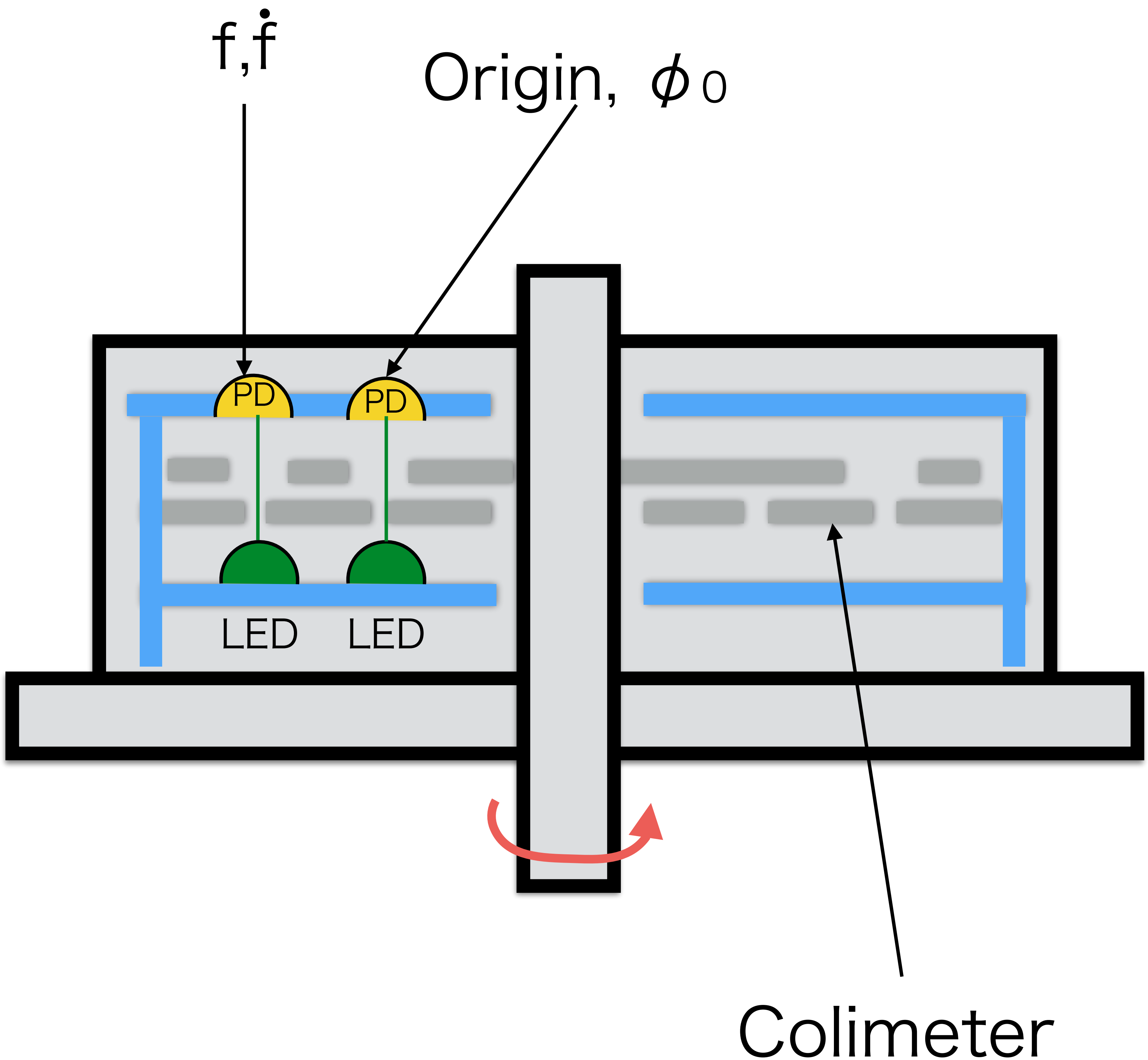


Sensors

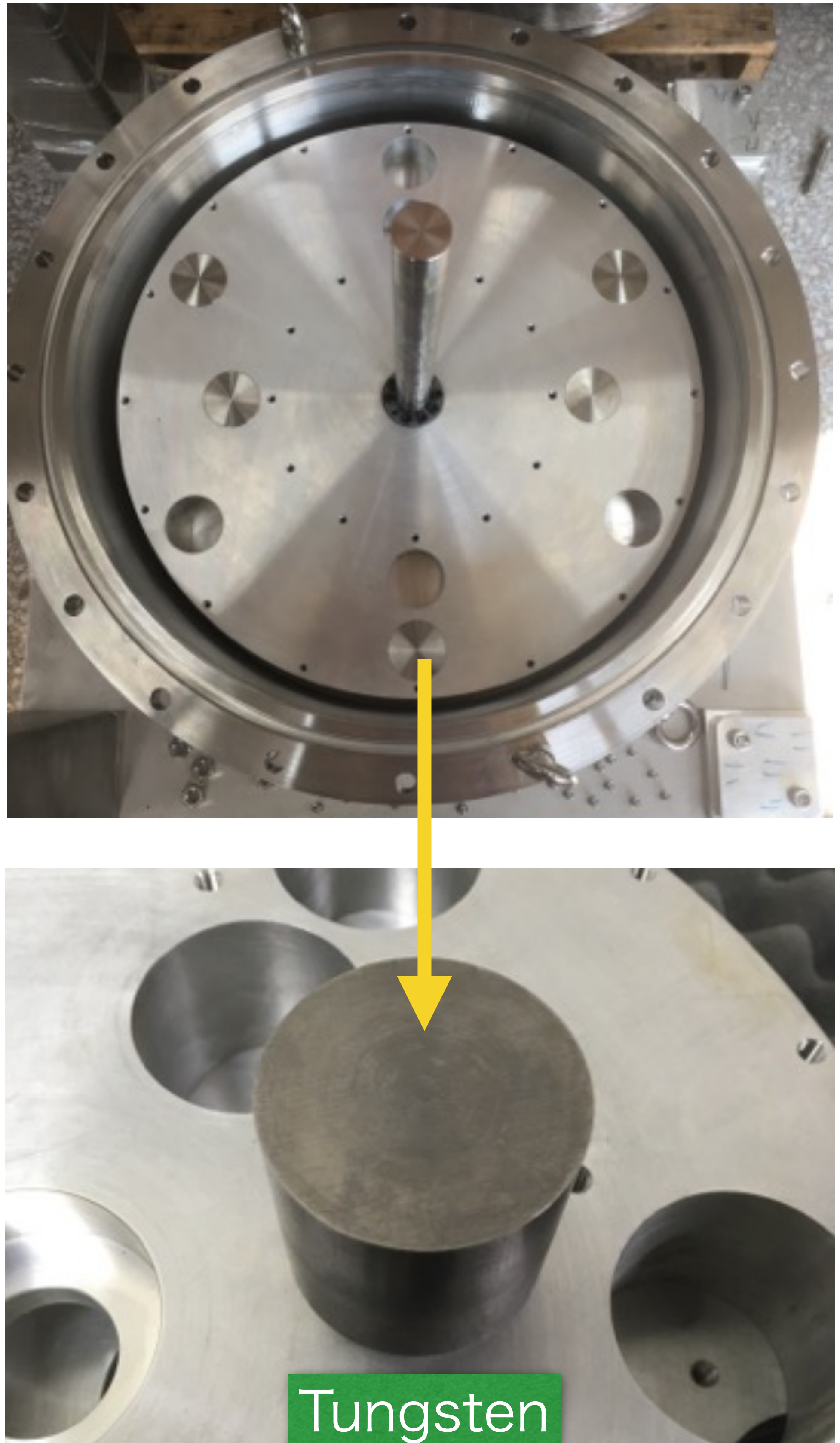
2.Encoder

$f, \dot{f}\ \phi_0$





An idea of quadrupole and hexapole method



	Value	Relative uncertainty
G	$6.67408 \times 10^{-11}\ \mathrm{m^3kg^{-1}sec^{-2}}$	0.0047 %
M	22.89 kg	0.02 %
m_{q}	4.485 kg	0.004 %
m_{h}	4.485 kg	0.004 %
r_{q}	0.200 m	0.010 %
r_{h}	0.125 m	0.016 %

$$x_{2\mathrm{f}} = 9\frac{GMm_{\mathrm{q}}r_{\mathrm{q}}^2}{d^4}s(\omega).$$

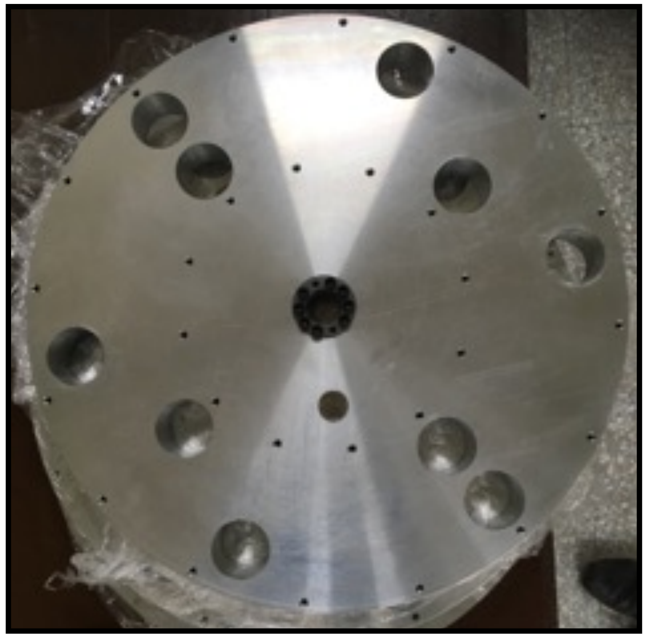
We can measure each parameter except for distance

$$x_{3\mathrm{f}} = 15\frac{GMm_{\mathrm{h}}r_{\mathrm{h}}^3}{d^5}s(\omega).$$

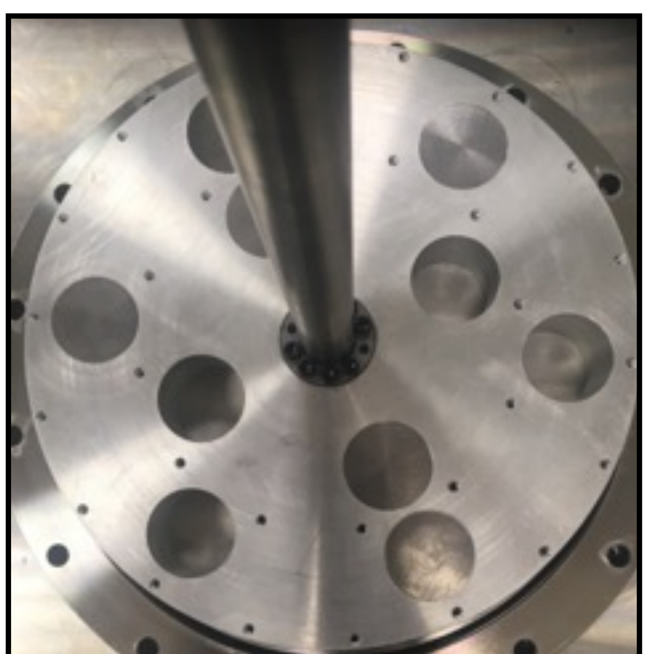
By using Hexapole, we can **measure** the distance!!

Basic parameter of KAGRA Gcal

520mm Gcal



350mm Gcal



	350mm	520mm
m_q	1889.5g	1889.5g
m_h	1889.5g	1889.5g
r_q	80mm	160mm
r_h	135mm	220mm
d	2.9m	4.0m
f	15Hz	15Hz

Hybrid method

Improving the absolute accuracy of the gravitational wave detectors by combining the photon pressure and gravity field calibrators

Yuki Inoue,[1,2] Sadakazu Haino,[1,2] Nobuyuki Kanda,[3] Yujiro Ogawa,[2,4] Toshikazu Suzuki,[2,5,6] Takayuki Tomaru,[2,4,5,6] Takahiro Yamanmoto,[7] and Takaaki Yokozawa[7]

Photon Calibrator

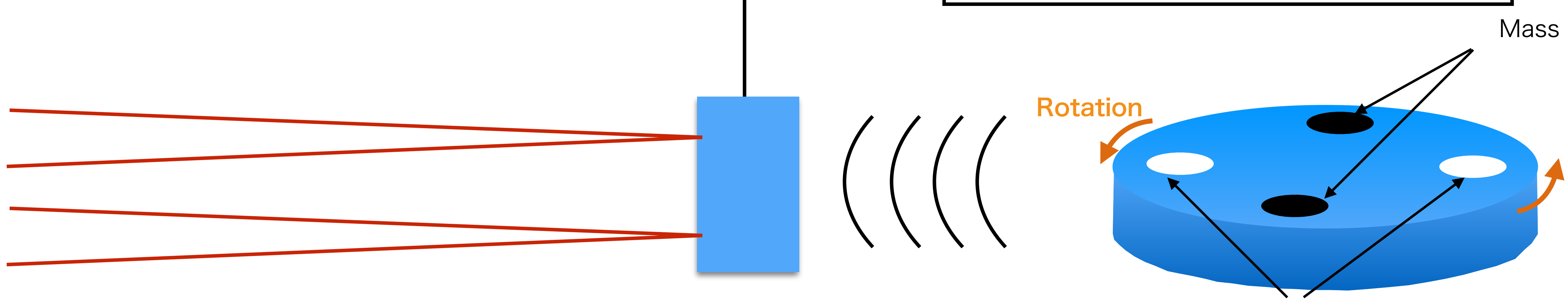
$$dx = \frac{2P\cos\theta}{c} s(f) \left(1 + \frac{M}{I}\vec{a}\cdot\vec{b}\right)$$

Cancelation

Gravity field Calibrator

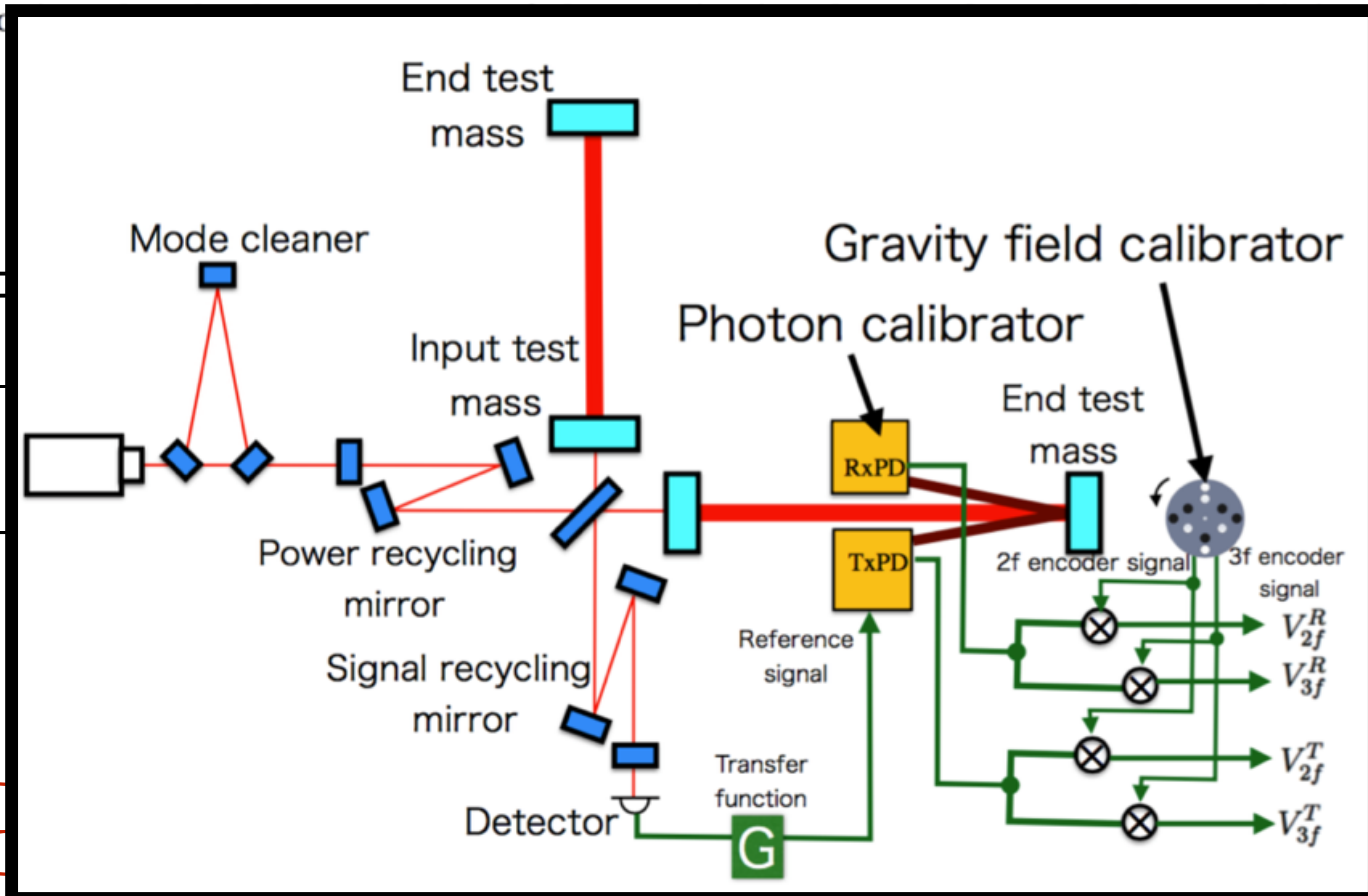
$$x_{2\mathrm{f}} = 9\frac{GMm_{\mathrm{q}}r_{\mathrm{q}}^2}{d^4}s(\omega).$$

$$x_{3\mathrm{f}} = 15\frac{GMm_{\mathrm{h}}r_{\mathrm{h}}^3}{d^5}s(\omega).$$



Hybrid method

Impr… the



$dx =$

Hybrid method

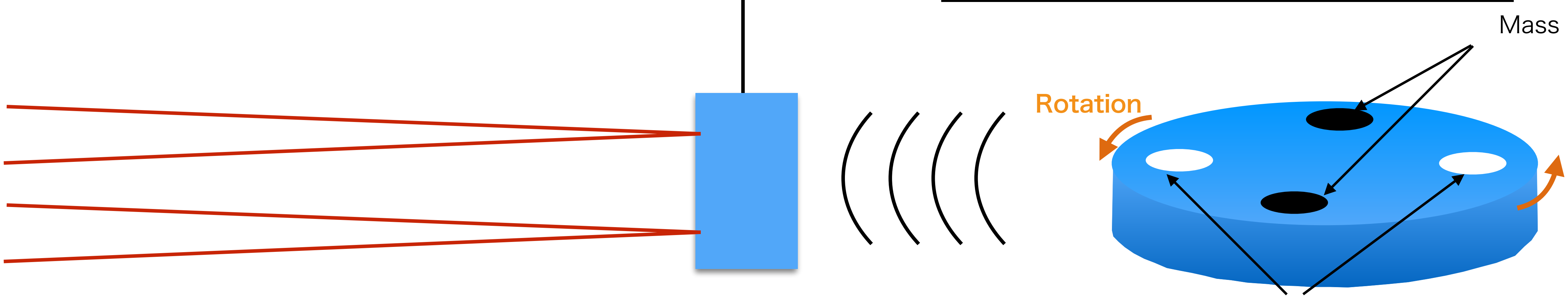
Photon Calibrator | Gravity field Calibrator

$$dx = \frac{2P\cos\theta}{c} s(f) \left(1 + \frac{M}{I}\vec{a}\cdot\vec{b}\right)$$

Cancelation

$$x_{2\mathrm{f}} = 9\frac{GMm_{\mathrm{q}}r_{\mathrm{q}}^2}{d^4}s(\omega).$$

$$x_{3\mathrm{f}} = 15\frac{GMm_{\mathrm{h}}r_{\mathrm{h}}^3}{d^5}s(\omega).$$



Technological limit

limited by geometry

$$\left(\frac{\delta x}{x}\right)^2 \sim \left(\frac{\delta V_{in}}{V_{in}}\right)^2 + \left(\frac{\delta s(\omega)}{s(\omega)}\right)^2 + 25\left(\frac{\delta V_{2\mathrm{f}}^{\mathrm{R}}}{V_{2\mathrm{f}}^{\mathrm{R}}}\right)^2 + 16\left(\frac{\delta V_{3\mathrm{f}}^{\mathrm{R}}}{V_{3\mathrm{f}}^{\mathrm{R}}}\right)^2 + \left(\frac{\delta x_{\mathrm{sys}}}{x_{\mathrm{sys}}}\right)^2$$

~0.17%

$$\frac{\delta x_{\mathrm{sys}}}{x_{\mathrm{sys}}} = \frac{\delta G}{G} + \frac{\delta M}{M} + \frac{12}{\sqrt{6}}\frac{\delta r_{\mathrm{h}}}{r_{\mathrm{h}}} + \frac{10}{2}\frac{\delta r_{\mathrm{q}}}{r_{\mathrm{q}}} + \frac{5}{2}\frac{\delta m_{\mathrm{q}}}{m_{\mathrm{q}}} + \frac{4}{\sqrt{6}}\frac{\delta m_{\mathrm{h}}}{m_{\mathrm{h}}}.$$

Application for Power calibration

Photon Calibrator

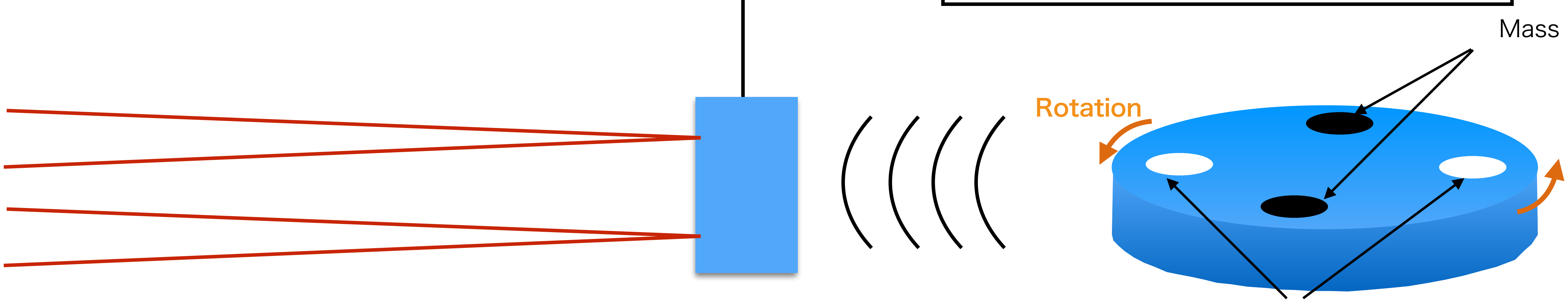
$$dx = \frac{2P\cos\theta}{c} s(f)\left(1 + \frac{M}{I}\vec{a}\cdot\vec{b}\right)$$

Cancelation

Gravity field Calibrator

$$x_{2\mathrm{f}} = 9\frac{GMm_{\mathrm{q}}r_{\mathrm{q}}^2}{d^4}s(\omega).$$

$$x_{3\mathrm{f}} = 15\frac{GMm_{\mathrm{h}}r_{\mathrm{h}}^3}{d^5}s(\omega).$$

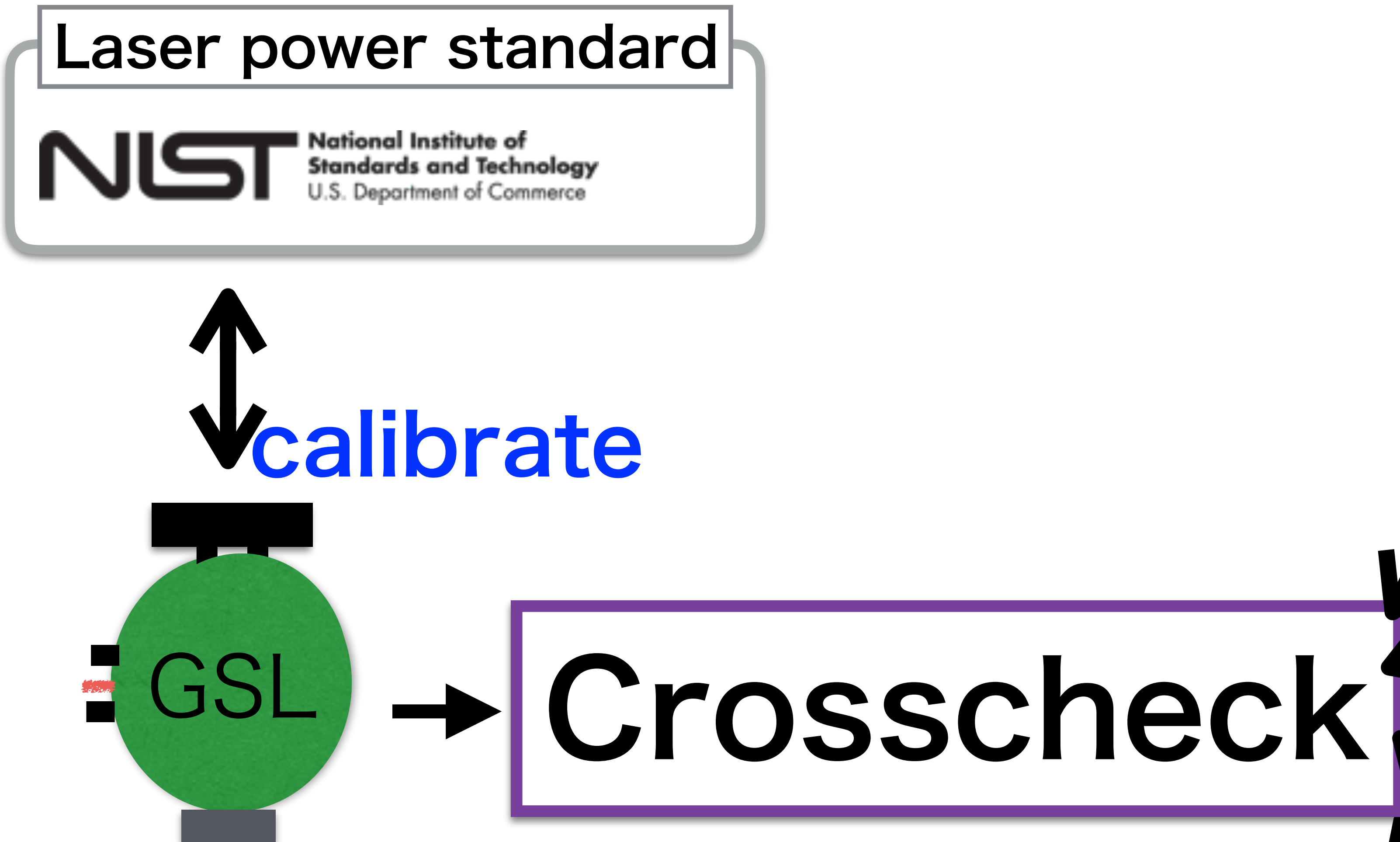


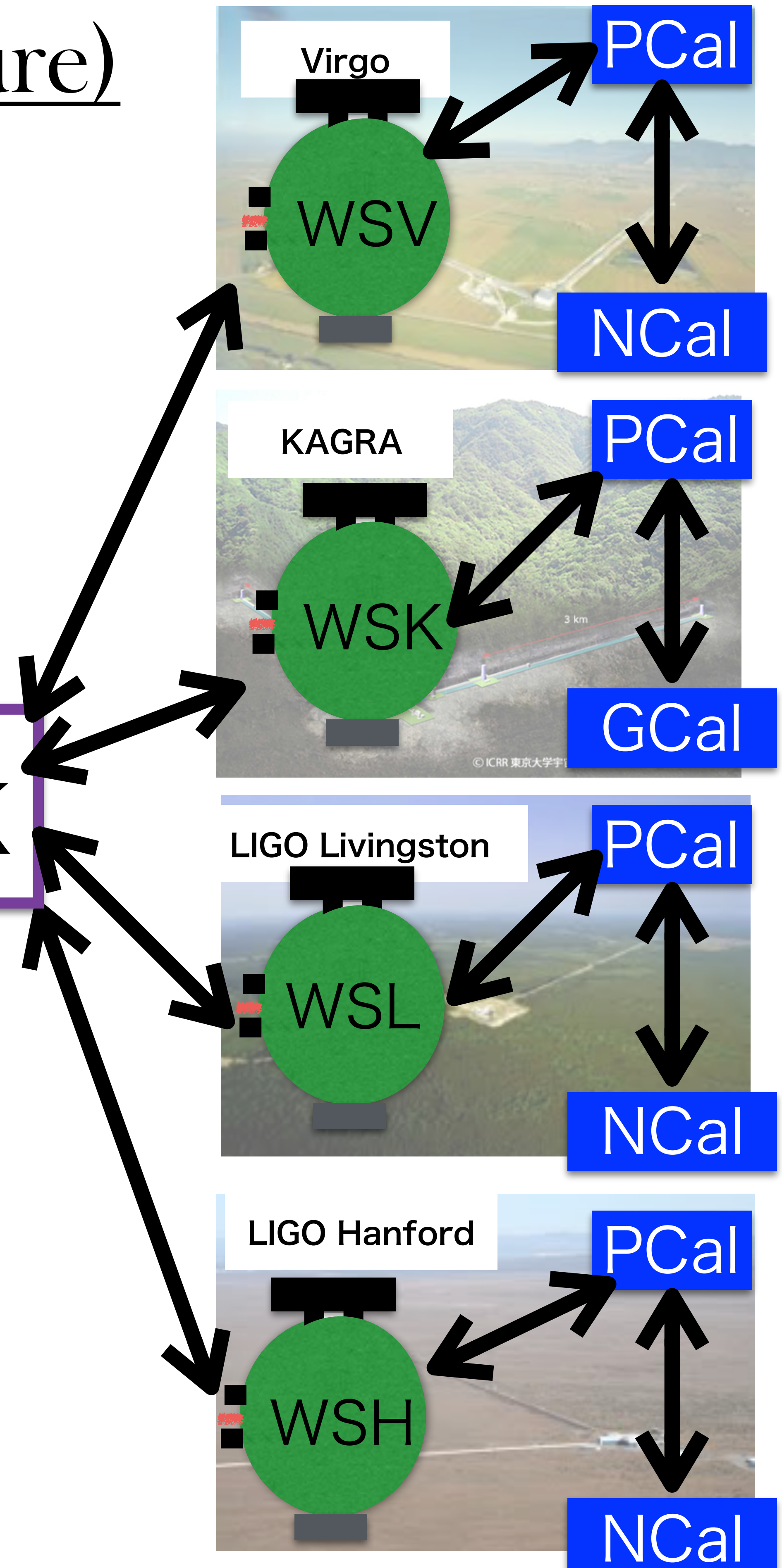
$$P_{2\mathrm{f}} = \frac{9}{2}\frac{Gcm_{\mathrm{q}}Mr_{\mathrm{q}}^2}{d^4 cos\theta}\frac{1}{1+\frac{M}{I}\vec{a}\cdot\vec{b}},$$

$$P_{3\mathrm{f}} = \frac{15}{2}\frac{Gcm_{\mathrm{h}}Mr_{\mathrm{h}}^3}{d^5 cos\theta}\frac{1}{1+\frac{M}{I}\vec{a}\cdot\vec{b}}.$$

We can define the power as geometrical factor and Gravity constant!

Calibration standard (More future)



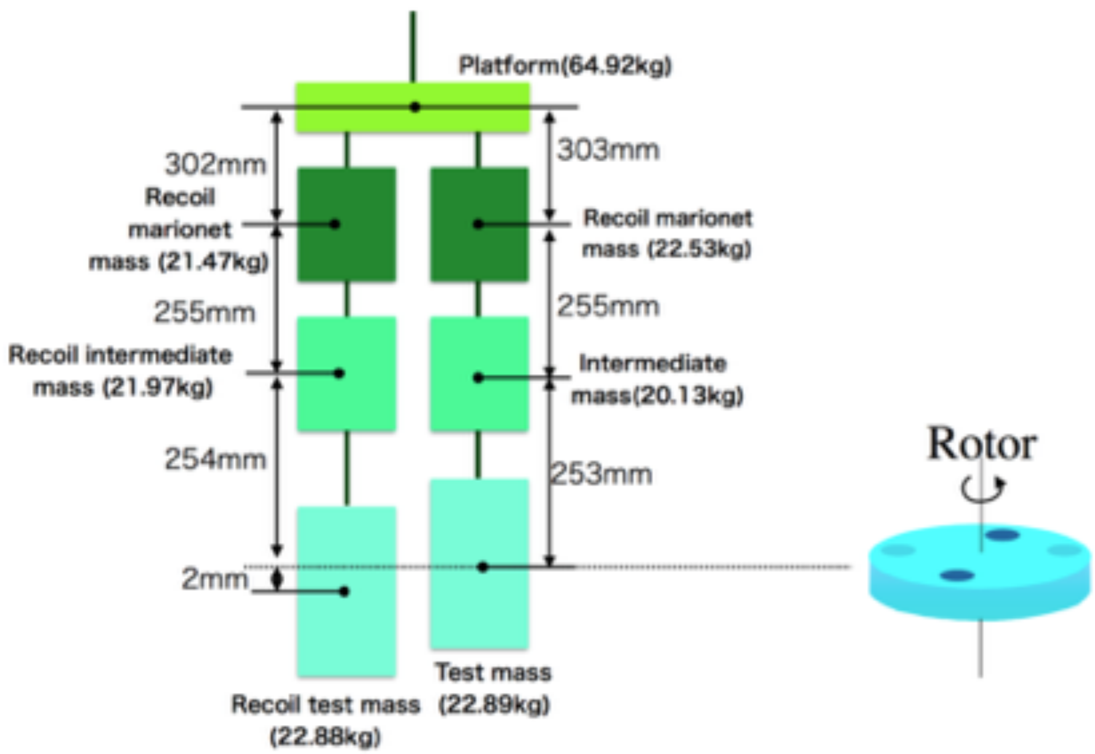


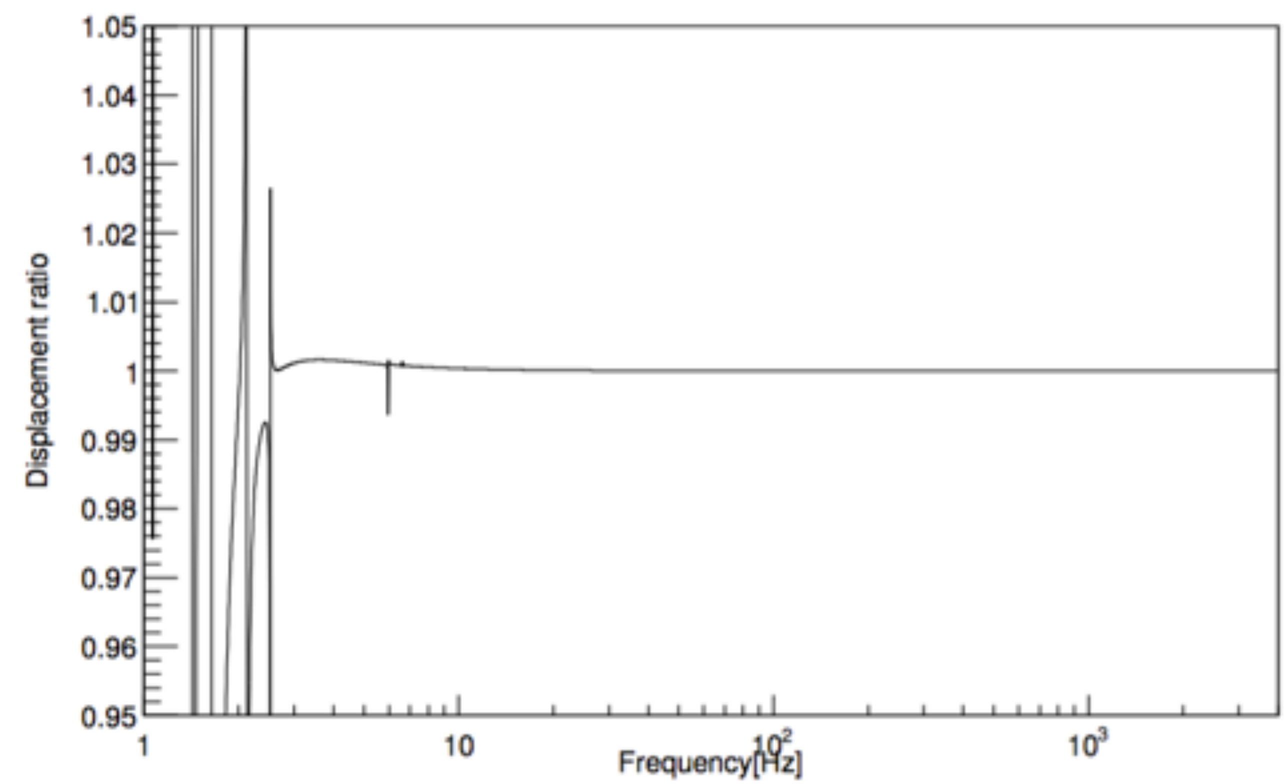
We can crosscheck each other for verification of Calibration uncertainty

Summary

- Gcal/Ncal are one of the new calibrators for interferometric gravitational wave detector.
- LVK are developing the new calibrators.
- KAGRA start the development and characterization of Gravity field calibrator from 2016. KAGRA will install the system after O3.
- Hybrid method can probably give an independent measurement method of the absolute laser power.

1.Coupling of other suspension





2.Higher Harmonics

TABLE III. Calculated quadrupole ($N = 2$) displacement. n is the order of the Legendre polynomial, where $\omega = n\omega_{\rm rot}$.

	n=1	n=2	n=3	n=4	n=5	n=6	n=7
1-f	0	0	0	0	0	0	0
2-f	0	$9\frac{Gmr^2}{d^4\omega^2}$	0	$\frac{25}{4}\frac{Gmr^4}{d^6\omega^2}$	0	$\frac{735}{128}\frac{Gmr^6}{d^8\omega^2}$	0
3-f	0	0	0	0	0	0	0
4-f	0	0	0	$\frac{175}{16}\frac{Gmr^4}{d^6\omega^2}$	0	$\frac{273}{32}\frac{Gmr^6}{d^8\omega^2}$	0
5-f	0	0	0	0	0	0	0
6-f	0	0	0	0	0	$\frac{1617}{128}\frac{Gmr^6}{d^8\omega^2}$	0

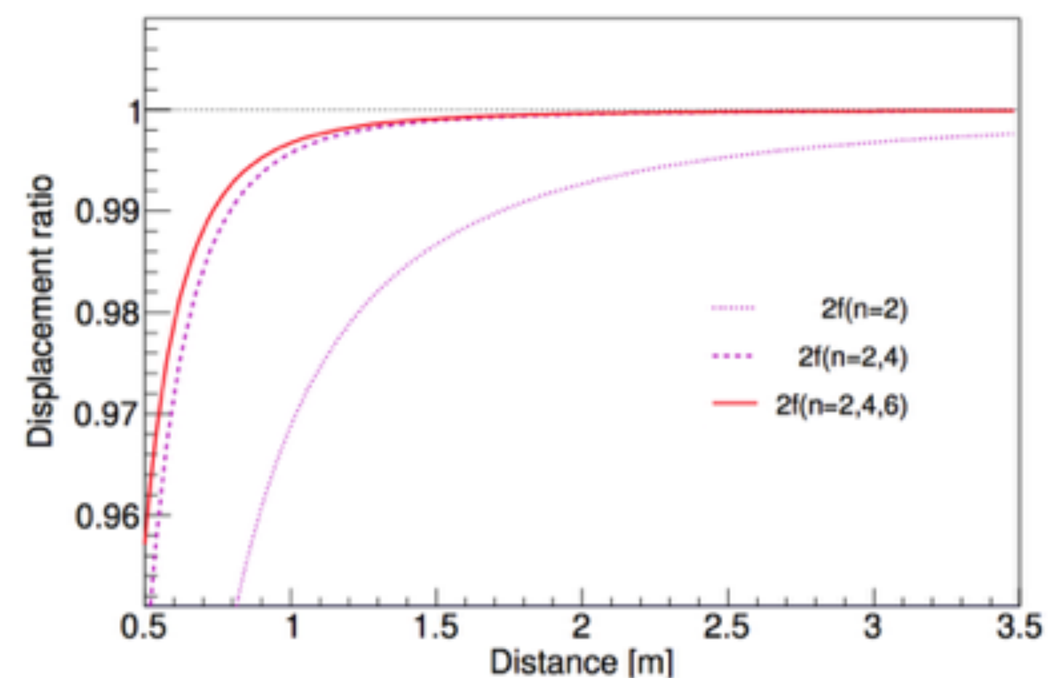
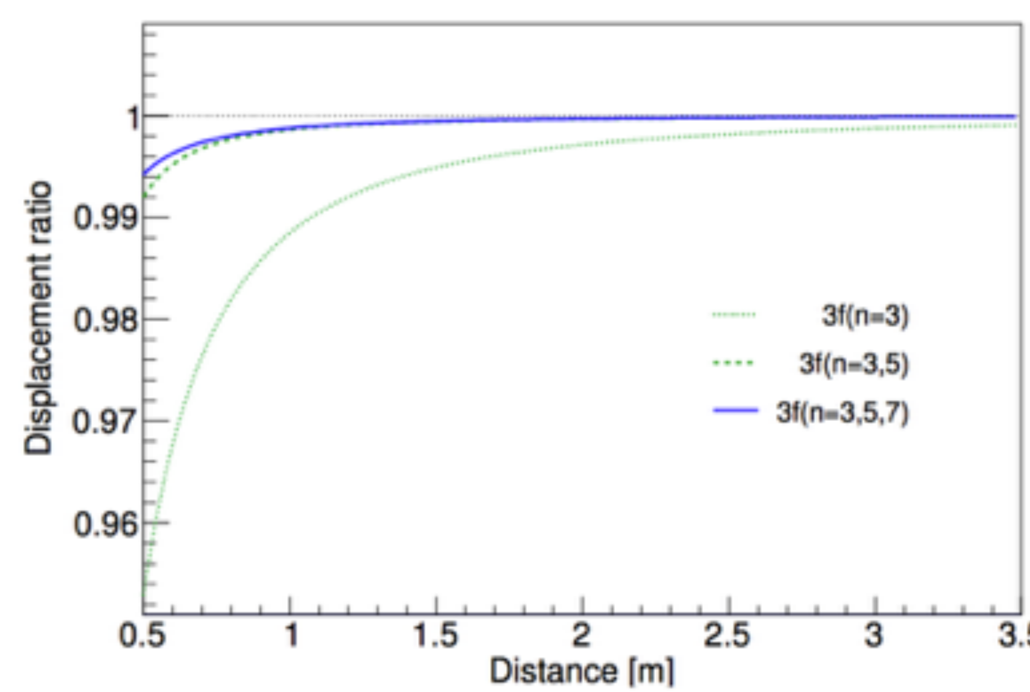


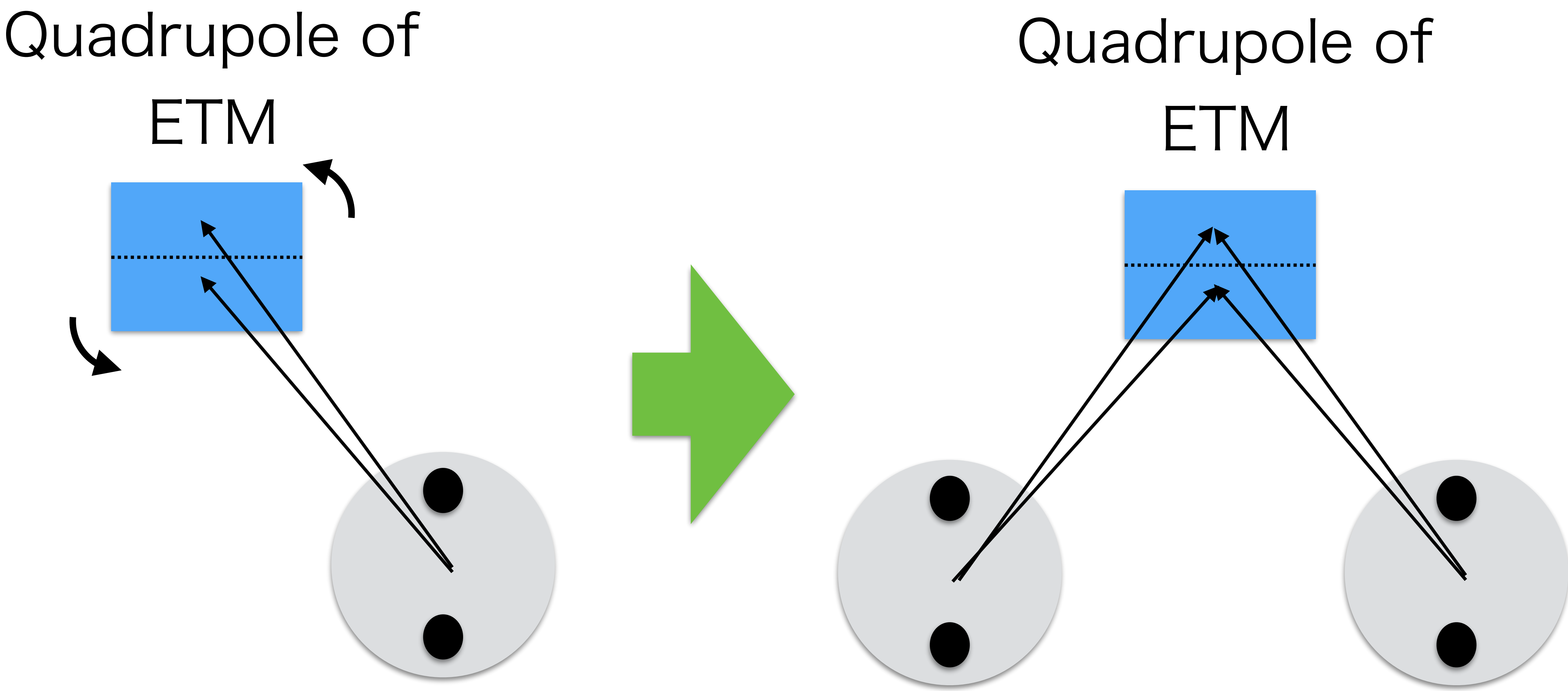
TABLE IV. Calculated hexapole ($N = 3$) displacement. n is the order of the Legendre polynomial, where $\omega = n\omega_{\rm rot}$.

	n=1	n=2	n=3	n=4	n=5	n=6	n=7
1-f	0	0	0	0	0	0	0
2-f	0	0	0	0	0	0	0
3-f	0	0	$15\frac{Gmr^3}{d^5\omega^2}$	0	$\frac{315}{32}\frac{Gmr^5}{d^7\omega^2}$	0	$\frac{567}{64}\frac{Gmr^7}{d^9\omega^2}$
4-f	0	0	0	0	0	0	0
5-f	0	0	0	0	0	0	0
6-f	0	0	0	0	0	$\frac{4851}{256}\frac{Gmr^6}{d^8\omega^2}$	0



$$Ma = \frac{2GMm_{\rm q}}{d^2}\Sigma_{n=0}^{\infty}(n+1)\left(\frac{r_{\rm q}}{d}\right)^n \times\Sigma_{i=0}^{1}P_n\left(\cos\left(\omega_{\rm rot}t+\pi i\right)\right).$$

3.Rotation

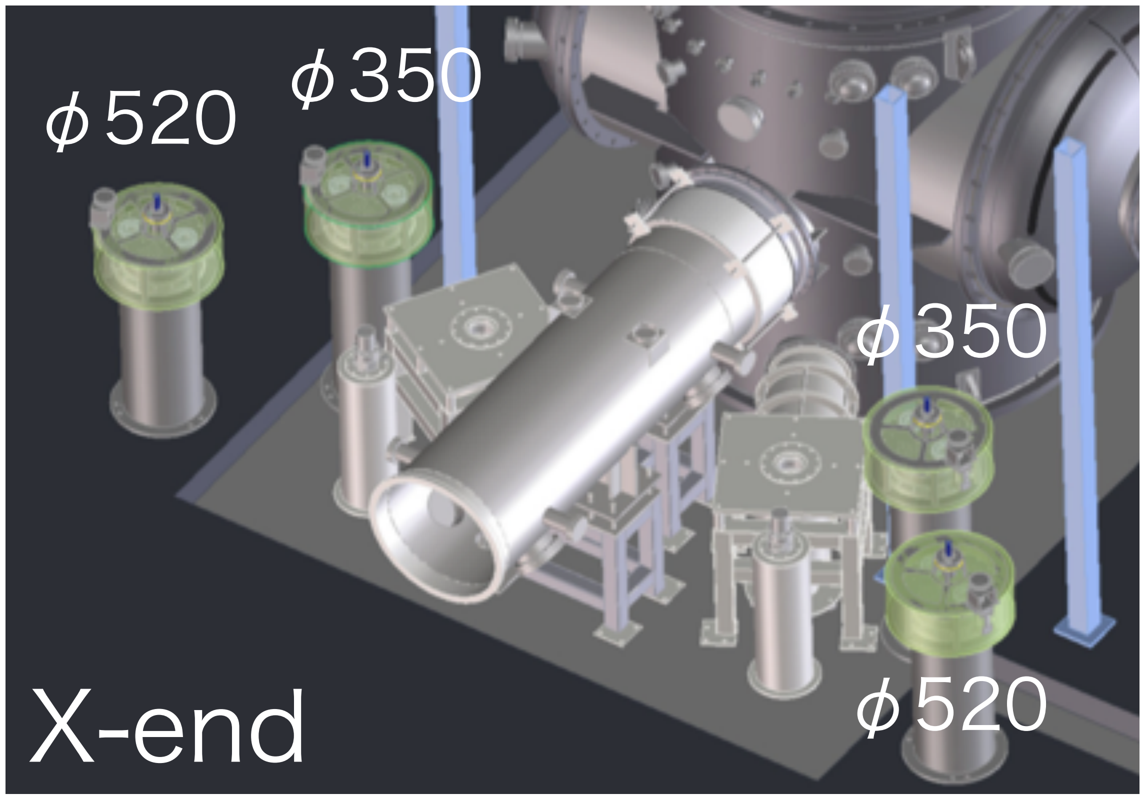


Cancellation!

Observation plans and verification of calibration

The verification of sub% error is very hard!

Target: <1% uncertainty



- We will operate the gravity field calibrator from O4.
- We plan to verify the calibration uncertainty by comparing four calibrators.