Thoughts on Damping the Sidles-Sigg torques on the Testmass

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1 Summary

The Sidles-Sigg torques on the arm cavity mirrors create a ‘hard’ mode in the arms with pitch and yaw frequencies over 3 Hz. These modes are currently damped by the wavefront sensors, and this damping is one of the drivers for the upper unity gain frequency of those loops. If one were able to damp the hard-mode motion with some other technique, that could enable us to lower the bandwidth of the hard-mode WFS control, and perhaps lower the noise coupling into DARM in the 10-30 Hz band.

Here, we present 2 possible damping schemes for the hard-mode suspension modes. These are meant as plausibility arguments only; clearly much work remains. The first technique is to sense the angular motion of each mirror directly and apply a control force at the penultimate mass (PUM). The second technique is to sense and drive at the PUM.

The plant we use is a modified version of the SUS model. This version was developed Brett Shapiro. In this instance we have 370 kW circulating in the arm cavity. This is more power than O3, but less than the final design. It was chosen for convenience; the hard mode is dominated by the optical torque, and the soft mode is still stable, so the calculations are more clear. We are also using the PUM damping described in T1800504, which is useful to control a 1 Hz mode which appears between the PUM and mirror.

The code is available in the SeismicSVN in the Common/Documents/T1800504 folder

2 Sense the Test-mass, Drive the PUM

This is the technique we mention in the 2018 proposal with Leo Hollberg T1800500. For this, we assume an angular sensor with a noise of $10^{-13}\,\mathrm{rad}/\sqrt{\mathrm{Hz}}$ of noise at 10 Hz. This is presumably mounted to the optical table of the ISI. Figure 1 shows the motion of the Suspension point for ITMX on Dec 19, 2018. That motion is comparable with this noise level, although the angular motion might need to be improved a bit.

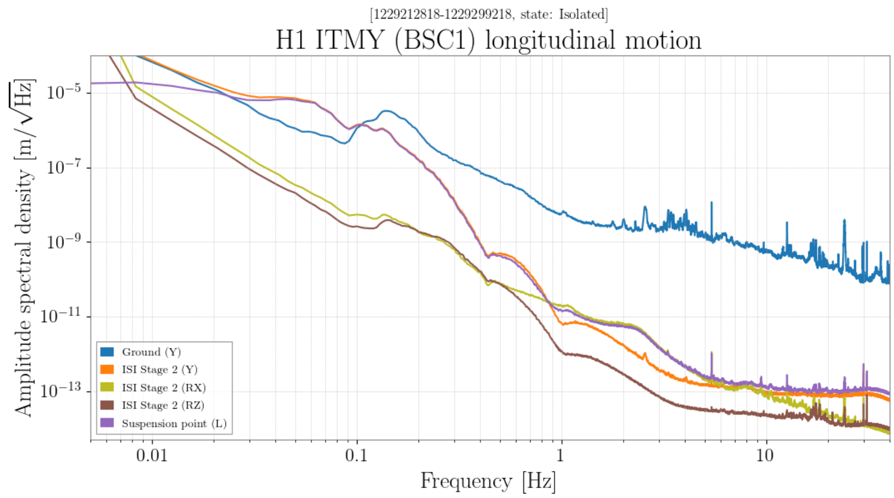

Figure 1: Motion of the ISI suspension point on Dec 19, 2018. These show the contributions to the SUSpoint motion, so the curve for the RX (pitch) is the (angular motion of the table) * (the lever arm from the coordinate center to the top of the suspension). The lever arm is about 22 cm.

For this damping loop, we choose to sense the mirror, but apply damping at the PUM. This has 2 benefits. First, one gets some isolation of control noise by driving at the PUM, and second, the motion is much easier to sense at the testmass. Figure 2 compares the coupling of PUM drive to PUM motion vs. PUM drive to mirror motion. The peak at ~ 2.8 Hz is the mode which is coupled to the hard-mode of the Sidles-Sigg torque. It is clear that the coupling to the mode is much better when one senses the the mirror (blue curve) than if one senses the PUM (orange curve).

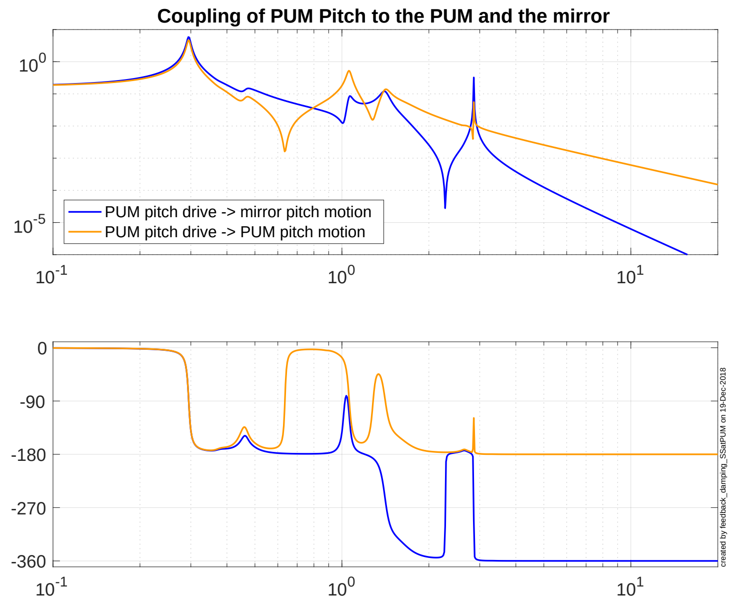

Figure 2: Comparison of the Transfer function when driving the PUM in pitch. The orange curve show the pitch response of the PUM, and the blue curve shows the pitch response of the mirror. We are trying to damp the peak at ~ 2.8 Hz. That mode is much more visible in the blue curve, which is why we model a case where the sensor is measuring the mirror.

We design a controller to damp the peak at 2.8 Hz, but not affect the other modes (at least not much). This controller employs some plant-inversion, in particular, matching the poles of the controller to the zero in the plant at 2.28 Hz. This is not a simple control; the plant changes with optical power, and clearly more work is required here. Never-the-less, in figure 3 we see that shape is

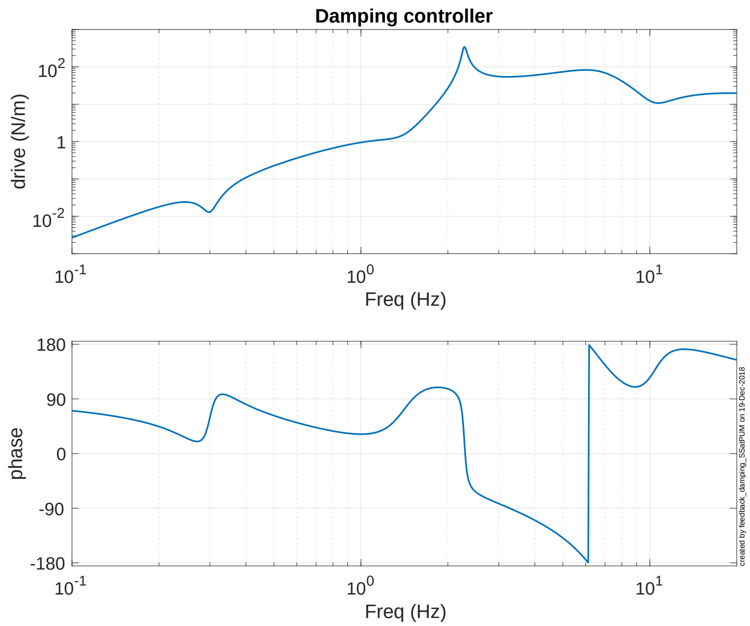

Figure 3: Controller used to damp the SS mode at 2.8 Hz. This controller is tailored to the plant - the peak at 2.28 Hz matches a zero in the plant, shaping below 2 Hz minimizes impact on other modes, and the dip at 10 Hz reduced noise coupling to DARM.

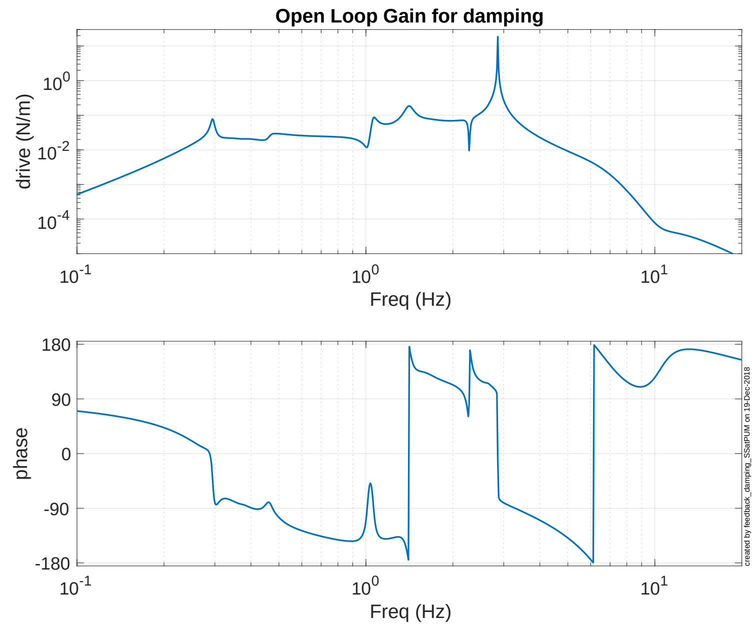

Figure 4: Open loop gain for the 2.8 Hz damping control. This is the product of the PUM actuator to Testmass pitch plant in figure 2 and the controller in figure 3. The gain is > 1 only around the 2.8 Hz mode.

When this controller is applied to the model, the Q of the 2.8 Hz mode is reduced to about 20, as shown in figure 5.

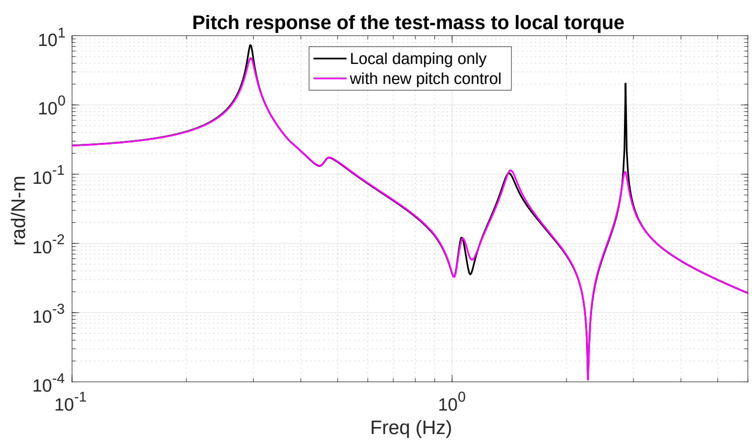

Figure 5: Pitch of the test-mass in response to a torque on the test-mass. Compare the 2.8 Hz mode when there is only local damping from the top OSEMs and and the UIM dampers, compared with the addition of local damping from a sensor on the test mass and control applied to the PUM (magenta). The additional local path can damp the mode.

The final plot shows the pitch motion of the test-mass in response to a sensor with an angular noise of $10^{-13}\,\mathrm{rad}/\sqrt{\mathrm{Hz}}$. The noise coupling to the mirror at 10 Hz is about 1e-4. This coupling is the closed loop transmission drive at the PUM to motion at the mirror * the controller. When the Open loop gain is large, this is approximately -1, and when the loop gain is small, this is about the same as the open loop gain.

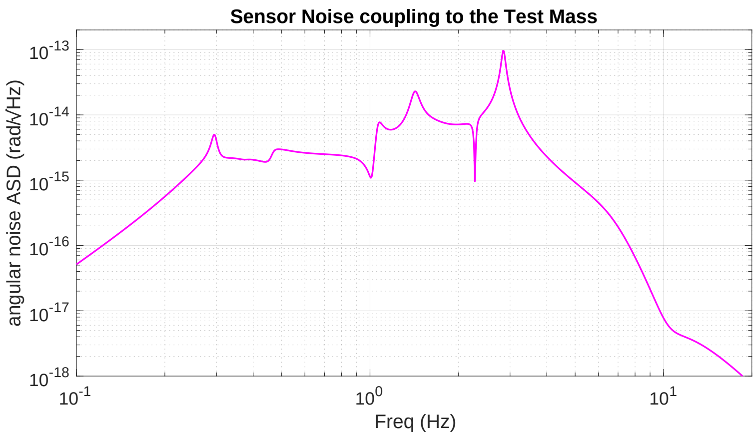

Figure 6: Pitch of the test-mass as driven by sensor noise from the angular sensor used for the damping. This assumes sensor noise of $10^{-13}\,\mathrm{rad}/\sqrt{\mathrm{Hz}}$. At 10 Hz, it meets the requirement of $10^{-17}\,\mathrm{rad}/\sqrt{\mathrm{Hz}}$.

This is the result which was presented in the proposal.

3 Parametric Study - Introduction

Edgard Bonilla has also conducted a parametric study on a potential feedback controller that uses sensing and actuation on the Penultimate Mass (PUM) of a Quadruple suspension (QUAD) to try and damp the Sigg-Sidles mode down to a Q of less than 20.

The calculations were made with the aid of the QUAD Matlab Model, augmented with the optical spring that represents the laser power circulating in the cavity.

This section intends to show the results of the parametric study, to show a couple of ideas worth investigating further and to point to the resources necessary to replicate its findings. It starts by providing basic information on the Matlab model, then explaining our metrics for evaluating the effectiveness of the feedback loop and finally shows results on a reasonable configuration that meets our needs.

4 Summary of Methods

4.1 Radiation Pressure Model

The radiation pressure model can be found in the Sus SVN repository under `'sus/trunk/QUAD/Common/MatlabTools/Q`

The script outputs both an Input Test Mass and End Test Mass model, as well as the connected Radiation Pressure model that we use for most results here.

The parametric study assumes 70 Watts of input laser power.

4.2 Q values and FWHM

We will estimate the Q value of the Sigg-Sidles resonance by using two different methods. The first one is by grabbing the resonance's damping ratio ζ and calculate

$$Q = \frac{1}{2\zeta} \tag{1}$$

If this quantity is positive, we expect the pole to be stable.

The other method consists of measuring the width of the peak on the transfer function. This is done by dividing the resonance frequency by the Full-Width at Half-Maximum (FWHM) of the resonance in the power spectrum for the signal. We calculate the FWHM by looking at the Test Mass to Test Mass transfer function magnitude around the Sigg-Sidles resonance and measuring the width at $\frac{1}{\sqrt{2}}$ of the maximum magnitude (the square root is to scale for amplitude instead of power).

The nuance and difference from these two methods becomes apparent when there are two resonant frequencies close together. If no such case is encountered, we will limit ourselves to measuring the Q by using the analytic equation above.

4.3 Sensor noise Requirements

We would like the coupling of the sensor noise through the loop to be below $10^{-17}\text{rad}/\sqrt{\text{Hz}}$ at 10 Hz. We opt for showing the maximum allowed sensor noise that would satisfy that requirement at 10 Hz.

5 Preliminary Parametric Study

For the parametric study we chose a very simple damping filter to serve as a proof-of-concept to what can be done by damping on the PUM. The filter is just a zero at DC and two poles that can be varied in frequency. The gain of the filter can be adjusted to find an optimal (Q-minimizing) configuration given the shape of the filters. Finally, the sensor noise requirement can be also plotted vs. frequency and gain.

Figure 7 shows an example of the study results, for all of the different filter shapes, the Q factor of the Sigg-Sidles mode undergoes a 'U' shaped curve. At the low magnitude end the system is practically undamped. At the high frequency end the PUM does not participate on the motion, and it causes the resonance to be undamped again.

We chose the two filter designs highlighted in figure 7 from the 2 poles at 4.2 and 8.2 Hz curves to perform sanity checks. The results are shown in figures 8 and 9. Figure 8 shows that both loops are stable, although the phase margin is better for the one with poles at higher frequencies.

The projected performance can be further explored by looking at the sensitivity functions for both filters, as shown in Figure 10. The peak for the filter with two poles at 4.2 Hz has a magnitude of about 3, which we consider to be right on the limit of being acceptable.

Finally, we can compare the different filters with the undamped plant using figure 9. It can be appreciated that both schemes significantly improve the Q of the resonance.

The sensor noise requirements for the filters shown in figures 8 and 9 are $2 \times 10^{-14}\text{rad}/\sqrt{\text{Hz}}$ and $8 \times 10^{-15}\text{rad}/\sqrt{\text{Hz}}$ at 10Hz for the 4.2 and 8.2 configurations respectively.

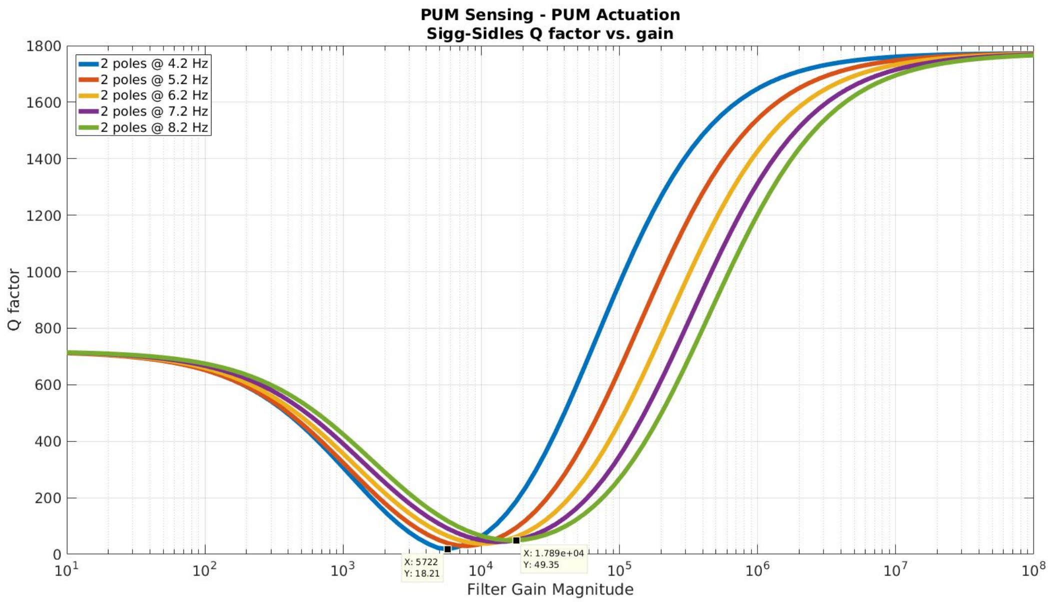

Figure 7: *Q factor of the Sigg-Sidles resonance, as seen in the Test Mass pitch drive to Test Mass pitch motion transfer function. The systems are damped with a PUM pitch sensing and actuation loop. The control filter has one zero at DC and two poles at the specified locations. All the curves have a very clear minimum, and lower frequency poles seem to be favored.*

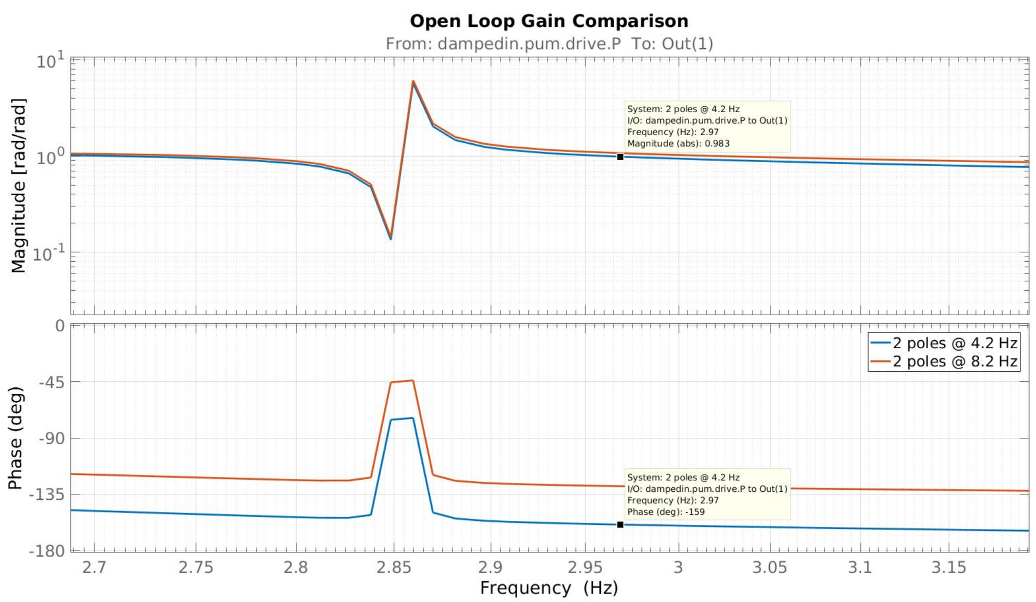

Figure 8: *Open Loop Gain for the two different controllers. We can appreciate that while both of them are stable, the phase margin might be reason enough for selecting one over the other (20 degrees for 4.2 vs 50 degrees for 8.2).*

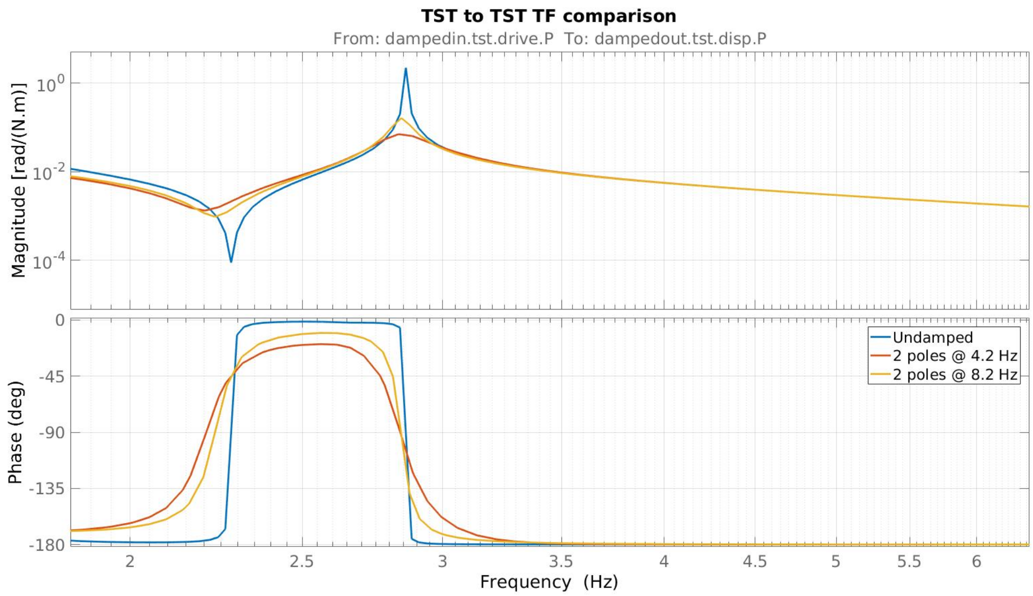

Figure 9: *Test Mass pitch drive to Test Mass pitch motion transfer function. The comparison of the undamped plant to the two damping schemes shows that placing the two poles closer (4.2 Hz vs. 8.2 Hz) to the Sigg-Sidles resonance (≈3 Hz) improves the Q of the mode appreciably.*

6 Discussion

The preliminary study seems to suggest a number of things that we will use as stepping stones for a more detailed study of damping the Sigg-Sidles mode from the PUM. We will list the most important results and next steps below:

1. It seems possible to lower the Q of the resonance below 20 by using a local PUM controller. In fact we believe that by locating the two poles closer to the resonance can achieve better results.

2. The performance of this kind of controller depends on a trade-off between how strongly we damp the PUM and the ability of the PUM to participate in the resonance we are trying to damp. This idea will be investigated further to gain better intuition on how to tune the gain.

3. The parametric study showed that placing the two poles at lower frequencies achieves better performance, but the filter generated is at risk of being unstable. We need to find a compromise for what a good phase margin should be and constrain our search to filters that satisfy that requirement.

4. To achieve a motion of less than $10^{-17}\text{rad}/\sqrt{\text{Hz}}$ at 10 Hz imposes a requirement for the PUM sensor noise to be $10^{-14}\text{rad}/\sqrt{\text{Hz}}$ or less at 10 Hz.

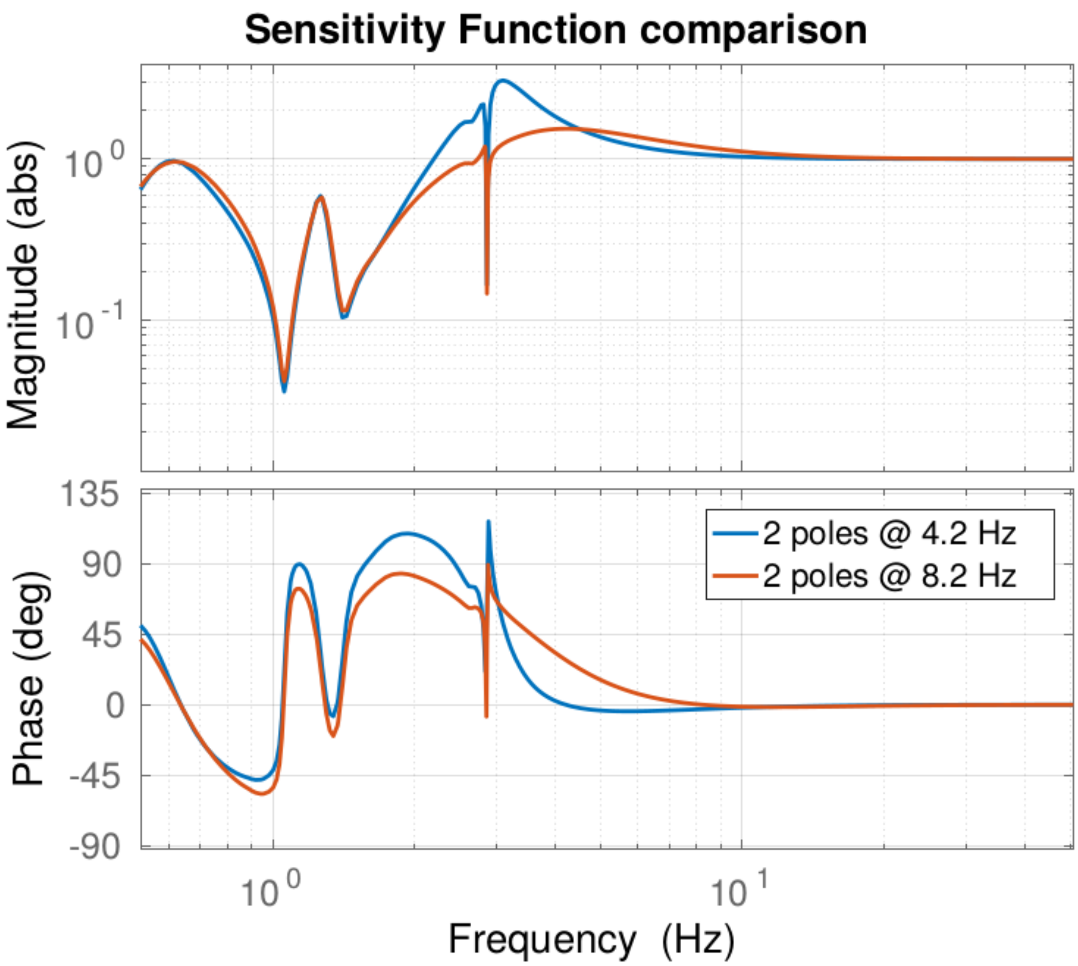

Figure 10: *Sensitivity function for the two damping schemes. The controller with two poles at 4.2 Hz has a higher peak, meaning that it amplifies the PUM motion. However, its performance in damping the Sigg-Sidles mode at the test mass is better, as shown in figure 9*