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Filter Cavity Baffle Diffraction

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1 Baffle Diffraction

The following equations are based off of T950101 eq. 4, which gives the scatter coupling into strain including phase factors for the light. Just following that equation is a section that outlines all of the parameters, and the equation can be converted to the following form. In practice, the most convenient parameters available to characterize the scattering from an optic is its Bi-directional reflectance distribution (BRDF). For this document, only one-direction is necessary as the optic is illuminated by a coherent laser beam, so the BRDF, $B_O(\theta, \phi)$ indicates the fractional power scattered into unit solid angle at radial angle θ and azimuthal angle ϕ. The azimuthal angle is often dropped as it averages to a constant assuming azimuthally symmetric randomness of mirror surfaces.

The important point of T950101 is that the phasing of the scattered light can be expected to be slowly varying across the baffle edge, due to diffraction smoothing out the phase front from the optic. Due to this, motion of the baffle can strongly modulate the coupling from one mirror to another from the variations in the overlap integral of the scattered light. Serrations may be added to generate phase modulations that average away the overlap integral of a modulating aperture.

This document primarily derives the noise component without serrations, using the BRDF. This allows a quick calculation of the magnitude of the problem if serrations are omitted. The BRDF is additionally useful because it is related to the total scatter loss of the optic, and expectations of that loss constrain scattering models.

The length noise from a single baffle between two optics is:

$$\delta l_{\mathrm{FC}} \leq \frac{\lambda R \sqrt{B_o(\theta_n) B_o(\theta'_n)}}{\pi l_n (L - l_n)} \delta X_{\mathrm{baffle}} \tag{1}$$

where l_n is the distance down the tube of the baffle. R is the radius of baffle aperture. $L = 300\mathrm{m}$ is (approximately) the tube length and $B_o(\theta_n)$ is the BRDF of the optic surface for the opening angle of the n'th baffle. The angles θ_n and θ'_n are the polar angles of the baffle apertures from each respective optic and baffle faces.

$$\theta_n = \frac{R}{l_n} \qquad\qquad \theta'_n = \frac{R}{300\mathrm{m} - l_n} \tag{2}$$

Dennis's document T2000280 outlines that the horizontal motion of the tubes/baffles can be expected to be in the few $10^{-8}\frac{\mathrm{m}}{\sqrt{\mathrm{Hz}}}$, so we will assume $\delta X_{\mathrm{baffle}} \leq 10^{-7}\frac{\mathrm{m}}{\sqrt{\mathrm{Hz}}}$. Given expected backscatter, the filter cavity has the length noise requirement $\delta l_{FC} \leq 2{\cdot}10^{-16}\frac{\mathrm{m}}{\sqrt{\mathrm{Hz}}}$ at which optical modulations are sufficiently hidden from the interferometer. This factor includes a safety factor of 10 and the squeezing level. This gives the requirement:

$$\frac{\delta l_{\mathrm{FC}}}{\delta X_{\mathrm{baffle}}} \leq S_{\mathrm{diffraction}} \leq 2{\cdot}10^{-9} \tag{3}$$

The formula for $S_{\mathrm{diffraction}}$ ultimately depends on all of the baffles down the tube, and can be considered as a worst case, where all of the baffles coherently add

$$S_{\text{diffraction}} = \sum_{n=0}^{N} \frac{\lambda R \sqrt{B_o(\theta_n) B_o(\theta'_n)}}{\pi l_n (300\text{m} - l_n)} \quad \text{or} \quad S_{\text{diffraction}} = \sqrt{\sum_{n=0}^{N} \left| \frac{\lambda R \sqrt{B_o(\theta_n) B_o(\theta'_n)}}{\pi l_n (300\text{m} - l_n)} \right|^2} \tag{4}$$

1.1 Plots assuming different BRDFs of the mirrors

In this section, various BRDF models are plugged into the formulas above to determine the apparent length noise arising from baffle modulations. All models use a θ^{-2} dependence at low frequencies, as this dependence can be considered maximally bad. Any faster θ^{-3} or more and the rolloff is so fast that the $theta'$ contribution is minimal (or the total scatter becomes unrealistic). Any slower like $\theta{-}1$ and the θ contribution is very small for realistic total scatter. In any case, random mirror maps of the optic surface seem to indicate that $theta^{-2}$ is a reasonable angle dependence.

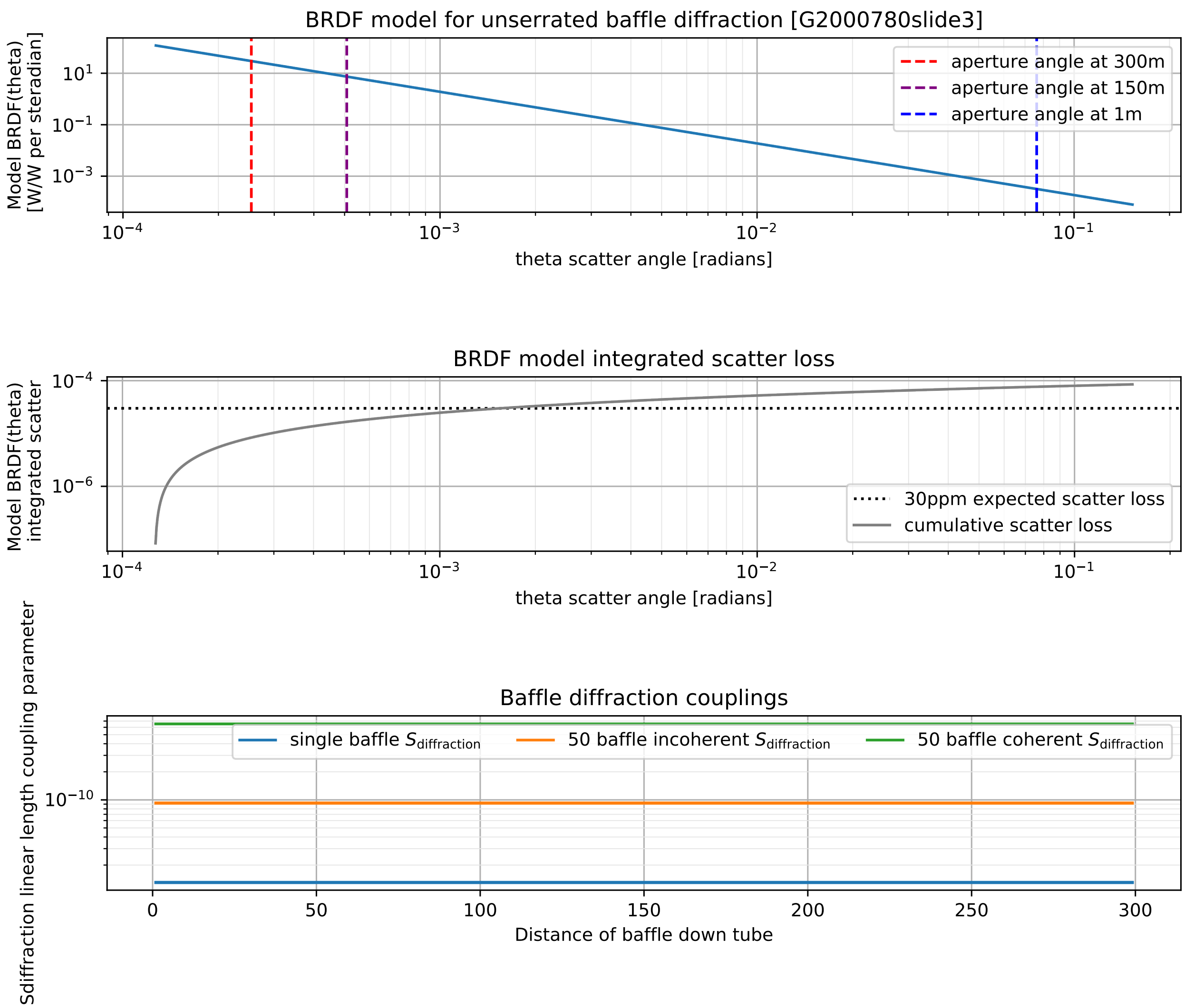

Figure 1: This model uses $B_O(3\text{in}/300\text{m}) \approx 80$ as shown in slide 3 of G2000780, which also appears to model a similar θ^{-2} dependence to the scatter power.

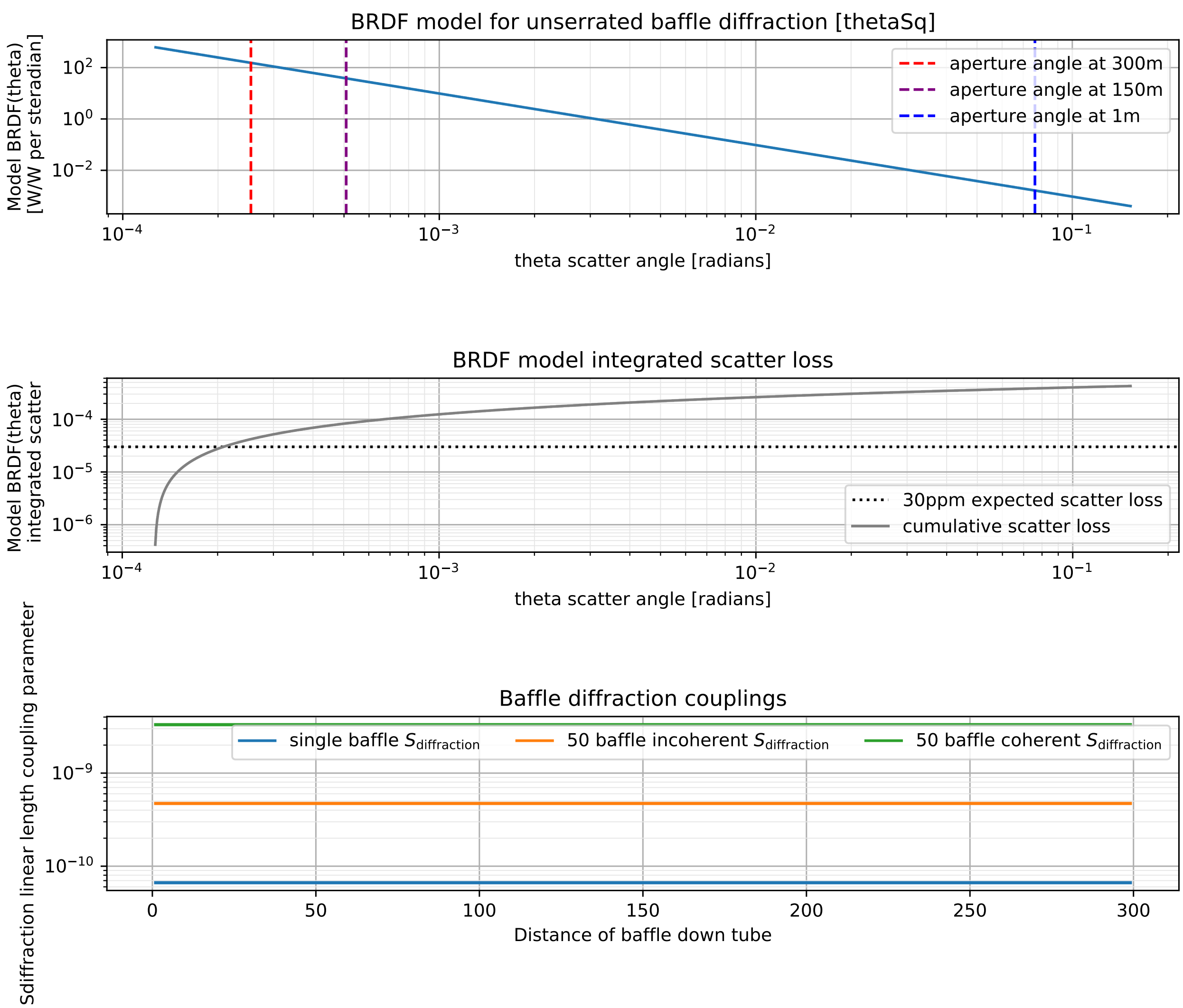

Figure 2: This BRDF uses a worst case BRDF of 30ppm/$\pi\theta^2$, which causes any/all θ to saturate the expected loss.

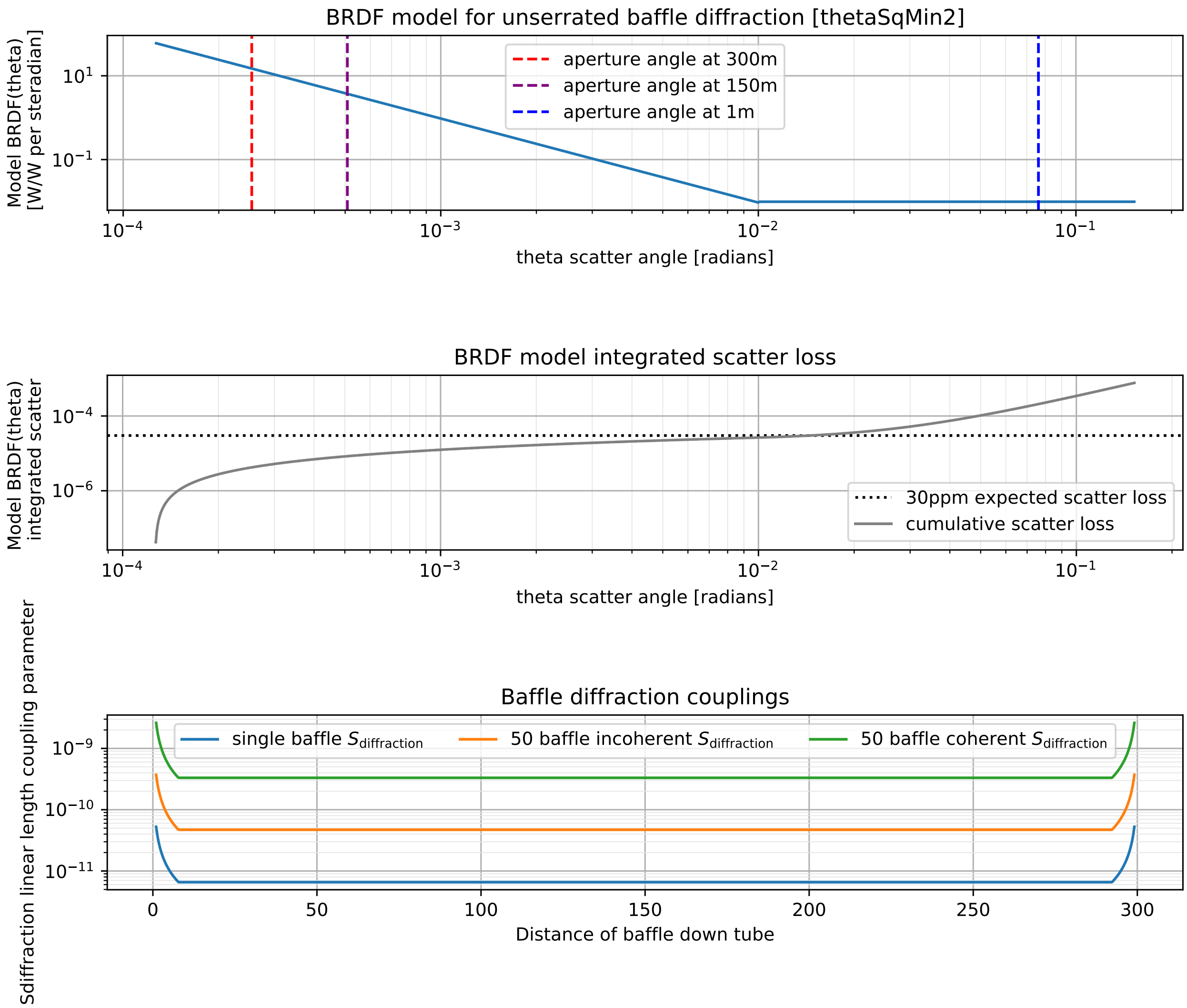

Figure 3: This BRDF uses a BRDF of $30\text{ppm}/10\pi\theta^2$, scaled to not violate the loss expectation. Then it cuts off the roll off for a constant reflectance. This causes it to violate the loss expectation, but in a manner that investigates the influence of intermediate angle scattering.

1.2 Derivation

This section derives the equations used above by relating the field distribution along any z-plane down the tube from the input optic FC1 $U_{FC1}(x, y, z)$ back to the BRDF of the optic.

The field overlap of the field plane is calculated using an overlap integral with a masking function $m(x, y)$ that depends on the modulating lateral displacement δX of the baffle.

$$A = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} m(x, y)\mathrm{d}x\mathrm{d}y = 4R\delta X \tag{5}$$

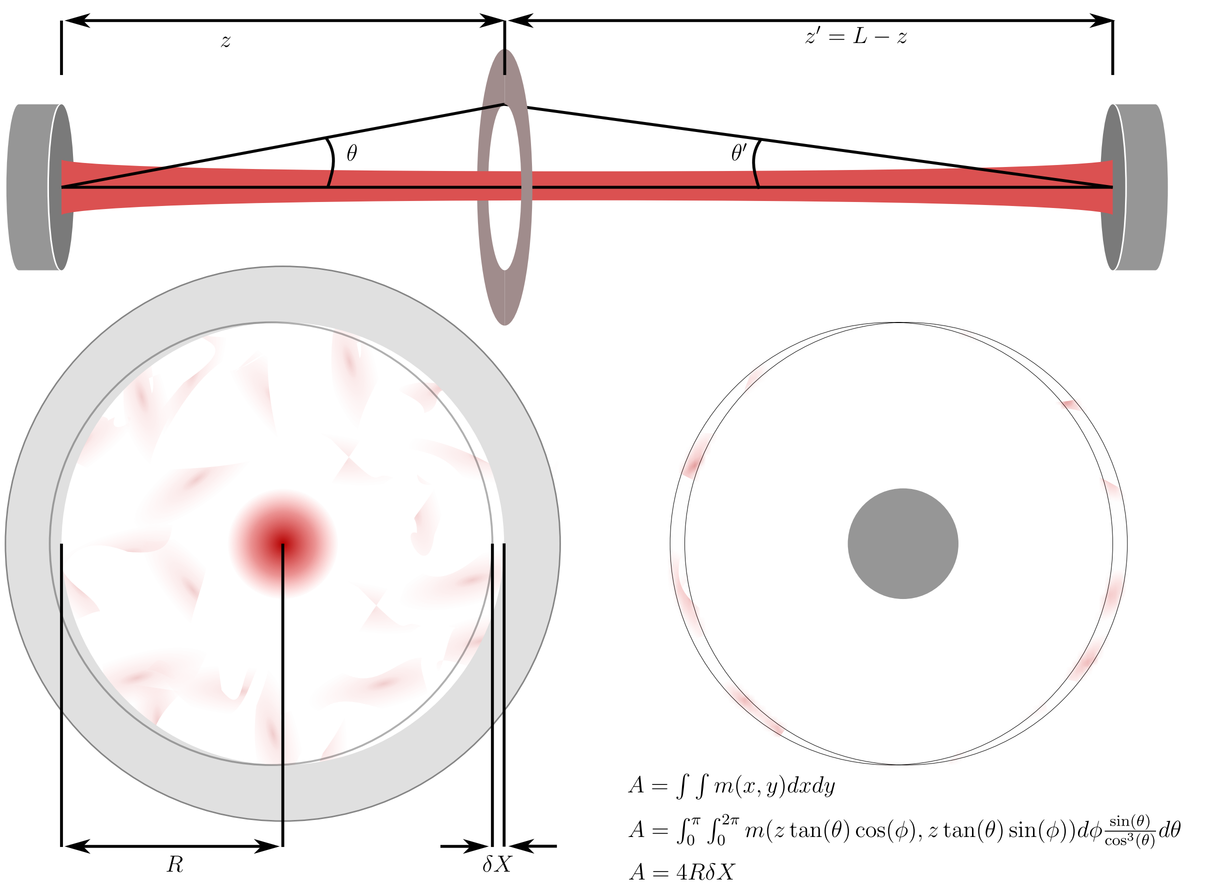

Figure 4:

Down the tube there is the beam field profile for each mirror $U_{FC1}(x, y, z)$ and $U_{FC2}(x, y, z')$. These fields can be related to a nominal fundamental mode U_{00} and the scatter is related to deviation of the field from the fundamental, which is expressed as U_{diff1} and U_{diff2}.

$$U_{FC1}(x, y, z) \approx U_{00}(x, y, z) \tag{6}$$

$$1 = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |U_{FC}(x, y, z)|^2\mathrm{d}x\mathrm{d}y \tag{7}$$

$$1 = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |U_{00}(x, y, z)|^2\mathrm{d}x\mathrm{d}y \tag{8}$$

$$U_{\mathrm{diff1}}(x, y, z) = U_{FC1}(x, y, z) - U_{00}(x, y, z) \tag{9}$$

To relate this field deviation profile to the BRDF, we must change coordinates and account for the volume elements since the field profile and BRDF are both densities. The coordinate transformation is:

$$x = z\tan(\theta)\cos(\phi) \qquad\qquad y = z\tan(\theta)\sin(\phi) \tag{10}$$

And the two are then related by:

$$B_o(\theta,\phi)\sin(\theta)\mathrm{d}\phi\mathrm{d}\theta = |U_{FC1}(x,y,z) - U_{00}(x,y,z)|^2\mathrm{d}x\mathrm{d}y \tag{11}$$

$$B_o(\theta,\phi)\sin(\theta) = \left|U_{\text{diff1}}\left(z\tan(\theta)\cos(\phi), z\tan(\theta)\sin(\phi), z\right)\right|^2 z^2\frac{\sin(\theta)}{\cos^3(\theta)} \tag{12}$$

$$B_o(\theta,\phi) \approx \left|U_{\text{diff1}}\left(z\tan(\theta)\cos(\phi), z\tan(\theta)\sin(\phi), z\right)\right|^2 z^2 \tag{13}$$

With the BRDF and field profile established, we can write the exact modulation coupling, c, of the fields from the two optics from the masked overlap integral. This coupling can then be related to an equivalent effective length modulation.

$$c = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} m(x,y)U_{\text{diff1}}(x,y)U^*_{\text{diff2}}(x,y)\mathrm{d}x\mathrm{d}y \tag{14}$$

$$\delta L = \frac{\lambda}{4\pi}|c| \tag{15}$$

Of course, we do not know U exactly, so we can set an estimate, C, using the expected magnitude of U. This removes the phase modulation that causes cancellations in the overlap integral.

$$|c| \leq C = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} m(x,y)\left|U_{\text{diff1}}(x,y)U^*_{\text{diff2}}(x,y)\right|\mathrm{d}x\mathrm{d}y \tag{16}$$

This can then be related back the BRDF and simplified.

$$C = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} m(x,y)\sqrt{\frac{B_o(\theta(x,y,z),\phi(x,y))}{z^2}}\sqrt{\frac{B_o(\theta'(x,y,z),\phi(x,y))}{(L-z)^2}}\mathrm{d}x\mathrm{d}y \tag{17}$$

$$\approx \frac{4R\delta X}{z(L-z)}\sqrt{B_o(\theta)B_o(\theta')} \tag{18}$$

Finally, the coupling limit C is exactly $S_{\text{diffraction}}$ from the first section.

$$\delta L \leq \frac{\lambda R\sqrt{B_o(\frac{R}{z})B_o(\frac{R}{L-z})}}{\pi z(L-z)}\delta X \tag{19}$$

1.3 Required Serration Scale

Assuming serrations are wanted, we can attempt to determine how long they must be to attenuate the coupling through cancellation. Here we have to assume that $\sqrt{B_o(\theta,\phi)B_o(\theta',\phi)}$ is slowly varying envelope in ϕ. The serration length is $Y(\phi)$ and the baffle is assumed to be at a 45° angle, although the final expression does not depend on this. Using these parameters, the coupling, c averages down to become approximately:

$$c \approx \frac{4R\delta X}{z(L-z)} \int_0^{2\pi} e^{i2kD(\phi)} \sqrt{B_o(\theta,\phi)B_o(\theta',\phi)} |\sin(\phi)| \mathrm{d}\phi \tag{20}$$

$$D(\phi) = \sqrt{(L-z-Y(\phi))^2 + (R+Y(\phi))^2} + \sqrt{(z+Y(\phi))^2 + (R+Y(\phi))^2} \tag{21}$$

$$= z'\sqrt{\left(1-\frac{Y(\phi)}{z'}\right)^2 + \left(\frac{R}{z'}+\frac{Y(\phi)}{z'}\right)^2} + z\sqrt{\left(1+\frac{Y(\phi)}{z}\right)^2 + \left(\frac{R}{z}+\frac{Y(\phi)}{z}\right)^2} \tag{22}$$

$$\approx L + \frac{Y^2(\phi) + R^2/2 + RY(\phi)}{z'} + \frac{Y^2(\phi) + R^2/2 + RY(\phi)}{z} \tag{23}$$

$$\approx L + \frac{LR}{z(L-z)}(R/2 + Y(\phi)) \tag{24}$$

The number of fringe wraps implies the number of "attempts" at a phase cancellation around the slowly varying wavefront envelope. Since this is generating an incoherent process, the average cancellation will be inversely proportional to the square-root of the number of cancellations. For a factor of 10 reduction, this requires 100 wavelengths of phase difference for the traversal distance from the baffle edge to the two mirrors.

$$D_{\mathrm{span}} \sim 100 \cdot 10^{-6}\mathrm{m} \tag{25}$$

$$Y_{\mathrm{span}} = \frac{z(L-z)}{LR} D_{\mathrm{span}} \leq \frac{L}{4R} D_{\mathrm{span}} = 0.1\mathrm{m} \tag{26}$$

This is larger than the baffle itself. Serrations of 1cm are more likely, but will only achieve an attenuation of the order of 3x.

2 Baffling

This section is included from the FC DRD, T18000447, since it uses a similar formulation.

The Filter cavity (FC) will require some level of baffling to the possibility that ground motion in the vacuum enclosures couples optically to either the backscatter and sensing fields. The requirements are set from the backscatter performance, as well as the total length RMS of the cavity. The cavity enhances optical couplings through the round-trip gain of the cavity, which can modulate the backscatter field to cause noise, or it can modulate the sensing field to generate sensing noise, which is then injected through the control loops.

The calculate this we need to estimate the BRDF of the mirrors, the BRDF of the surrounding baffle or beamtube material, and kinematic factors for the scattering off of mirrors to baffles, off of baffles to mirrors, and finally from mirrors into the beam. All of these factors can be wrapped into a triple integral and reduced down to the condensed form used in eq. 17 of T940063 as

$$S^2_{\mathrm{diffuse}} = \int_0^{2\pi} \int_0^{\frac{\pi}{2}} \frac{\lambda^2}{r^2(\theta)} B_o^2(\theta) B_t(\theta + X) \sin(\theta) \mathrm{d}\theta \, \mathrm{d}\phi \tag{27}$$

where S^2_{diffuse} is the *power* coupling of the beam in the cavity, off of surroundings and back into the beam. The factor B_o is the BRDF of the optic, and B_t is the BRDF of the tube or baffle. The additional angle X is the angle between the tube/baffle and the beamline. $r(\theta)$ is the distance of the scattering surface (tube/baffle) from the optic.

Requirements on the diffuse scattering are set from the noise they introduce on the backscatter, as well as length RMS introduced into the length sensing. Since the sensing co-propogates with the squeezing field in the cavity, it is likely that introduced phase noise is common between then and should not affect the phase noise requirements where the loop is active, but it is worth checking how much noise is introduced nevertheless.

For the length sensing, the scale is (from eq. 18 of T940063 converted from strain to length)

$$\delta l_{\mathrm{scatter}} = S_{\mathrm{diffuse}} \delta l_{\mathrm{ground}} \tag{28}$$

which for the backscatter must be below the limits set in sec **??**. For length noise RMS, the integrated rms of this noise within the control bandwidth must fall below the limits set in **??**.

$$S_{\mathrm{diffuse}} \leq 2.1{\cdot}10^{-16} \frac{\mathrm{m}}{\sqrt{\mathrm{Hz}}} \frac{1}{\delta l_{\mathrm{ground}}(f)} \left(\frac{P_{\mathrm{leak}}}{50\mathrm{pW}}\right)^{-1/2} \left(\frac{f_{\mathrm{FC}}}{50\mathrm{Hz}}\right) \qquad \text{for } f > 10\mathrm{Hz} \tag{29}$$

and then for the total RMS within the control bandwidth, the requirements are

$$S_{\mathrm{diffuse}} \leq \frac{7.9{\cdot}10^{-13}\mathrm{m}}{\sqrt{20\mathrm{Hz}}} \frac{1}{\delta l_{\mathrm{ground}}(f)} \leq 1.8{\cdot}10^{-13} \frac{\mathrm{m}}{\sqrt{\mathrm{Hz}}} \frac{1}{\delta l_{\mathrm{ground}}(f)} \qquad \text{for } f < 20\mathrm{Hz} \tag{30}$$

Using ground motion with $1{\cdot}10^{-7} \frac{\mathrm{m}}{\sqrt{\mathrm{Hz}}}$ from T2000280.

$$S_{\mathrm{diffuse}} \leq 2.1{\cdot}10^{-6} \left(\frac{P_{\mathrm{leak}}}{50\mathrm{pW}}\right)^{-1/2} \left(\frac{f_{\mathrm{FC}}}{50\mathrm{Hz}}\right) \text{ from backscatter} \tag{31}$$

$$S_{\mathrm{diffuse}} \leq 1.8{\cdot}10^{-7} \text{ from phase noise} \tag{32}$$

2.1 Scatter light level estimates

The BRDF of the mirrors may be estimated using the total scattering loss expected of the mirrors, assuming all/most of the power is distributed into some small angular scale $\theta_{\mathrm{minscale}}$ and the remainder is uniform (Lambertian) into wide scales. The wide angle assumptions are approximately true from existing FC scatter measurements [**?**] and the numbers used here are pessimistic in any case. The narrow angle assumption is used since existing BRDF measurements cannot capture small angle scattering, which is known to give substantial contribution in super polished optics. The underlying phase noise of the polish has some length scale for its largest fluctuations. It may be that with polishing as good as the LIGO test-masses, that the narrow angle length scale is below the divergence angle of the FC beam. The aim of this analysis is to show that reasonable baffling strategies do not depend on that small angle scattering since we can put limits on the total mirror loss.

$$B_o(\theta) = \begin{cases} \alpha & \theta < \theta_{\mathrm{minscale}} \\ \beta \cos(\theta) & \theta > \theta_{\mathrm{minscale}} \end{cases} \tag{33}$$

Where we are assuming that the mirrors have low scattering loss, so

$$2\pi \int_0^{\frac{\pi}{2}} B_o(\theta)\sin(\theta)\mathrm{d}\theta \ < \Lambda_{\mathrm{rt}}/2 < 50{\cdot}10^{-6} \tag{34}$$

which sets upper limits for the scaling factors

$$\alpha < \frac{50{\cdot}10^{-6}}{\pi\theta_{\mathrm{minscale}}^2} \qquad\qquad \beta < \frac{50{\cdot}10^{-6}}{\pi} \tag{35}$$

Now to evaluate the scattering noise limits we can consider total scattering amplitudes for a near baffle wall, a tube, and a rear wall. These three cases simply change the formulation of $r(\theta)$ and the minimum and maximum radial angles used in eq. 27.

Near wall

$$r(\theta) = \frac{d_{\mathrm{wall}}}{\cos\theta} \qquad\qquad \theta_{\mathrm{minscale}} < \theta < \pi/2 \tag{36}$$

$$S_{\mathrm{diffuse}} < \sqrt{\frac{\pi B_t}{4}}\beta\frac{\lambda}{d_{\mathrm{wall}}} < 4{\cdot}10^{-5}\sqrt{B_t}\frac{\lambda}{d_{\mathrm{wall}}} \tag{37}$$

which easily meets the requirements for a near wall of any distance and diffuse reflection.

Beam tube

$$r(\theta) = \frac{d_{\mathrm{tube}}}{2\sin\theta} \qquad\qquad 0 < \theta < \pi/2 \tag{38}$$

$$S_{\mathrm{diffuse}} < \sqrt{\frac{\pi B_t}{4}}\frac{\lambda}{d_{\mathrm{tube}}}\sqrt{\beta^2 + \alpha^2\theta_{\mathrm{minscale}}^4} < 6{\cdot}10^{-5}\sqrt{B_t}\frac{\lambda}{d_{\mathrm{tube}}} \tag{39}$$

which easily meets the requirements for a beam tube of any diameter and diffuse reflection.

Far wall

$$r(\theta) = \frac{L_{\mathrm{FC}}}{\cos\theta} \qquad\qquad 0 < \theta < \theta_{\mathrm{minscale}} \tag{40}$$

$$S_{\mathrm{diffuse}} < \sqrt{\frac{\pi B_t}{4}}\alpha\theta_{\mathrm{minscale}}^2\frac{\lambda}{L_{\mathrm{FC}}} < 2{\cdot}10^{-13}\sqrt{B_t} \tag{41}$$

which is extremely small.

Altogether, these calculations suggest that only wide angle scattering contributes to length noise, and so a simple near wall baffle should be installed.

2.2 Specular scatterering

Given the apparent lax requirements for baffling, the only remaining concern may be specular scattering, which can couple large amounts of light between the mirrors through external surfaces. Specular scatter from some distance must refocuse the beam back to the original point, this implies (by unit analysis) a surface with a BRDF that is enhanced to cancel the mirror cross section factor $\frac{\lambda}{r(\theta)}$.

$$S^2_{\text{specular}} = \int_0^{2\pi} \int_0^{\frac{\pi}{2}} B_o^2(\theta) B_s(\theta + X) \sin(\theta) \mathrm{d}\theta \, \mathrm{d}\phi \tag{42}$$

Where $B_s(\theta)$ is the reflectance function for specular scatterers. It is order 1 where such scattering exists and 0 elsewhere. It primarily measures the area of specular scatterers. Assuming that the only specular scattering at narrow angles is the optic, then the total coupling for specular scatter is:

$$S^2_{\text{specular}} < A_{\text{specular}} (50{\cdot}10^{-6})^2 \tag{43}$$

and so the total area of specular scattering from vacuum seams or baffle edges must be smaller than $\frac{1}{50}$ of a steradian to meet the requirements. This should be easily met.

2.3 Baffle Implementation

Given the rather minimal requirements for scatter from the beam tube, we propose that a limited number of baffles to be installed only to prevent specular reflection down the tubes. For baffles occupying 1/4 of the tube radius, say 1in baffles in a 8in diameter tube, only 5 baffles should be needed to prevent the beam from seeing the far mirror via reflections from the tube. This configuration will allow the beam to see the first half of the tube, which may have seems from vacuum connections. As long as these take up sufficiently small area specular reflections from seams will not impact the FC operation.

Since the scattering on near surfaces comes the closest to violating the requirement, we recommend a simple mirror baffle mounted on the ISI in front of each cavity optic with an aperture equal to the mirror diameter.