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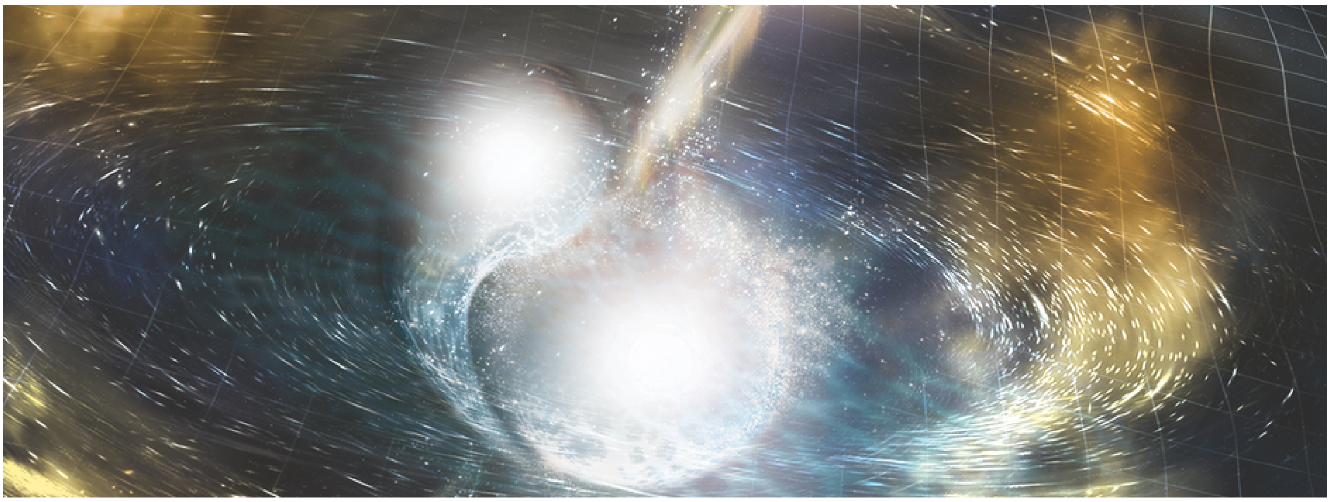
φTEM

LIGO LABORATORY

M1 PHYTEM Internship report

Study of the gravitational waveform created by a Binary Neutron Star (BNS) merger



Artist's view of the merging of two neutron stars from [1]

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Summary

1 Introduction

The study of gravitational waves can help us learn more about the universe. Their existence was conjectured by Henri Poincaré in 1905 [2], then predicted by Albert Einstein in 1916 as a consequence of his general theory of relativity [3], and they were later observed indirectly [4]. But their first direct observation was in September 2015 [5]. Their observation can contribute to astronomy, the study of gravity, but also nuclear physics by studying neutron stars : in neutron stars, all four fundamental forces are at their most extreme, and studying how matter behaves inside neutron stars will give insight into the strong nuclear force at supranuclear densities [6].

1.1 Gravitational waves

1.1.1 What are gravitational waves ?

General Relativity describes gravity as the curvature of space-time with the Einstein Field Equations

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}T_{\mu\nu}$$

where $G_{\mu\nu}$ is the curvature of space-time, $T_{\mu\nu}$ is the energy-momentum tensor, and $g_{\mu\nu}$ is the metric, which describes the structure of space-time. Gravitational waves are ripples in the curvature of space-time, which propagate at the speed of light and are generated by accelerated masses. They can be represented by a small perturbation $h_{\mu\nu}$ of the Minkowski flat space-time metric $\eta_{\mu\nu}$: $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ with $|h_{\mu\nu}| \ll 1$ [7].

1.1.2 Detecting gravitational waves

Space-time is very stiff, so even at its strongest a gravitational wave has a very small amplitude, which decreases as it propagates over a bigger region of space. This is why we can currently only detect waves created by really massive accelerated objects, like black holes or neutron stars [7]. Furthermore, black holes and neutron stars are compact objects : two larger objects of comparable mass would merge when their centers are much further from each other, which means that they would orbit each other more slowly according to Kepler's third law, creating gravitational waves with a much longer wavelength currently not detectable. The waves detected in the last three years were generated by the coalescence of two compact objects : the objects orbit around each other and lose energy by emitting gravitational waves, which causes them to get closer to each other and therefore go faster and faster, until they merge. The inspiral part of the generated wave (i.e., before the merger) looks like a chirp : the amplitude and frequency increase when the objects accelerate as seen in fig. 1.1. Then there is the merger, right at the highest amplitude, and then the ringdown, when the resulting system settles.

The detectors are giant Michelson interferometers with orthogonal arms several kilometers long (4km for the LIGO detectors in the United States and 3km for the VIRGO detector in Europe). They are sensitive to gravitational waves stretching one arm and shortening the other. The effective length of the arms is increased by Fabry-Pérot interferometers to increase the signal as shown in fig 1.2. With 4 km-long arms, the difference in length caused by a currently detectable gravitational wave is about a thousandth of the size of a proton.The presence of several detectors at different locations on Earth allows to estimate the direction of the source via triangulation [1].

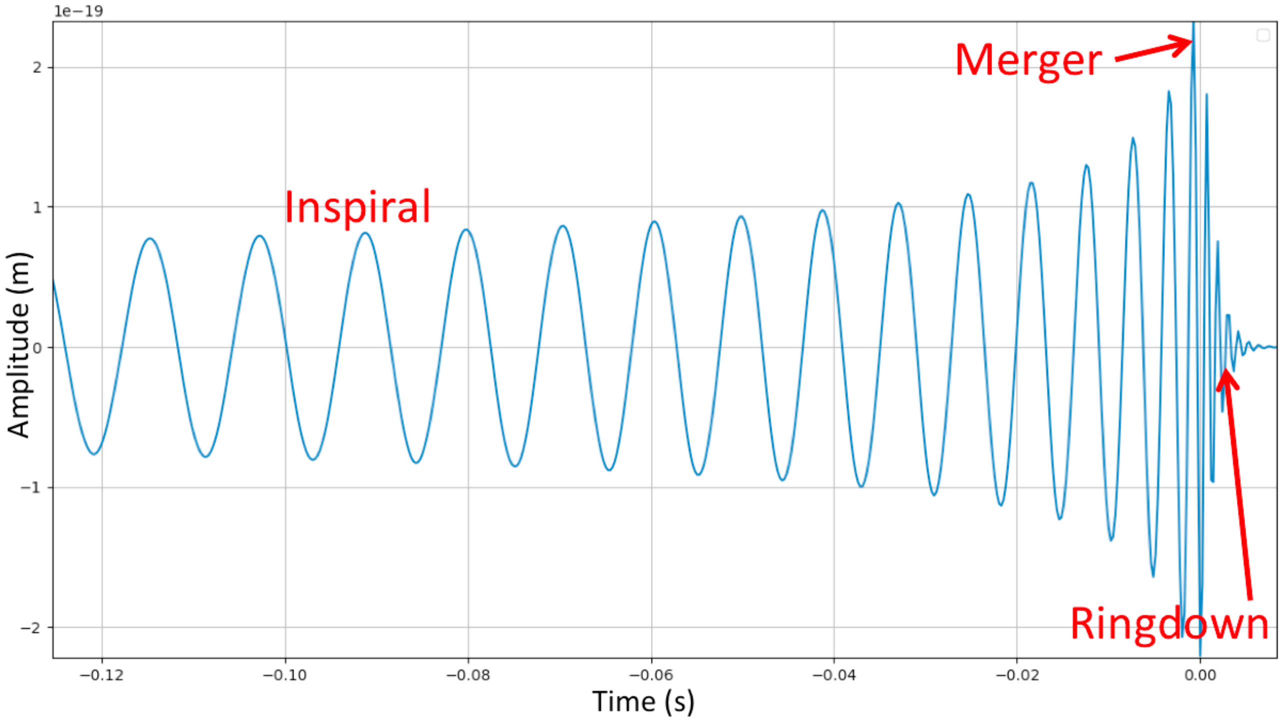

FIGURE 1.1 – Model of the signal created by the merging of two black holes

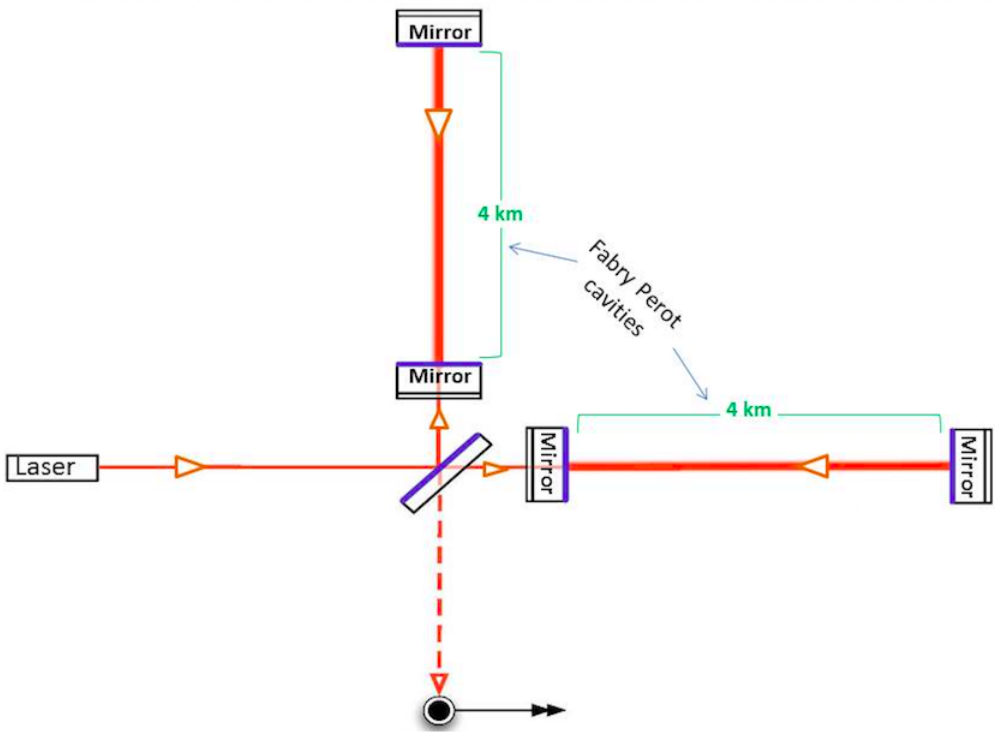

FIGURE 1.2 : Gravitational wave detector from [1]

1.1.3 Analyzing the signal

The shape of the detected signal contains information about many macroscopic properties of the system, most notably the masses and radii of the objects, and the 'chirp mass' of the system, which is the parameter that governs the energy loss through the emission of gravitational waves, but also the spins of the objects, their deformability, ... To get this information, it is necessary to create models taking into account the desired parameters, then numerically create the corresponding waveform, and fit it to the signal in noisy data.

1.2 LIGO Laboratory and detectors

The Laser Interferometer Gravitational-Wave Observatory (LIGO) Laboratory is split between the California Institute of Technology and the Massachusetts Institute of Technology and is part of the LIGO Scientific Collaboration (LSC) which counts over 80 scientific institutions world-wide. The LSC designed and built two gravitational waves detectors in the United States : in Hanford, Washington and Livingston, Louisiana. The French-Italian VIRGO near Pisa in Italy is a similar detector. The three can be used together to locate the direction of the source. The LSC also analyses the data from the detectors. There are also detectors under development or construction in several countries.

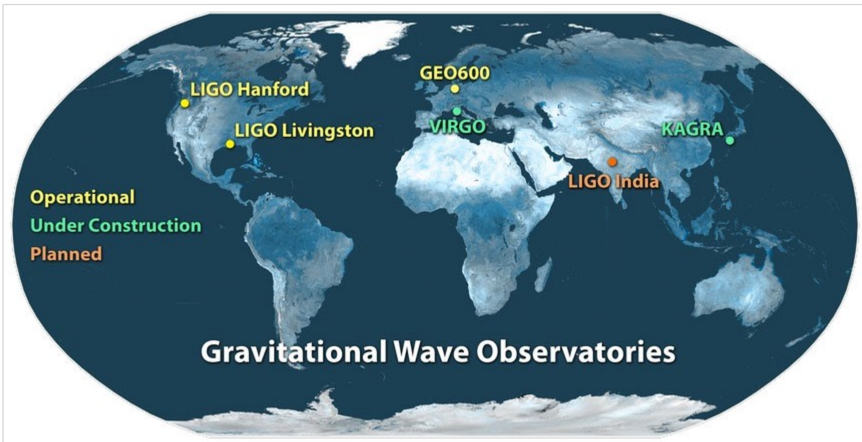

FIGURE 1.4 : Current and future detectors in the world from [1]

LIGO Laboratory operates the LIGO detectors, researches and develops ways to further improve the capabilities of the LIGO detectors, and researches the fundamental physics of gravitation, astronomy, and astrophysics. The laboratory is also involved in public education and outreach. LIGO is funded by the U.S. National Science Foundation and operated by the California Institute of Technology (Caltech) and the Massachusetts Institute of Technology (MIT) [1].

1.3 Gravitational waves from a Binary Neutron Star system

1.3.1 Differences from a Binary Black Hole system

For a binary black hole merger, the waveform is completely predicted by solving Einstein's equations. However neutron stars involve nuclear physics, and unlike black holes, neutron stars can be distorted by tidal effects. In a binary neutron star merger, both stars attract each other, creating tidal effects which increase and induce a quadrupole moment as they get closer. The quadrupole moment generates more gravitational waves, making the stars lose more energy and causing them to inspiral faster. The faster inspiral and the deformation cause the stars to make contact earlier, i.e., at a lower orbital frequency, than two black holes with the same mass would. There is a distortion and then a disruption of the stars right before they merge as seen in fig. 1.5.

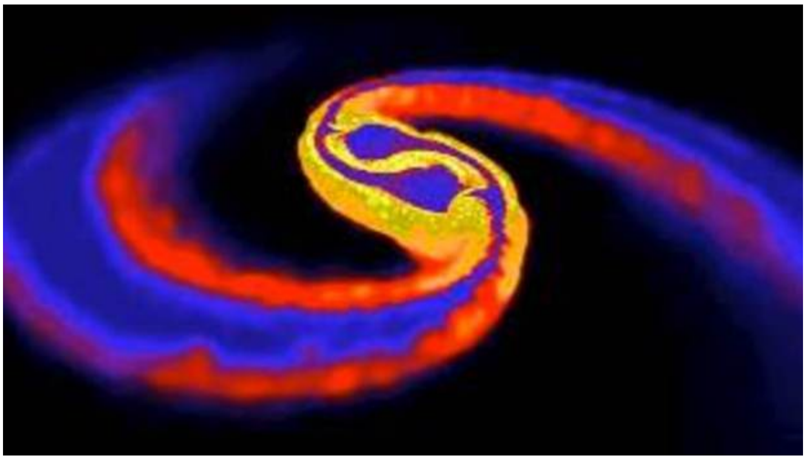

FIGURE 1.5 : Merging neutron stars from [1]

1.3.2 Identifying the equation of state of nuclear matter

Conditions of density and pressure inside the core of a neutron star are too extreme to be observed in a laboratory, and most observations of neutron stars give information about their macroscopic properties and not their microscopic properties, so how can we know how matter behaves inside a neutron star ? It is necessary to link the microscopic properties of the star to its macroscopic properties. This is achieved through the Tolman-Oppenheimer-Volkoff (TOV) equations, which are derived from the Einstein Field Equation

$$\frac{dP}{dr} = -\frac{G(mc^2 + 4\pi r^3 P)(\epsilon + P)}{rc^4(r - \frac{2Gm}{c^2})}$$

ϵ being the energy density. This equation, along with $\frac{dm}{dr} = 4\pi\frac{\epsilon}{c^2}r^2$, models the hydrostatic equilibrium of the star [8].

The simplest model is to consider that the star contains only neutrons and is supported by the pressure of degenerate neutrons. Furthermore, after its creation the star cools to a negligible temperature when compared to its Fermi temperature and can therefore be considered to be at $0K$. However the star cannot contain only neutrons because of the β-decay equilibrium and because the nuclear force becomes repulsive at high densities. A neutron star most likely contains neutrons, electrons and nuclei, and there is a debate on the quantity of strange matter present. Many different models have been elaborated [8].

Observations help constrain these models. For example, if a neutron star with a mass higher than the maximum mass supported by a certain model is observed, then that model is not viable. Hypotheses of high quantities of strange matter at the core of neutron stars have been mostly disproven in this manner : these models do not support neutron stars of 2 solar masses, and one such neutron star has been observed [9].

An observable macroscopic parameter for a neutron star is its tidal deformability Λ which depends on the model and the compactness $\frac{M}{R}$ of the star. Λ represents the capacity of the star to be deformed by tidal effects. This is the parameter I focused on during my internship.

2 Work accomplished

Working on gravitational waves requires learning to use many different computing tools created by a vast and dynamic community, which means the documentation can sometimes be difficult to find. The primary goal for my internship was to make me proficient in the use of those tools, first to understand the effects of changing the tidal deformability Λ of the stars on the waveform, then to learn how to extract Λ from the waveform using parameter estimation .

2.1 Solving the TOV equations

To know the radius and total mass of the star for a specific model and core density, one needs to integrate the TOV equation numerically from the center of the star to an overestimated radius, and find the radius for which the pressure becomes $0Pa$, which is the external boundary of the star. Then integrate $\frac{dm}{dr} = 4\pi\frac{\epsilon}{c^2}r^2$ from the center to that radius to get the total mass. This gives one point of the mass versus radius profile for a specific model. Doing this with a coherent range of core densities gives the full profile.

2.1.1 For a pure ideal neutron gas

The equation of state for a pure ideal neutron gas is $P = K\rho^\Gamma$, K and Γ being constants. Unfortunately, I could not find the correct values of K and Γ in the literature. I then created a table of values of ρ, P and ϵ depending on the value of the dimensionless Fermi momentum $x = \frac{p_F}{m_e c}$ with p_F the Fermi momentum. For a specific value of x and at $T = 0K$, with $\lambda_n = \frac{h}{2\pi m_n c}$ the Compton wavelength of a neutron, [8]

$$\rho = \frac{m_n}{3\pi^2(\frac{x}{\lambda_n})^3}$$

$$P = \frac{m_n c^2}{{\lambda_n}^3 8\pi^2[x(\frac{2x^2}{3} - 1)\sqrt{1+x^2} + log(x + \sqrt{1+x^2})]}$$

$$\epsilon = \frac{m_n c^2}{{\lambda_n}^3 8\pi^2[x(2x^2 + 1)\sqrt{1+x^2} + log(x + \sqrt{1+x^2})]}$$

Using this table, I numerically solved the TOV equation for this equation of state and obtained the mass versus core density and radius profile shown in fig. 2.1. This model only allows for a maximum mass of 0.7 solar masses. Furthermore, neutron stars with a radius smaller than 9km are unstable : the mass increases with the radius instead of decreasing which indicates a core collapse due to gravity : the star becomes a black hole.

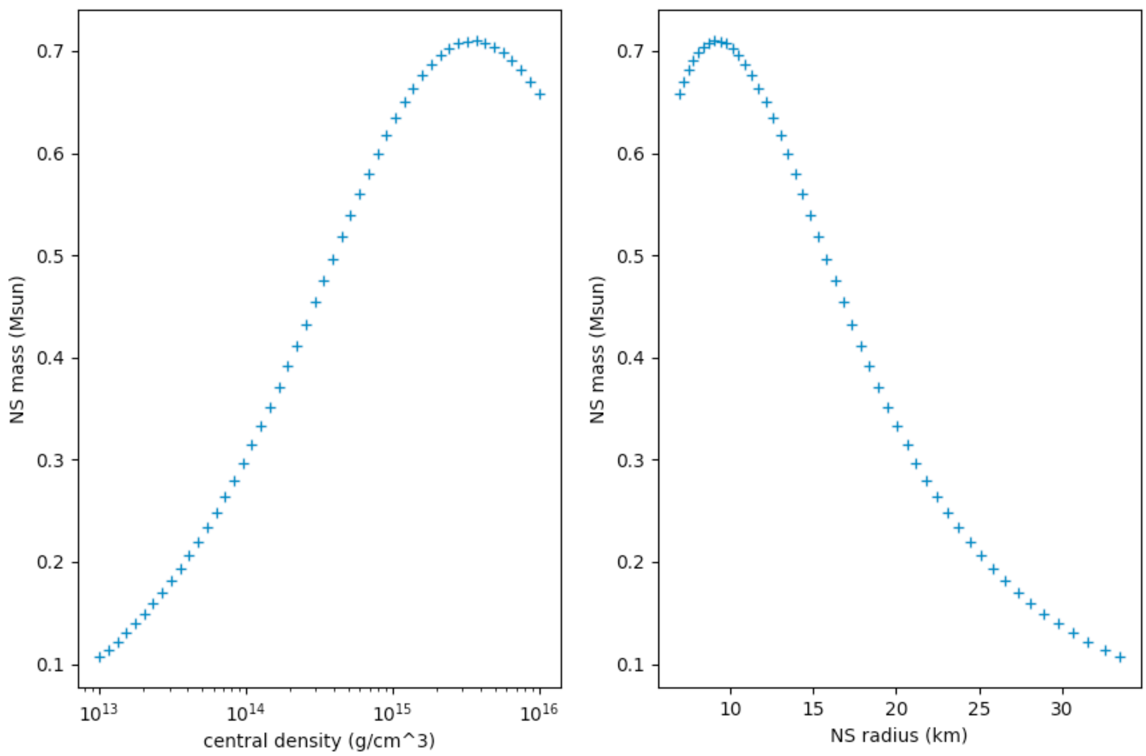

FIGURE 2.1 : Mass versus core density (left) and radius (right) profile for a pure ideal neutron gas

2.1.2 More complicated models

Most possible equations of state are published as multi-dimensional tables of parameter values and are accessible in several different ways. Using those tables it is possible to plot the mass versus radius profile for each different model.

Using published hdf5 tables

I used the tables accessible on the `https://stellarcollapse.org/equationofstate` website. They are available as hdf5 files. The data can be either read or interpolated from the recorded data using a fortran driver. I had to debug the driver file, which was corrupted, then modify it to create a file containing the data I needed to solve the TOV equations for all the equations of state with the python script used earlier. But the interpolation function in the driver did not converge for most of the needed data points for a reason I was unable to identify. I then started to learn how to access data from an hdf5 file directly using python but stopped when I found out there was another way to get the data.

Using LAL

Researchers have created two repositories of routines to analyse and generate waveforms : the LSC Algorithm Library Suite (LALSuite) in C, and Python Compact Binary Coalescence (PyCBC) in Python. LAL contains equations of state tables and routines to solve the TOV equations and calculate the values of certain parameters such as the deformability Λ, and there are routines to interface between LAL and a Python program. [10] [11] I ran my program on the shared servers and had to learn how to access them and how to edit a script directly from the command line. Using these routines, I plotted the mass versus radius and tidal deformability versus mass profiles for several models as shown in fig. 2.2 and 2.3. We can see in these figures that mass increases as radius decreases for all of these models. This is because when mass increases the gravitational attraction towards the core increases, which decreases the radius. Furthermore, for all of these models the radius is constant for a broad range of masses. These profiles are very useful for identifying the correct model : for example, some equations of state do not support neutron stars with a mass higher than 2 solar masses so if one such star is observed these equations will be reworked or discarded.

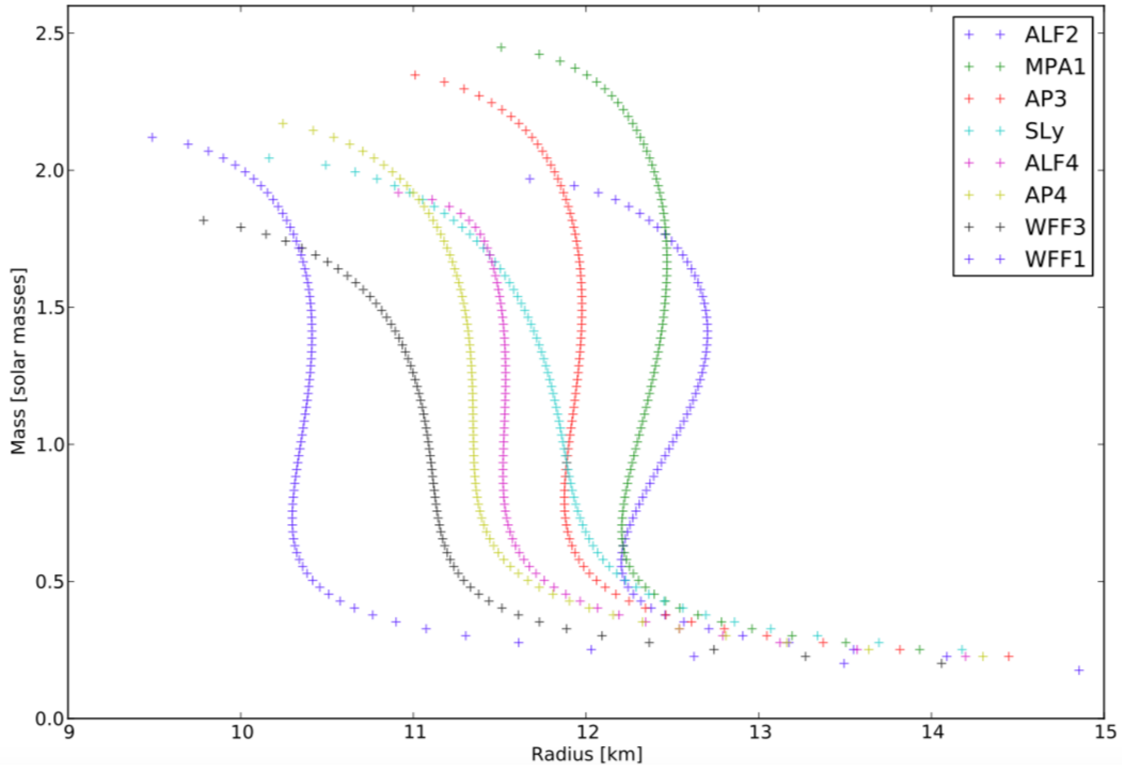

FIGURE 2.2 : Mass versus radius profile for several equations of state

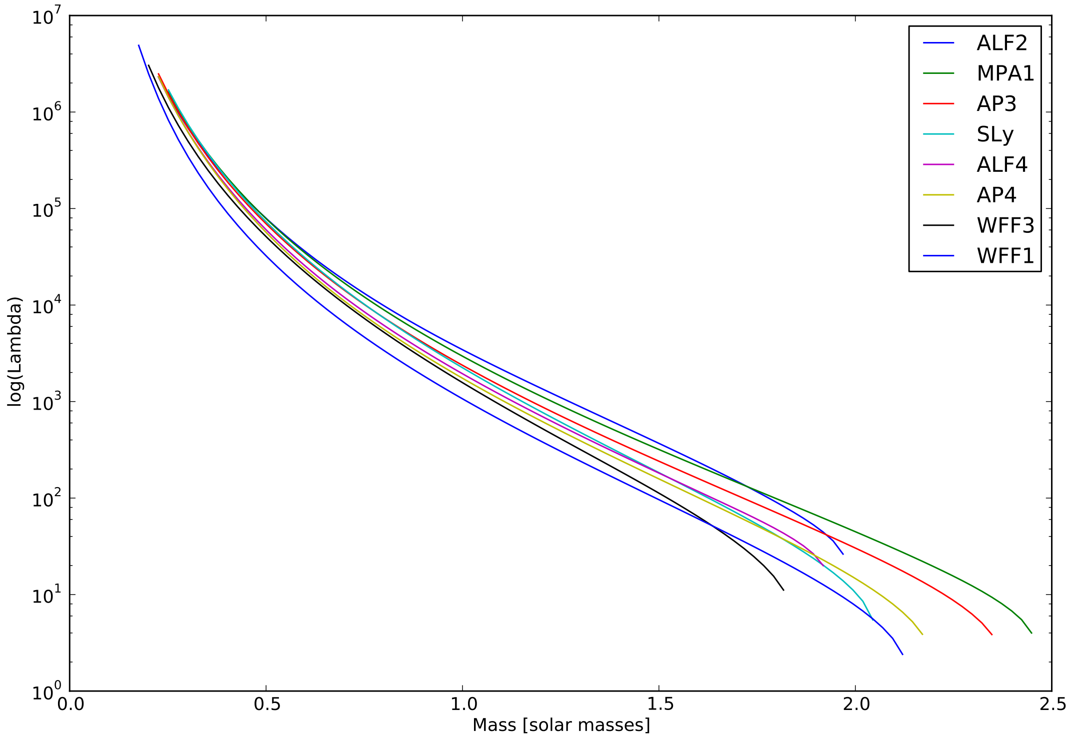

FIGURE 2.3 : Lambda versus mass for several equations of state

2.2 Generating the waveform

There are many different waveforms, each emphasizing the importance of some parameters over others. Waveforms also usually model one part of the phenomenon, such as the beginning of the inspiral, the end of the inspiral, or the merger, more accurately than the others : post-Newtonian or effective one body models work only for a system with a low velocity and are therefore inaccurate near the merger, whereas waveforms with 'NRTidal' in their name are are better for the inspiral right before the merger, which is where the tidal effects are most important.[12] I was supposed to use the waveform IMRPhenomPv2_NRTidal but its documentation took a long time to find so I started working with the waveform TaylorF2 to learn how to generate a waveform.

Waveforms are created through PyCBC or LAL routines as a specific class of objects : time series or frequency series, which consist of an array containing the data, the time (resp. frequency) between samples, and eventually the date of the first sample in seconds and the data type.[11]

2.2.1 TaylorF2

TaylorF2 is a post-Newtonian model. Post-Newtonian formalism is a calculational tool that expresses Einstein's (nonlinear) equations of gravity in terms of the lowest-order deviations from Newton's law of universal gravitation for non-relativistic objects. TaylorF2 does not contain the merger and ringdown. TaylorF2 does not generally use the tidal deformabilities $\Lambda_{1,2}$ of the objects as parameters, but there are routines to add parameters to waveforms. I created waveforms for two neutron stars with the same mass of 1.4 solar masses and therefore the same Λ to simplify the comparison. The value of Λ depends on the model, so changing Λ means considering a different model. I compared waveforms with $\Lambda_{1,2} = 0$ (the objects are black holes), $\Lambda_{1,2} = 100$, and $\Lambda_{1,2} = 1000$. The resulting waveforms in this case are accurate in terms of phase but not amplitude near the merger : the waveforms with added tidal effects should be smoother. We can see by looking at the end of the inspiral in fig. 2.4 a difference in phase compared to the waveform for a binary black hole system which increases progressively. Furthermore, as expected a bigger tidal deformability causes the merging to occur earlier.

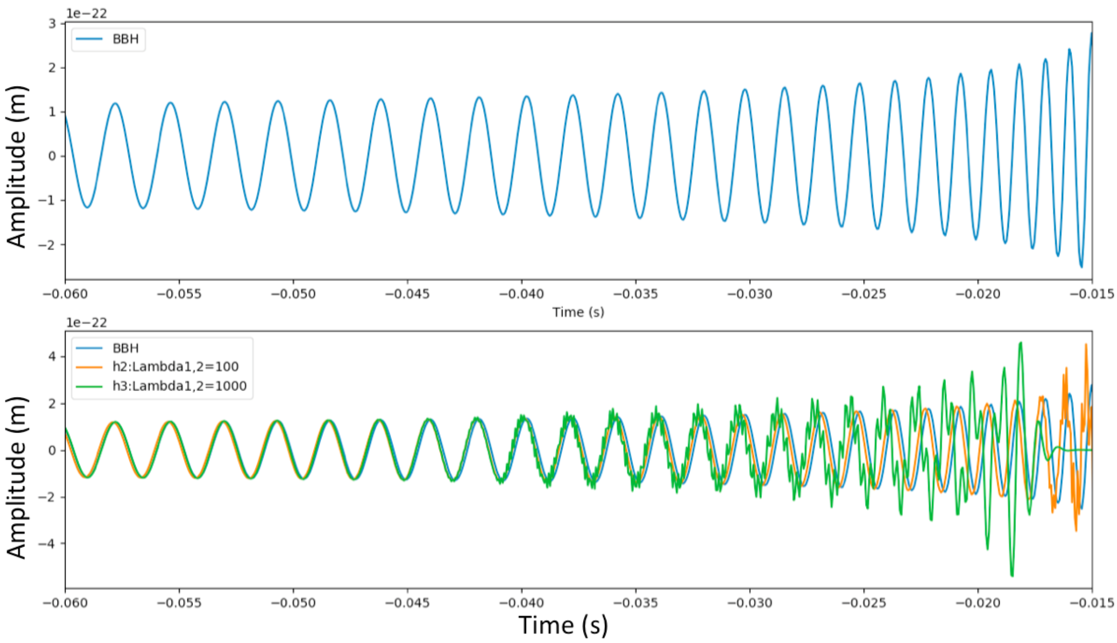

FIGURE 2.4 : End of the inspiral for a binary black hole system $\Lambda_{1,2} = 0$ (top) and for $\Lambda_{1,2} = 0$, $\Lambda_{1,2} = 100$, and $\Lambda_{1,2} = 1000$ (bottom)

2.2.2 IMRPhenomPv2_NRTidal

Unlike TaylorF2, IMRPhenomPv2_NRTidal is not accessible through the standard LALsuite package. Another version of the package, LALinference_O2 must be sourced to access the waveform.

Furthermore, TaylorF2 generates the waveform in the time domain while IMRPhenomPv2_NRTidal generates it in the frequency domain (see fig. 2.5). It should be noted that IMRPhenomPv2_NRTidal was made to study the tidal effects and probably tends to overestimate them [12]. The tidal effects are most visible right before the merger, which corresponds to the highest frequencies. As expected the waveform spectra differ at high frequency for different Λ as seen in fig. 2.5. Furthermore, unlike TaylorF2 the waveform does not have amplitude artefacts right before the merger as seen in fig. 2.6 but it still presents phase differences and earlier merging for a higher Λ.

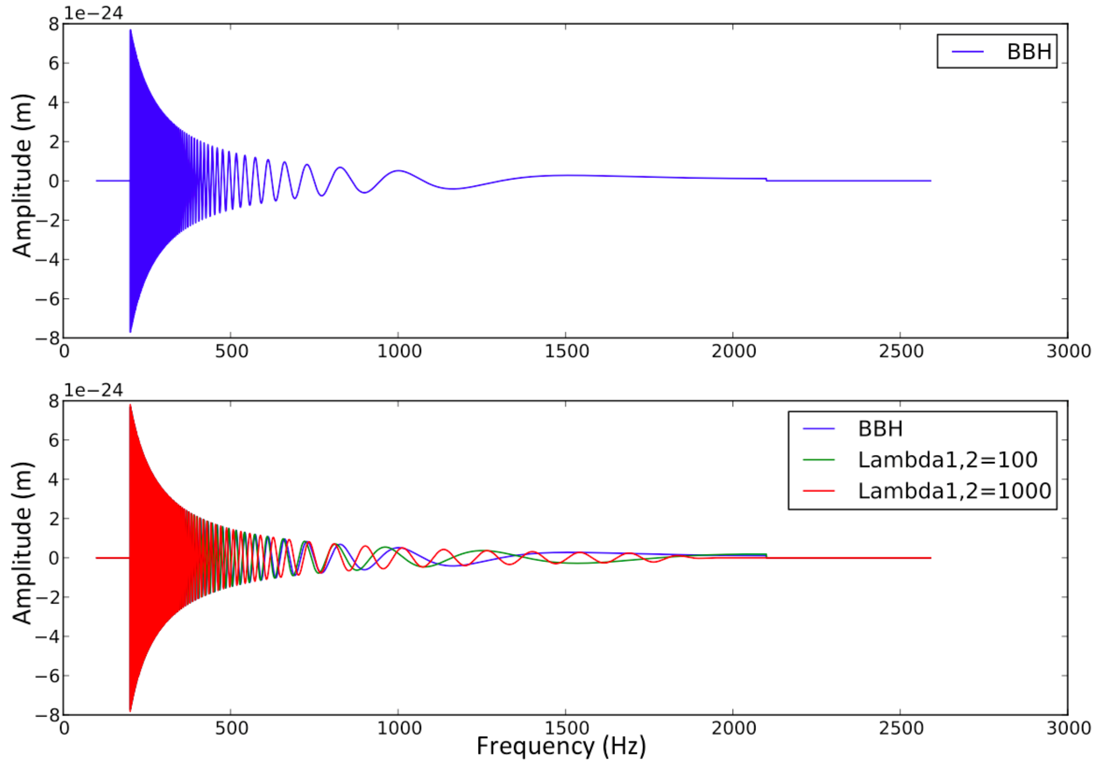

FIGURE 2.5 : Spectrum of the waveform for a binary black hole system $\Lambda_{1,2} = 0$ (top) and for $\Lambda_{1,2} = 0$, $\Lambda_{1,2} = 100$, and $\Lambda_{1,2} = 1000$ (bottom)

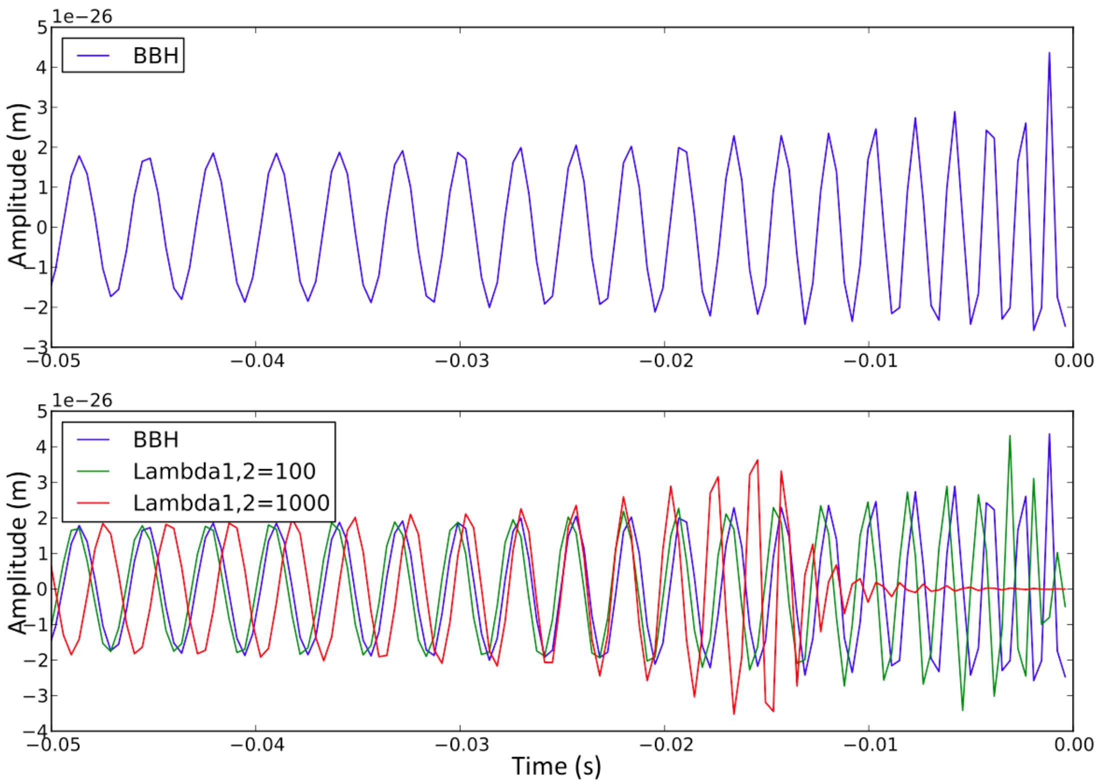

FIGURE 2.6 : End of the inspiral for a binary black hole system $\Lambda_{1,2} = 0$ (top) and for $\Lambda_{1,2} = 0$, $\Lambda_{1,2} = 100$, and $\Lambda_{1,2} = 1000$ (bottom)

2.3 Analyzing the waveform

The tidal effects are most visible in the phase of the waveform, and in the time at which the merging occurs : the objects merge earlier and the phase diverges from that of a binary black hole merger waveform earlier when the tidal effects are stronger as seen in fig. 2.7 to 2.10. Therefore, comparing the phase of a waveform with tidal effects to the phase of a waveform without tidal effects (corresponding to a binary black hole merger) is a very effective way of understanding the effect of the model on the waveform.

2.3.1 TaylorF2

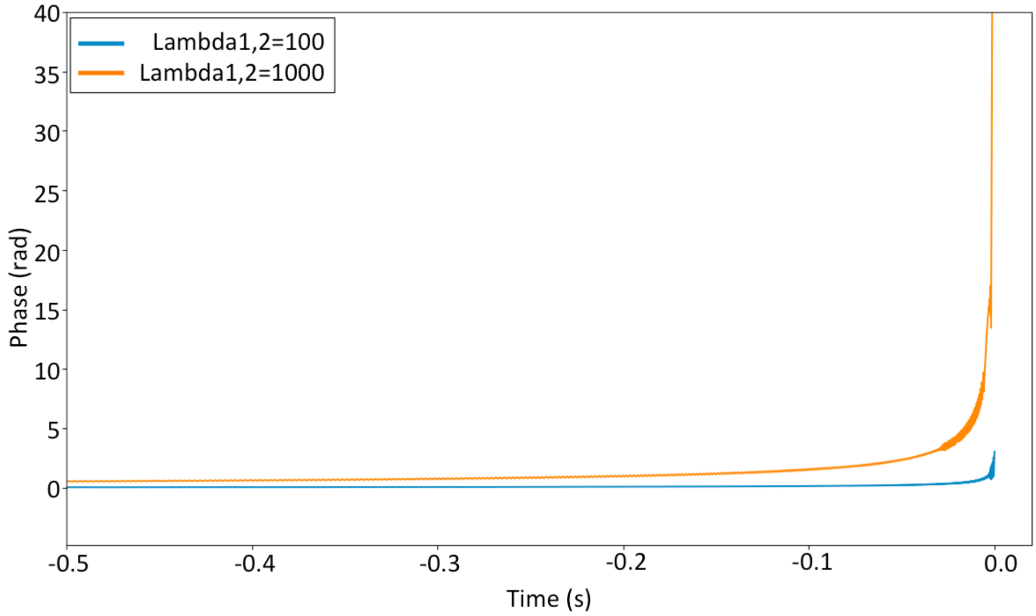

FIGURE 2.7 : Phase difference from a BBH for $\Lambda_{1,2} = 100$ and $\Lambda_{1,2} = 1000$ in the time domain

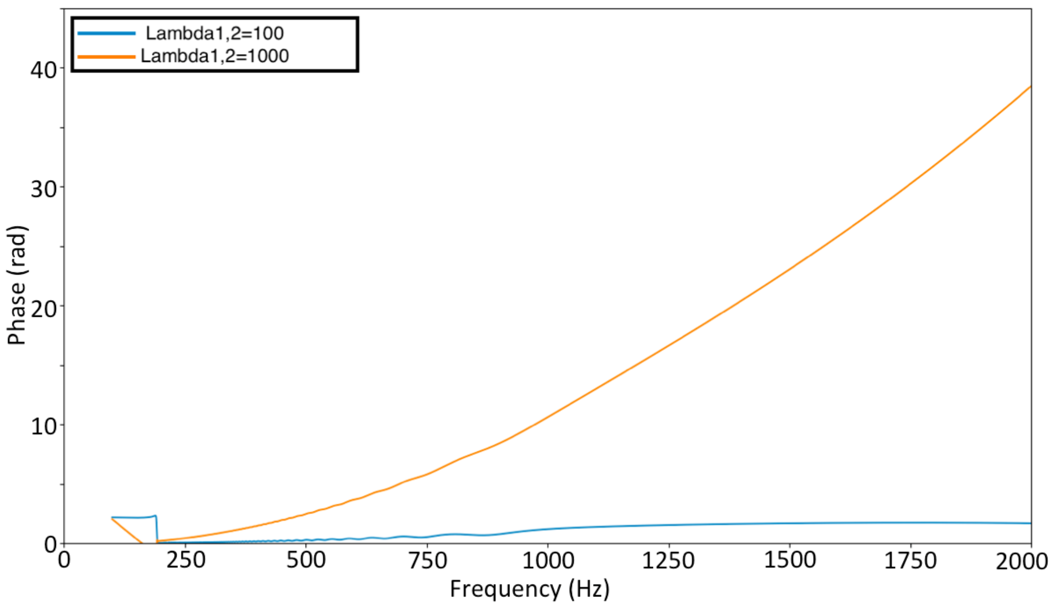

FIGURE 2.8 : Phase difference from a BBH for $\Lambda_{1,2} = 100$ and $\Lambda_{1,2} = 1000$ in the frequency domain

2.3.2 IMRPhenomPv2_NRTidal

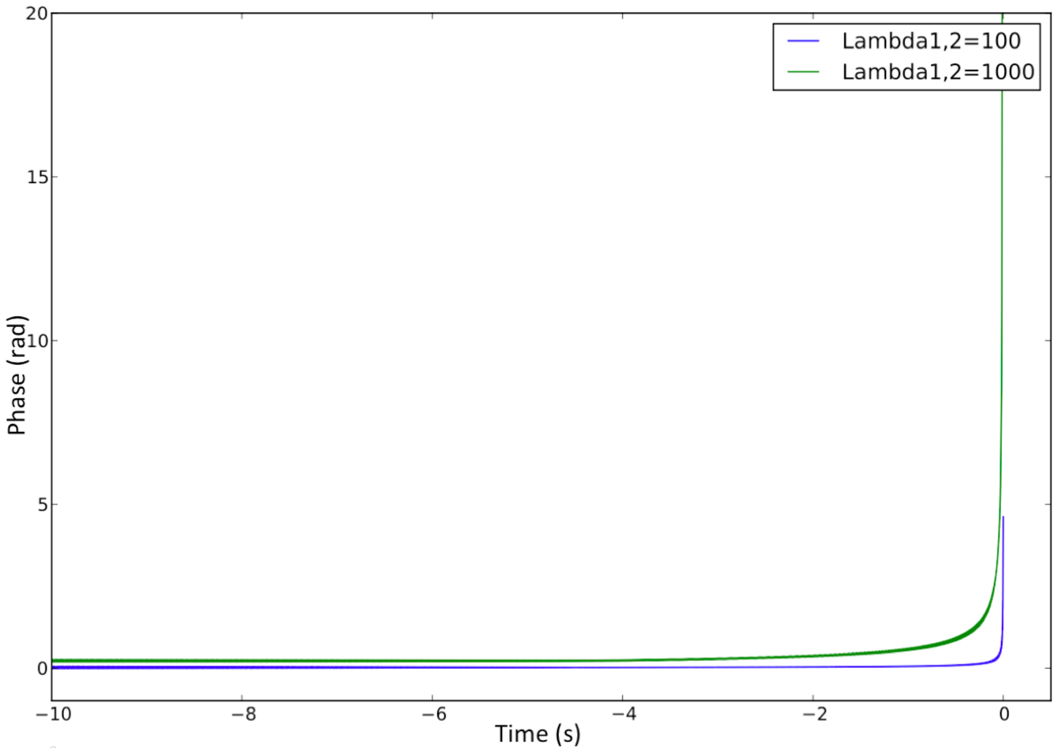

FIGURE 2.9 : Phase difference from a BBH for $\Lambda_{1,2} = 100$ and $\Lambda_{1,2} = 1000$ in the time domain

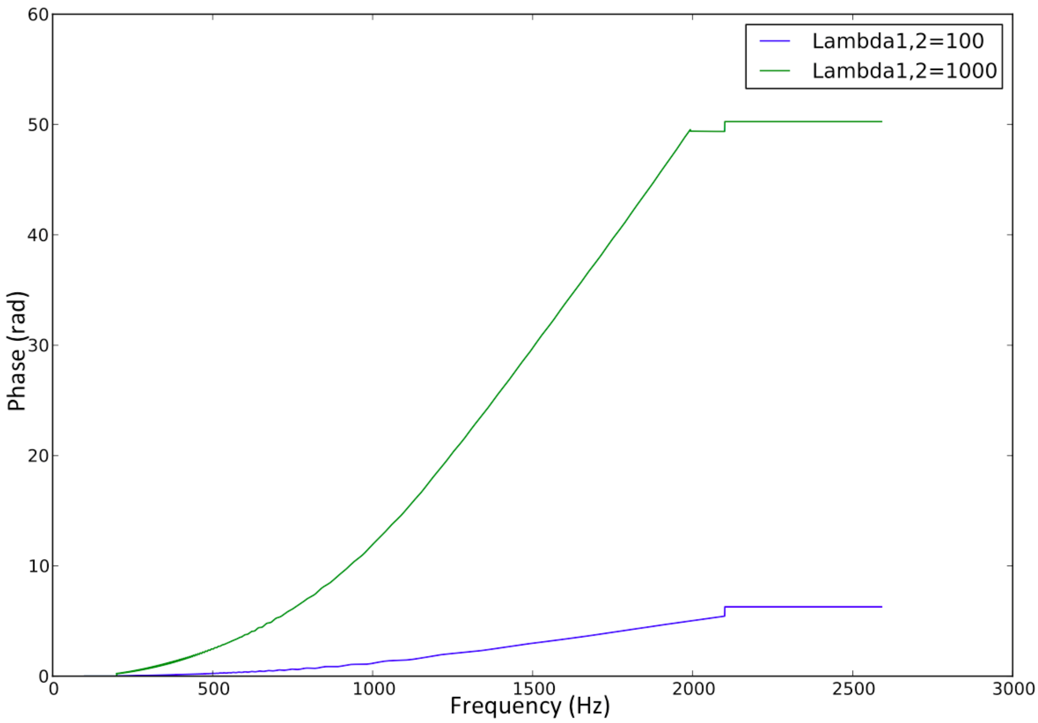

FIGURE 2.10 : Phase difference from a BBH for $\Lambda_{1,2} = 100$ and $\Lambda_{1,2} = 1000$ in the frequency domain

2.4 Parameter estimation

How do we estimate the different parameters of the binary system ? To estimate one parameter Λ knowing the exact values of all the other parameters, which is a one-dimensional parameter estimation, it is necessary to calculate the likelihood $L(\vec{d}|\Lambda)$ of a certain value of this parameter being the correct one for the data, then plot this likelihood for different values of Λ. The data $\vec{d}$ corresponds to the sum of the signal and the noise. The likelihood is

$$L(\vec{d}|\Lambda) = Nexp\left(\frac{-\langle\vec{d}-\vec{h}(\Lambda)|\vec{d}-\vec{h}(\Lambda)\rangle}{2}\right)$$

$\vec{h}(\Lambda)$ being the generated waveform for a particular value of Λ. The likelihood versus parameter plot should look like a gaussian centered around the most probable value of Λ as shown in fig. 2.11, with the peak being narrower and higher for a higher signal to noise ratio.

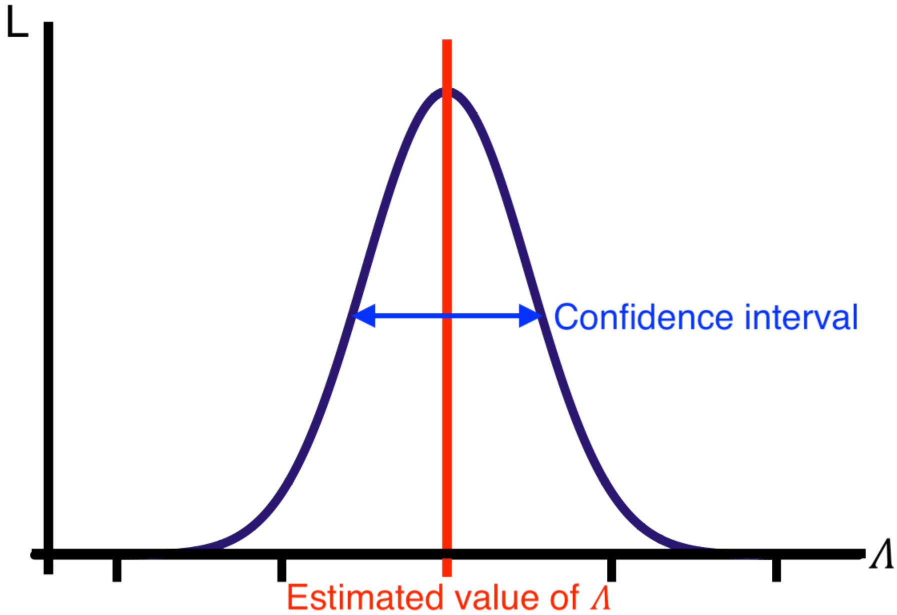

FIGURE 2.11 : Predicted likelihood versus parameter profile for a 1D parameter estimation

A multi-dimensional parameter estimation follows the same principle, but is often applied with a Markov chain Monte Carlo algorithm to minimise the computing time.

I first attempted to add noise to the signal to create the data $\vec{d}$, but LALinference_O2 seems to be incompatible with PyCBC. I then modified my program to create a file containing the signal, that I would then be able to use in a program using PyCBC but not LAL. I then wrote a separate program to generate the waveforms with a range of values of Λ using LALinference_O2 and write them in a file to be used in the parameter estimation. Unfortunately, the created file was empty unless I created only two waveforms at a time. This was done using the shared servers.

I then tried to do a 1D parameter estimation on my own computer using TaylorF2 instead of IMRPhenomPv2_NRTidal, using the LIGO Open Data Workshop tutorials [13]. Unfortunately there still seemed to be incompatibility problems between LAL and PyCBC and I was unable to get results.

3 Teaching

3.1 Outreach

Caltech regularly organises outreach events for the general public and the LIGO Laboratory participates often. The laboratory has a Michelson interferometer with a suspended mirror and a camera hooked to a speaker to convert the signal created by the movement of the mirror into sound which helps explain how the gravitational waves detectors work, and a 'fabric universe' which is a stretchable fabric on a frame, with different sized marbles to demonstrate general relativity. I participated in an event at a primary school and an event at a night market.

I really enjoy explaining complex scientific concepts to people in a simple way, and the tools the laboratory has are very useful for that. The fabric universe really appeals to children and makes explaining general relativity really straightforward and fun. Parents usually approach just to accompany their children but then really enjoy the explanations and ask questions. My favorite thing is the way people light up when they understand science even though it's supposed to be really difficult.

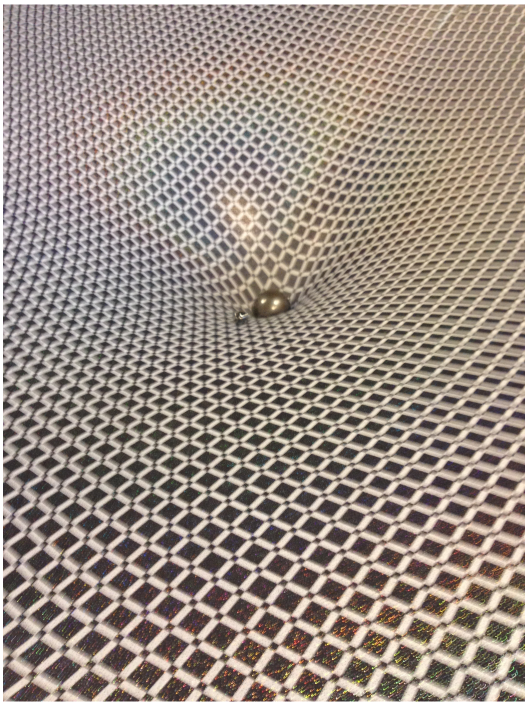

FIGURE 3.1 : 'Fabric universe' : Planet orbiting the sun

3.2 Helping other interns

Working on gravitational waves requires knowledge of many different computing tools such as working on shared servers and working from the command line. I struggled with this in the beginning and thought it would be useful for future interns to have a cheat sheet which I wrote. I also made a powerpoint presentation because my supervisor suggested I present it to the undergraduate summer interns.

I also wrote instructions to use IMRPhenomPv2_NRTidal with Python because the existing documentation was not very clear.

4 Conclusion

During this internship, I was able to link the microscopic and macroscopic properties of a neutron star, then to study the effect of these microscopic properties on the gravitational wave created by a binary neutron star merger through the parameter of their tidal deformability Λ. The work performed during this internship has also uncovered software problems in the newest version of LALinference, which seems incompatible with the software developed in the last twenty years to study and model gravitational waves. The powerpoint presentation and PDF instructions I wrote on how to use some computing tools, and the instructions to use the IMRPhenomPv2_NRTidal waveform with Python will hopefully be useful to future interns, especially in studying the effects of Λ on the waveform, then in studying how reliably we can extract Λ from the data depending on the distance of the source from Earth, and in constraining the models for the nuclear matter equation of state by measuring Λ for multiple events.

I learned a lot during this internship, most notably about computing, using shared servers, and many of the tools developed and used by the LIGO Scientific Collaboration. I also experienced life in a laboratory and as a researcher : I attended team meetings and presented my results regularly, participated in laboratory activities such as seminars, outreach, helping other interns, and social hour, and I struggled with my work which did not automatically give results unlike experiments done in a class setting.

I would like to thank everyone in the lab for their warm welcome, most notably my supervisor Alan who explained everything really well and patiently, and did his best to help me get results, and Michael who taught me how to use some of the software.

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