LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY
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Constructing a Homodyne Detector for Low Quantum Noise Gravitational Wave Interferometry

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1 Introduction and Background

Achieving more efficient detection of gravitational radiation is a goal of contemporary experimental physics, as it will enable novel tests of general relativity and provide information on astronomical bodies that are difficult to observe through the electromagnetic spectrum. The gravitational waves (GWs) that encompass this radiation are described by oscillatory perturbations to a background spacetime metric. These waves manifest themselves physically by altering displacements in spacetime, such as spatial distances and time durations. Current GW observatories, such as LIGO, use high precision laser interferometry to detect miniscule changes in the length of interferometer arms, indicating the passage of a GW. Since typical changes in the LIGO arm lengths induced by GWs are of the order of 10^{-18} m, incredibly precise measurements must be conducted to observe a GW. In particular, LIGO uses a large Michelson interferometer furnished with Fabry-Perot cavities and power recycling mirrors to optimize its sensitivity and ability to detect GWs.

Despite their intricate designs, interferometric GW detectors are subject to various sources of noise that limit their resolution. Some of this noise arises from external sources, like human activity and weather patterns. The resulting noise can be combated by numerous techniques, such as performing interferometry in vacuum chambers and employing vibration isolation systems. Furthermore, on atomic and subatomic scales, new sources of intrinsic noise arise as the laws of quantum mechanics take precedence over those of classical physics. For instance, in quantum electrodynamics (QED), the quantized electromagnetic field reveals the discrete photon nature of light. This phenomenon introduces shot noise and radiation pressure noise into the interferometer due to the fact that the electromagnetic field of a beam of light is not smooth and continuous, but rather is composed of individual photons. Noise that arises from quantum mechanical processes is known as quantum noise and owes its existence to the Heisenberg uncertainty principle and quantum fluctuations. Because of these immutable laws, sources of quantum noise dictate that the sensitivity of classical GW interferometers is bounded below by the Standard Quantum Limit (SQL). For example, in a GW interferometer with arm lengths L, test masses of mass m, and detecting a GW of frequency Ω, the noise spectral density of the GW strain, h, is bounded below by [7]

$$S_h^{\mathrm{SQL}}(\Omega) = \frac{2\hbar}{m\Omega^2 L^2}. \tag{1}$$

In general, the SQL will differ depending on the precise interferometric setup, but the limitations it conveys remain the same.

However, it turns out that the SQL only applies to interferometers when the sources of noise are uncorrelated, as they are classically. In fact, despite its counterintuitive name, the SQL can be surpassed by cleverly constructed interferometers that take into account quantum mechanics and correlated noise. One such method of beating the SQL utilizes squeezed light and balanced homodyne detection. A balanced homodyne detector (BHD) is composed of two photodiodes, a 50/50 beam splitter, and two sources of light: the signal and the local oscillator. An image of a BHD setup is displayed in Figure 1. The signal is the light that contains the desired information; for instance, it could be the light coming from the main interferometer that encodes the structure of a passing GW. On the other hand, the local oscillator is a stabilized source of light with its carrier frequency equal to that of the signal. In an interferometer, the local oscillator light can be obtained from the incident laser light before it reaches the main interferometer. In balanced homodyne detection, these two sources of light are first mixed by being sent through the beam splitter. Next, the two photodiodes measure the photocurrents induced by the two outgoing beams from the beam splitter. One can then measure and analyze these photocurrents, from which information about the quadratures can be extracted.

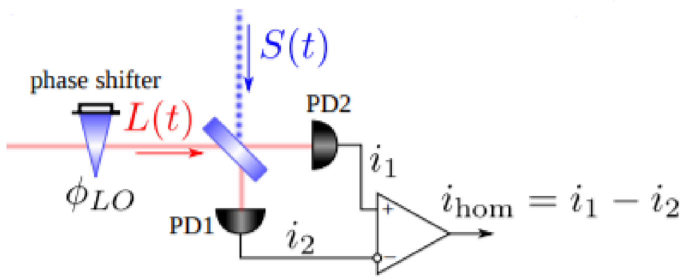

Figure 1: A standard BHD setup, obtained from [15].

Following the analysis presented in [15], we can describe this procedure mathematically. Let the electric fields of the signal and local oscillator light have quadratures $S_{c,s}(t)$ and $L_{c,s}(t)$, respectively, and carrier frequency ω:

$$S(t) = S_c(t)\cos(\omega t) + S_s(t)\sin(\omega t), \qquad L(t) = L_c(t)\cos(\omega t) + L_s(t)\sin(\omega t). \tag{2}$$

Due to inevitable quantum noise, the quadratures will contain terms due to noise. Denoting these by $n_{c,s}(t)$ and $l_{c,s}(t)$, respectively, the quadratures can be decomposed as

$$\begin{aligned} S_{c,s}(t) &= \text{signal} + \text{quantum noise} = G_{c,s}(t) + n_{c,s}(t) \\ L_{c,s}(t) &= \text{classical field} + \text{laser noise} = L^{(0)}_{c,s}(t) + l_{c,s}(t). \end{aligned} \tag{3}$$

Since the local oscillator is under the experimentalist's control, we will impose on it a phase shift, ϕ_{LO}, known as the homodyne angle: $L_c^{(0)}(t) = L_0\cos(\phi_{LO})$, $L_s^{(0)}(t) = L_0\sin(\phi_{LO})$. Such an alteration could be realized by changing the path length of the local oscillator. Additionally, we will assume that the local oscillator's amplitude is much greater than the other amplitudes in this scenario: $L_0 \gg G_{c,s}, n_{c,s}, l_{c,s}$. Under these assumptions, one can calculate the ideal photocurrents induced at the two photodiodes, which we denote by i_1 and i_2. In balanced homodyne readout, one chooses not to measure these currents, but instead measures the difference between them: $i_{\text{hom}} = i_1 - i_2$. To first order in $G_{c,s}, n_{c,s}$, and $l_{c,s}$, this is given by [15]

$$i_{\text{hom}} \propto L_0((G_c + n_c)\cos(\phi_{LO}) + (G_s + n_s)\sin(\phi_{LO})). \tag{4}$$

Evidently, this expression is independent of $l_{c,s}$, so the noise from the local oscillator does not factor into measurements of i_{hom}. In addition, Eq. (4) indicates that, by varying ϕ_{LO} and measuring i_{hom}, one can measure the signal's quadratures and linear combinations of them, with a precision limited only by the quantum noise of the signal.

In an interferometric GW detector, acquisition of the quadratures provides accurate information about the passing gravitational radiation. Thus, a properly constructed BHD presents the opportunity to probe exceptionally small length scales and improve GW detection. However, the real world is not so ideal. In the construction of a physical BHD, other sources of optical and electronic noise exist within the interferometer. The optical noise arises from noise present in the signal, and the electronic noise emerges from noise induced in the detector. For instance, an imperfect beam splitter will create an imbalance in the light beams emerging from it and introduce local oscillator noise into i_{hom}. In addition, this setup is susceptible to noise in its electronic circuits, such as thermal noise in the resistors and intrinsic $1/f$ noise. In order to analyze the noises within the electronic circuits, one must calculate the noise spectral density of each circuit element. Let the noise spectral density of the j^{th} circuit element be denoted by e_{n0j}. The exact form of e_{n0j} will differ for distinct circuit elements since their noise contributions will not be the same. Then, the noise voltage due to this element, denoted by E_{n0j}, is obtained via

$$E_{n0j}^2 = \int_0^\infty df \; |e_{n0j}|^2. \tag{5}$$

Generally, this integral will be limited by the finite bandwidth over which the circuit operates. Finally, the total noise voltage due to all the circuit elements, denoted by E_{n0}, can be obtained from an RMS summation:

$$E_{n0}^2 = \sum_j E_{n0j}^2. \tag{6}$$

Proper analyses of all of these noises must be incorporated in order to correctly interpret the data from a BHD.

Current gravitational wave observatories do not exploit balanced homodyne detection of this sort. Instead, these experiments primarily use DC readout [14], in which a single photodetector measures the light output from the main interferometer's beam splitter. As shown in [15], DC readout schemes are affected by the noise in the local oscillator, unlike ideal balanced homodyne readout. In addition, DC readout schemes are not as effective as balanced homodyne readout at measuring arbitrary quadratures of light in an interferometer. This is best done by using squeezed light and a BHD with a variable homodyne angle, which enables one to take advantage of the reduced quadrature uncertainties of the squeezed light. Therefore, it is believed that balanced homodyne detection will provide more precise interferometric measurements in GW detectors than the current DC readout schemes do. We hope that further research into this technology will lead to the implementation of balanced homodyne detectors in GW interferometers and improved detection of gravitational radiation.

2 Approach

The goal of this project will be to construct a balanced homodyne detector. Once completed, we will then analyze the electronic and optical noise that exists within the BHD. The BHD will be constructed from standard optical devices used in interferometry, including a laser, beam splitters, and photodiode detectors. More specifically, we will use InGaAs photodiodes. These are optimal for this setup because they have a high quantum efficiency and convert incident light into photocurrent very effectively. To amplify the interferometric signal and convert it from a current signal to a voltage signal, we will implement transimpedance amplifiers with low current noise. The specific transimpedance amplifiers to be used are homemade ones from the Adhikari lab, and it is desired that they contribute a minimal level of electronic noise. An image of a sample transimpedance amplifier circuit, which will be used as a guide in the BHD construction, is displayed in Figure 2.

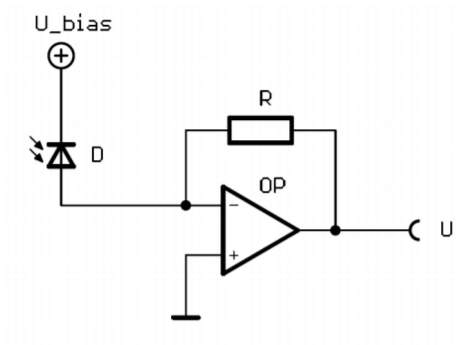

Figure 2: A basic transimpedance amplifier circuit, obtained from [5]. This circuit contains a photodiode (D), a resistor (R), and an operational amplifier (OP). U_bias and U denote the bias and output voltages, respectively.

Moreover, in order to analyze the optical and electronic noise in these devices, we will utilize two programs known as LISO and Finesse. LISO will be used to optimize electronic noise, and Finesse will be used to simulate the noise in the interferometer. Proper application of both programs is central to gauging the success of the final homodyne detector. Lastly, when a mathematical analysis of the noise in the BHD is required, Python Jupyter notebooks will be used to carry out analytic calculations and plot results.

3 Progress

3.1 Week 1

During my first week at Caltech, I began my project by studying additional material on gravitational wave interferometry, signal modulation, random processes, and noise. In particular, I read through sections of [15] and [11], which covered each of these topics. Learning more about these subjects has helped me better understand gravitational wave interferometry and the methods used to combat the noise in GW detectors. In addition, I familiarized myself with LTspice, a computer program used to model and simulate electronic circuits. LTspice can be employed to study the functionality of a circuit design and investigate the noise that exists within it. The ability to perform these operations easily via computer software will be of utmost importance to me when I analyze the electronic noise in the BHD. Lastly, I was taught about laboratory safety procedures, such as those pertaining to laser usage, so that I may work in the lab without causing harm.

3.2 Week 2

At the beginning of the next week, I started to work with photodiode amplifiers. These devices will be incorporated into the BHD in order to strengthen any optical signal detected. In addition to studying the noise produced by photodiode amplifiers in [9], I constructed sample photodiode amplifier circuits, and conducted measurements on them to ensure that they function properly. The amplifiers I built are akin to that in Figure 2; in my amplifiers, I incorporated an MTD5052W photodiode, an LT1028 op-amp, and 7.5 kΩ and 1 kΩ resistors. One of these amplifiers is intended to be used in the photodiode in the LIGO outreach miniature interferometer. In the coming weeks, I plan solder this amplifier onto a circuit board and encase it in a 3D printed container so that it can be easily incorporated into the outreach interferometer.

Furthermore, I began constructing the BHD during this time. This consisted of setting up a 1064 nm Nd:YAG laser, wave plates, a Faraday rotator, a beam splitter, and beam dumps. A picture of the current setup is displayed below in Figure 3. The wave plates and Faraday rotator are used to alter the polarization of the light emitted by the laser. Our setup of these elements forms an optical isolator, in which light is transmitted only in the forward direction. This ensures that no back-scattered light interferes with the laser light. By tweaking the relative phase shift imparted onto the light by the wave plates and the Faraday rotator, this setup allows us to block certain polarizations of light and control the amount of light that will enter the BHD. In effect, this gives us control over the strength of the light in the BHD interferometer. Next, the beam splitter splits the light that exits the Faraday rotator and will be an integral component of the interferometer. Lastly, the beam dumps capture scattered and reflected light produced by this setup, which would otherwise interfere with the light in the BHD and pose a threat to our safety.

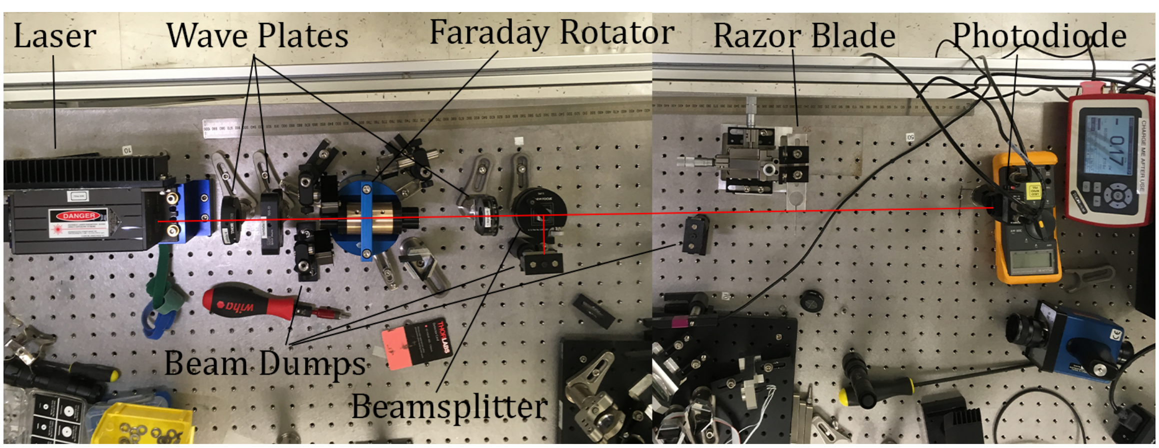

Figure 3: Our current BHD setup, including a laser, wave plates, a Faraday rotator, a beam splitter, and beam dumps. The razor blade and photodiode will be discussed in the next section.

3.3 Week 3

During my third week, I learned how to solder electronic circuits by constructing a simple RC lowpass filter that will be used in the LIGO outreach interferometer. This simple circuit consisted of a 430 Ω resistor in series with a 1.5 μF capacitor; a schematic of the circuit is pictured below in Figure 4. This circuit has a cutoff frequency of 250 Hz, and is designed to pass low frequency signals to an audio amplifier.

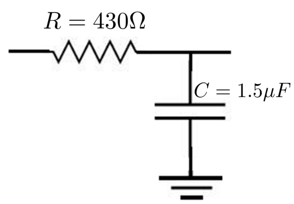

Figure 4: A schematic of the RC lowpass filter.

Moreover, I also studied Gaussian beams, which describe the electromagnetic waves emitted by lasers. In particular, I read Kogelnik and Li's article on Gaussian beams [6], which covered numerous concepts about the light produced by lasers and resonators. Mathematically, a Gaussian beam is a paraxial beam of light that can be expanded into transverse electromagnetic modes, characterized by $m, n \in \mathbb{N}_0$ and denoted by TEM_{mn}. The simplest mode, is the TEM_{00} mode. If the beam propagates along the z direction with a wave number, k, the x and y components of the electric field of the TEM_{00} mode are [2]:

$$E_{x,y}(\vec{x}, t) = E_{x,y0}\left(\frac{w_0}{w(z)}\right)e^{\frac{-r^2}{w(z)^2}}e^{i(kz-\omega t+\frac{kr^2}{2R(z)}+\alpha(z))}, \quad r^2 = x^2 + y^2. \tag{7}$$

Here, $w(z)$ is the beam waist, which takes a minimum value of w_0:

$$w(z) = w_0\sqrt{1 + (z/z_R)^2}; \tag{8}$$

z_R is the Rayleigh range:

$$z_R = \frac{1}{2}kw_0^2; \tag{9}$$

$R(z)$ is the beam's radius of curvature:

$$R(z) = z + z_R^2/z; \tag{10}$$

and $\alpha(z)$ is the Gouy phase:

$$\alpha(z) = -\arctan(z/z_R). \tag{11}$$

For our BHD setup, we desire the light entering the interferometer to be a stable TEM_{00} mode. In order to determine the identity of the light being emitted by the laser, I conducted beam profiling on our setup. Specifically, I first chose five locations along the lab bench, each equally spaced by 10 cm and located at different distances from the laser. At each point I placed a horizontal razor blade in front of the beam. I then varied the height of the razor blade so that it blocked part of the beam, and simultaneously measured the voltage produced

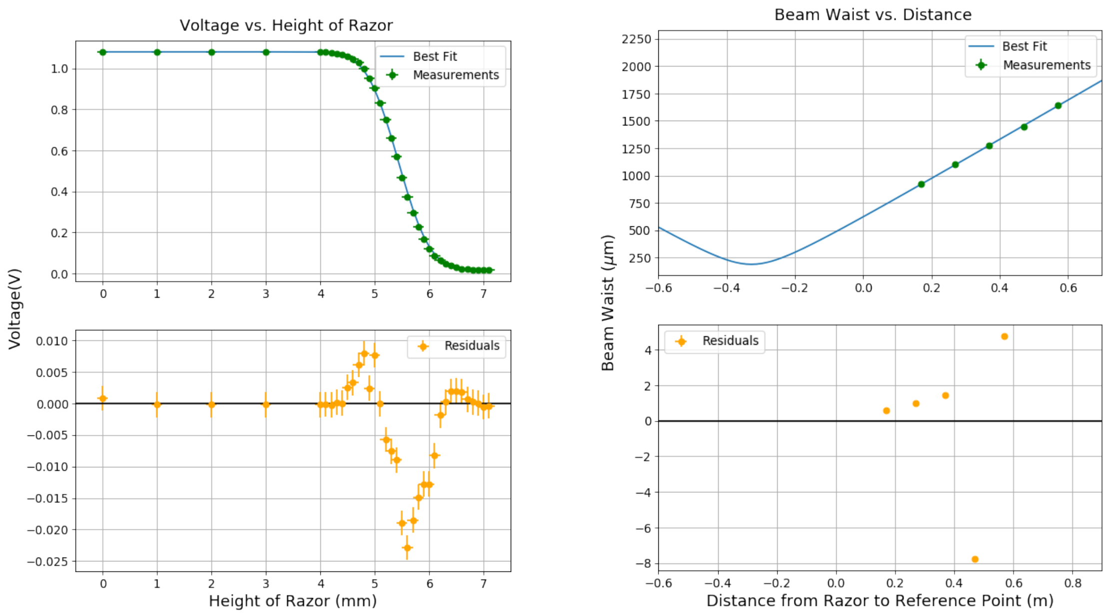

Figure 5: Left: Voltage measurements and line of best fit at the first measurement location. Right: Beam waist measurements and line of best fit.

by a photodiode in the path of the beam. The placements of the razor and photodiode are shown in Figure 3.

The photodiode responds to the beam by generating a voltage in response to the power it receives from the beam. With the razor blade at height u from the center of the beam, the power received by the photodiode is (ignoring diffraction effects)

$$P(z) = \int I(\vec{x})dA \propto \int_{-\infty}^{\infty}\int_{u}^{\infty} dxdy\Big(\frac{w_0}{w(z)}\Big)^2 e^{-\frac{2r^2}{w(z)^2}} \propto w_0^2\Big(1 - \text{erf}\Big(\frac{\sqrt{2}u}{w(z)}\Big)\Big). \qquad (12)$$

Next, we expect the voltage produced by the photodiode to be linear in the power it receives. However, I must note that the height of the razor blade that I recorded, denoted by h, is not equal to the height above the center of the beam, which is u. Instead, there is a constant difference between these: $u = h + b$, where b is parameter corresponding to the height of $h = 0$ above the center of the beam. Taking this into account, we expect

$$V(h, z) = V_0 + a\Big(1 - \text{erf}\Big(\frac{\sqrt{2}(h + b)}{w(z)}\Big)\Big), \qquad (13)$$

where V_0, a, b, and $w(z)$ are variable parameters at each location. I then fit the data to this curve in a Jupyter notebook by minimizing $\chi^2 = \sum_i \Big(\frac{V_i - V(h_i,z)}{\sigma_i}\Big)^2$ with respect to the parameters. The voltage data and the curve of best fit obtained at the first location are displayed above in Figure 5. The fits at the other four locations were quite similar. In general, the fits appear pretty good and the residuals are low in magnitude. However, the residuals display a distinct pattern and are not randomly distributed. Therefore, it is likely that there is an effect not accounted for in this fit, such as diffraction or a non-TEM_{00} mode contaminating the laser beam.

Once these fits were completed, I had obtained five empirical values of $w(z)$ at different distances from the laser. Specifically, I recorded the distances from the razor blade to a reference point near the laser, which I chose to be the base of a beam dump, and denoted this quantity by D. This distance differs from z in our coordinate system by a constant: $z = D + c$. With this fact noted, I then fit these empirical values to $w(z)$ specified in Eq. (8) by treating w_0 and c as fitting parameters and minimizing χ^2. Doing so yielded a minimum beam waist of $w_0 = 1.871 \cdot 10^{-4} \pm 1.419 \cdot 10^{-9}$ m and a constant offset of $c = 0.3274 \pm 4.557 \cdot 10^{-6}$ m, both of which are reasonable values for this laser and setup. The data and fit for the beam waist are displayed above in Figure 5.

3.4 Future Work

My present work involves constructing more of the BHD, such as its photodiodes and electronic circuits. I anticipate that this will be the most difficult part of my project. Thus, I will continue to focus on constructing the BHD in the coming weeks. Once the BHD is completed, I plan to conduct measurements on it and analyze its noise.

Simultaneously, I will be working on making improvements to the LIGO outreach interferometer. I have been tasked with constructing a new photodiode and a new laser for the interferometer. At the moment, the photodiode is set up on a breadboard; I aim to solder it to a circuit board and encase it in a 3D-printed box next week. Likewise, I have been given the laser pointer to use in the construction of the new laser, and I plan to begin setting it up next week.

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