LIGO SURF

Second Interim Report

Name: **Alicia Lima** (alima@bowdoin.edu.edu)
Mentor: **Davide Gerosa** (dgerosa@caltech.edu)

Visualizing 2PN Binary Black Hole Spin Precession

From 2D Parameter spaces-(S, ξ), (J, ξ) and (S, J)- to 3D (ξ, J, S)

Merging of Binary Black Holes (BBH) are one of the most promising source of gravitational waves (GW) for detection at LIGO and VIGO [1]. BBH have complicated dynamics,in particular when both black-holes are spinning. Spinning black-hole Binaries have three angular momenta: the two spins, $\mathbf{S_1}$ and $\mathbf{S_2}$, and the orbital angular momentum, $\mathbf{L}$. On top of the binary's orbital motion, spin-spin and spin-orbit couplings make the detection and characterization of GW from spinning binaries more challenging since they cause the three angular momenta to precess [2, 4].

Given a BH binary with mass ratio $q = m_2/m_1 < 1$, total mass $M = m_1 + m_2$, spins $\mathbf{S_1}$ and $\mathbf{S_2}$, separation r, and angular momentum $\mathbf{L}$, the time evolution of $\mathbf{S_1}$, $\mathbf{S_2}$ and $\mathbf{L}$ is a nine parameter space in an inertial frame. But using geometry and constants of motion in the post-Newtonian regime (PN), one can reduce the parameter space to three. The relative orientations of the two spins and angular momentum can be described using either:

$$\cos(\theta_1) = \mathbf{\hat{S}_1} \cdot \mathbf{\hat{L}}, \tag{1}$$

$$\cos(\theta_2) = \mathbf{\hat{S}_2} \cdot \mathbf{\hat{L}}, \tag{2}$$

$$\cos(\Delta\Phi) = \frac{\mathbf{\hat{S}_1} \times \mathbf{\hat{L}}}{|\mathbf{\hat{S}_1} \times \mathbf{\hat{L}}|} \cdot \frac{\mathbf{\hat{S}_2} \times \mathbf{\hat{L}}}{|\mathbf{\hat{S}_2} \times \mathbf{\hat{L}}|} \tag{3}$$

or,

$$\xi = M^{-2}[(1+q)\mathbf{S_1} + (1+q^{-1})\mathbf{S_2}] \cdot \mathbf{L} \tag{4}$$

$$J = |\mathbf{S_1} + \mathbf{S_2} + \mathbf{L}|. \tag{5}$$

$$S = |\mathbf{S_1} + \mathbf{S_2}|. \tag{6}$$

The two descriptions, in terms of $(\theta_1, \theta_2, \Delta\Phi)$ and (ξ, J, S), are equivalent, but the latter is a nice set of parameter since,on the precession timescale, both ξ and J are constant, and only S varies. Therefore, the entire dynamics can be parametrized with S.

The analysis of the dynamics of spinning, precessing BBH in term of this second set of variables was done in [2], and as mentioned in my First Interim Report, I spend the first half of the past month studying that paper in details and familiarizing with the public code PRECESSION [3], and enrich python coding skill. At the same time,I reproduced FIG 4 and FIG 5 of [2], which are plots in the $(S,\,\xi)$ and in the $(J,\,\xi)$ parameter space for BBHs, respectively.In additon, I created a new graph in the $(S,\,J)$ parameter space. For the second half of the past month, I explored 3D plotting packages, and study how we can combine all three plots in the $(S,\,\xi)$, $(J,\,\xi)$ and $(S,\,J)$ parameter spaces.

Ideally, we would like to visualize and analyze of the orientation of the three angular momenta, in such a way that is both intuitive and informative. Given this difficulty, Davide Gerosa and I aim at developing an interactive web interface to facilitate the exploration of spin precession in merging BHs. Using the PRECESSION, which is a PYTHON module, we started to explore variety of 3D plot packages, principally those which are interactive: Mayavi, VPython, Plotly and Matplotlib. So far, I have used Plotly and Matplotlib for plotting, but have faced some problems. Matplotlib is not interactive but it is still useful to allow me to see the 3D plot shape, and it is easier to use. One of the biggest problem is finding the best method to interpolate 3D surfaces. For instance, for 3D surface interpolation, Plotly offers functions like go.Surface and go.Mesh3D. I have used both methods but have not been successful so far: go.Surface ”crashes” when O(1000) points are used, and with go.Mesh3D I have not gotten any surface display. Go.Mesh3D uses an interpolation method called triangulation, and this method is promising because it is proven mathematically that every finite set of points, which are not all collinear, has a triangulation.Development is still in progress and we plan to leave results using go.Mesh3D by the end of the summer. Below is an example of 3D plot I created using go.Surface. For this family of BH binaries, I set $q = 0.2, \chi_1 = 1.0, \chi_2 = 1.0$ and $r = 10$, and I calculated J, ξ and S.

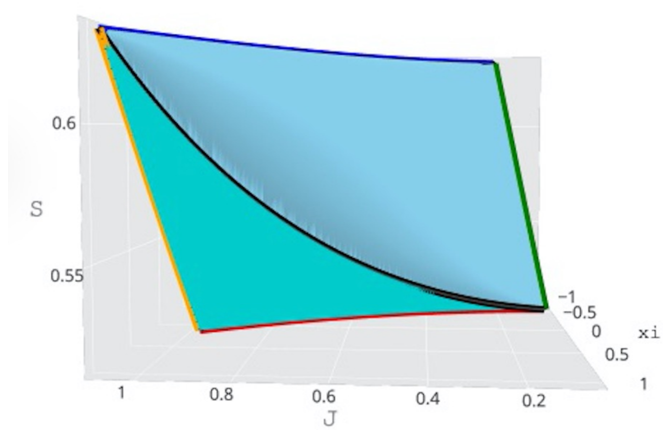

Figure 1: The 3D plot of BBHs spin precession dynamics with $q = 0.2, \chi_1 = 1.0, \chi_2 = 1.0$ and $r = 10$ in the (ξ, J, S) parameter space.The edge lines, which are colored in yellow, red, green and blue, correspond to the configuration where $S = S_{\min}$ or $S = S_{\max}$. The black line correspond to all the values of S when ξ is equal to $\xi_{\max}$, which is the red line of the middle panel of FIG 5 in [2]. The light blue and cyan surface represent all possible configuration when $S = S_+$ and $S = S_-$ respectively

.

Figure 1 is the 3D plot when seen from one angle, but one can rotate and analyze the plot from different angles. Below I provided more screenshots of the same plot as viewed from different angles.

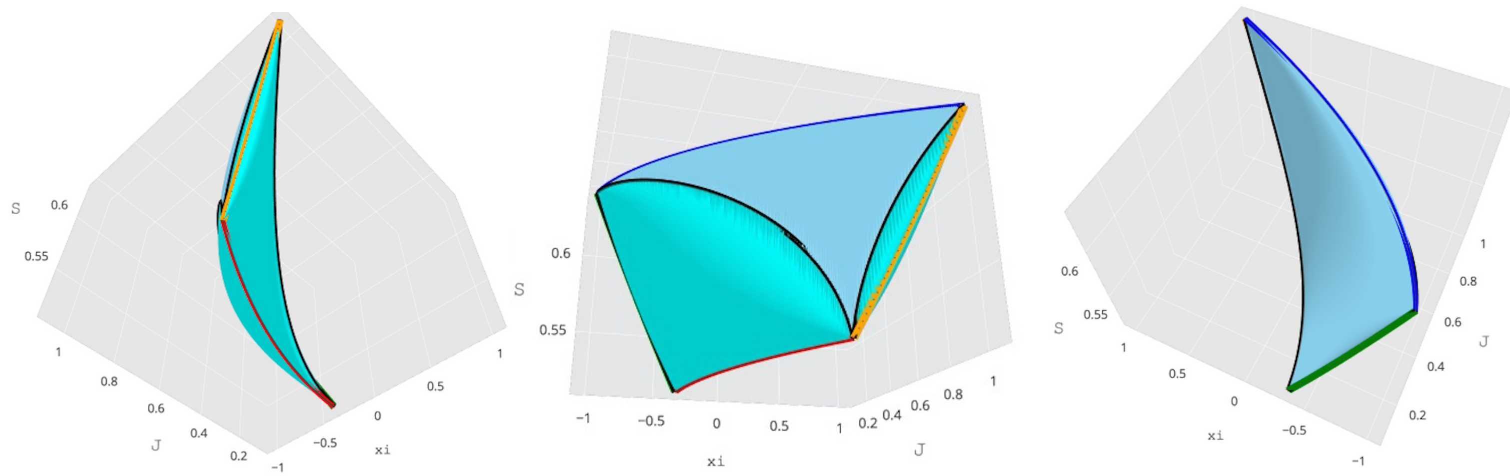

Figure 2: The same 3D plot as Figure 1, but view from different angles.

We Have Seen That Shape Before

Now that we have a 3D plot of the spin precession dynamics, how can we verify if the shape of 3D surface is correct? How can the 2D plots in [2] help us understand the 3D plot above? For instance, FIG 4 of [2] are plots in the $(J,\,\xi)$ parameter space for BBHs, which correspond to the projection of our 3D plot in the J-ξ plane. In fact, this seems to be the case: the first plot below is middle plot in FIG 4 of [2] and the second plot is a section in the J-ξ plane of the 3D plot above.

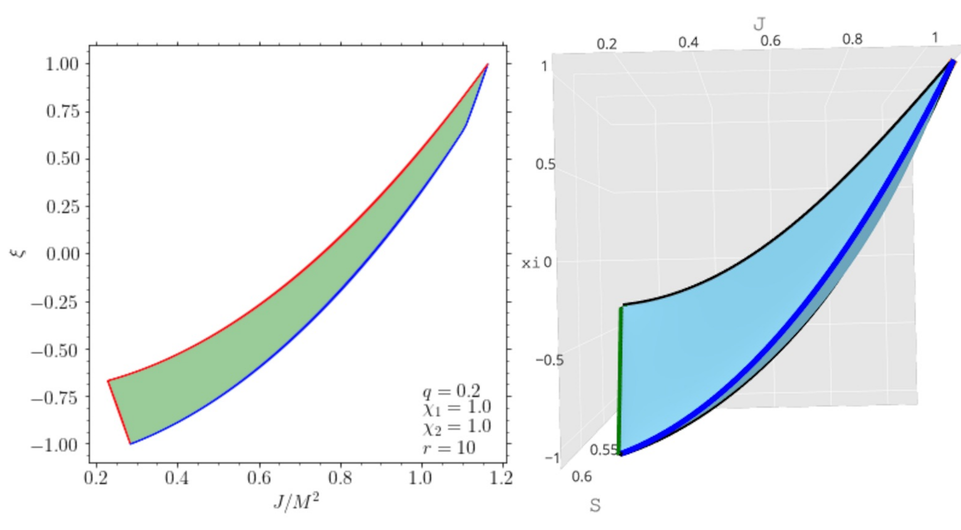

Figure 3: The left panel is a plot in the (J,ξ) parameter space for BBHs with $J_{\rm min} = |S_1 - S_2| - L$, and the right panel is a screenshot of Figure 1 as viewed from the J-ξ plane.

From Figure 3, one can see that those two plots agree, which is a good indicator that our 3D plot is correct. But is that a sufficient indicator? In addition to the agreement between plots in Figure 3, the edge lines colored in blue, yellow, green, red, and black are good indicators that our 3D plot is correct. Surprisingly,it was not the first time that we had seen those lines; we had saw them before in the J-ξ plane, as soon below.

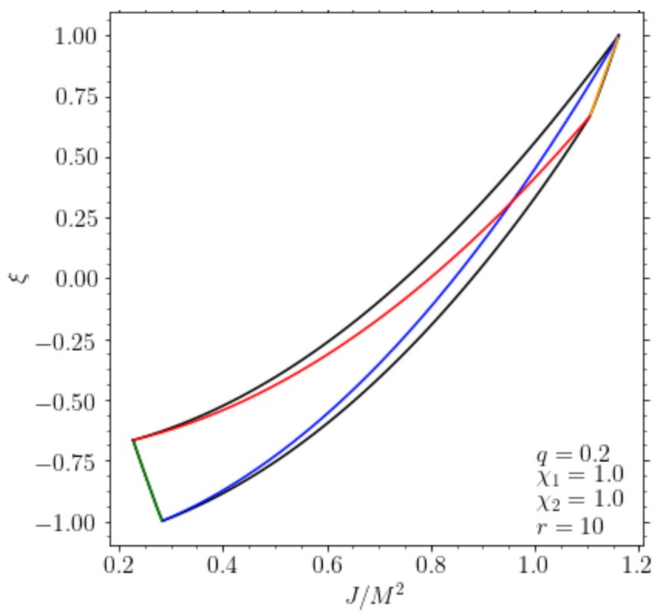

Figure 4: the (J, ξ) parameter space for BBHs with $J_{\min} = |S_1 - S_2| - L$, and the parameter q, χ_1, χ_2, and r are shown in the legend. The top black line corresponds to $\xi = \xi_{\min}$ and the bottom black line corresponds to $\xi = \xi_{\max}$. The yellow and the red lines correspond to $S = |J - L|$ and $S = |S1 - S2|$, respectively. The green and the blue lines correspond to $S = J + L$ and $S = S1 + S2$, respectively.

Before plotting those color lines on the 3D plot, Davide and I predicted that the yellow, red, green, and blue lines would be the edge of the 3D plot, and that the black lines would connect S_+ and S_- surface, as one can verify from Figure 1 and 2.

For future work, Davide and I will explore how to include time evolution to describe the inspiral of BBHs on the radiation-reaction timescale and include the spin morphology on the 3D plot.

References

[1] B. Abbott, et al., Phys. Rev. Lett. 116, 061102 (2016)

[2] D. Gerosa, M. Kesden, U. Sperhake, E. Berti, and R. OShaughnessy, PRD 92, 064016 (2015), arXiv:1506.03492 [gr-qc].

[3] Gerosa,Davide, and Michael Kesden. Precession: Dynamics of Spinning Black Hole Binaries with Python. Physical Review D, vol. 93, no. 12, 27 Dec. 2017, doi:10.1103/physrevd.93.124066.

[4] T. A. Apostolatos, Phys. Rev. (1994)