LIGO-G1801355-v1

Noises in gravitational wave detectors

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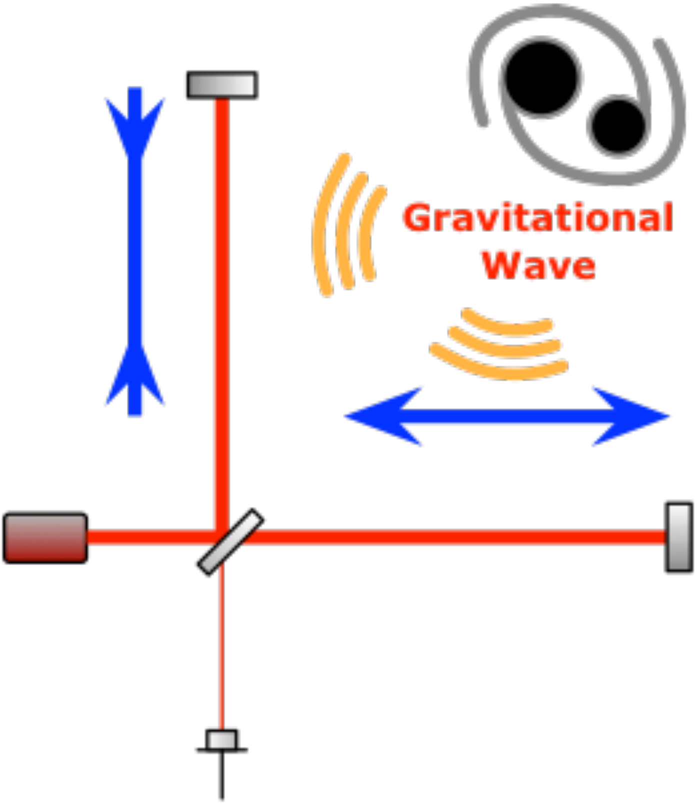
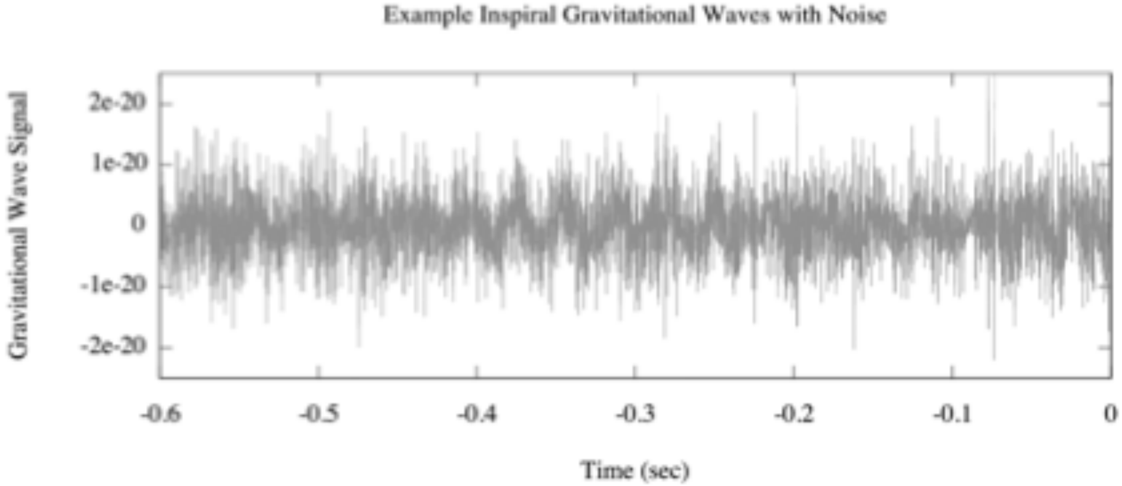
Detector output signal

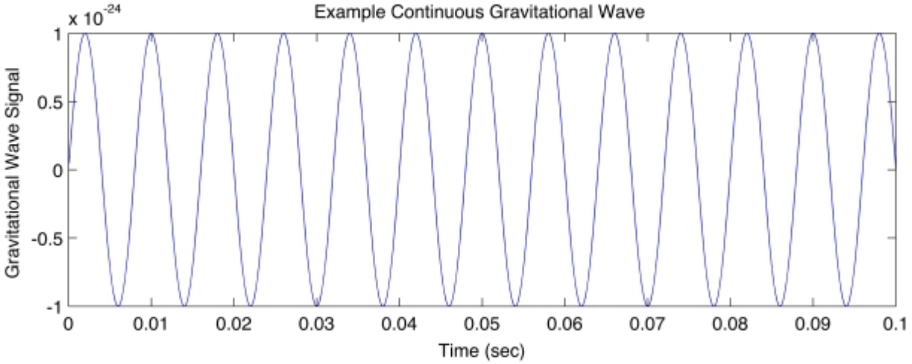
- h(t) = differential arm strain signal
- Single time series data



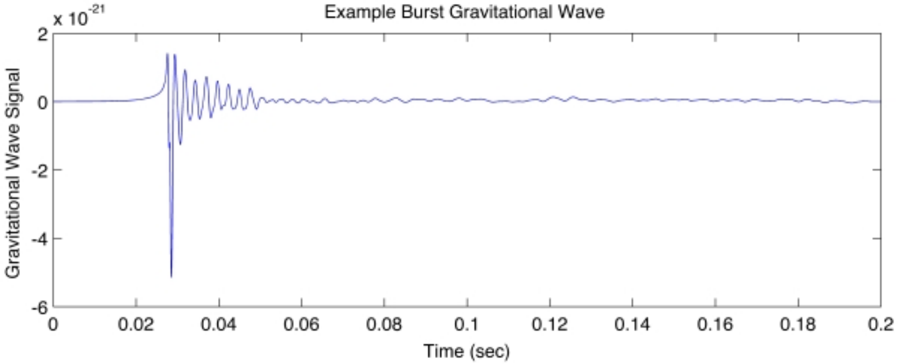


Signal vs Noise

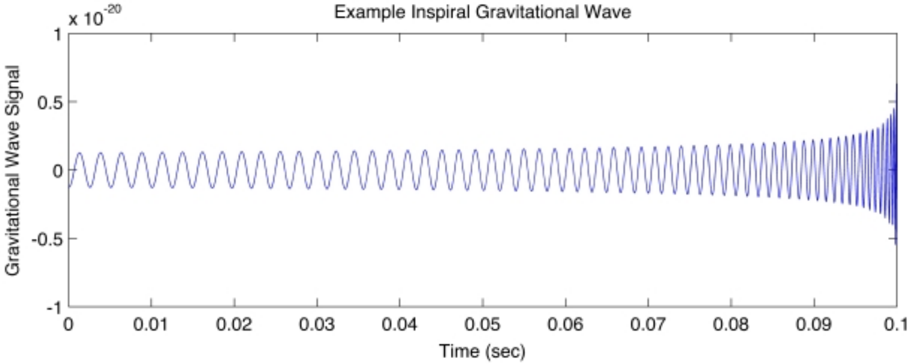
- **Waveform of GW signals**
- **Continuous**



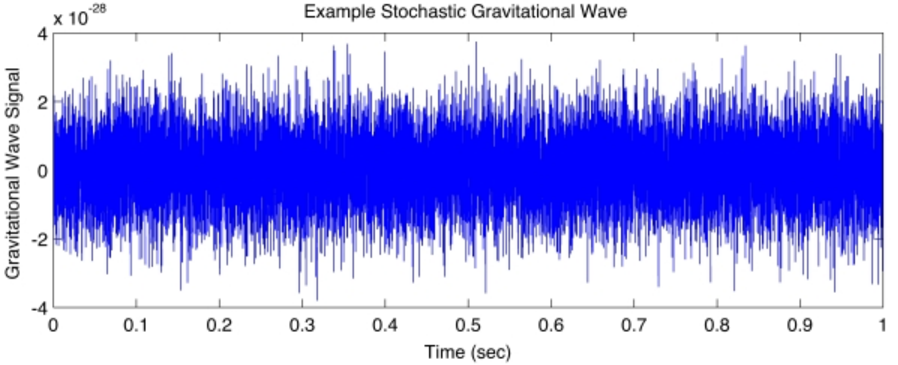
- **Burst** (unpredictable w.f.)



- **Compact Binary Coalescence**

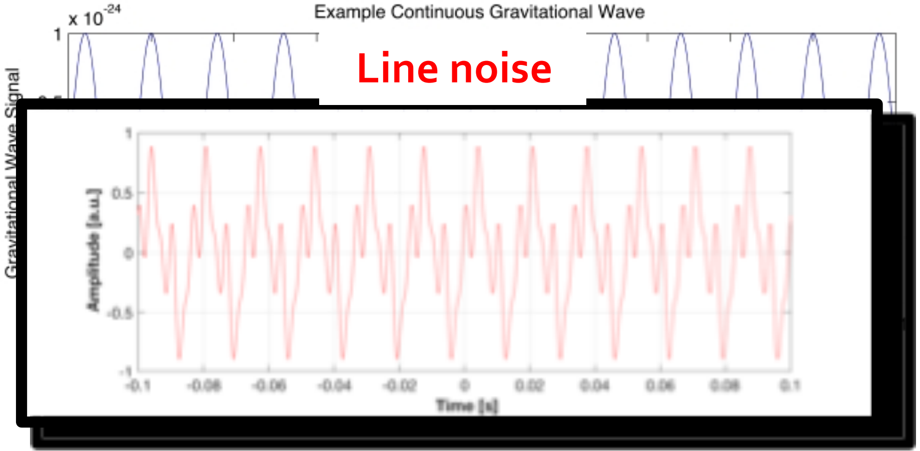


- **Stochastic**



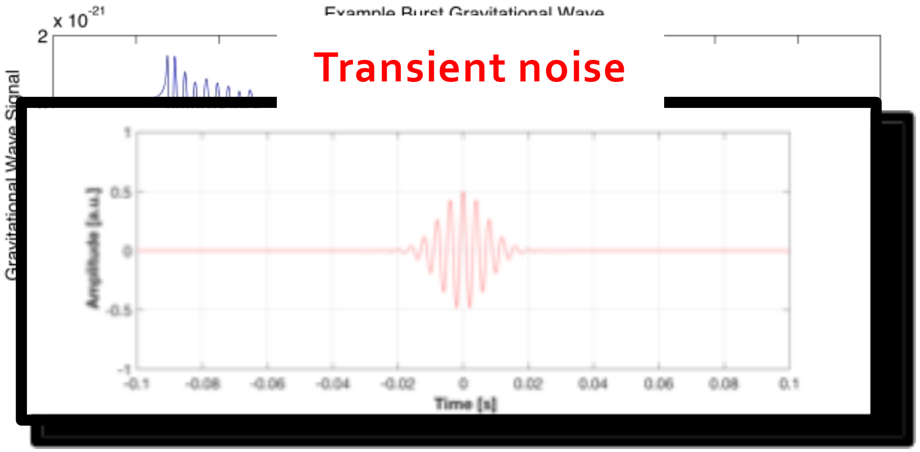
Signal vs Noise

- **Waveform of GW signals** vs noises
- **Continuous**



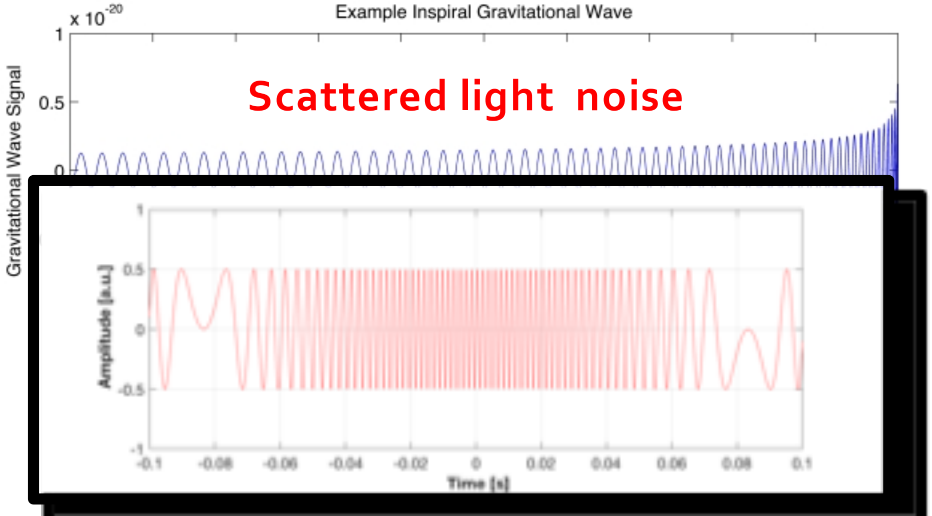
Line noise

- **Burst** (unpredictable w.f.)



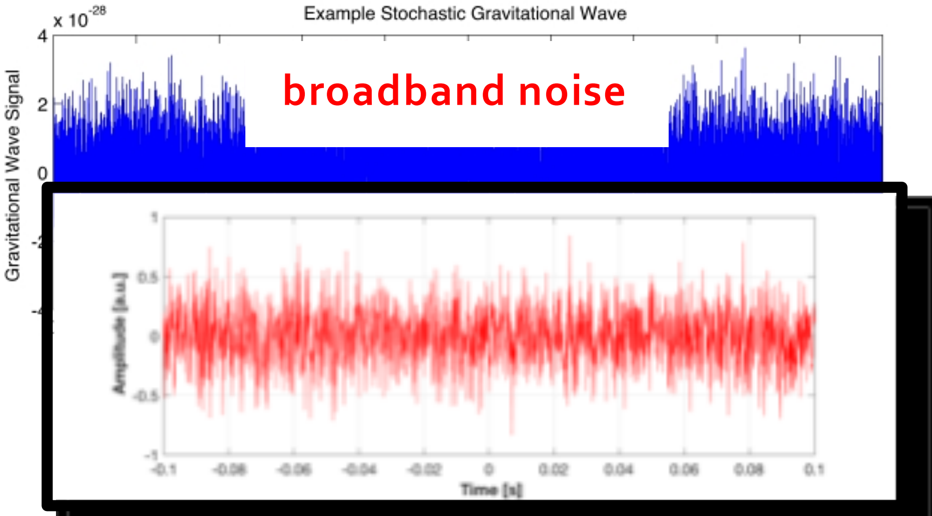
Transient noise

- **Compact Binary Coalescence**



Scattered light noise

- **Stochastic**

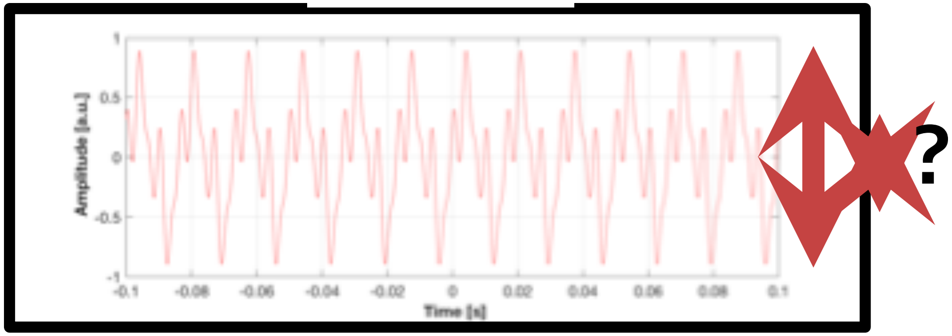


broadband noise

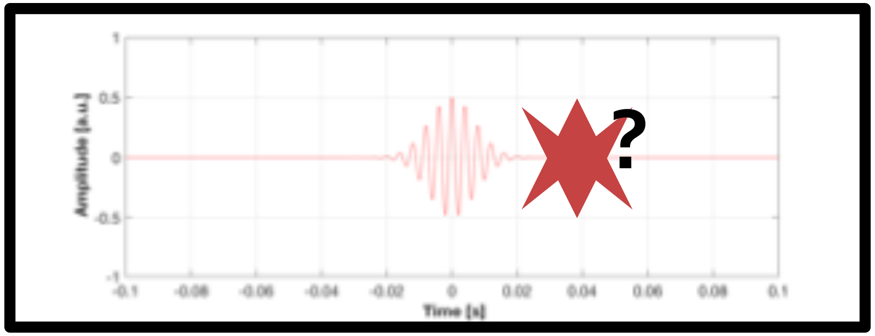
Characterization of time-series data

- **Amplitude**

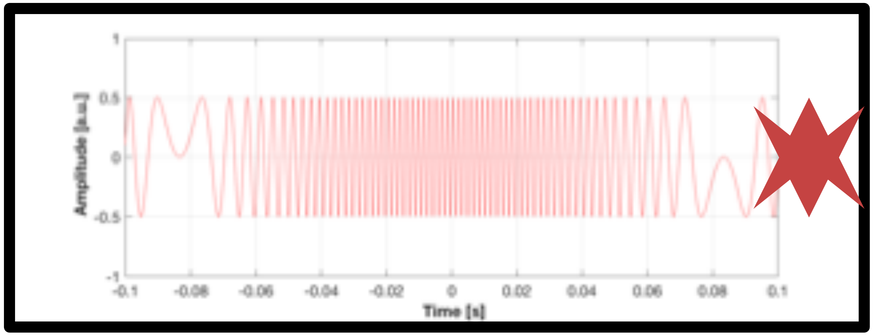
Line noise



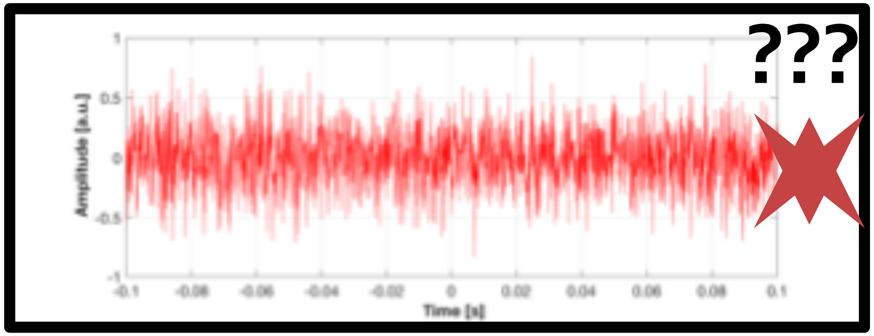
Transient noise



Scattered light noise



broadband noise

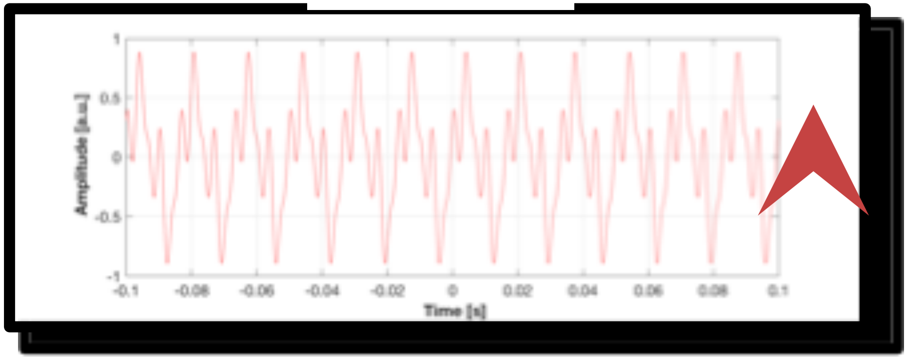


Characterization of time-series data

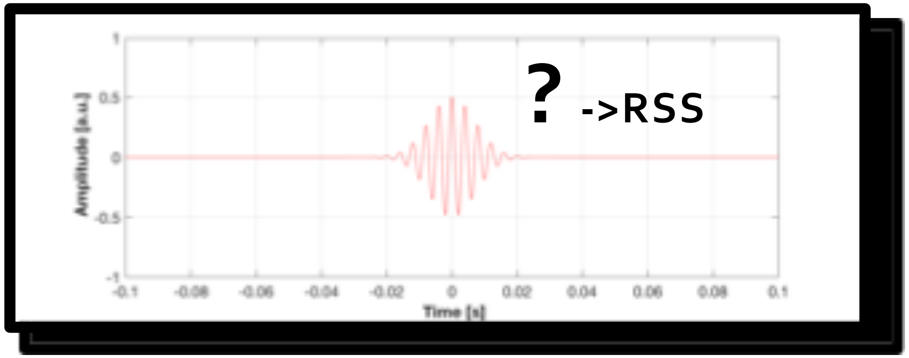
- **RMS (Root Mean Square)**

$$x_{\mathrm{RMS}} = \sqrt{\frac{1}{T}\int_{T_1}^{T_2} [x(t)]^2 \, dt}$$

Line noise

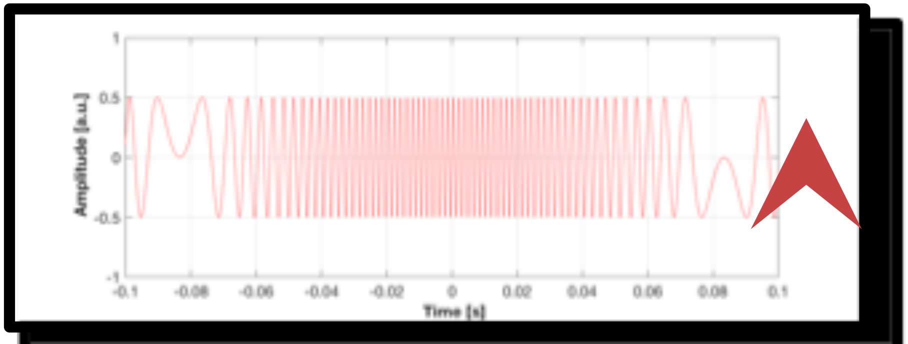


Transient noise

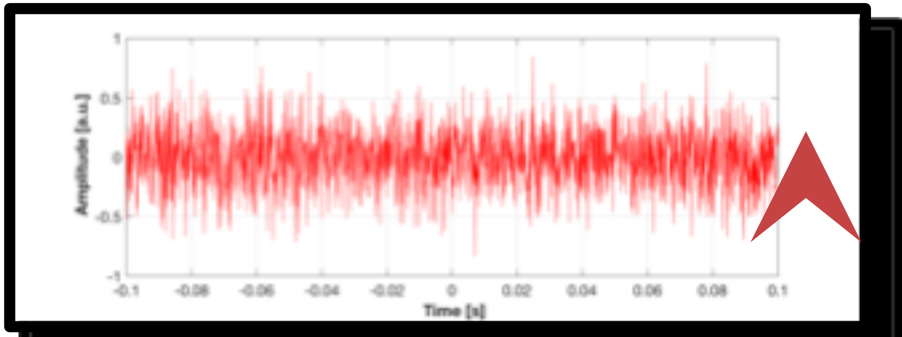


? ->RSS

Scattered light noise



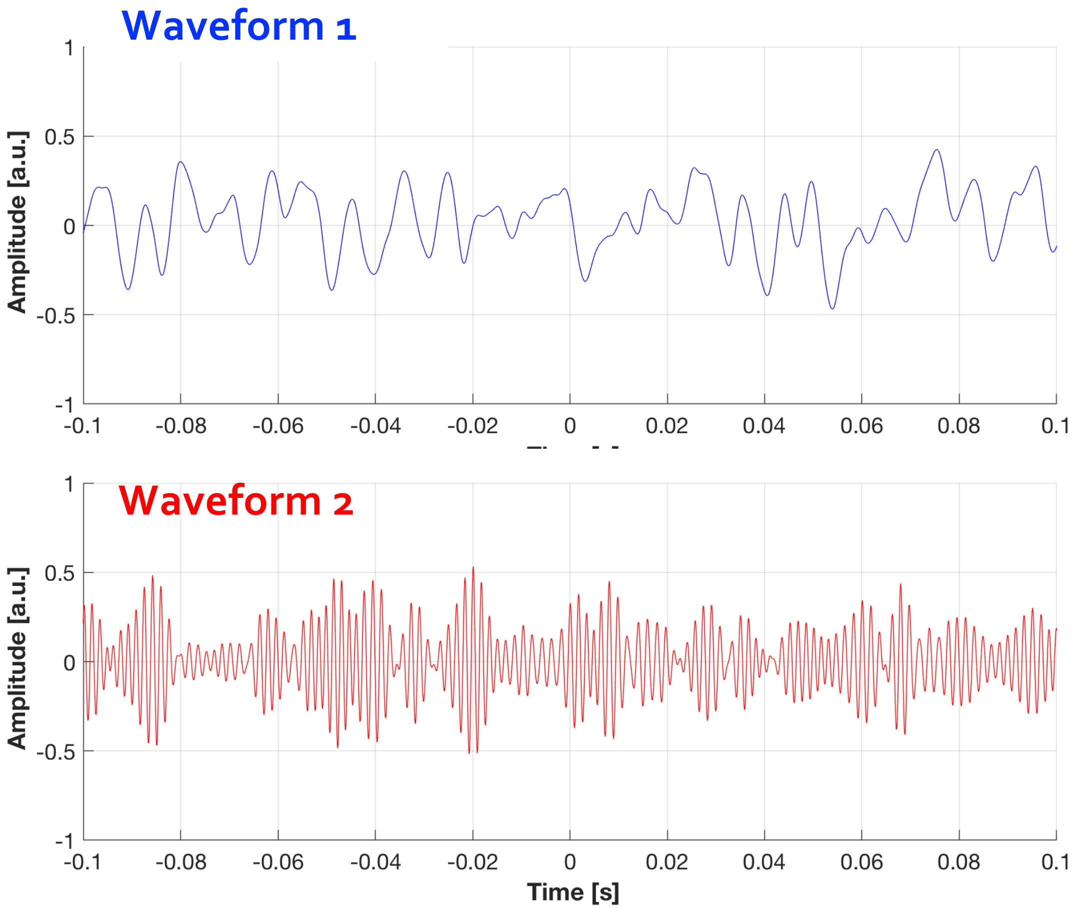
broadband noise



Characterization of time-series data

- **Same RMS (~0.2), but they look different**

$$x_{\mathrm{RMS}} = \sqrt{\frac{1}{T}\int_{T_1}^{T_2} [x(t)]^2 \, dt}$$



Freq domain: Power Spectral Density

- **Double sided PSD (-Infinity < f < Infinity)**

$$S_{\mathrm{DS}}(f) = \lim_{T\to\infty} \frac{1}{T} \left| \int_{-T/2}^{T/2} x(t) e^{-2\pi i f t} \mathrm{d}t \right|^2$$

- **Single sided PSD (0 <= f < Infinity)**

$$S_x(f) = 2 S_{\mathrm{DS}}(f)$$ [x_{unit}^2 / Hz]

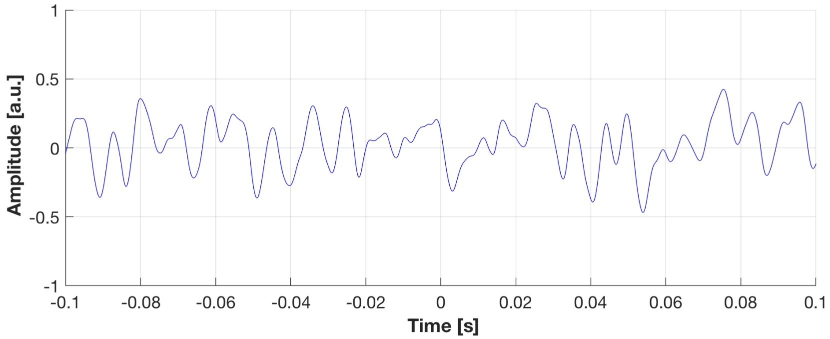
- **Linearized PSD:**

$$G_x(f) = \sqrt{S_x(f)}$$ [x_{unit}/ sqrtHz]

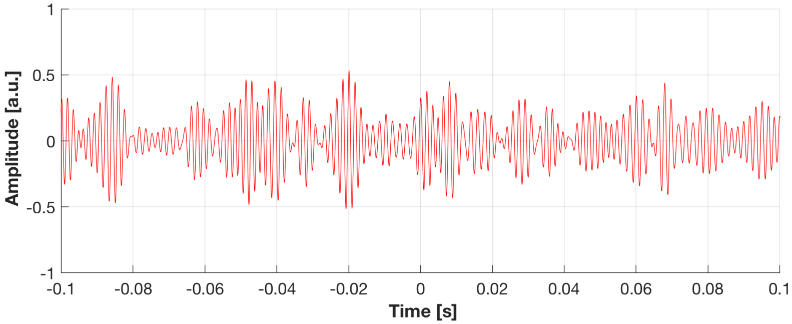
Characterization of time-series data

- **Same RMS (~0.2), but they look different**

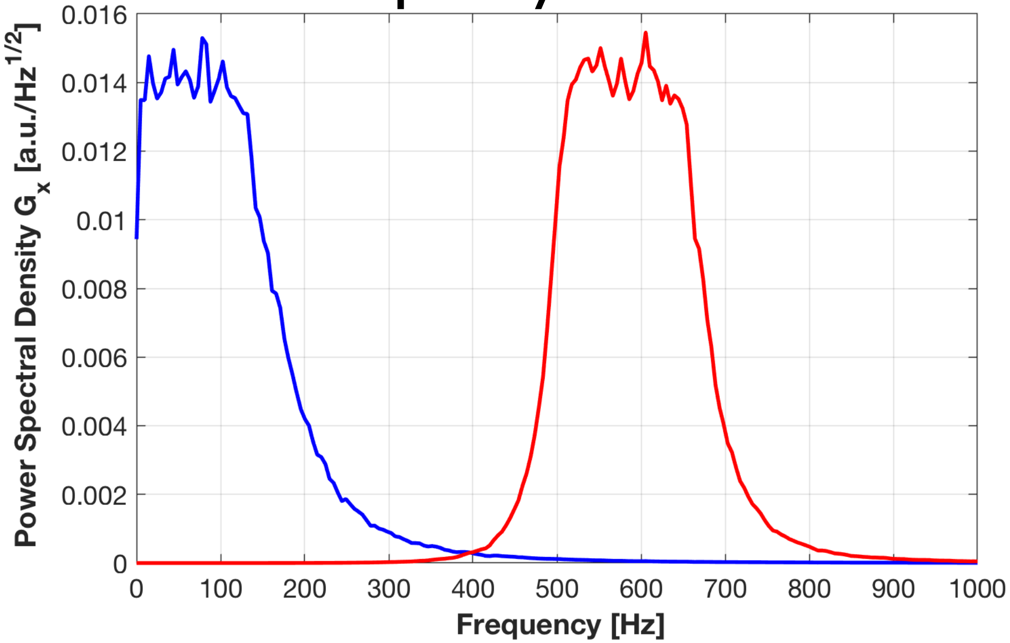
Waveform 1



Waveform 2



Frequency Domain



PSD <-> RMS

- **Parseval's Theorem** for signal RMS and PSD

$$\overline{x^2(t)} = \int_0^\infty S_x(f)\,df$$

$$\equiv x^2_{\mathrm{RMS}}$$

Root Mean of *x*(*t*):

average signal power density (per sec)
(cf. variance, std deviation)

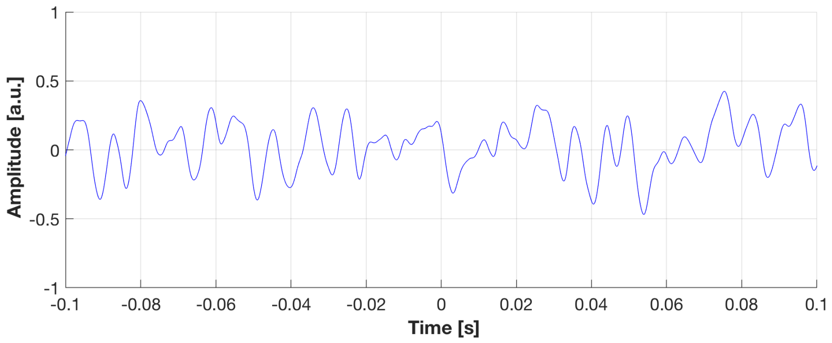
PSD *Sx*(*f*):

power density per frequency (per sec)

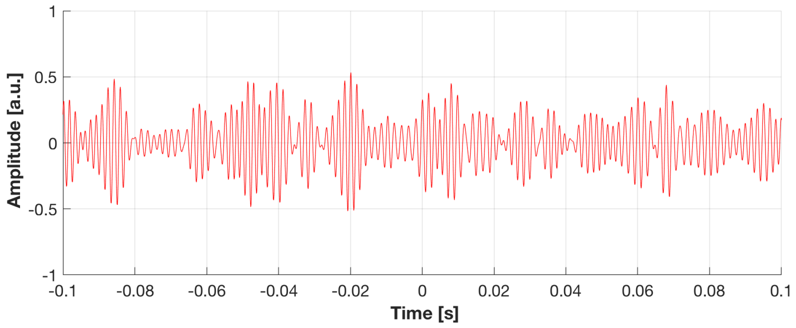
PSD <-> RMS

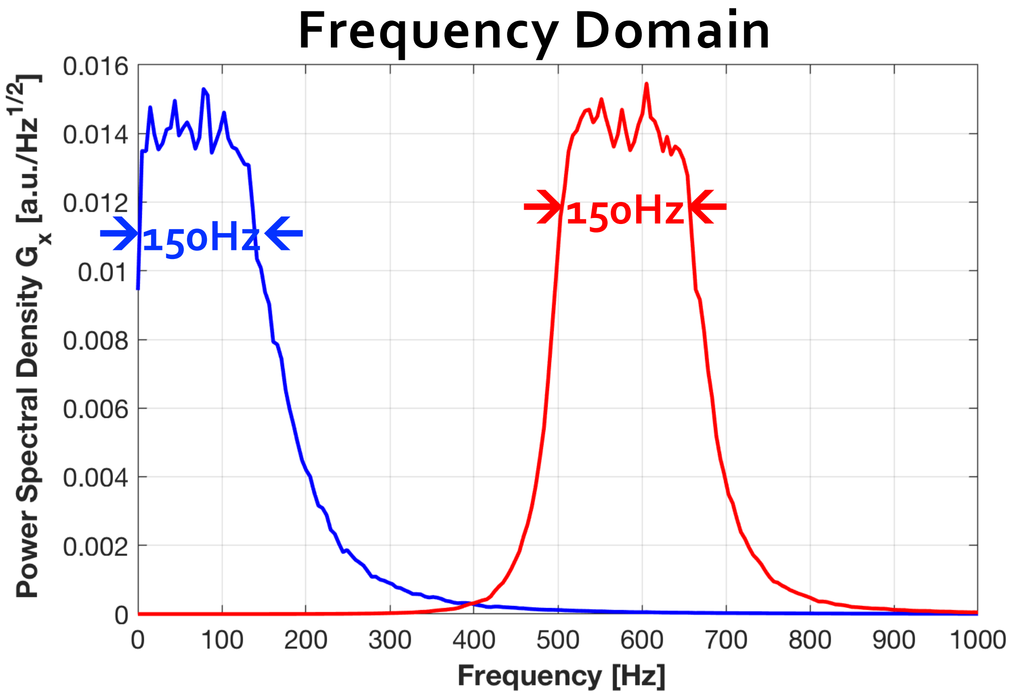
- **Same RMS (~0.2), but they look different**

Waveform 1: RMS~0.18



Waveform 2: RMS~0.18





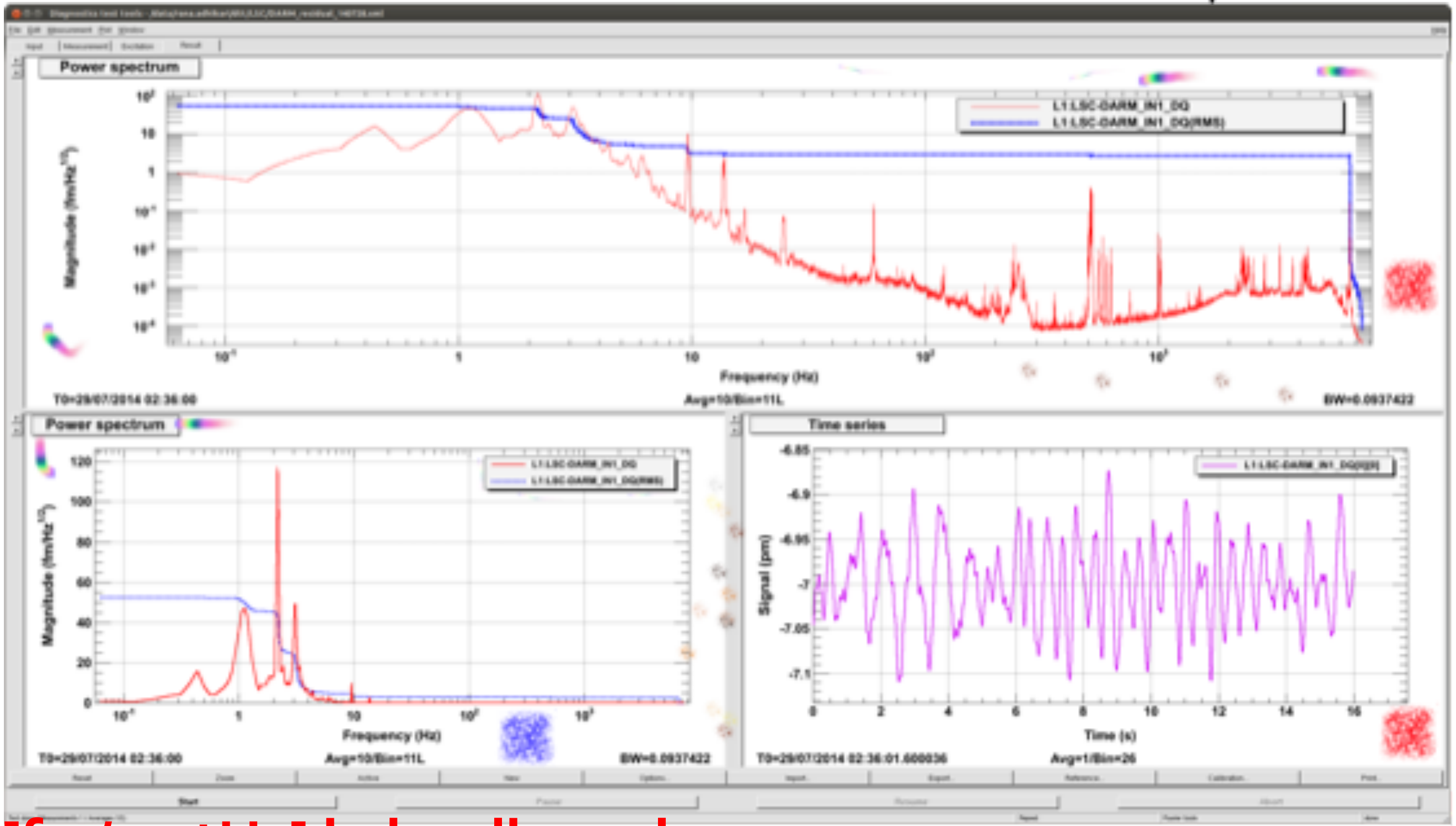
$\mathrm{Sqrt}(0.014^2 * 150)$
$=\sim 0.17$

Time series, PSD, and RMS

Example

PSD [fm/sqrtHz] in log-log scale,

$$x_{\mathrm{RMS}}(f) = \sqrt{\int_f^{\infty} S_x(f')\,df'}$$



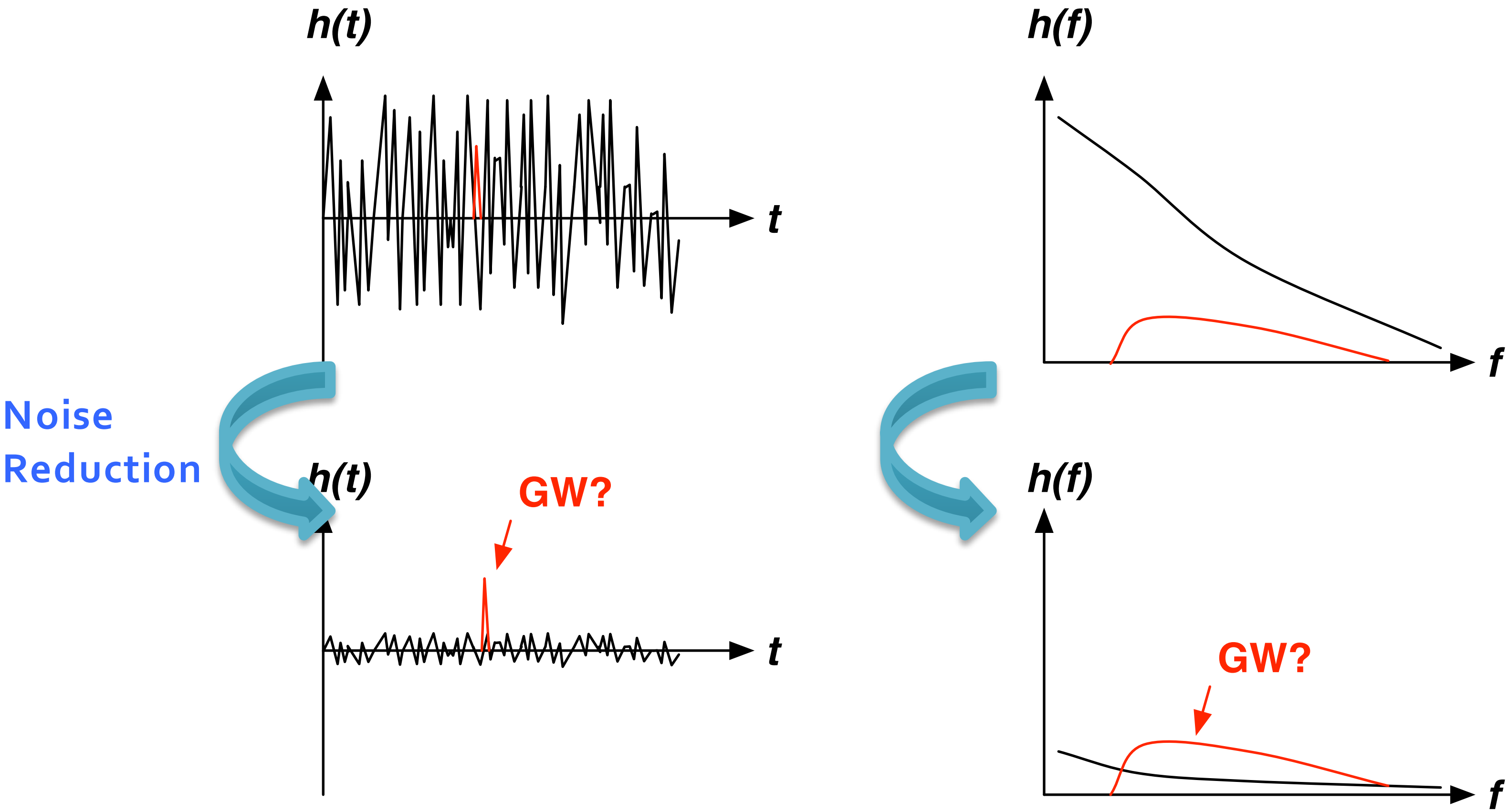
PSD [fm/sqrtHz] in log-lin scale

RMS [fm] ~ 50fm = 0.05pm

Time series [pm]

Time domain vs frequency domain

- Time domain vs frequency domain



- Time domain: transient noises
  Frequency domain: stationary noises

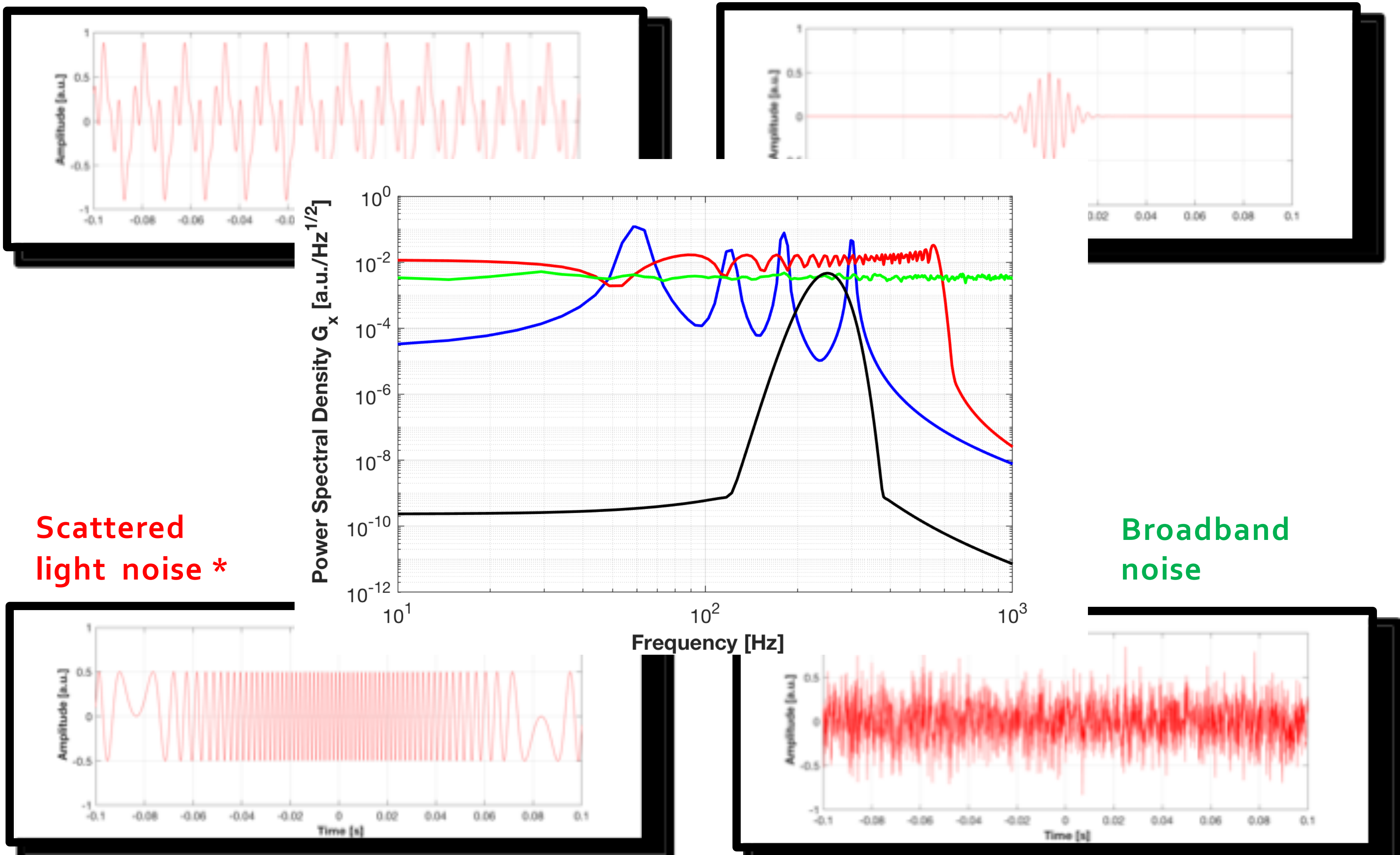
Time domain vs frequency domain

Line noise *

Transient noise *

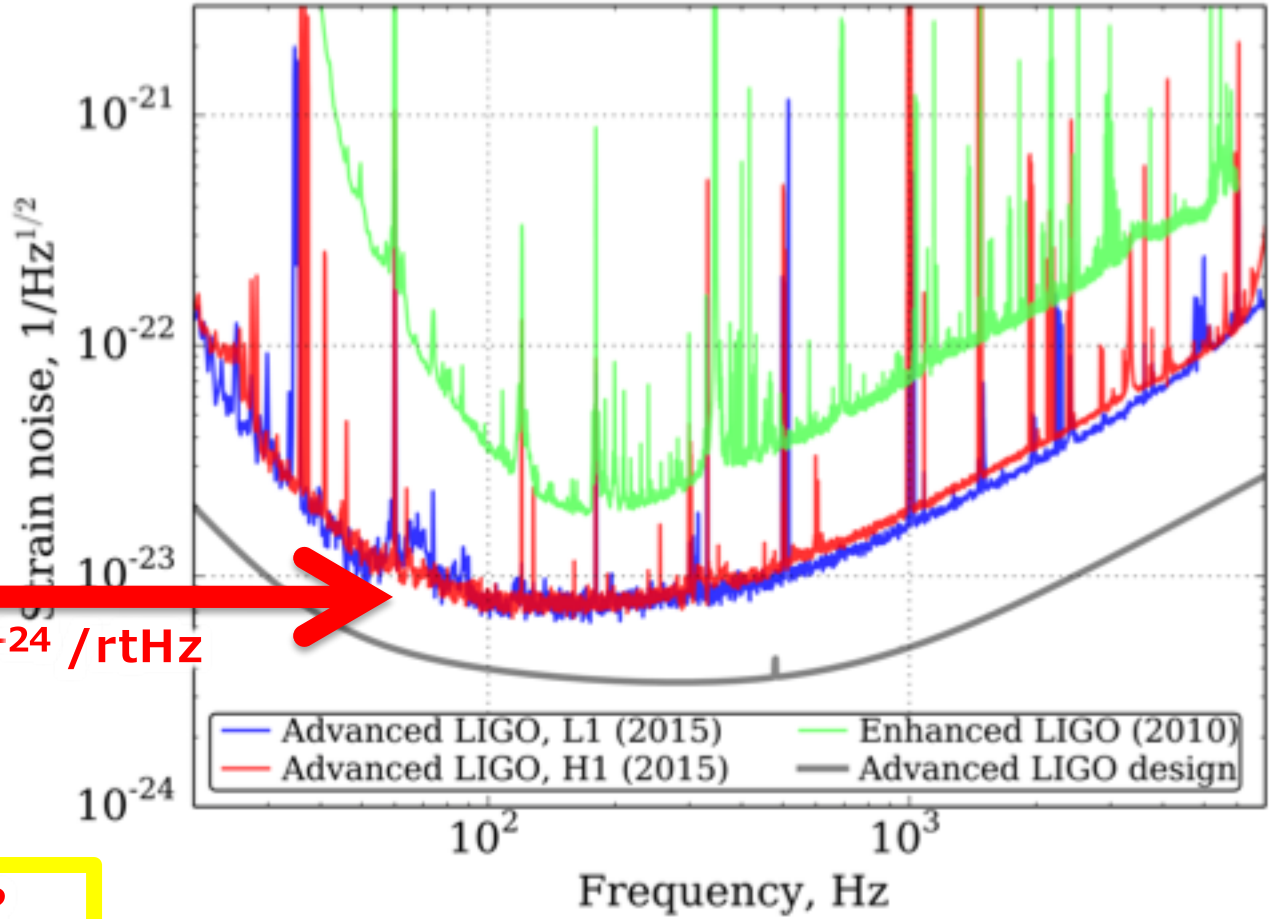
Scattered light noise *

Broadband noise



Sensitivity and noise

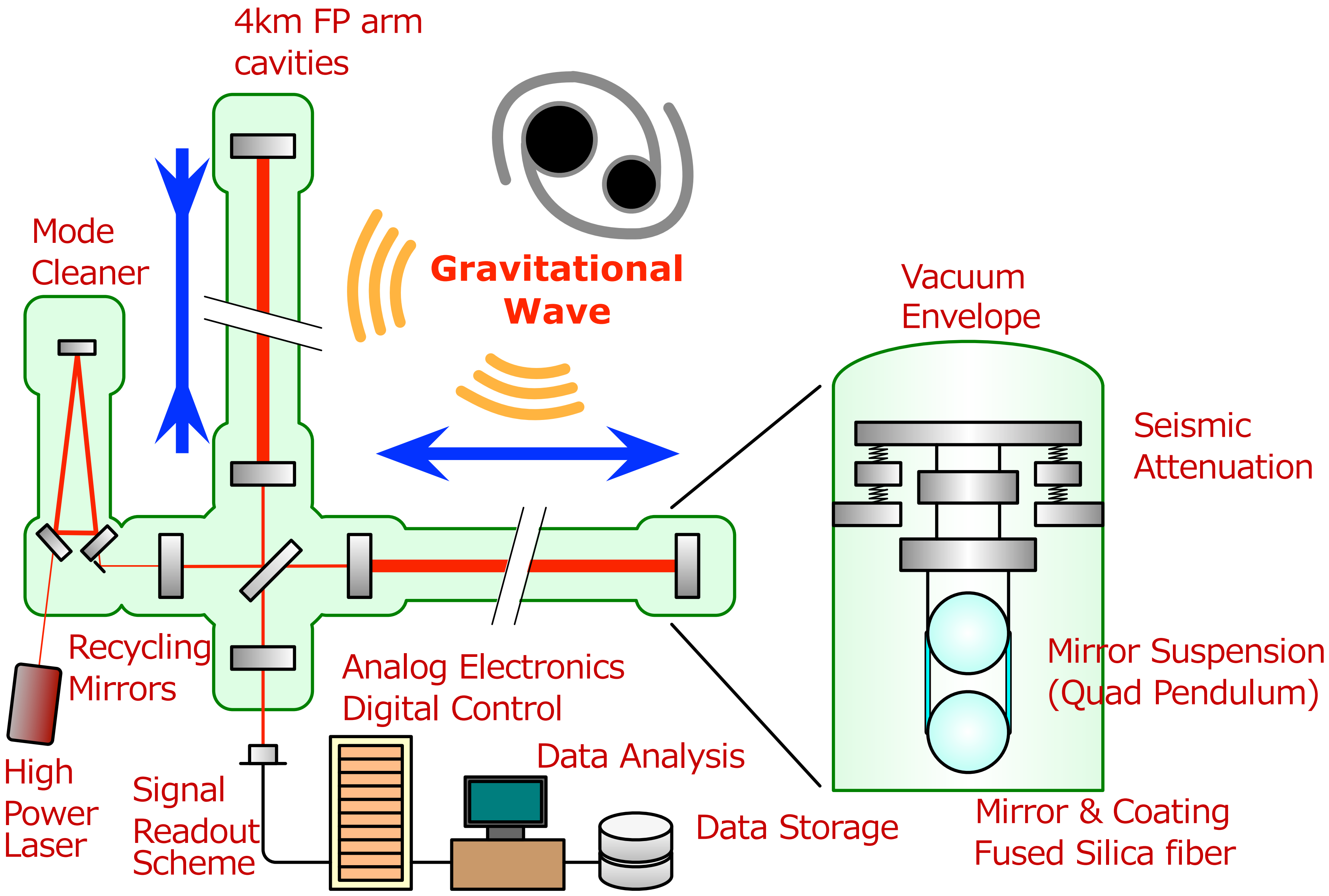
- Sensitivity (=noise level) of Advanced LIGO
- Current sensitivity



h= 8×10^{-24} /rtHz

RMS?

Actual GW detector



4km FP arm cavities

Mode Cleaner

Gravitational Wave

Vacuum Envelope

Seismic Attenuation

Recycling Mirrors

Analog Electronics

Digital Control

Mirror Suspension (Quad Pendulum)

Data Analysis

High Power Laser

Signal Readout Scheme

Data Storage

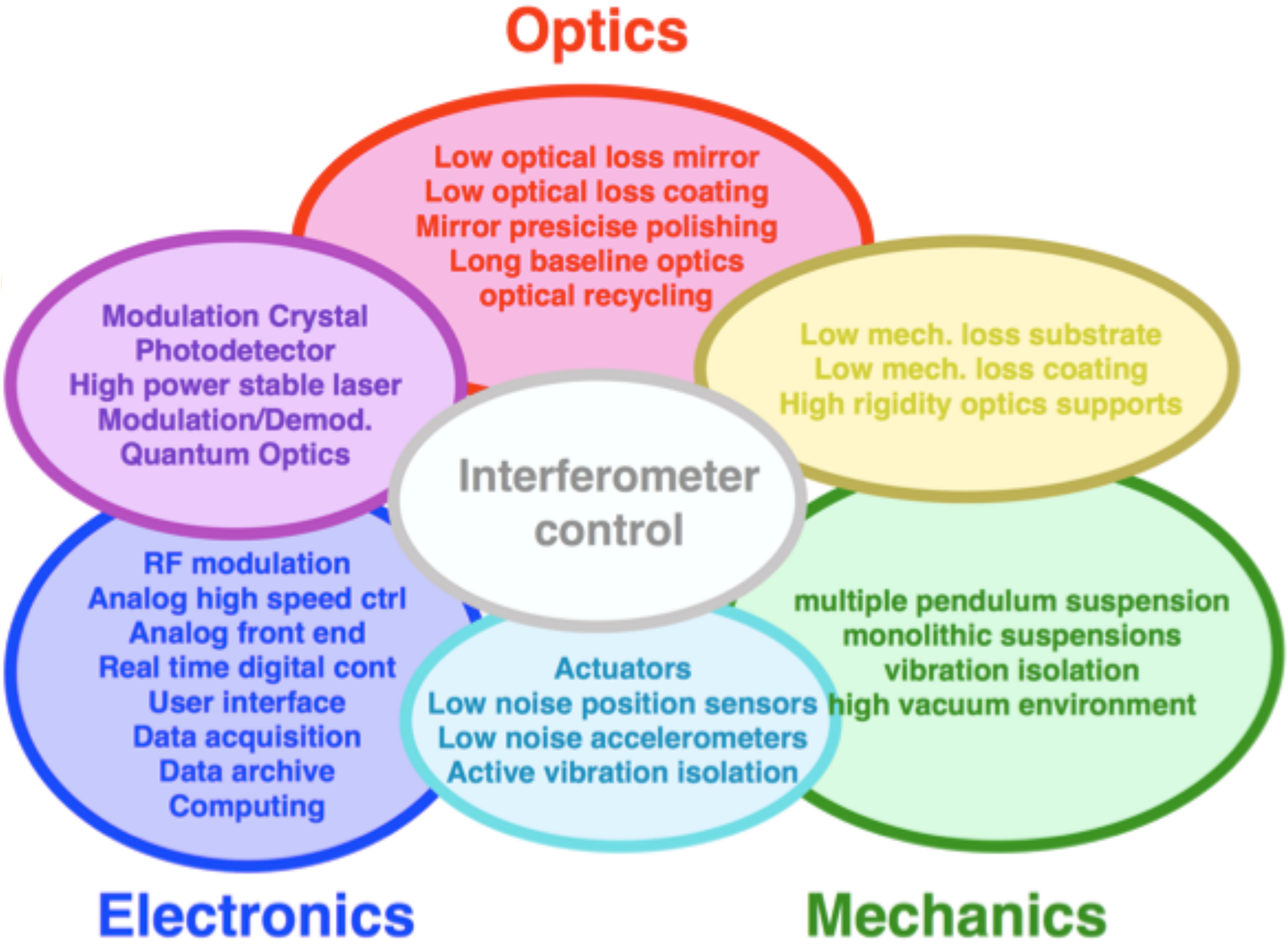
Mirror & Coating Fused Silica fiber

Components of the interferometer

- 3 elements of a GW detector
  - Mechanics
  - Optics
  - Electronics

Components of the interferometer

- 3



Optics

Low optical loss mirror
Low optical loss coating
Mirror presicise polishing
Long baseline optics
optical recycling

Modulation Crystal
Photodetector
High power stable laser
Modulation/Demod.
Quantum Optics

Low mech. loss substrate
Low mech. loss coating
High rigidity optics supports

Interferometer control

RF modulation
Analog high speed ctrl
Analog front end
Real time digital cont
User interface
Data acquisition
Data archive
Computing

Actuators
Low noise position sensors
Low noise accelerometers
Active vibration isolation

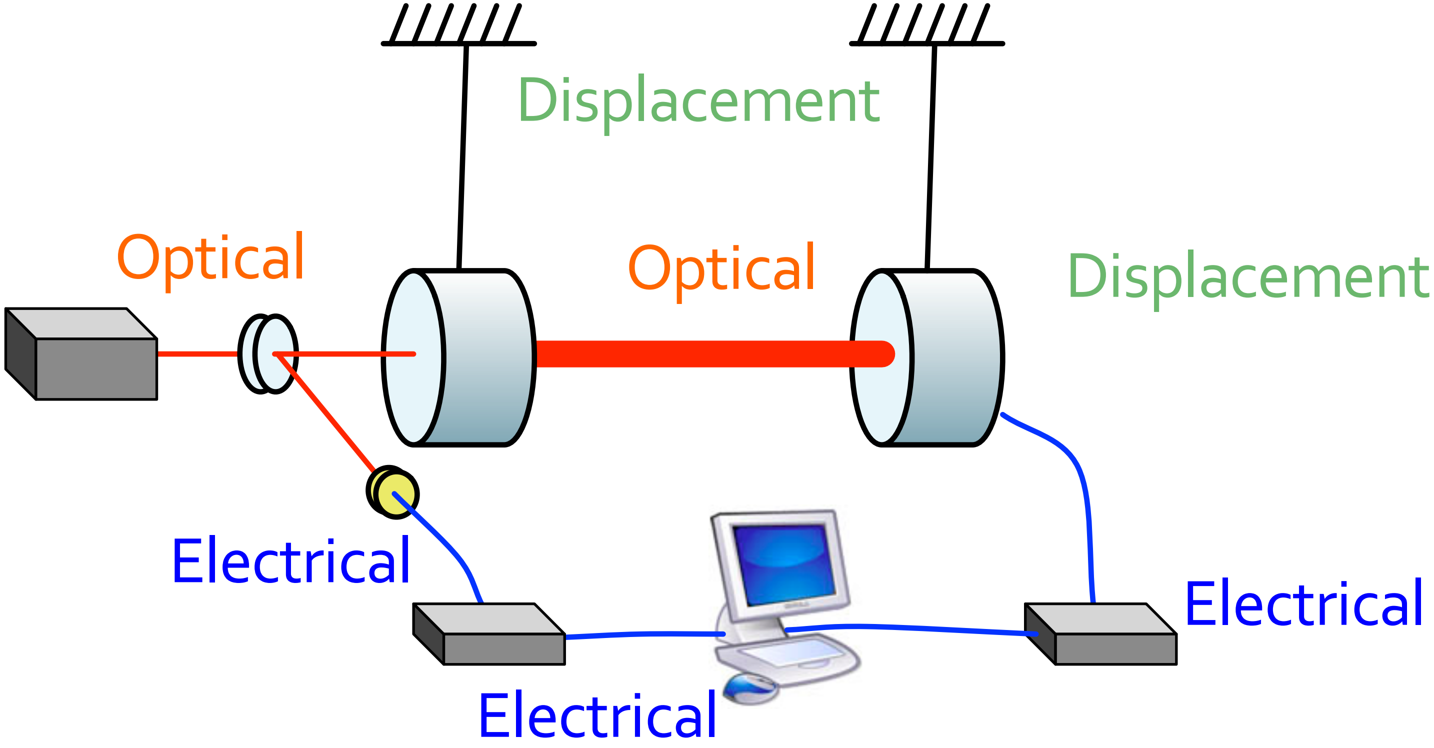
multiple pendulum suspension
monolithic suspensions
vibration isolation
high vacuum environment

Electronics

Mechanics

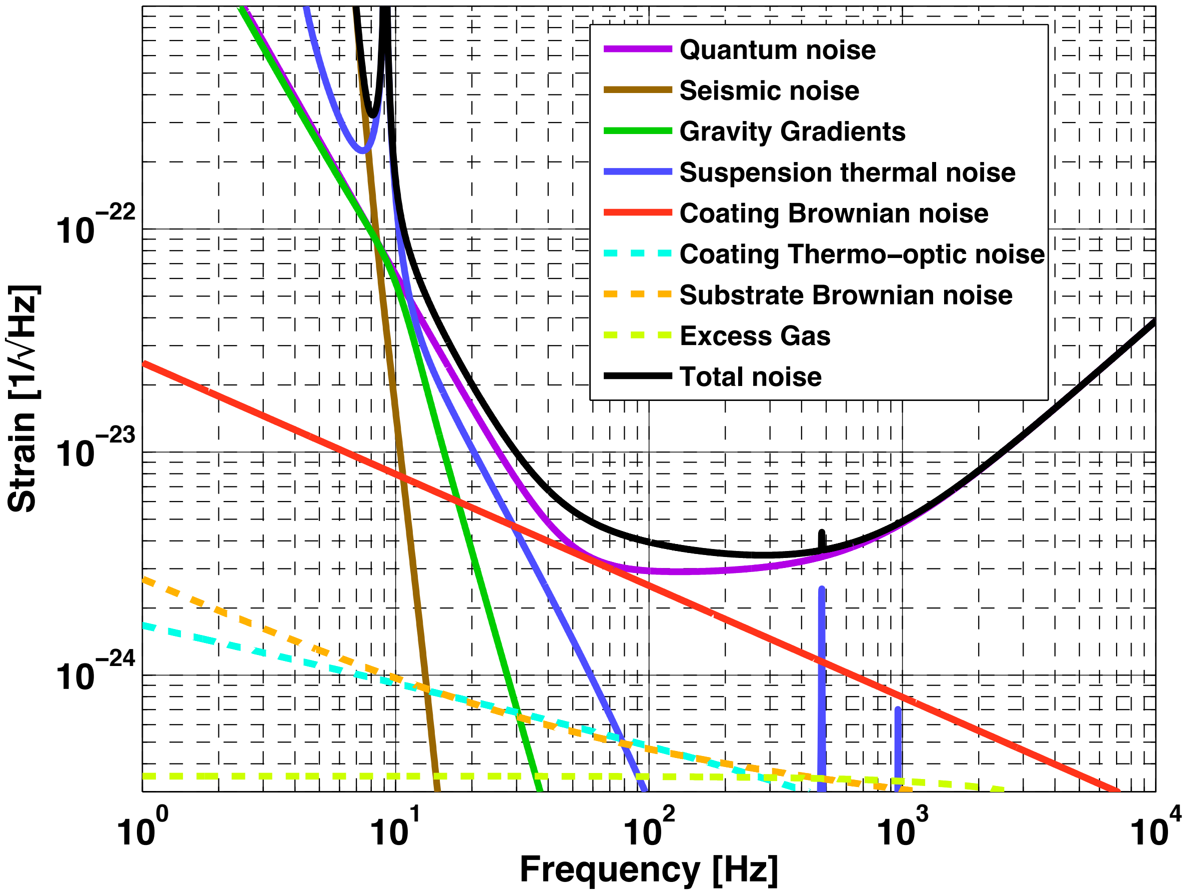
Noise categories

- 3 fundamentals of the GW detector
- Mechanics -> Displacement noises
- Optics -> Optical noises
- Electronics -> Electrical noises



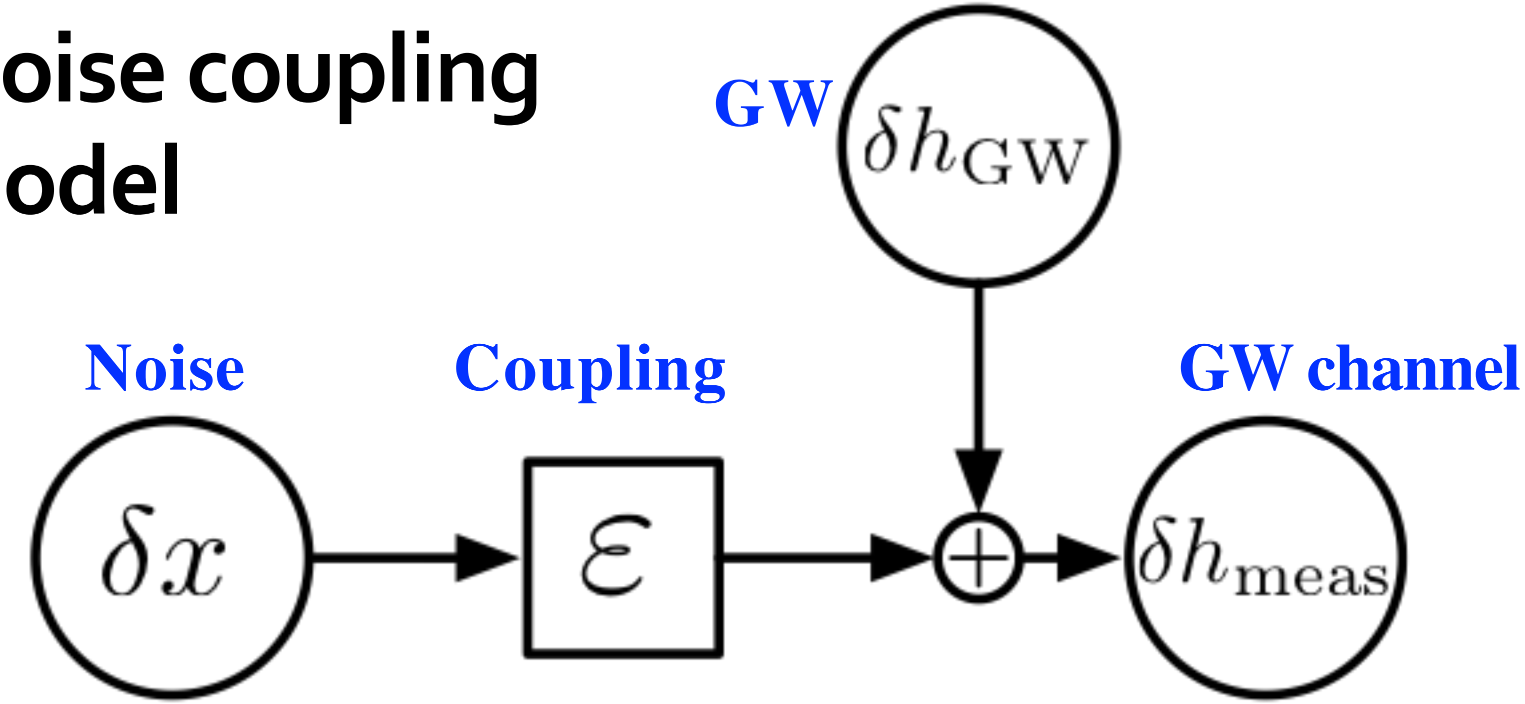
Sensitivity and noise

- Sensitivity (=noise level) of Advanced LIGO
- Design



Noise coupling to the GW channel

- Noise coupling model

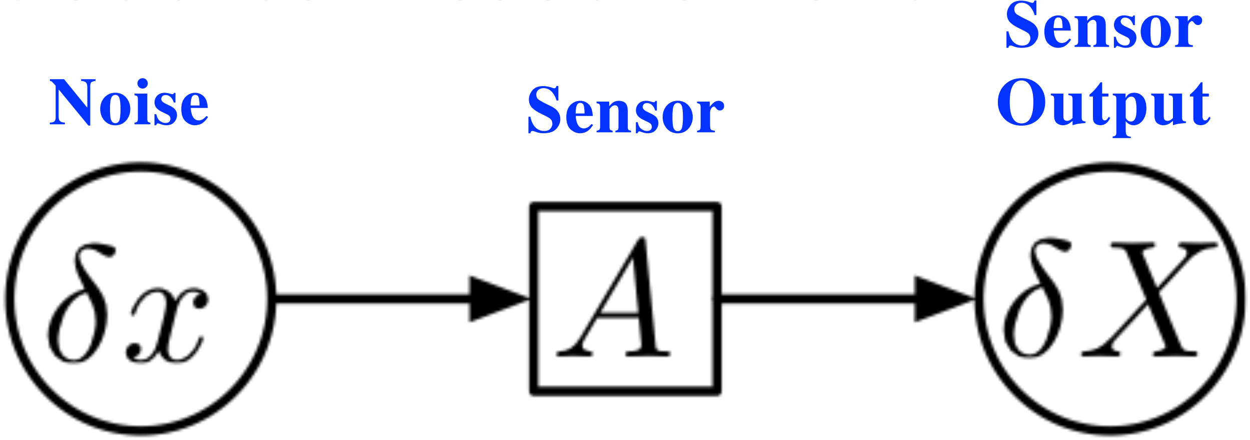


- Make the noise (dx) small
- Make the coupling (ε) small

- It is simple… isn't it?

Noise coupling to the GW channel

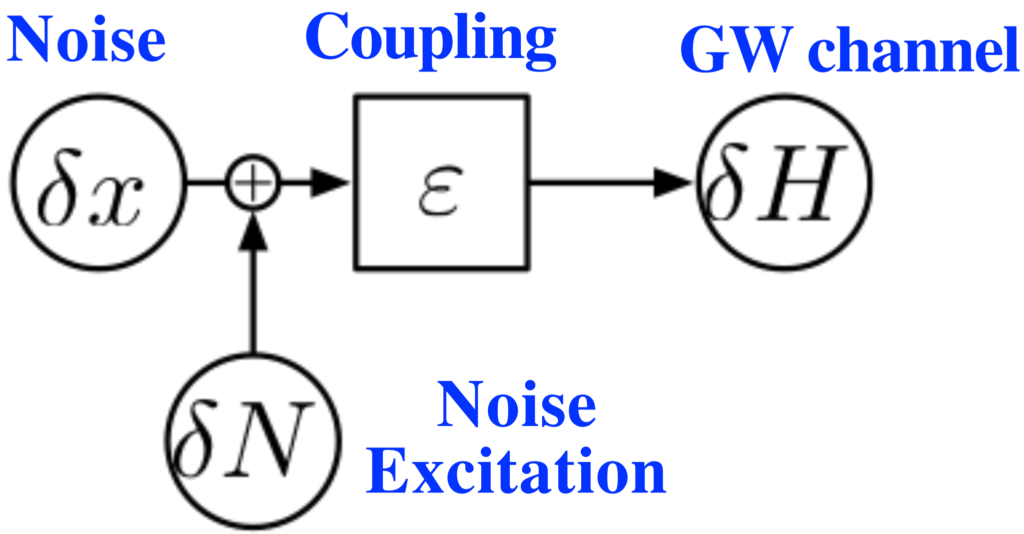
- Noise source measurement



- Use a sensor that is specifically sensitive to dx.
- This measurement itself is often difficult.

Noise coupling to the GW channel

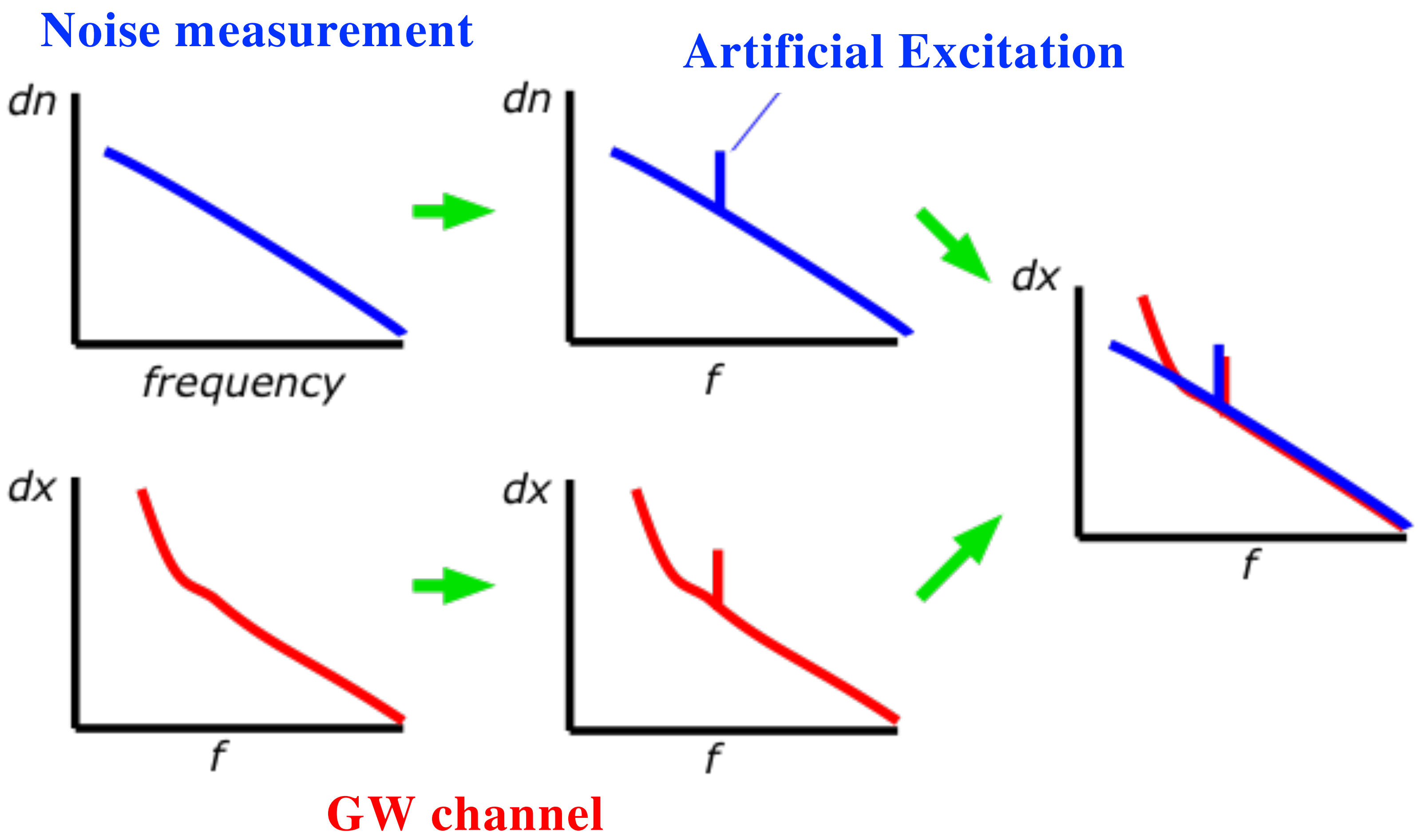
- Coupling measurement



- dH = ε dN.
- The system should be designed to allow the excitation.

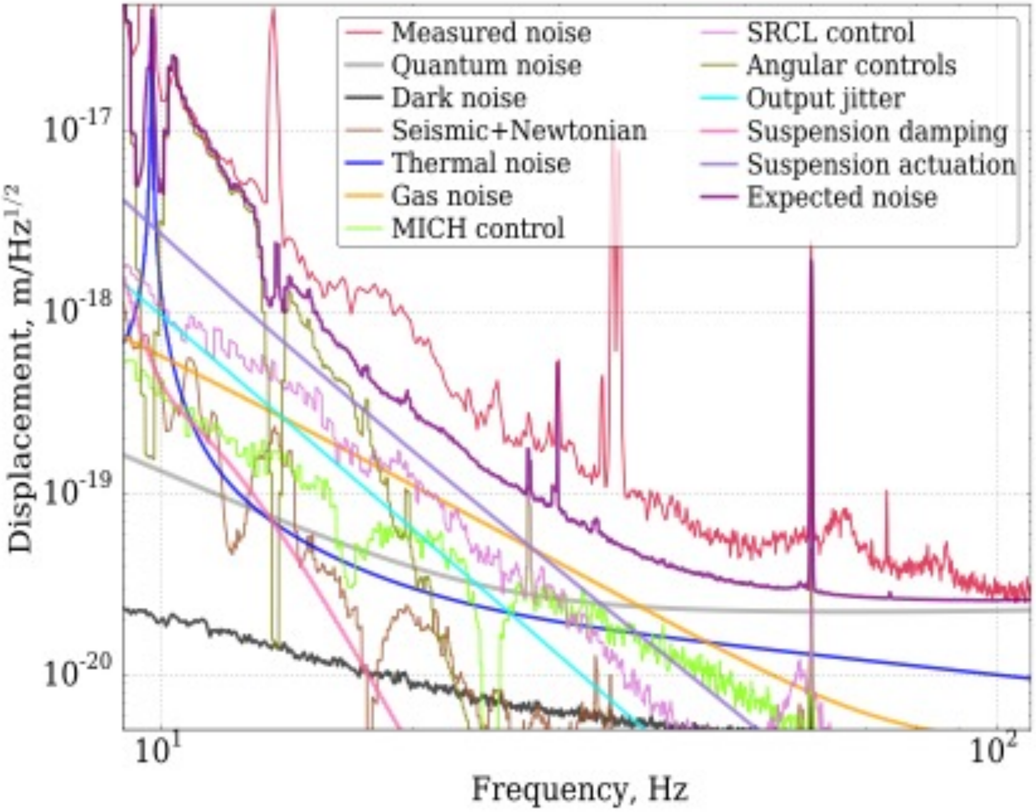
Noise coupling to the GW channel

- Noise budget

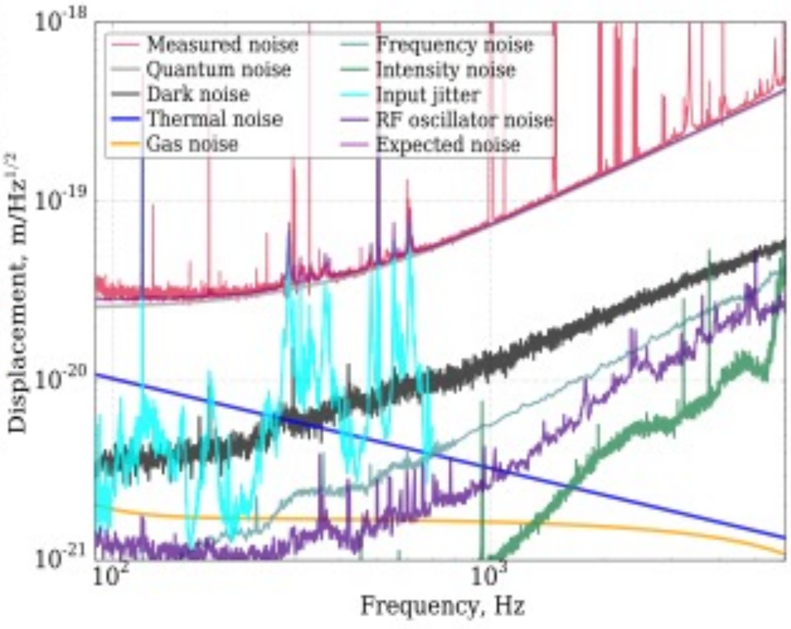


Sensitivity and noise

- Sensitivity (=noise level) of Advanced LIGO
- Noise budget



(a) LIGO Livingston Observatory

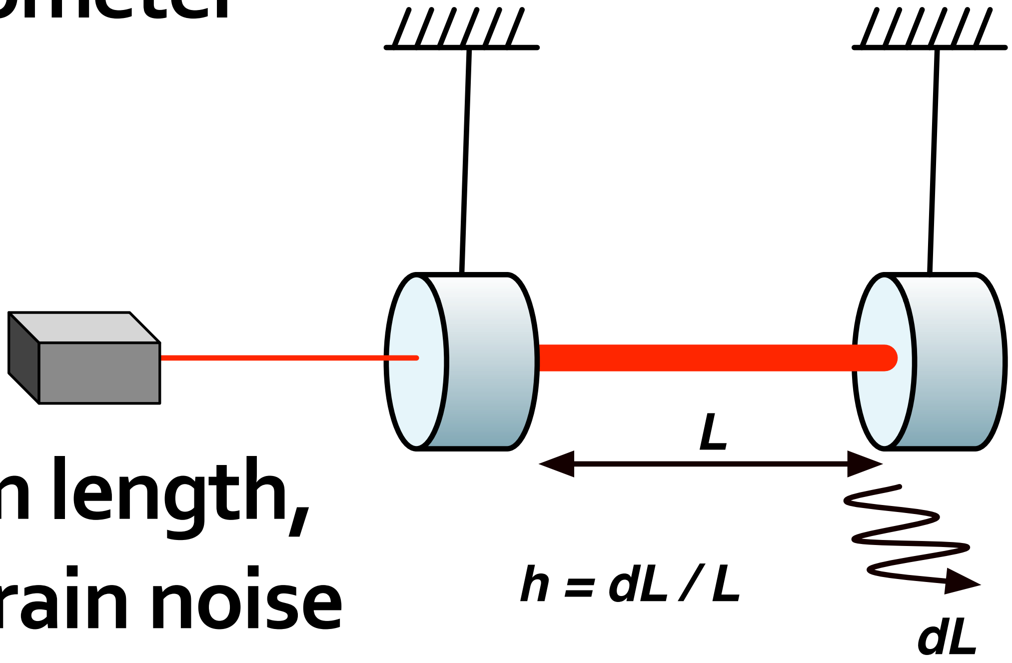


(b) LIGO Hanford Observatory

Displacement noises

Displacement noise

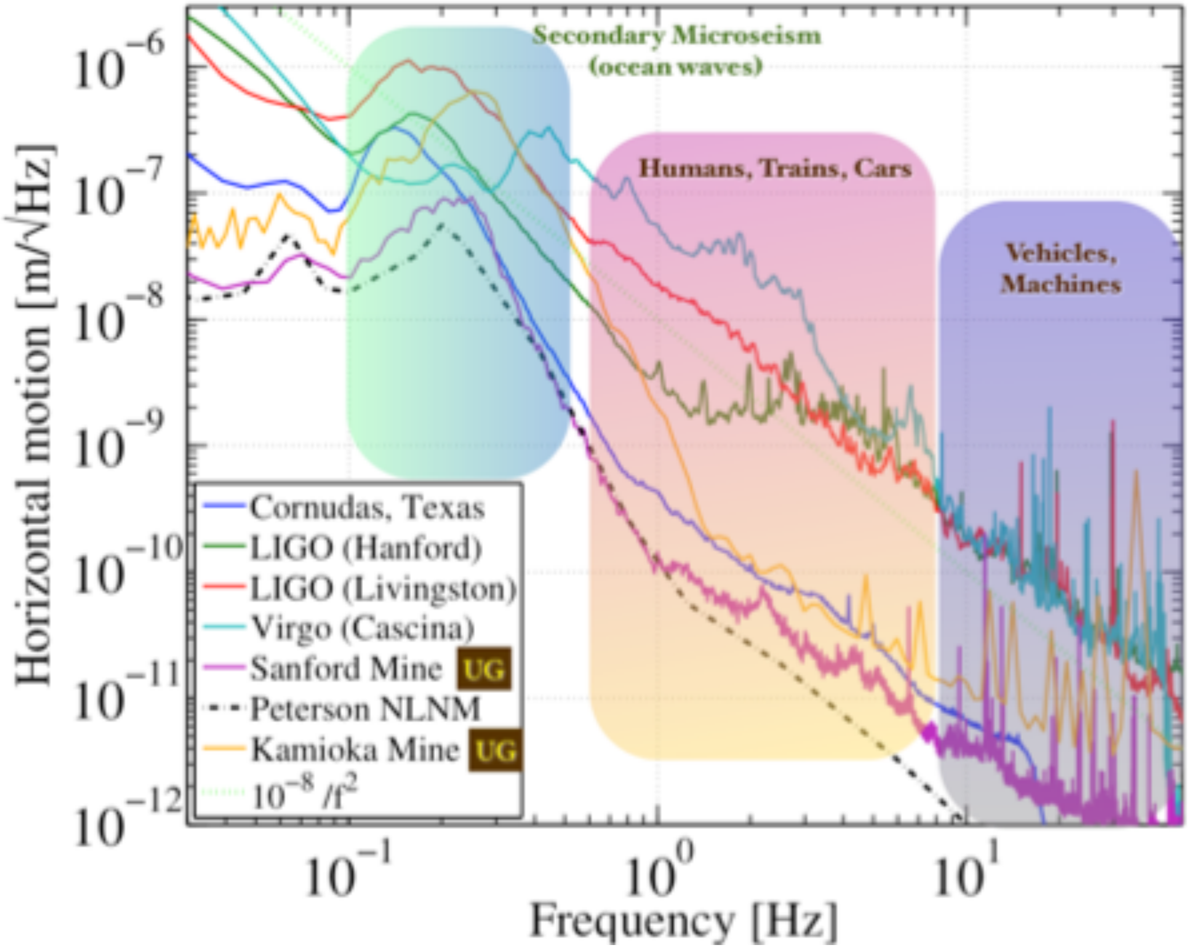
- Mechanical displacement sensed by a laser interferometer



- The longer the arm length, the smaller the strain noise
  - Seismic noise
  - Thermal noise
  - Newtonian Gravity noise

Displacement noise

- **Seismic noise**
  - Even when there is no noticeable earth quake…

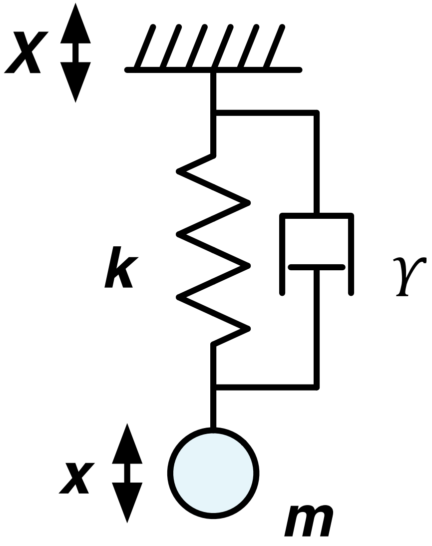


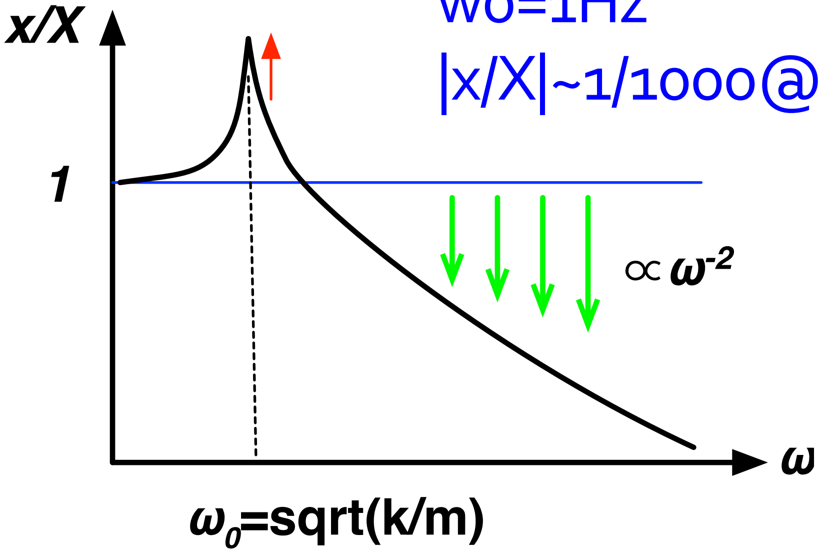
Target
disp. noise
10^{-20}m/rtHz

http://link.aps.org/doi/10.1103/RevModPhys.86.121 (http://arxiv.org/abs/1305.5188)

Displacement noise

- Vibration isolation ~ utilize a harmonic oscillator
  - A harmonic oscillator provides vibration isolation above its resonant frequency





wo=1Hz
|x/X|~1/1000@30Hz

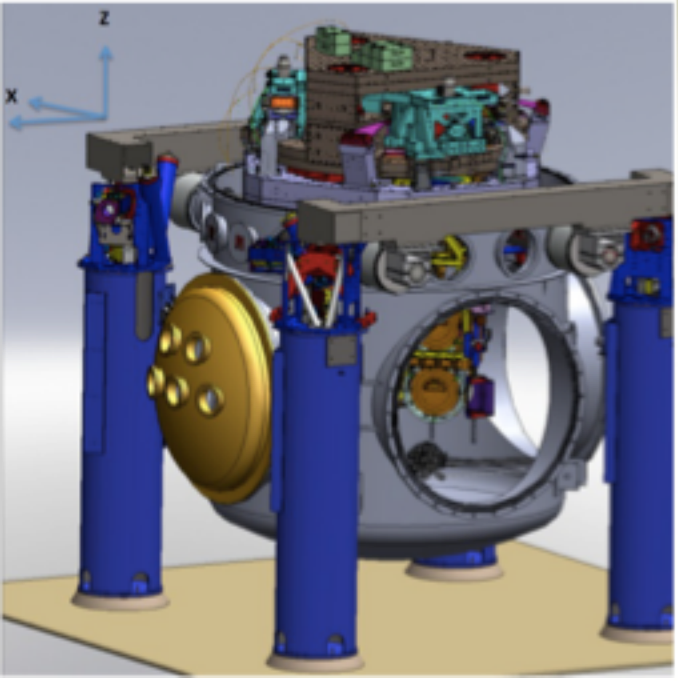
$$m\ddot{x} = -k(x - X) - \gamma(\dot{x} - \dot{X})$$

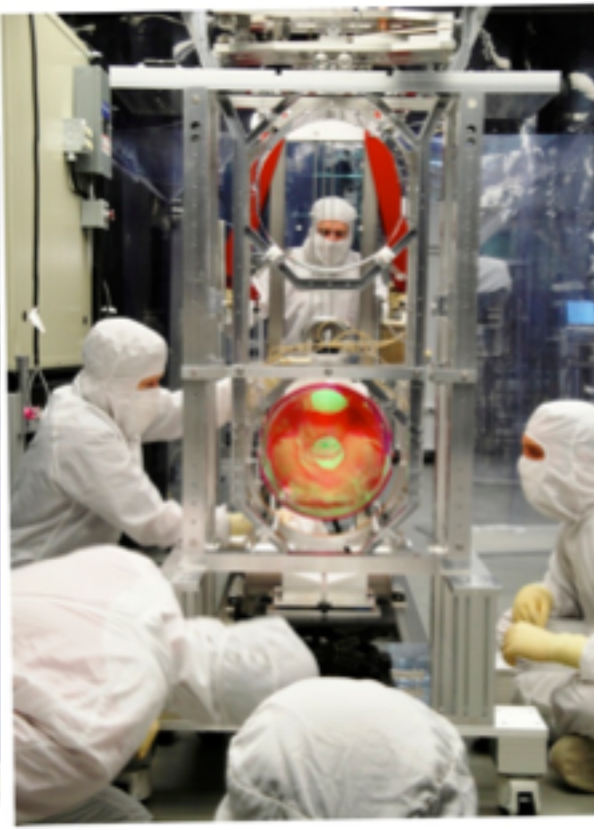
$$\left(\omega_0^2 + i\frac{\gamma}{m}\omega - \omega^2\right)\tilde{x} = \left(\omega_0^2 + i\frac{\gamma}{m}\omega\right)\tilde{X}$$

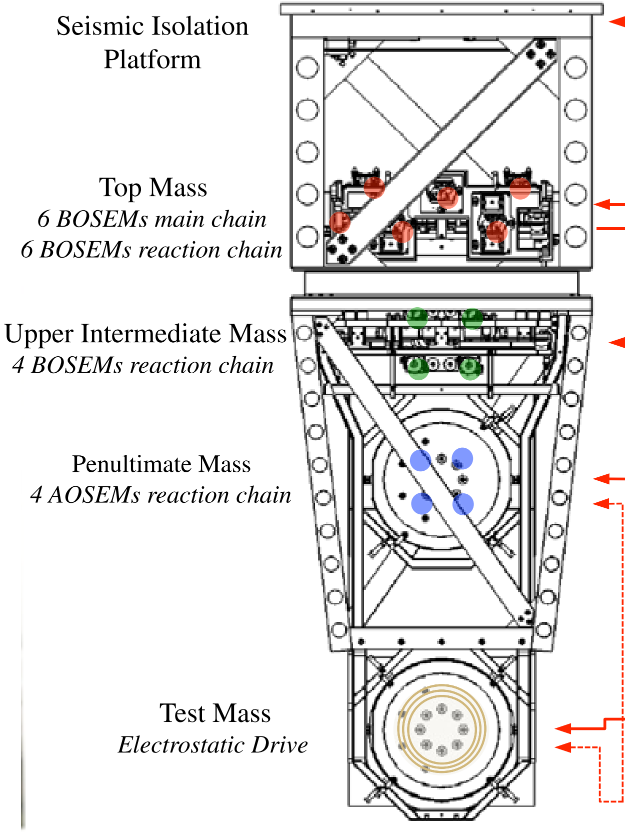
$$\frac{\tilde{x}}{\tilde{X}} = \frac{\omega_0^2 + i\frac{\gamma}{m}\omega}{\omega_0^2 + i\frac{\gamma}{m}\omega - \omega^2}$$

Displacement noise

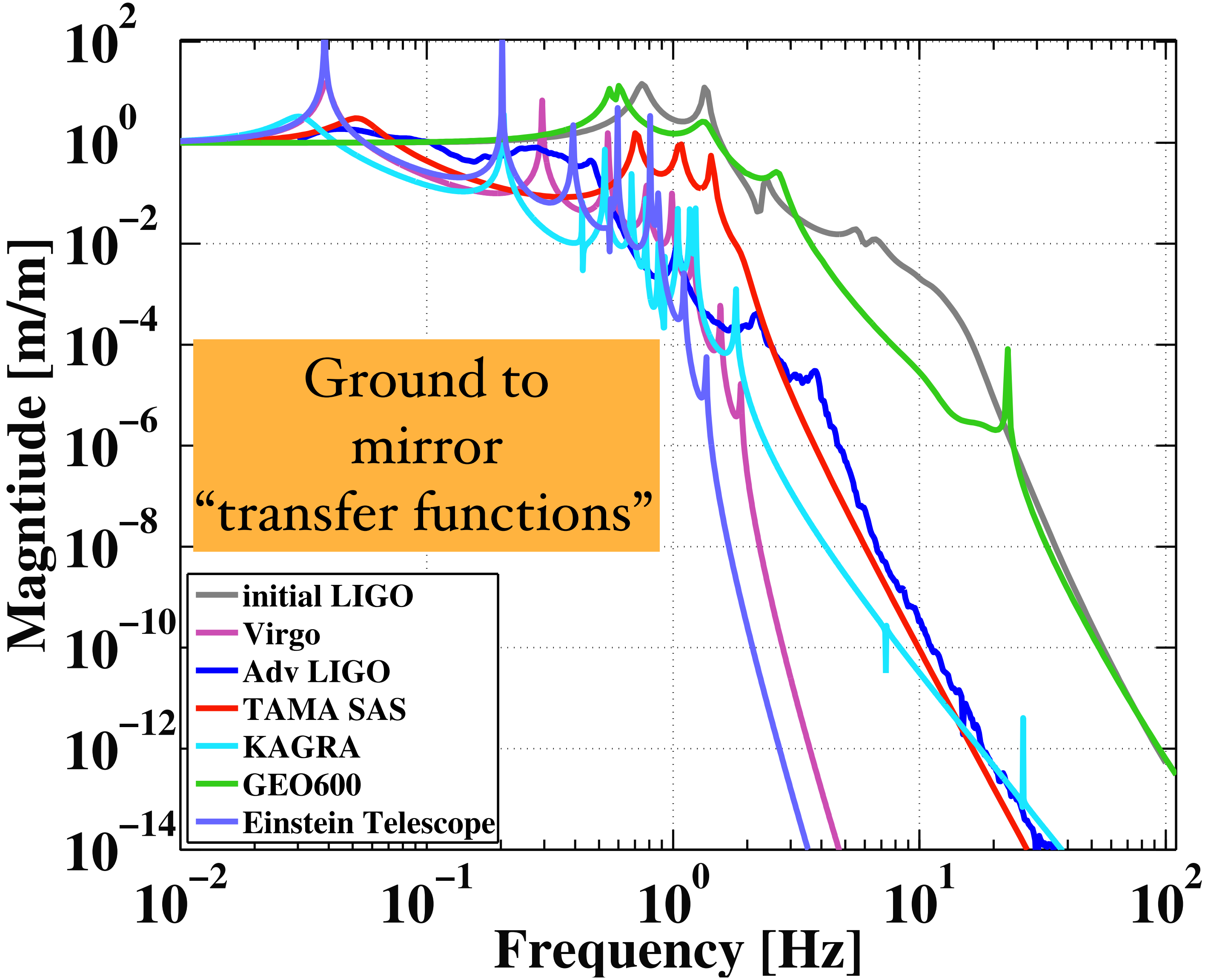
- aLIGO vibration isolation
- Hydraulic active isolation / Invacuum Active Isolation Platforms / Multiple Pendulum







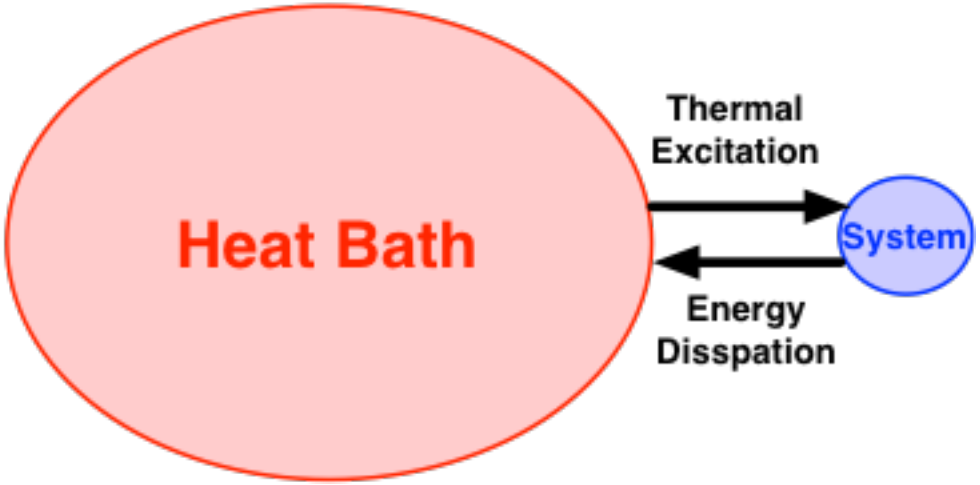
Displacement noise



http://link.aps.org/doi/10.1103/RevModPhys.86.121 (http://arxiv.org/abs/1305.5188)

Displacement noise

- Thermal noise:
- System in thermal equilibrium
  - the system can dissipate its energy to the heat bath
  - the system is thermally excited by thermal fluctuation



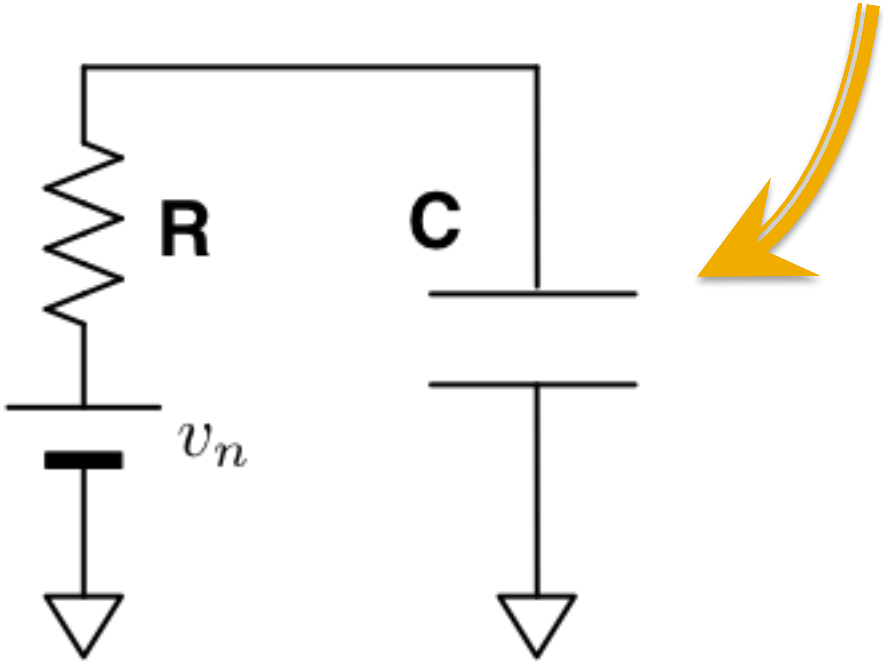
  - The level of thermal excitation can be evaluated using the energy dissipation of the system (e.g. friction, resistance) "Fluctuation Dissipation Theorem"

Displacement noise

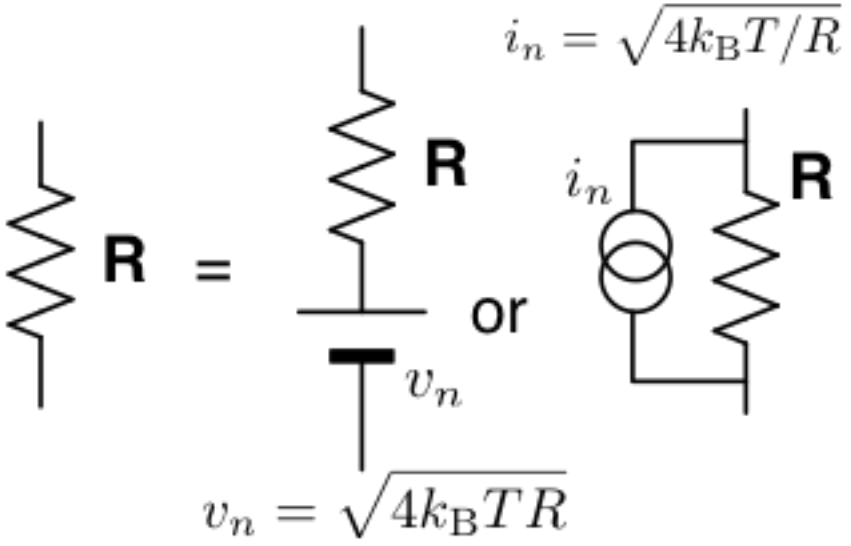
- **Equipartition Theorem**

In thermal equilibrium, each d.o.f. has the energy of

$$\langle E \rangle = \frac{1}{2} k_{\mathrm{B}} T$$



Johnson noise



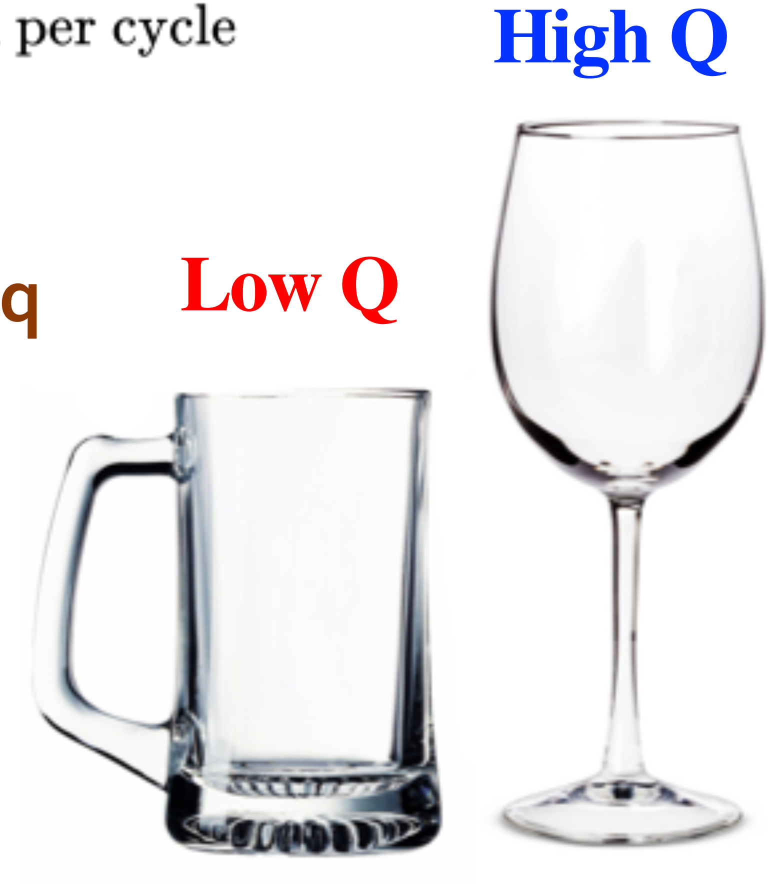
Displacement noise

- Mechanical resonance
- Quality factor Q

$$Q \overset{\text{def}}{=} 2\pi \times \frac{\text{energy stored}}{\text{energy dissipated per cycle}}$$

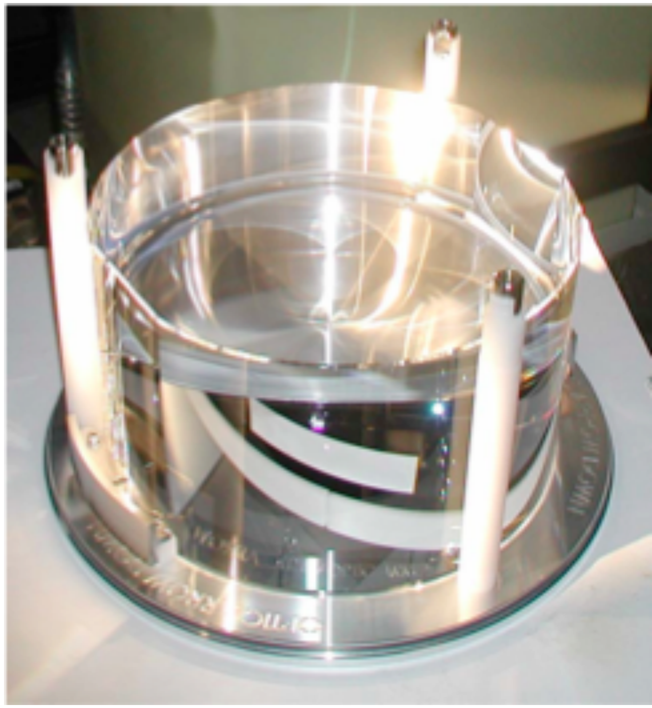
- High Q
  => most of the kBT/2 energy contained in the resonant freq

  => Lower thermal motion at off-resonance



Displacement noise

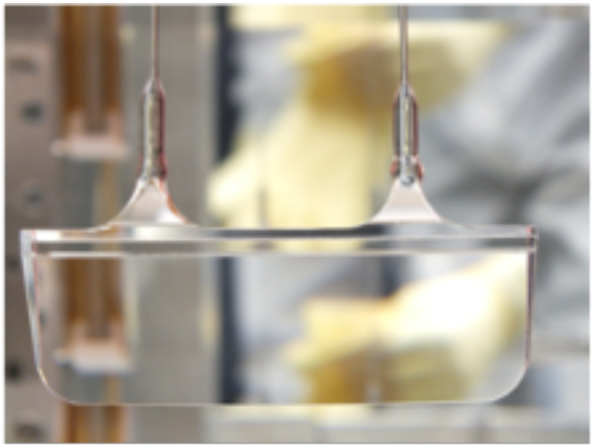
- **Mirror substrate thermal noise**
  - Brownian motion
    Mechanical loss associated
    with the internal friction
    ⬄Thermally excited body modes
    Optical coating (higher mech. loss)
    will be limiting noise source in aLIGO



  - Thermo elastic noise
    Elastic strain & thermal expansion coefficient
    => cause heat distribution & flow in the substrate
    ⬄Temperature fluctuation causes mirror displacement
  - Thermo-refractive noise
    ⬄Temp. fluctuation causes fluctuation of refractive index

Displacement noise

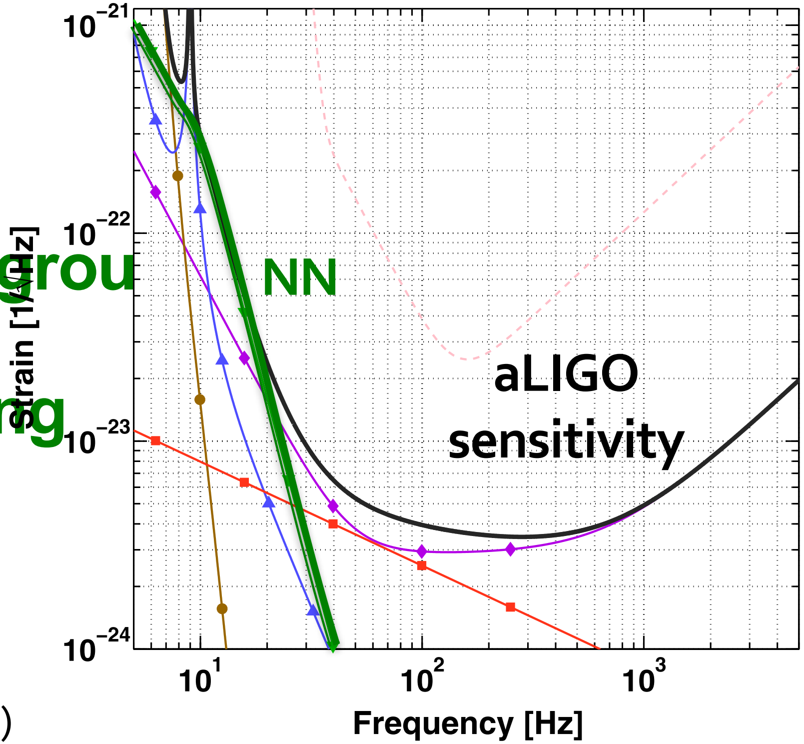
- **Suspension thermal noise**
  - Brownian motion
    Mechanical loss of the suspension fiber
    ⇔Thermally excited pendulum modes
  - Thermo elastic noise
    Elastic strain of the fiber & thermal expansion coefficient
    => cause heat distribution & flow in the fiber
    ⇔Temperature fluctuation causes mirror motion



<- Monolithic suspension for high pendulum Q

Displacement noise

- **Newtonian Gravity noise**
  - Mass density fluctuations around the test masses
    => test mass motion via gravitational coupling
  - Dominant source of Newtonian noise
    = Seismic surface wave
  - **Mitigation**
    1) Going to quiet place (undergrou
    2) Feedforward subtraction
    3) Passive reduction by shaping local topography

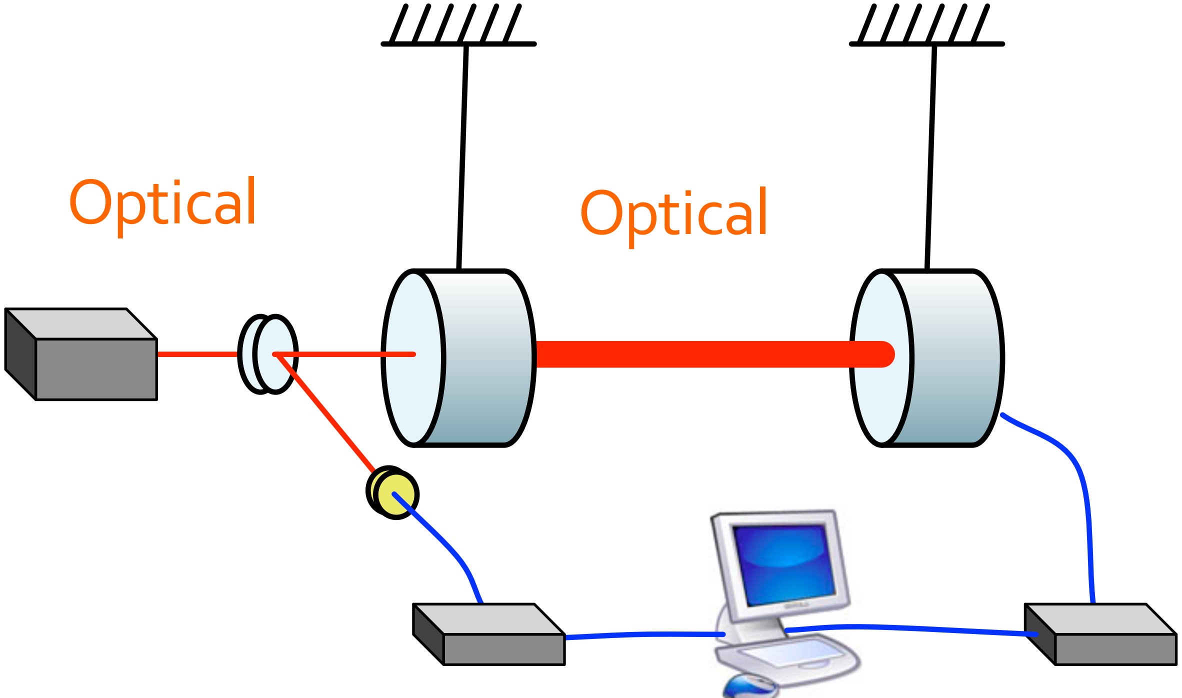


J Driggers, et al, PRD 86, 102001 (2012)
J Harms, et al, Class. Quantum Grav. 31 185011 (2014)

Optical noises

Optical noises

- Noises that contaminate the readout signal
  - Quantum noises (shot noise, radiation pressure noise)
  - Laser technical noises (frequency/intensity noise)
  - Modulation noises
  - Scattered light noise



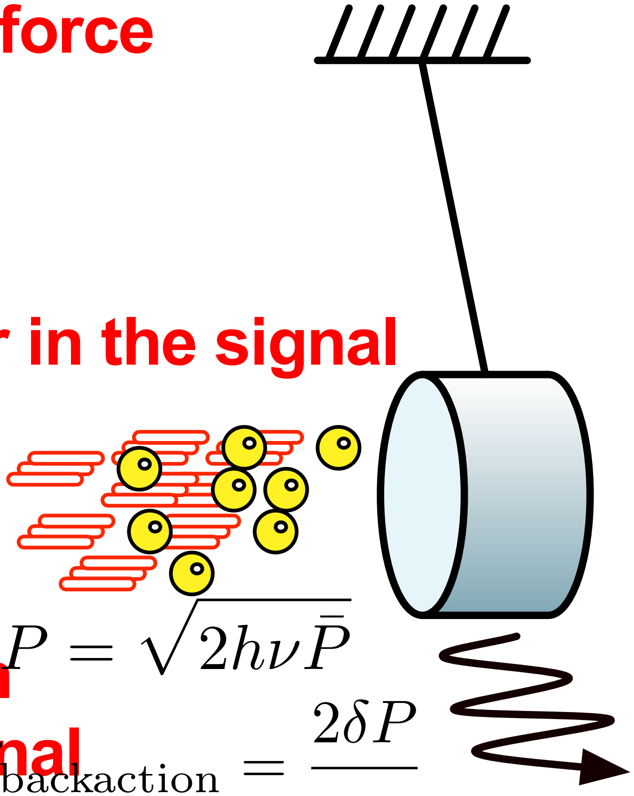
Optical noises

- Quantum noises: Shot noise
  - Noise due to photon counting statistics
  - N detected photon => standard deviation $\sqrt{N}$
  - Increasing the incident power P_{in},
    => The shot noise is increased by $\sqrt{P_{in}}$
    => The signal amplitude is increased by P_{in}
  - In total, the signal-to-noise ratio is improved by

$$\mathrm{SNR} \propto \sqrt{P_{\mathrm{in}}}$$

Optical noises

- **Quantum noises ~ Radiation pressure noise**
  - **Photon number fluctuation in the arm cavity**
    => Fluctuation of the back action force
  - **Quantum noise of the input laser**
    => Common noise for two arms
    => cancelled and does not appear in the signal
  - **Vacuum fluctuation injected from the dark port**
    => Differentially power fluctuation
    => Cause the noise in the GW signal



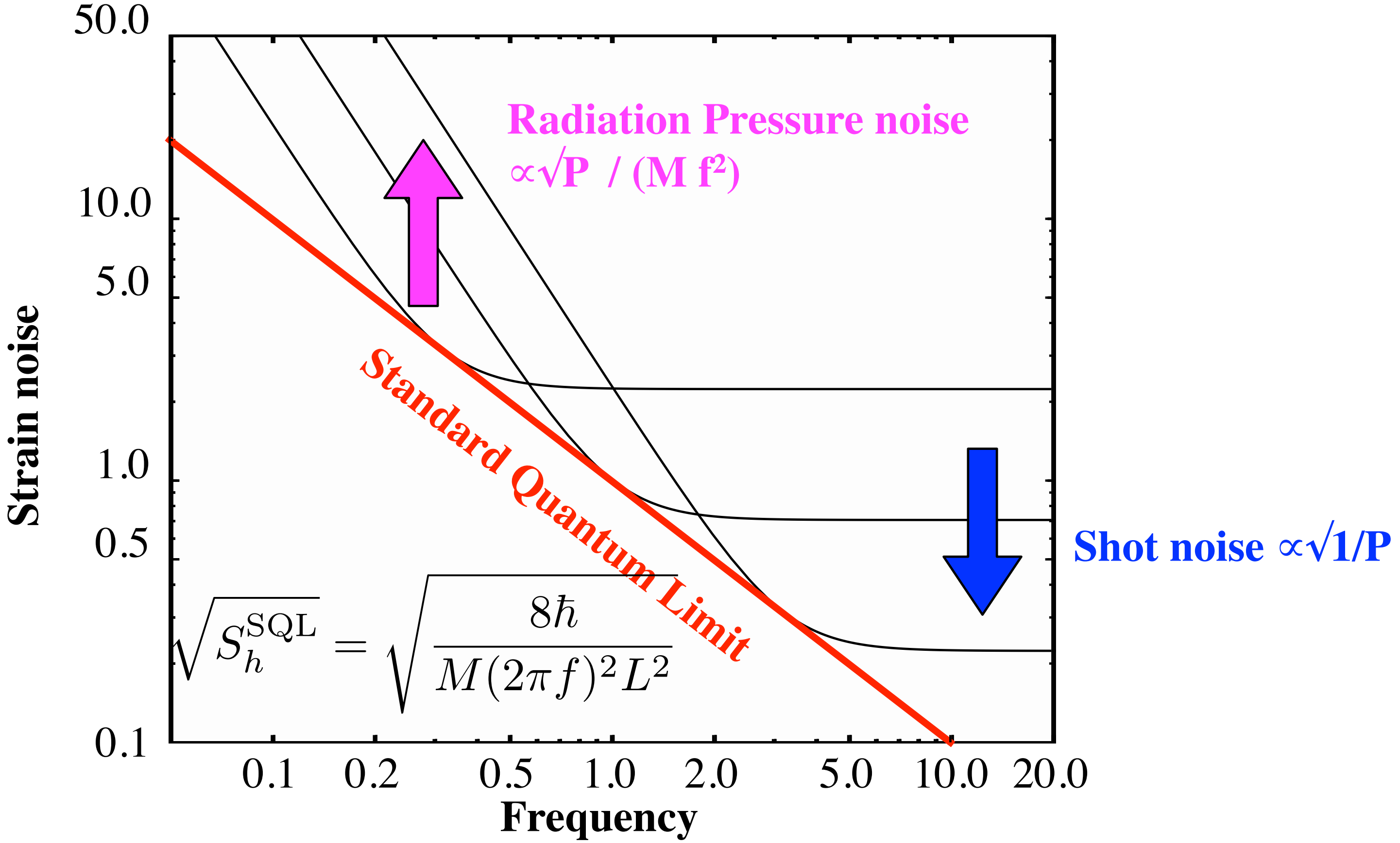
$$\delta P = \sqrt{2h\nu\bar{P}}$$

$$f_{\mathrm{backaction}} = \frac{2\delta P}{c}$$

$$\tilde{x} = \frac{f_{\mathrm{backaction}}}{M\omega^2}$$

Optical noises

- **Quantum noises**
  - **Standard Quantum Limit (SQL)**



$$\sqrt{S_h^{\mathrm{SQL}}} = \sqrt{\frac{8\hbar}{M(2\pi f)^2 L^2}}$$

- Trade-off Between Shot Noise and Radiation-Pressure Noise
- Uncertainty of the test mass position due to observation

Optical noises

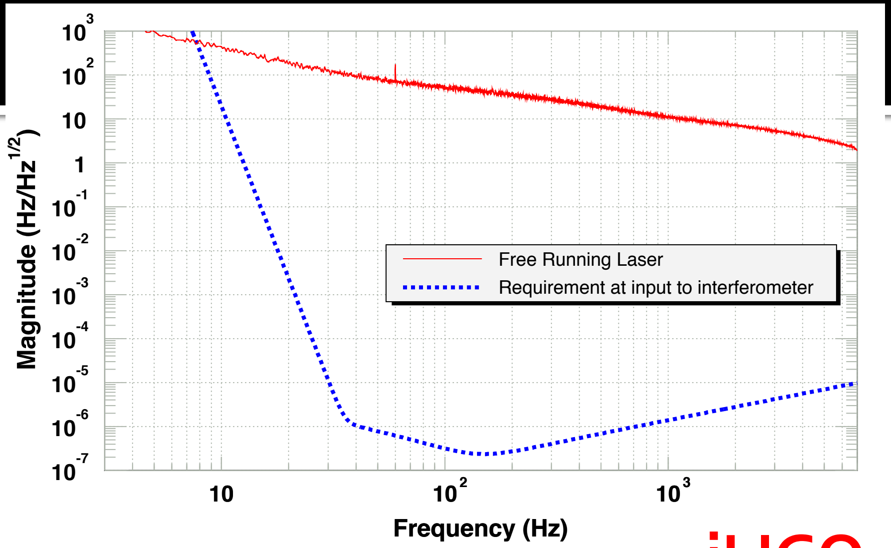
- Laser frequency noise
  - Laser wavelength ($\lambda = c / v$)
    = reference for the displacement measurement
  - Optical phase $\phi = 2 \pi v L / c$
    $d\phi = 2 \pi / c (L\, dv + v\, dL)$ <= indistinguishable

$$\boxed{\frac{dL}{L} = \frac{d\nu}{\nu}}$$

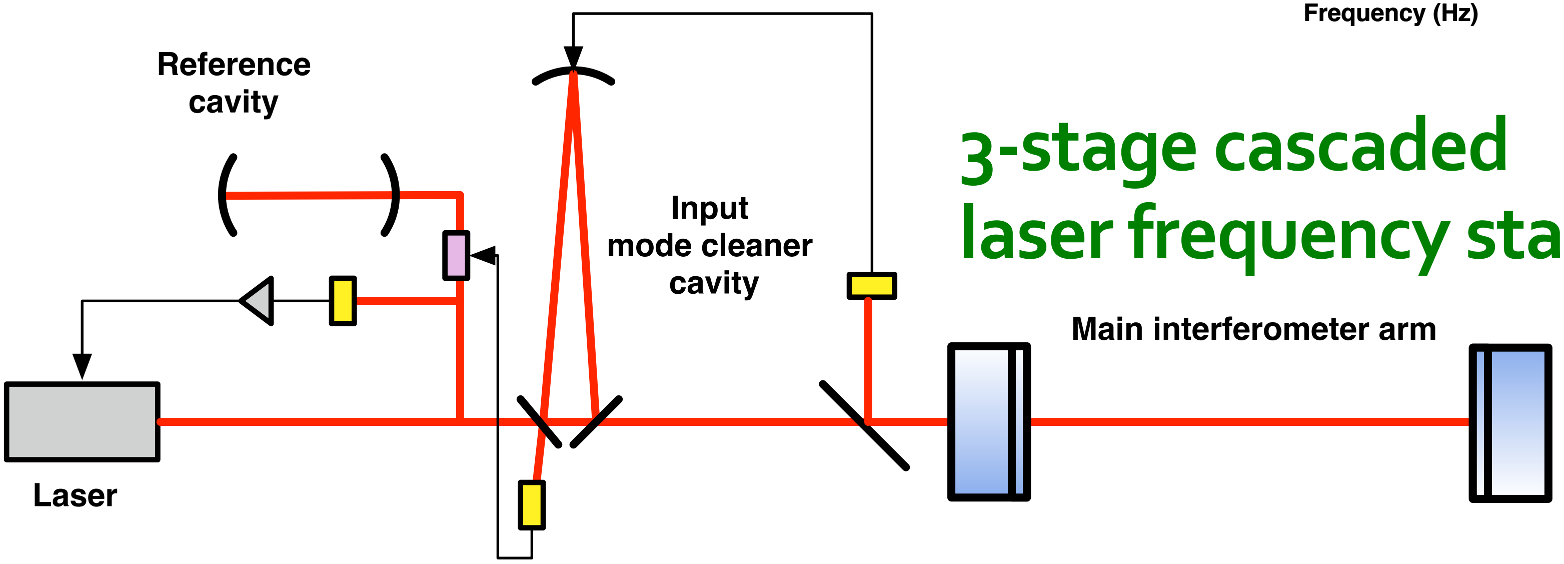
  - dL/L target 10^{-24}
    => $dv = 10^{-24} \times 300$ THz (1064nmYAG laser)
    $= 3 \times 10^{-10}$ Hz/rtHz

Optical noises

- **Laser frequency noise**
  - Target: $dv_{eff} = 3 \times 10^{-10}$ Hz/rtHz
  - Laser stability
    dv = 10~100 Hz/rtHz @100Hz



iLIGO



3-stage cascaded
laser frequency stabilizatio

Michelson's differential sensitivity provides
Frequency noise cancellation of 1/100~1/1000
"Common Mode Rejection"

Optical noises

- **Laser intensity noise**
  - **Relative Intensity Noise (RIN): dP/P**
  - **Sensor output V = P x**
    => dV = P dx + x dP <= indistinguishable

$$\boxed{\frac{dx}{x_{\mathrm{offset}}} = \frac{dP}{P}}$$

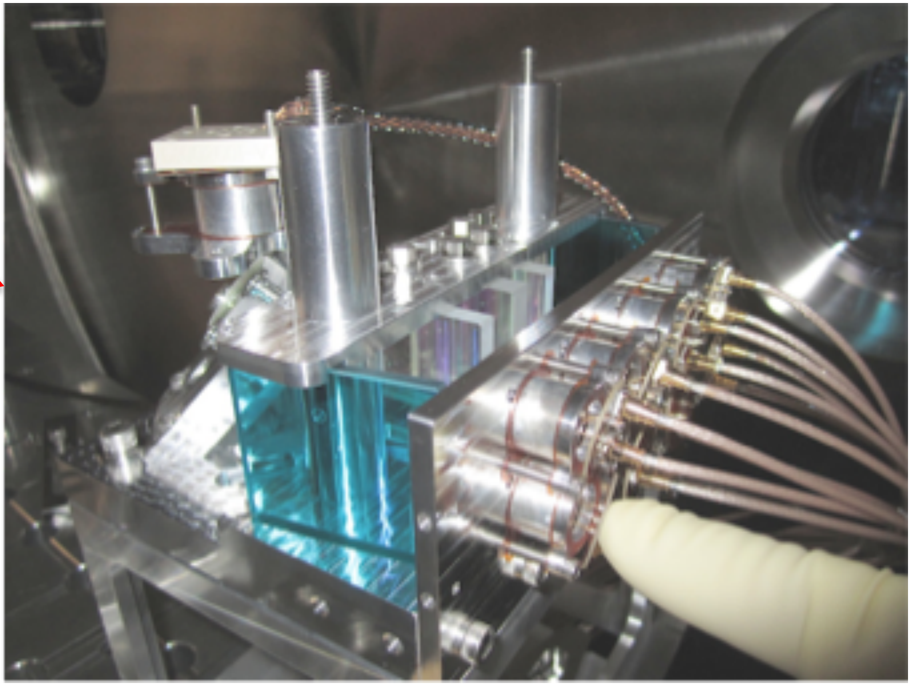
  - **Requirement: RIN = 10^{-9} 1/√Hz**
    x_{ofs}=10e-12 (DC Readout)
    => dx=1e-20 m/√Hz

Optical noises

- **Laser intensity noise ~ intensity stabiliaztion**
  - **Requirement: RIN = 10^{-9} 1/√Hz**
  - **2-stage cascaded intensity stabilization control**
  - **Challenge: requires 300mA of photodetection**

Shot noise limited RIN $$\frac{i_{\rm shot}}{i_{\rm DC}} = \frac{\sqrt{2ei_{\rm DC}}}{i_{\rm DC}} = \sqrt{2e/i_{\rm DC}}$$

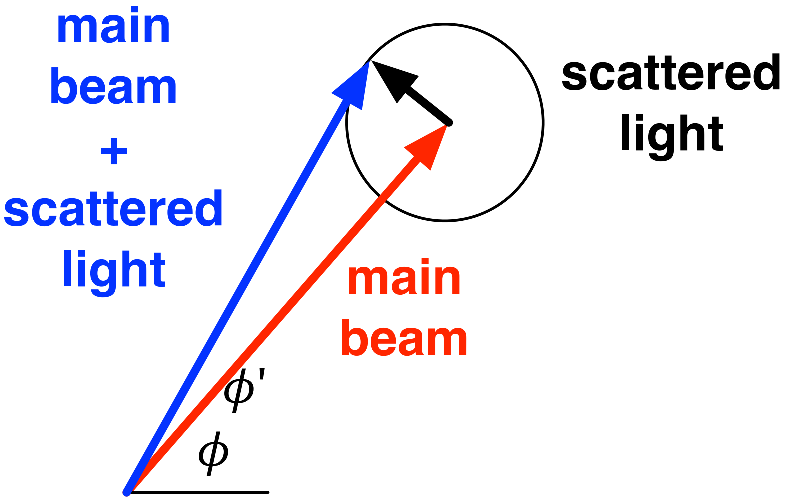
  - **In-vacuum 8-branch Photodiode array**



P. Kwee et al,
Optics Express **20** 10617-10634 (2012)

Optical noises

- **Scattered light noise**
  - Scattered light recouples to the interferometer beam with an arbitrary phase
    => causes amplitude and phase fluctuation
  - Two effects:
    1. Small motion regime: linear coupling of the phase fluctuation
    2. Large motion regime: low freq large motion of the scattering object
    => upconversion via fringe wrapping
  - Mitigation
    - Reduce scattered light
    - Vibration isolation of the scattering object



Electrical noises

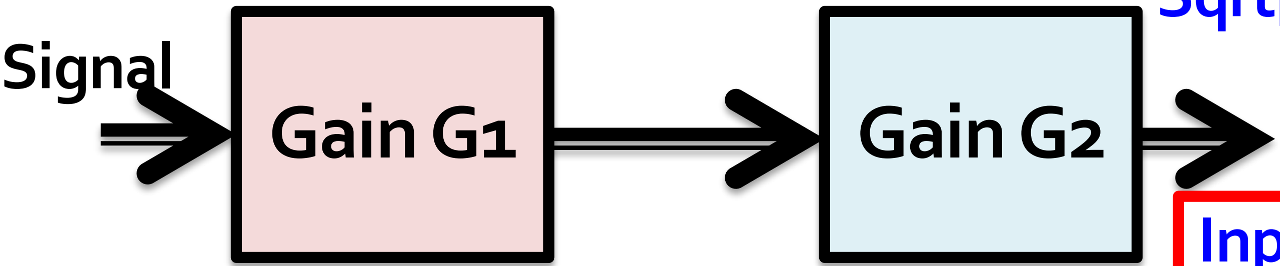
General rules for electrical noises

- **Low noise amplification at the beginning**
- **Give necessary gain as early as possible**
- **Don’t attenuate (and amplify again)**

input noise: v1

input noise: v2

output noise Vout:
$\mathrm{Sqrt}[(v1\ G1\ G2)^2+(V2\ G2)^2]$



output noise: v1 G1

Input equivalent noise
Vout / (G1 G2)
$= \mathrm{Sqrt}[v1^2+(v2/G1)^2]$

- Lessons
  - The input referred noise is determined by v1
  - It won’t become better by the later stages
  - If G1 is big enough, we can ignore the noise of later stages

Digitization (Quantization) noise

- **Analog signals (~+/-10V) -> Digital signal**
  - **Digitized to a discrete N bit integer number**

quantization

quantization error

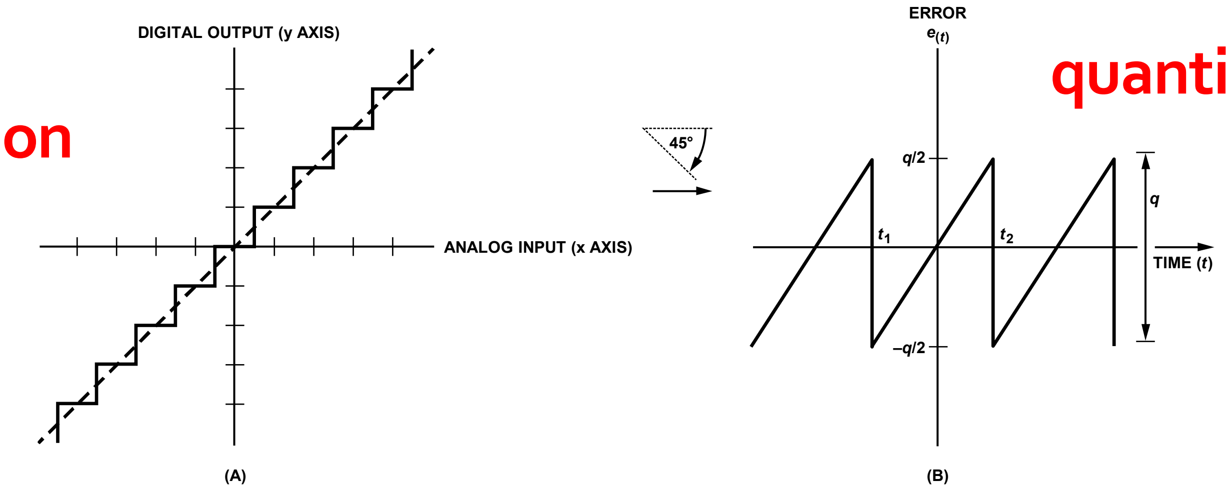


Figure 1. Ideal ADC Transfer Function (A) and Ideal N-Bit ADC Quantized Noise (B)

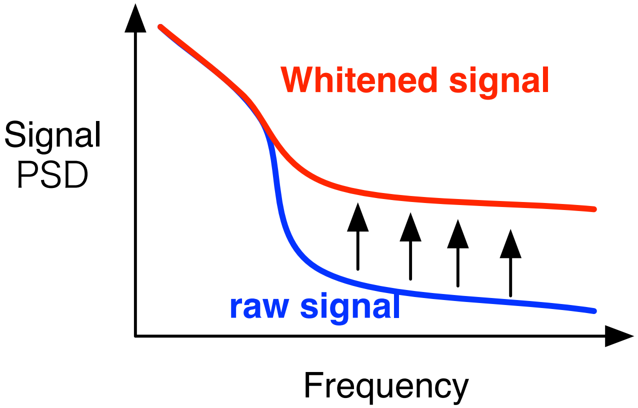
http://www.analog.com/static/imported-files/tutorials/MT-229.pdf

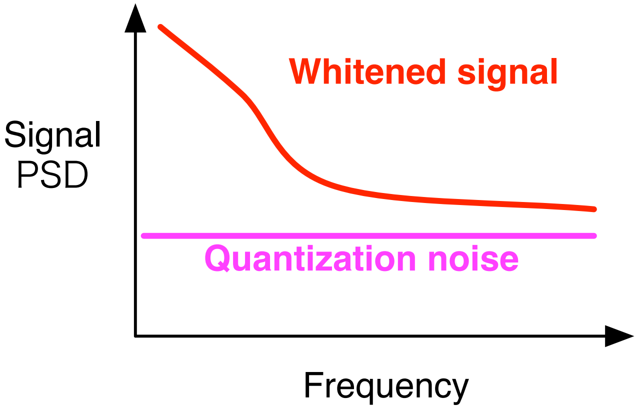
$$V_n = \frac{\Delta}{\sqrt{12}} \quad [\mathrm{V}/\sqrt{\mathrm{Hz}}]$$

- **Quantization causes a white noise**
  e.g. +/-10V 16bit => Δ = 0.3mV => Vn ~ 100 μV/sqrtHz
  cf. Input noise of a typical analog circuit 10nV/sqrtHz

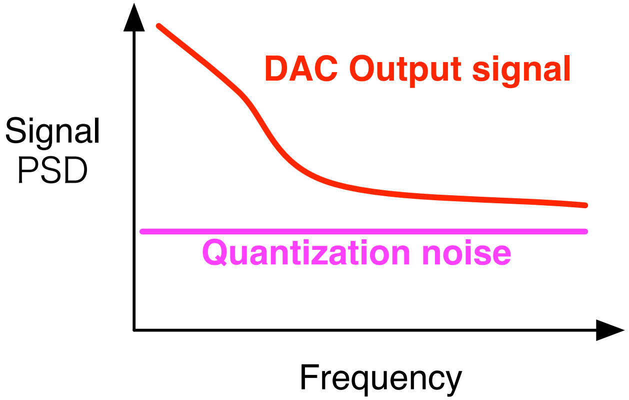
Digitization (Quantization) noise

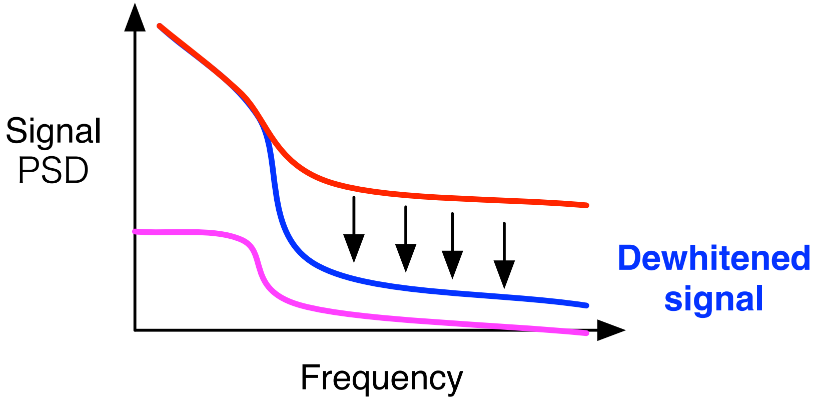
- **Whitening**
  - Amplify a signal in the freq band where the signal is weak





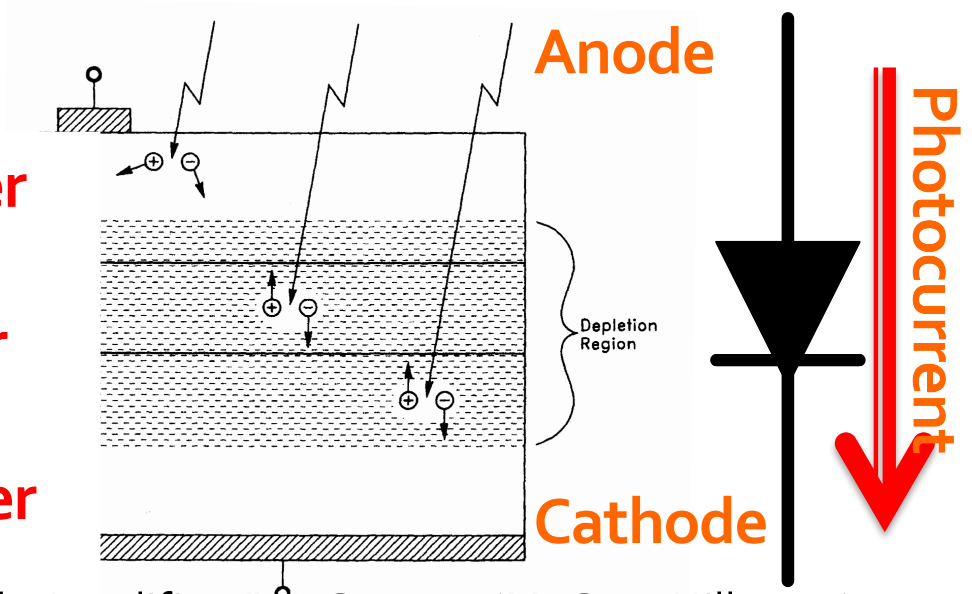
- **Dewhitening**
  - Amplify a signal in the freq band where the signal is weak





Noise in photodetectors

- Photodiodes
  - PIN photodiodes (InGaAs for near IR, Si for visible)
    - Good linearity
    - Low noise
    - High Quantum Efficiency (>90%)



P-layer

I-layer

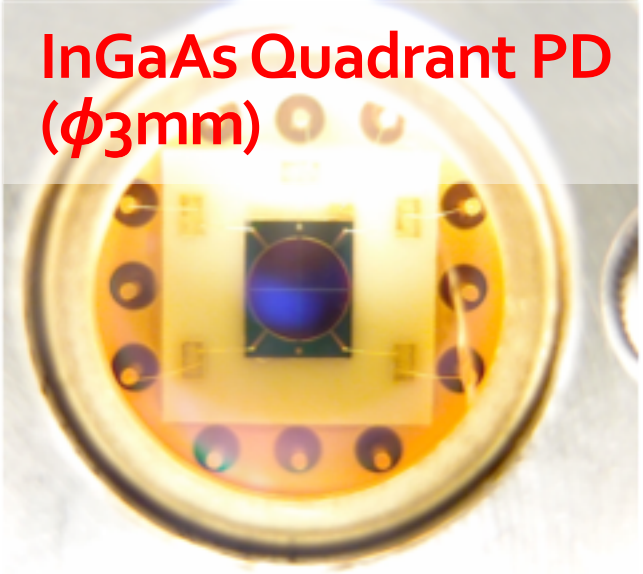
N-layer

Anode

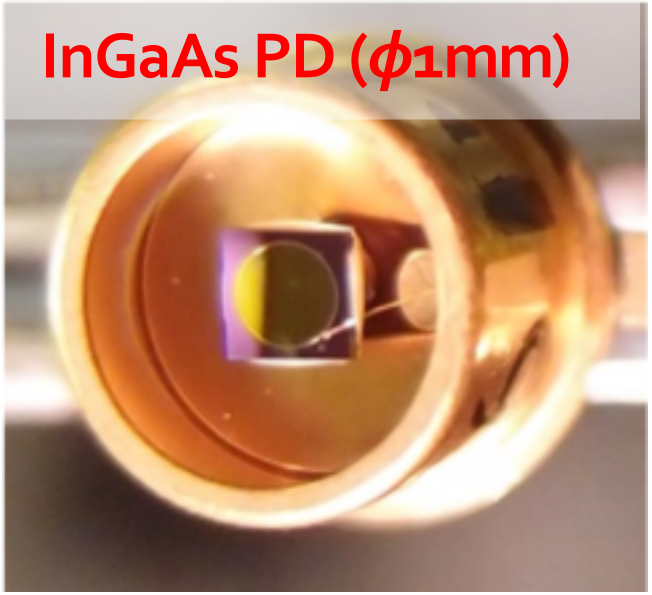
Cathode

Photocurrent

“Photodiode Amplifiers”, J. Graeme (McGrawHill 1995)



InGaAs Quadrant PD (ϕ3mm)



InGaAs PD (ϕ1mm)

Noise in photodetectors

- Noise in photodiodes
  - Photodiode equivalent circuit
    - Shunt Capacitance R_D (~100MΩ) Usually not a problem
    - Junction Capacitance C_D (1pF~1nF)
    - Series Resistance R_S (1Ω~100Ω)

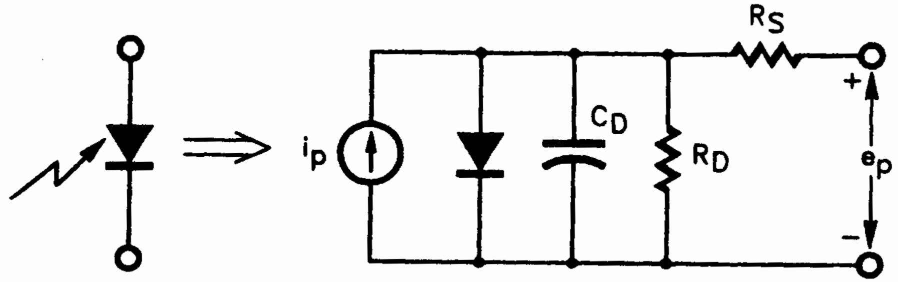


Figure 1.3 The circuit model of a photodiode consists of a signal current, an ideal diode, a junction capacitance, and parasitic series and shunt resistances.

input referred noise current

$$i_{Rs} \sim \omega C_d \sqrt{4 k_\mathrm{B} T R_s}$$

The diode aperture size needs to be ~mm => Cd tends to be big.

2mm InGaAs PD: Rs~10Ω, Cd~100pF

=> i_Rs = 20 pA/sqrtHz @100MHz

(equivalent to the shot noise of 1mA light ~ 1.3mW@1064nm)

Summary

Summary

- Summary
  - There are such large number of noises
  - They are quite omnidisciplinary
  - Only one untamed noise can ruin GW detection
  - GW detection will be achieved by
    - Careful design / knowledge / experience
    - Logical, but inspirational trouble shooting
  - Noise “hunting”
    - Systematic approach: noise budget