LIGO-G1801277-v1

Gravitational wave detection with laser interferometers

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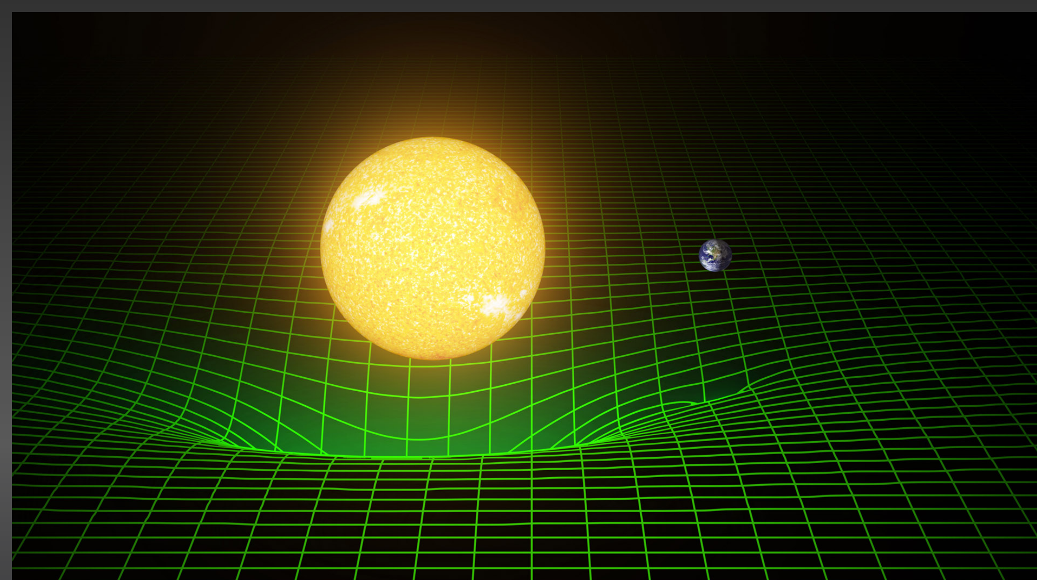
Gravity = Spacetime curvature

Albert Einstein: General Relativity

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

spacetime curvature

energy density (e.g. mass)



Credit: Caltech/MIT/LIGO Lab

Gravitational waves

Small perturbation of the spacetime

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

spacetime metric | flat spacetime | perturbation

$$\Rightarrow \quad \Box h_{\mu\nu} = 0$$

wave equation

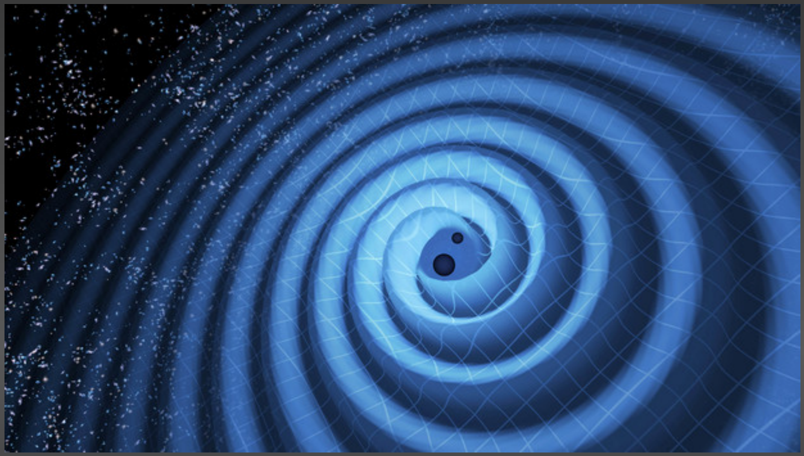
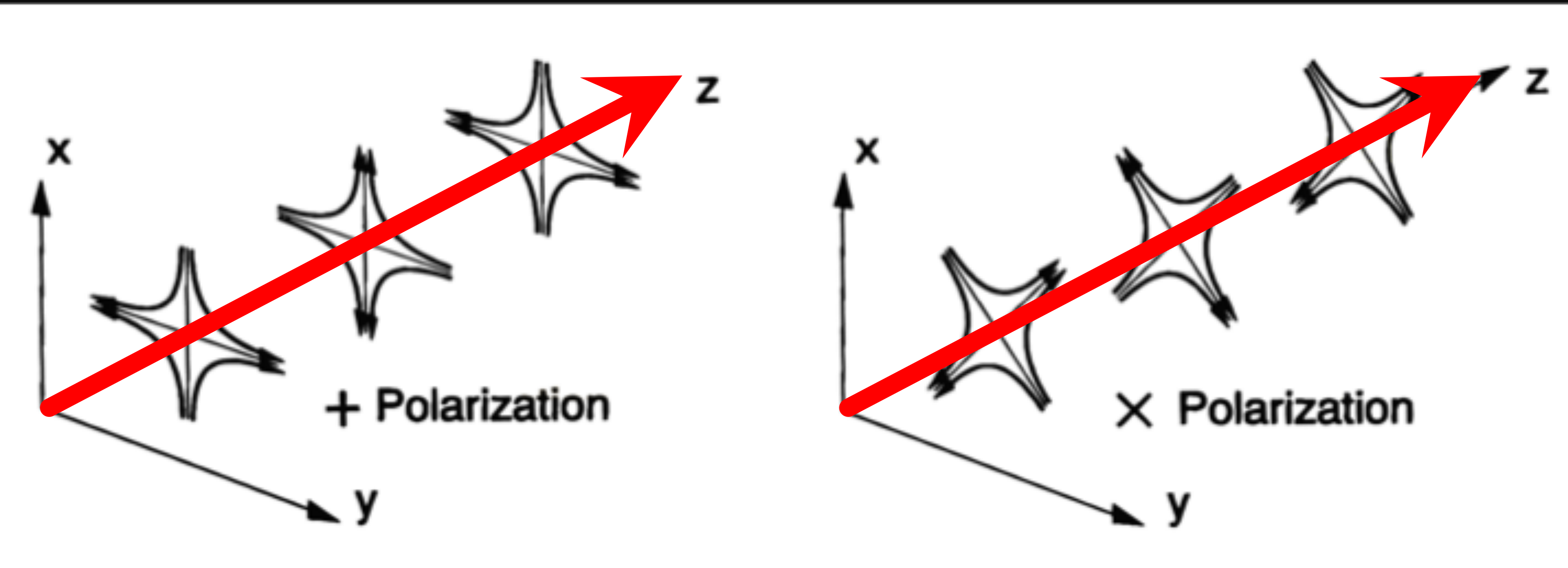


Image credit: LIGO/T. Pyle

Ripples in specetime

Two modes of gravitational waves



Quadrupole strain pattern

GW Detection

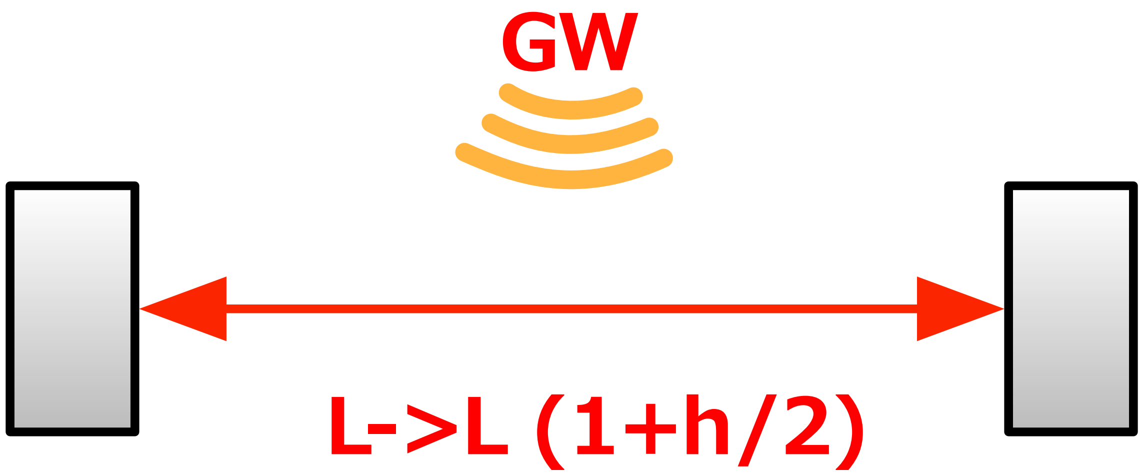
- Two free-falling masses

- Free fall? Free mass?

=> Equivance principle

GW Detection

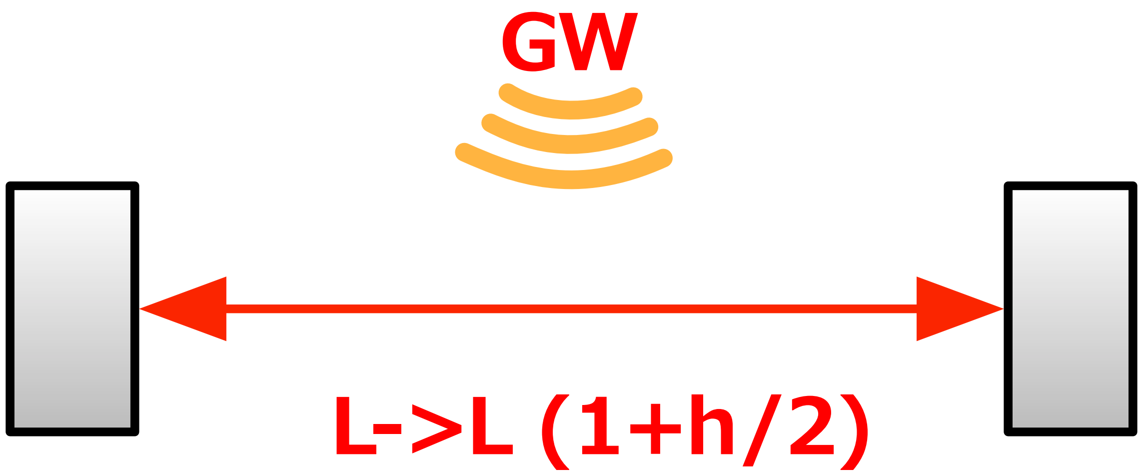
- Optically measure strain between free masses



  - GWs do not appear in the local motions
  - The effect appears in the optical distance between the masses

GW Detection

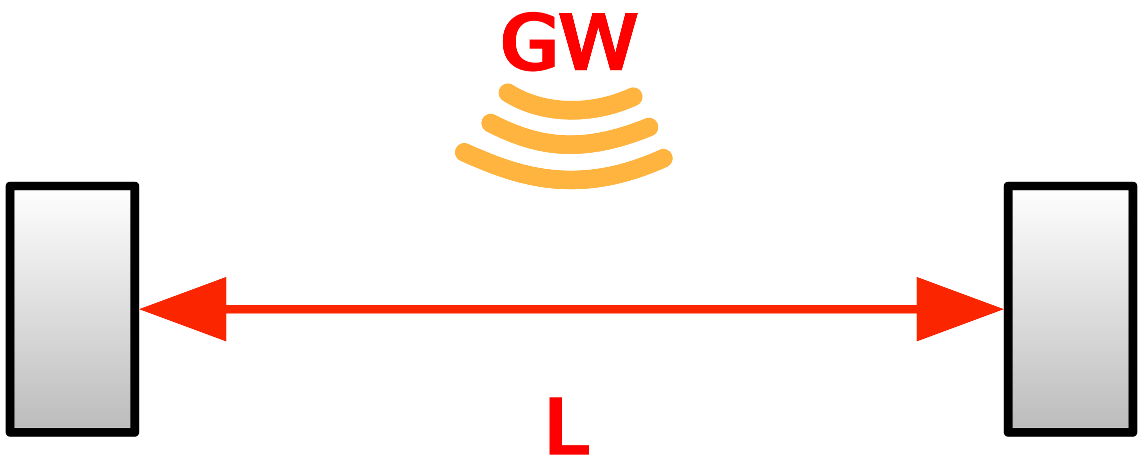
- Measure strain between free masses



  - GW does not appear in the local motion
  - Changes optical distance between the masses

- Longer the baseline, the bigger change
  - (displacement dx) = (Strain h) x (baseline L)

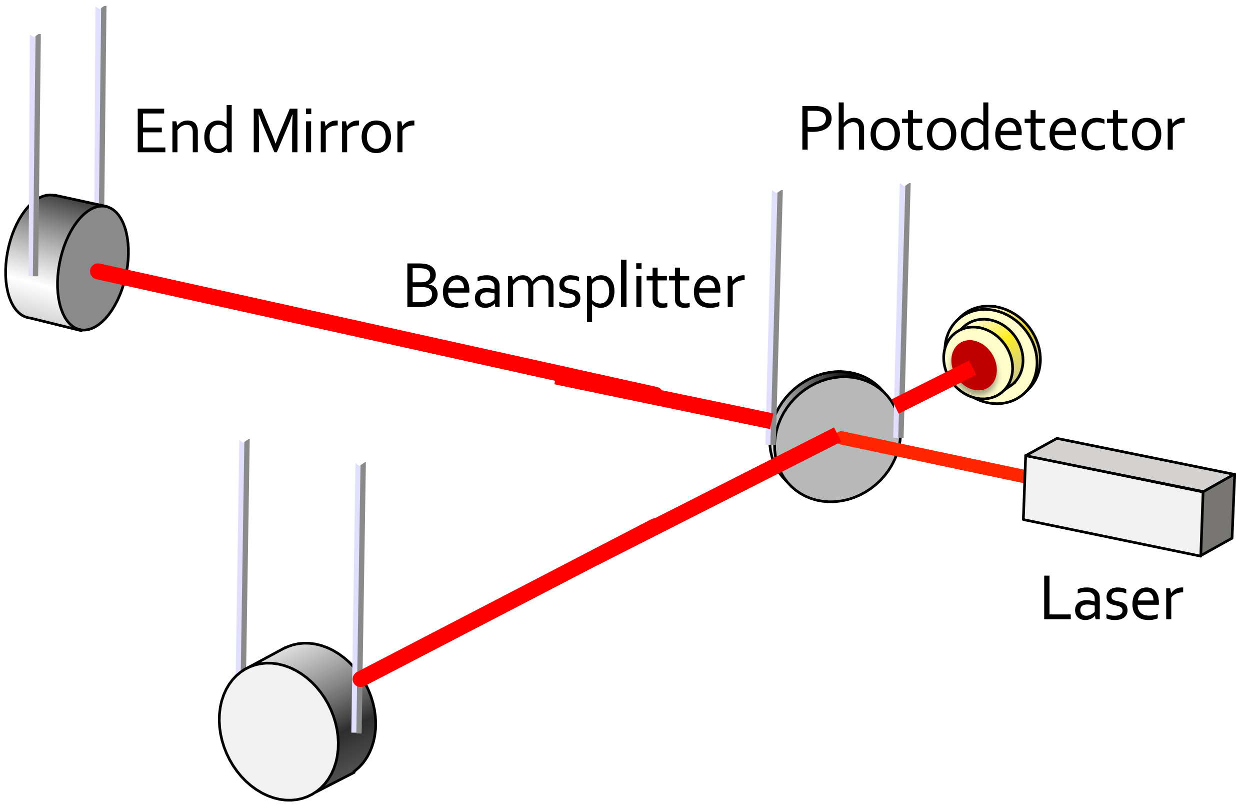
GW Detection



- Q: What happens if
  - The mirrors are not free-falling
  - The distance is measured with a tape measure with infinite precision

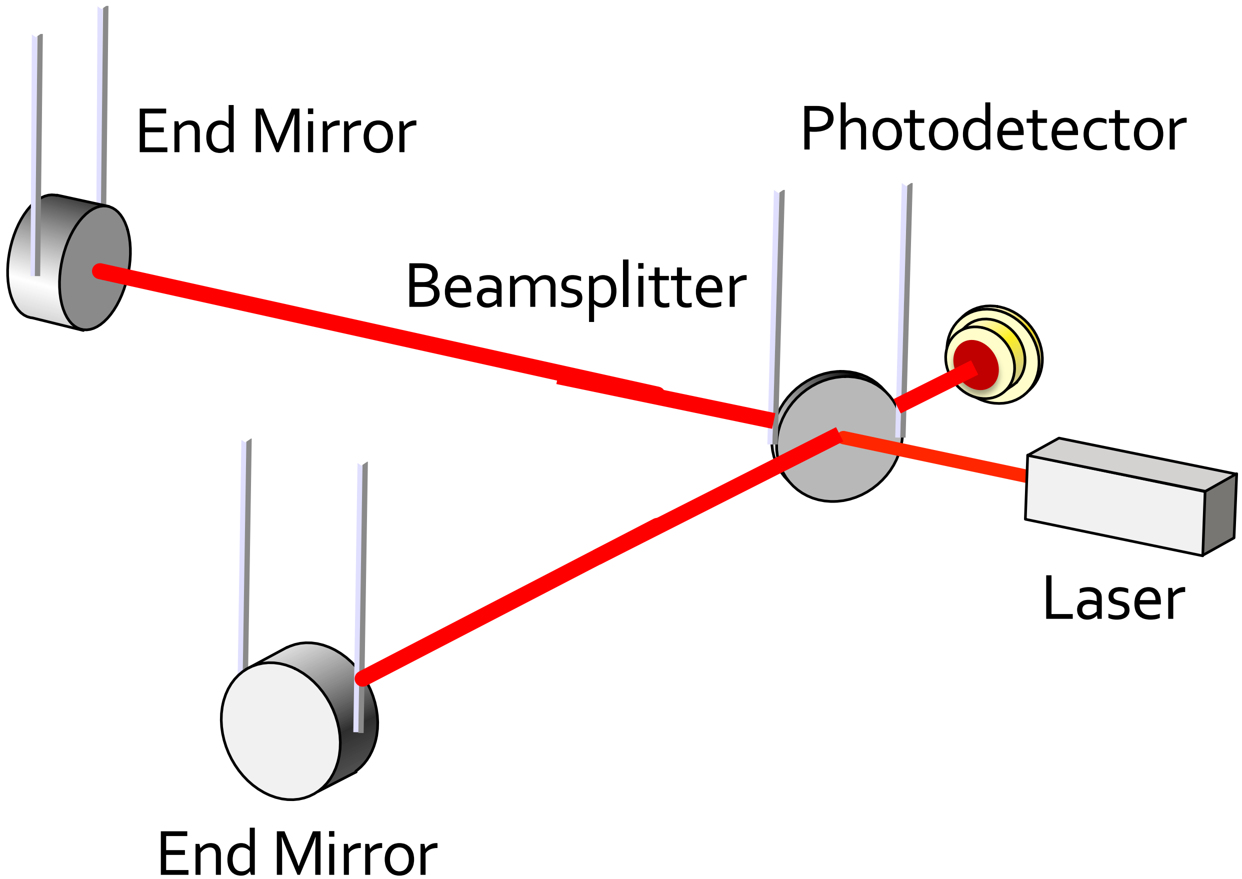
GW detection with a laser interferometer

- **Michelson-type interferometer**
  - With suspended mirrors



GW detection with a laser interferometer

- **Q: Why is the Michelson interferometer suitable for GW detection?**



GW detection with a laser interferometer

- Michelson interferometer =

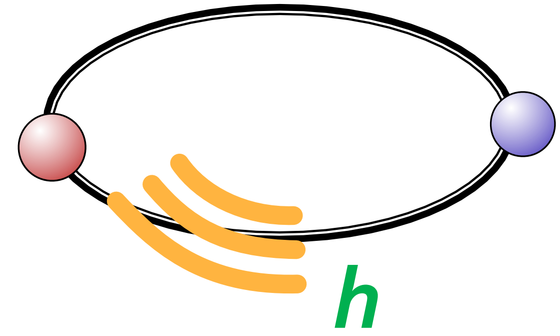
  Optical phase meter

  sensitive to differential arm displacement

  - The effect of GWs have the nature of differential strain
  - Insensitive to the wavelength (=frequency) change of the laser
  - Laser interferometry allows us to read the phase of the light
  - Laser beams can be propagated long distance without power loss

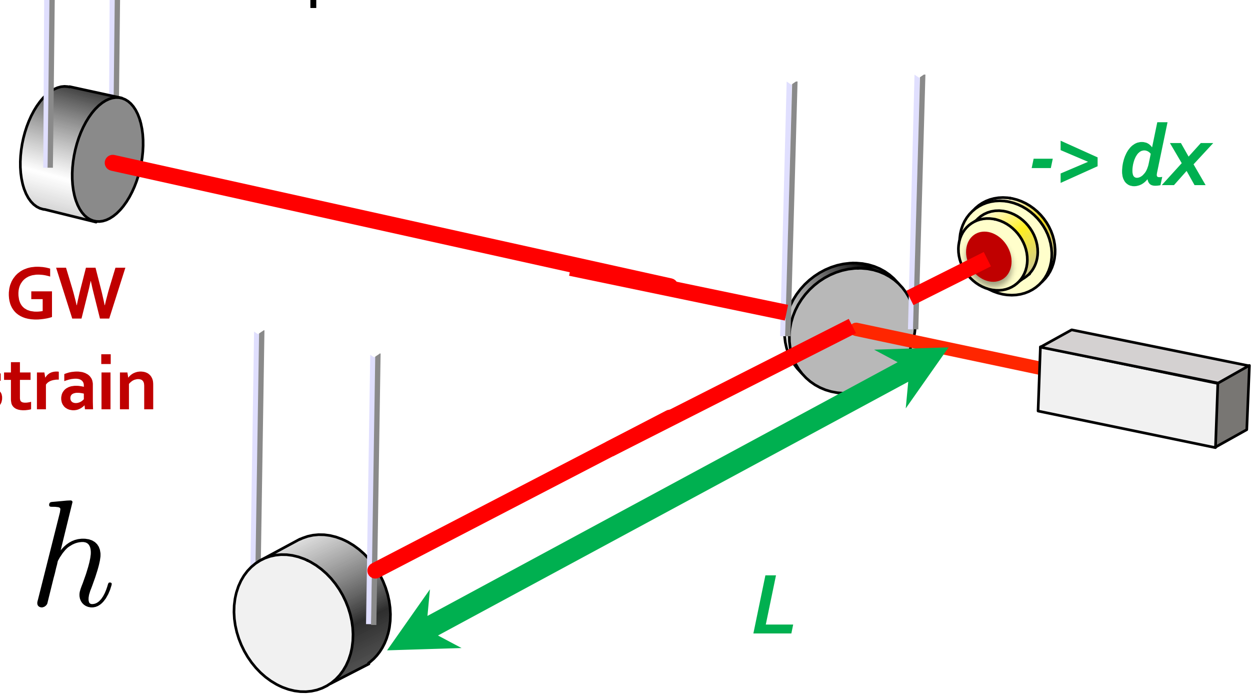
GW detection with a laser interferometer

- **Michelson-type interferometer**
  - With suspended mirrors
- **Differential path length change**
  - Intensity change at the output



Path length change — Arm length — GW strain

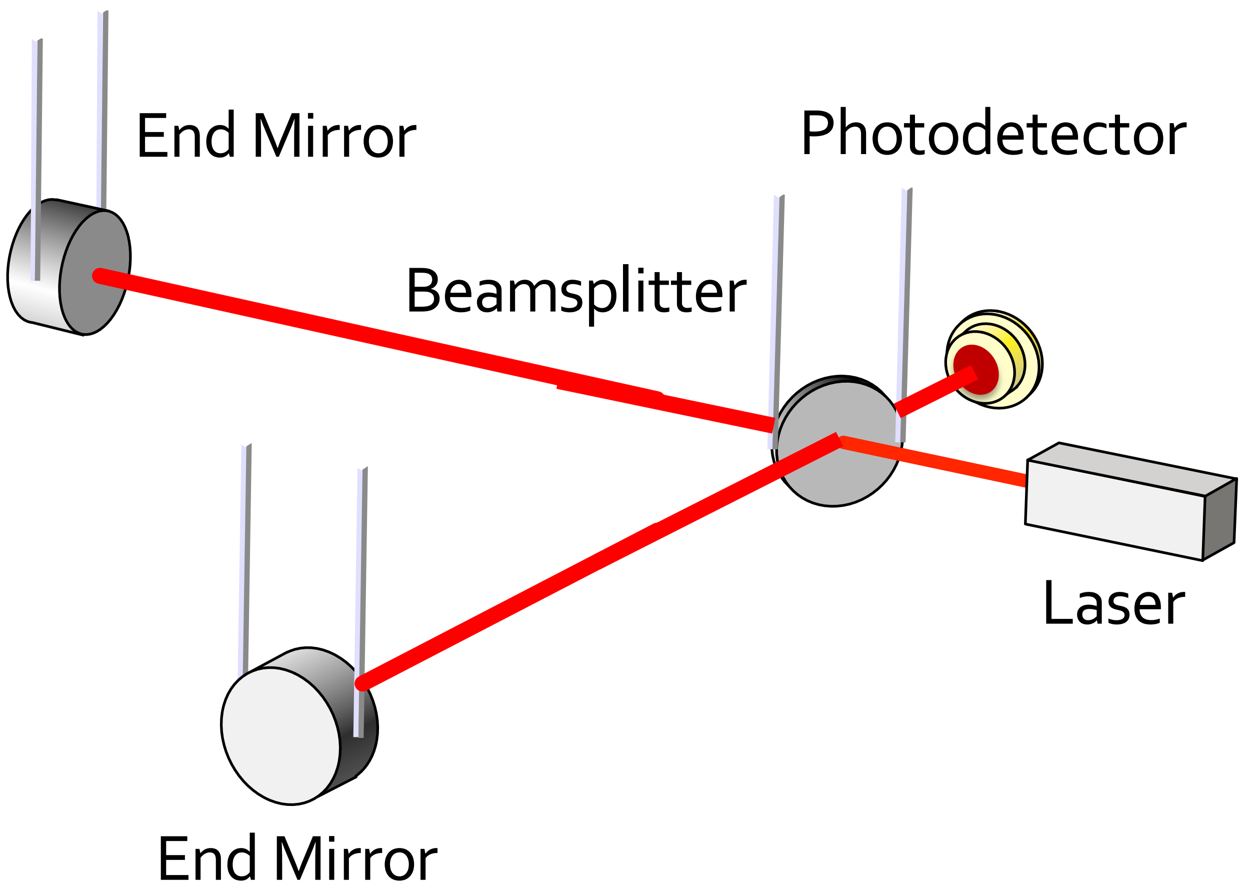
$$\delta x = L \times h$$



- **The longer the arms, the better!**

GW detection with a laser interferometer

- **Q: Why are the mirrors suspended?**



GW detection with a laser interferometer

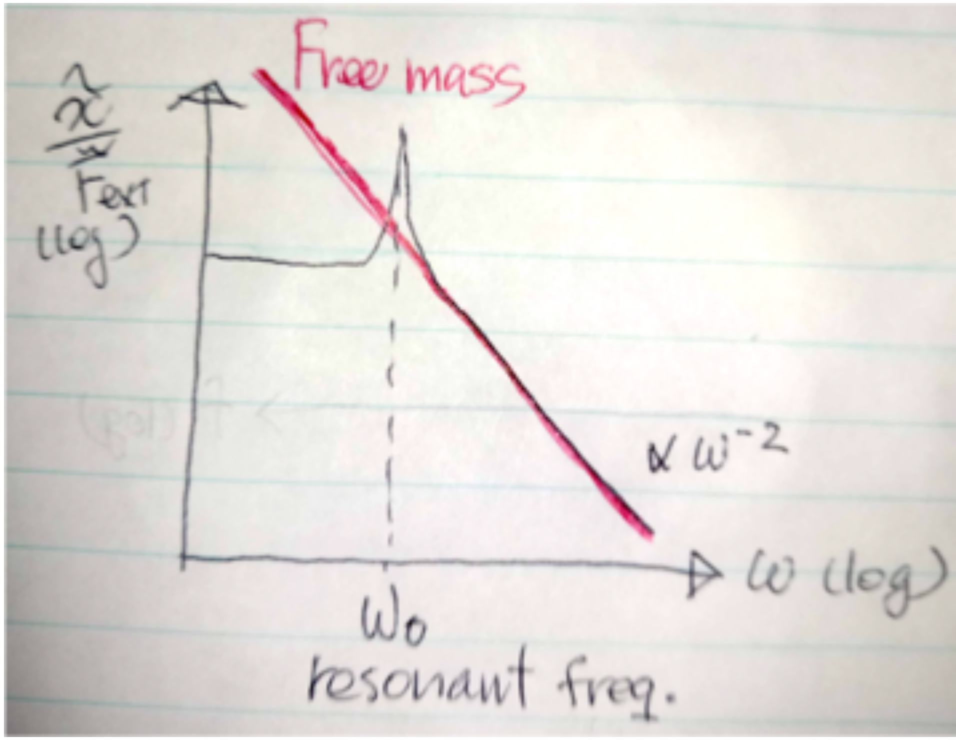
- **Why are the mirrors suspended?**

$$m\ddot{x} = -\frac{mg}{l}x + F_{\text{ext}}$$

$$(\text{F.T.}) \Rightarrow -\omega^2\tilde{x} = -\omega_0^2\tilde{x} + \tilde{F}_{\text{ext}}/m$$

$$(\omega_0 = \sqrt{g/l})$$

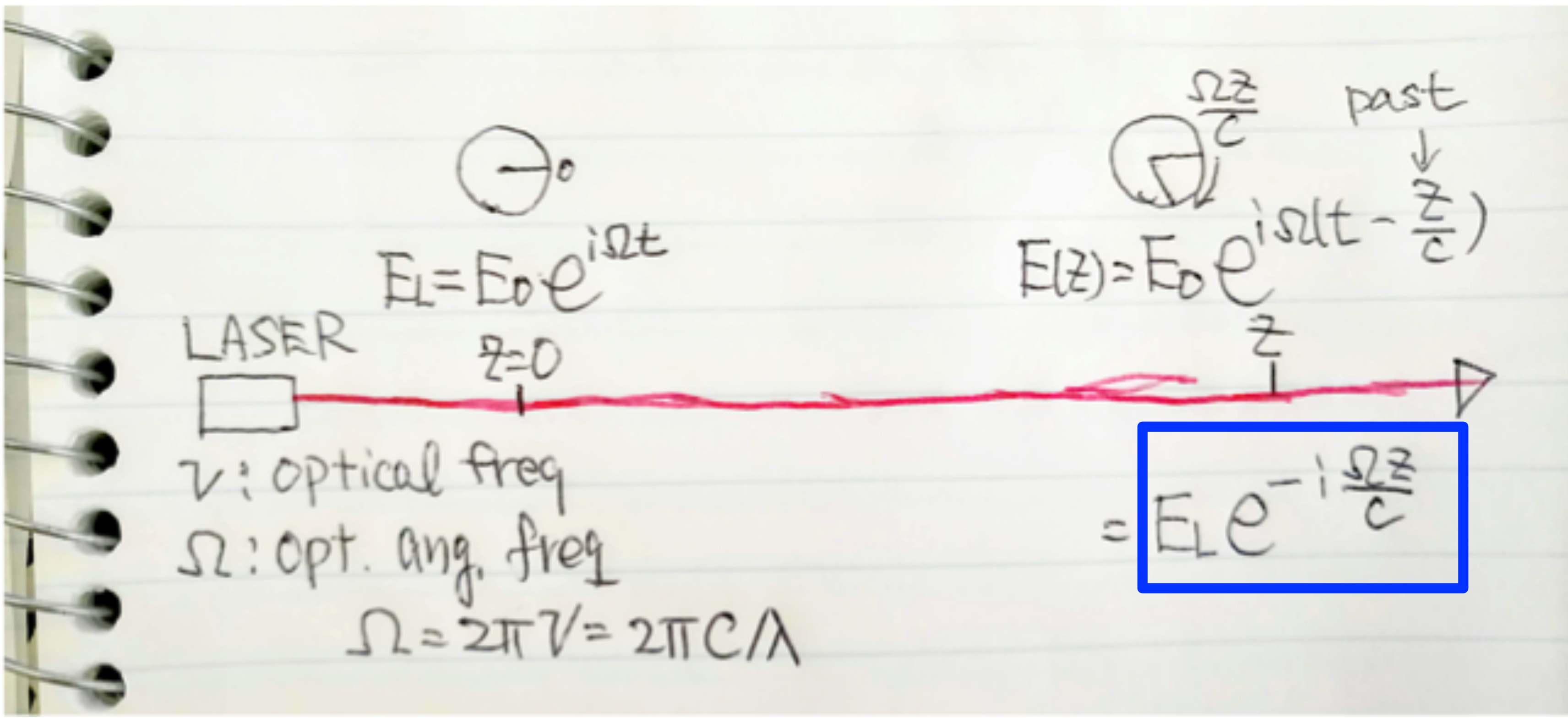
$$\Rightarrow \frac{\tilde{x}}{\tilde{F}_{\text{ext}}} = \frac{1}{m(\omega_0^2 - \omega^2)}$$



Above the resonant (pendulum) frequency, a suspended mirror behaves as a free mass At least in horizontal directions.

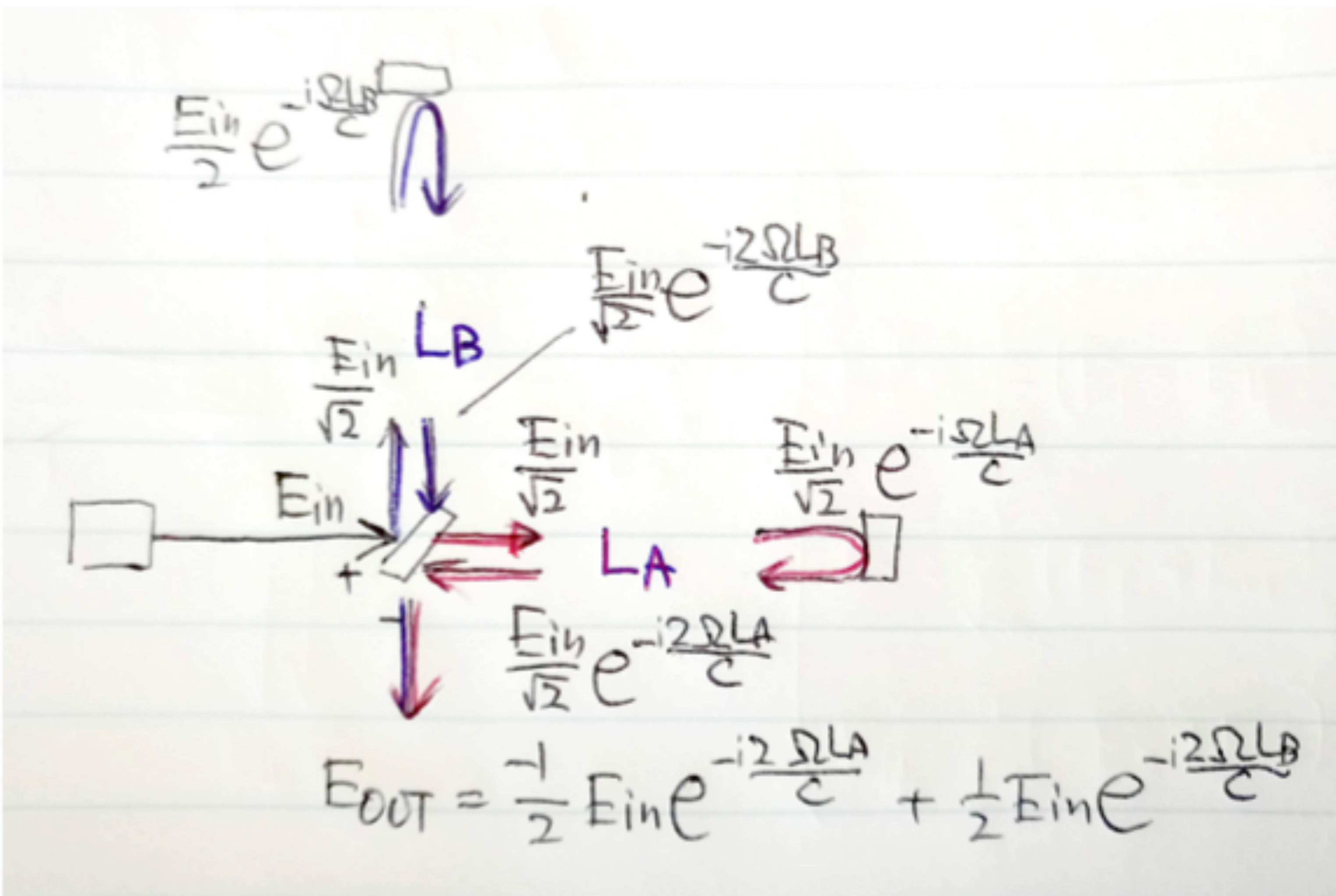
GW detection with a laser interferometer

- **Plane light wave & propagation**
  - **In a scaler model, the light field of a laser beam can be assumed as plane waves.**



Michelson interferometer

- **Electric field in a Michelson interferometer**



Michelson interferometer

- **Light intensity at the output port**
  - **Difference of the electric fields from the arms**

$$E_{\rm out} = \frac{1}{2}\left(e^{-i\phi_{\rm B}} - e^{-i\phi_{\rm A}}\right) E_{\rm in}$$

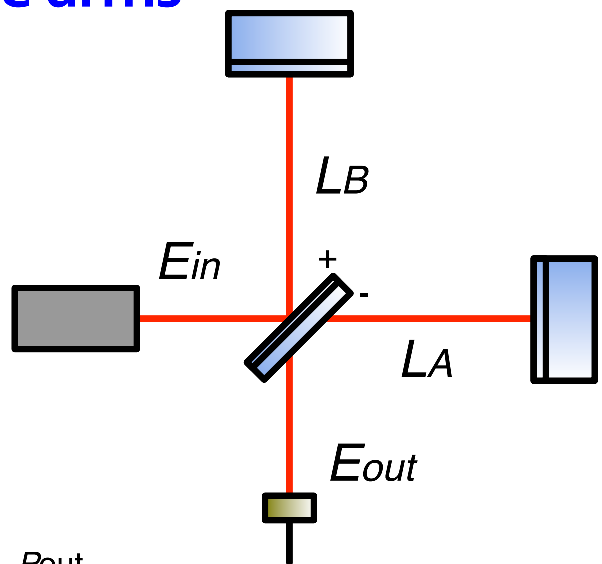
$$(\text{Roundtrip phase: } \phi_x = 4\pi\nu L_x/c)$$

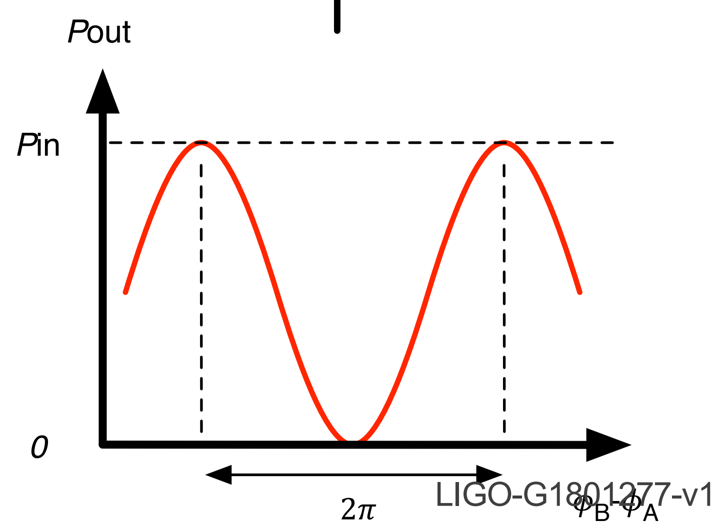
$$E_{\rm out} = \left[ie^{-i(\phi_A+\phi_B)/2}\sin\frac{\phi_A-\phi_B}{2}\right] E_{\rm in}$$

$$P_{\rm out} = E_{\rm out}E^*_{\rm out} = \left(\sin^2\frac{\phi_A-\phi_B}{2}\right) E_{\rm in}E^*_{\rm in}$$

$$= [1-\cos(\phi_A-\phi_B)]\frac{P_{\rm in}}{2}$$

Output intensity is sensitive to the differential phase





Michelson interferometer

- **Frequency response of the Michelson to GWs**

$$\phi_A - \phi_B = \int_{t-2L/c}^{t} \Omega \left[1 + \frac{1}{2}h(t)\right] dt - \int_{t-2L/c}^{t} \Omega \left[1 - \frac{1}{2}h(t)\right] dt$$

$$= \int_{t-2L/c}^{t} \Omega h(t) dt$$

Frequency response of the Michelson interferometer

$$h(t) = h_0 e^{i\omega t}$$

$$\phi_A - \phi_B = \boxed{\frac{2L\Omega}{c} e^{-iL\omega/c} \frac{\sin(L\omega/c)}{L\omega/c}} \cdot h_0 e^{i\omega t}$$

$$= \frac{4\pi L}{\lambda_{\mathrm{opt}}} e^{-i2\pi L/\lambda_{\mathrm{GW}}} \frac{\sin(2\pi L/\lambda_{\mathrm{GW}})}{2\pi L/\lambda_{\mathrm{GW}}} \cdot h_0 e^{i\omega t}$$

Ω: optical angular frequency, λ_{OPT} laser wavelength
ω: angular frequency of GW, λ_{GW} wavelength of GW

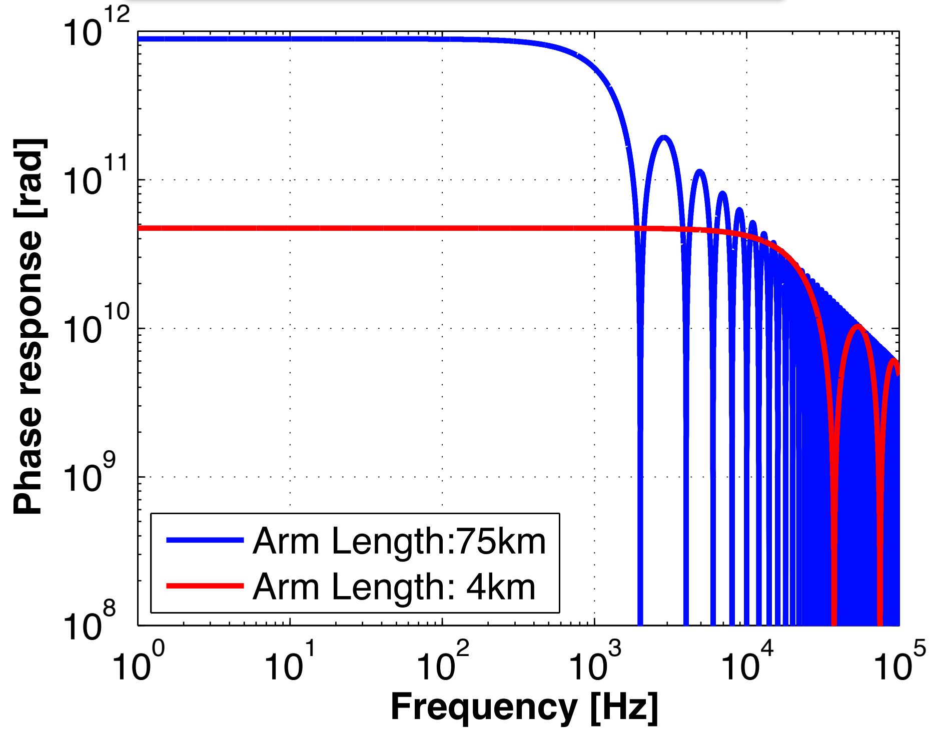
Jean-Yves Vinet, et al
Phys. Rev. D 38, 433 (1988)

Michelson interferometer

- **Frequency response of the Michelson to GWs**

$$\Delta\phi = \boxed{\frac{2L\Omega}{c} e^{-iL\omega/c} \frac{\sin(L\omega/c)}{L\omega/c}} \cdot h_0 e^{i\omega t}$$

DC Response longer -> larger



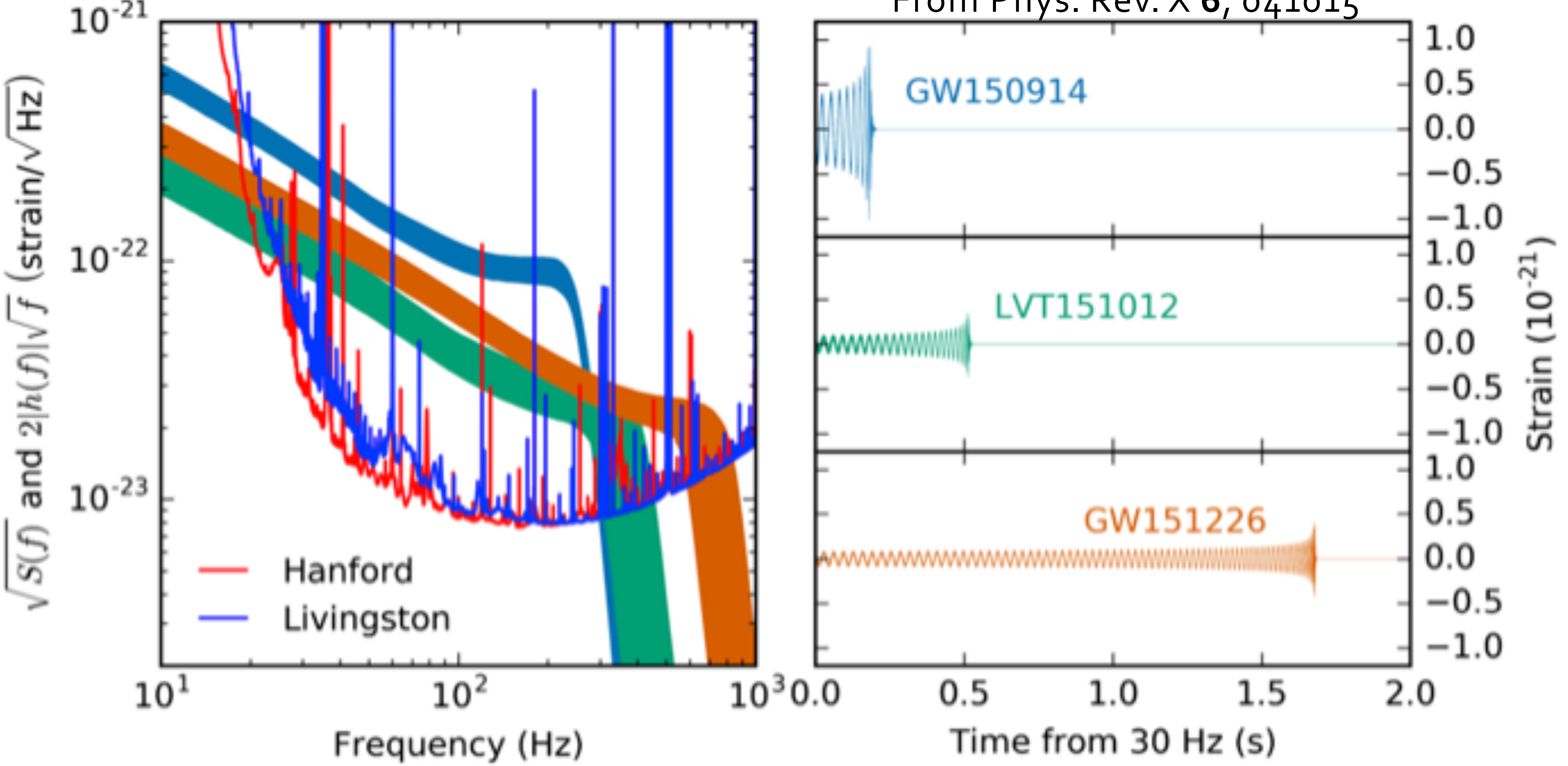
Cut off freq longer -> lower

Notch freq f = n c / (2 L)

Michelson arm length optimized for 1kHz GW -> 75km, too long! => Fabry-Perot cavity

Comparison: Michelson sensitivity and GW amplitudes

From Phys. Rev. X **6**, 041015

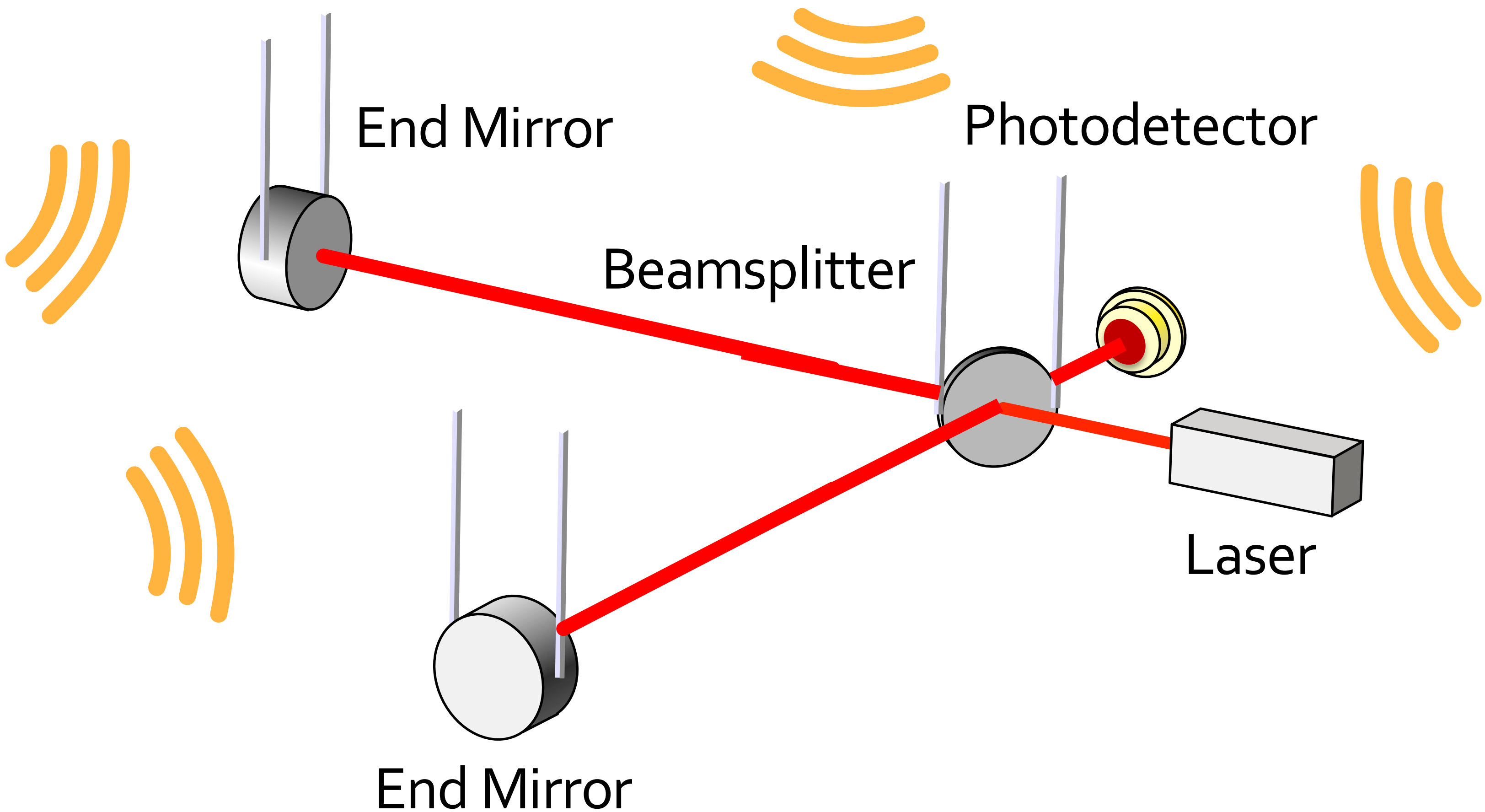


Michelson sensitivity (4km 1W): h ≈ 1.3×10^{-20} /√Hz

Not enough at all!! What to do? (talk about it later)

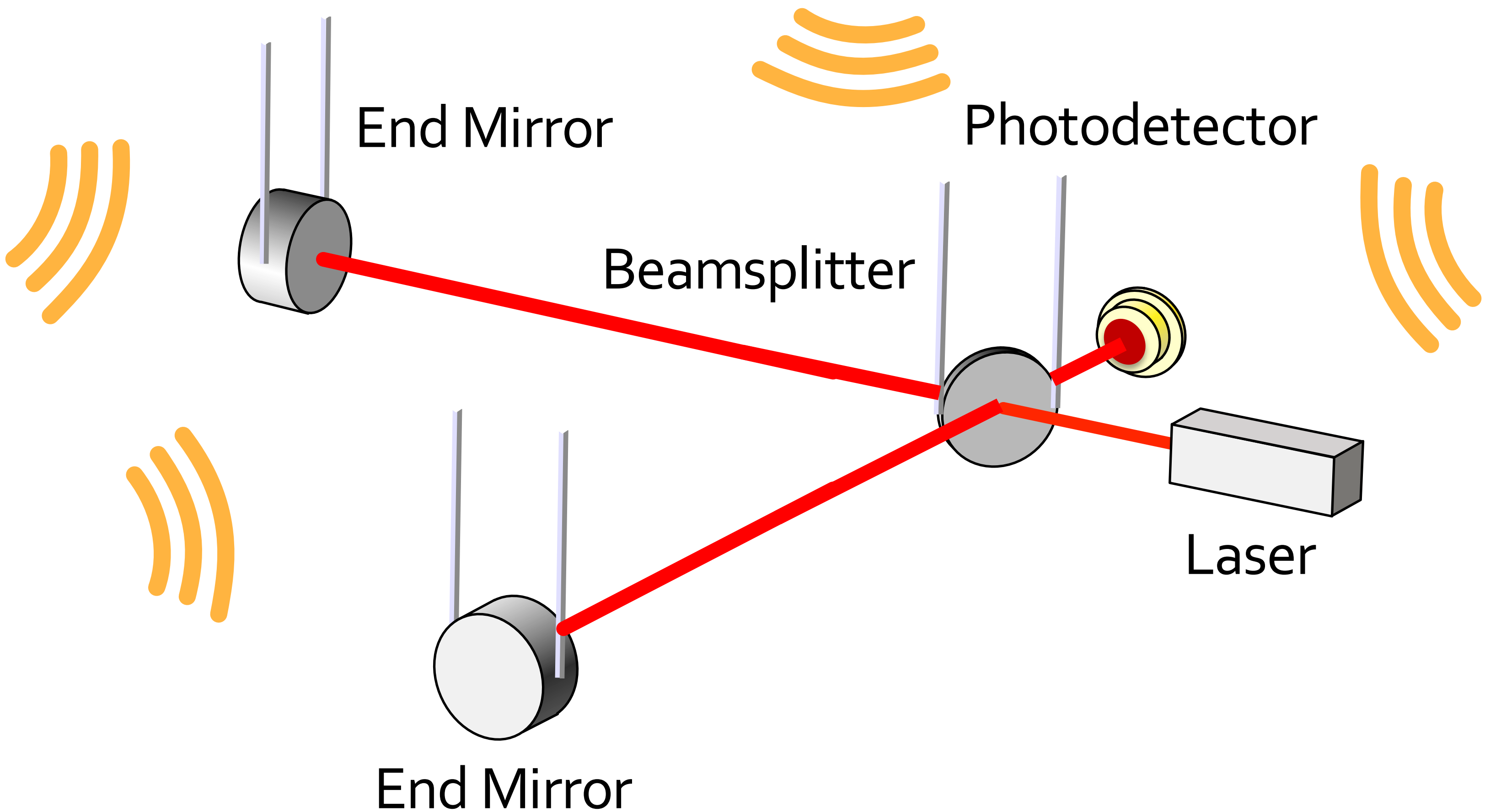
Antenna pattern of a Michelson interferometer

- Q: How does a Michelson interferometer reacts to GWs coming from various directions?



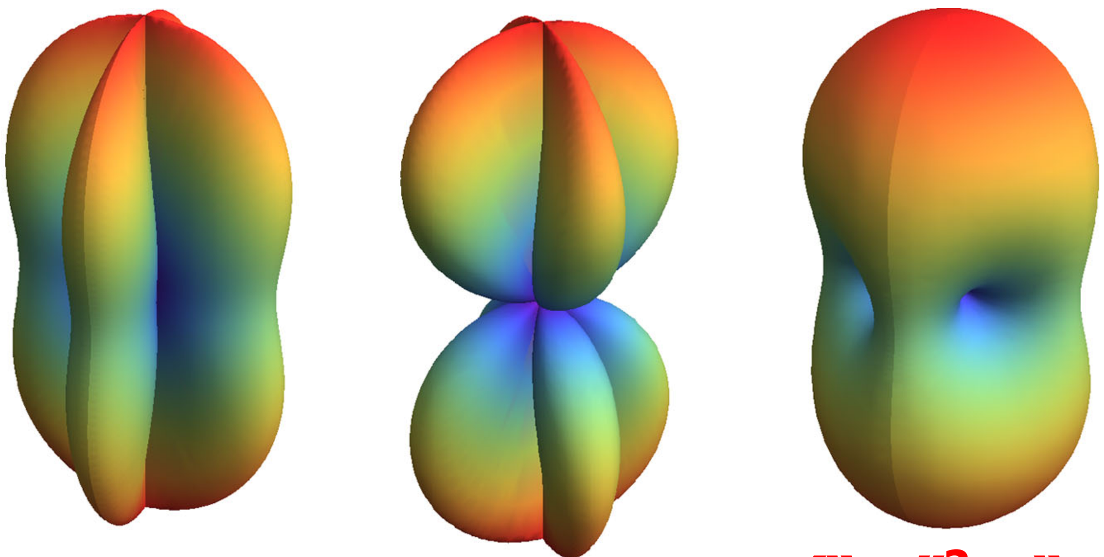
Antenna pattern of a Michelson interferometer

- Q: How does a Michelson interferometer reacts to GWs coming from various directions?



Antenna pattern of a Michelson interferometer

- **Response of the interferometer to GWs with + and x polarizations from various directions**



+ x sqrt("+"^{2} +"x"^{2})

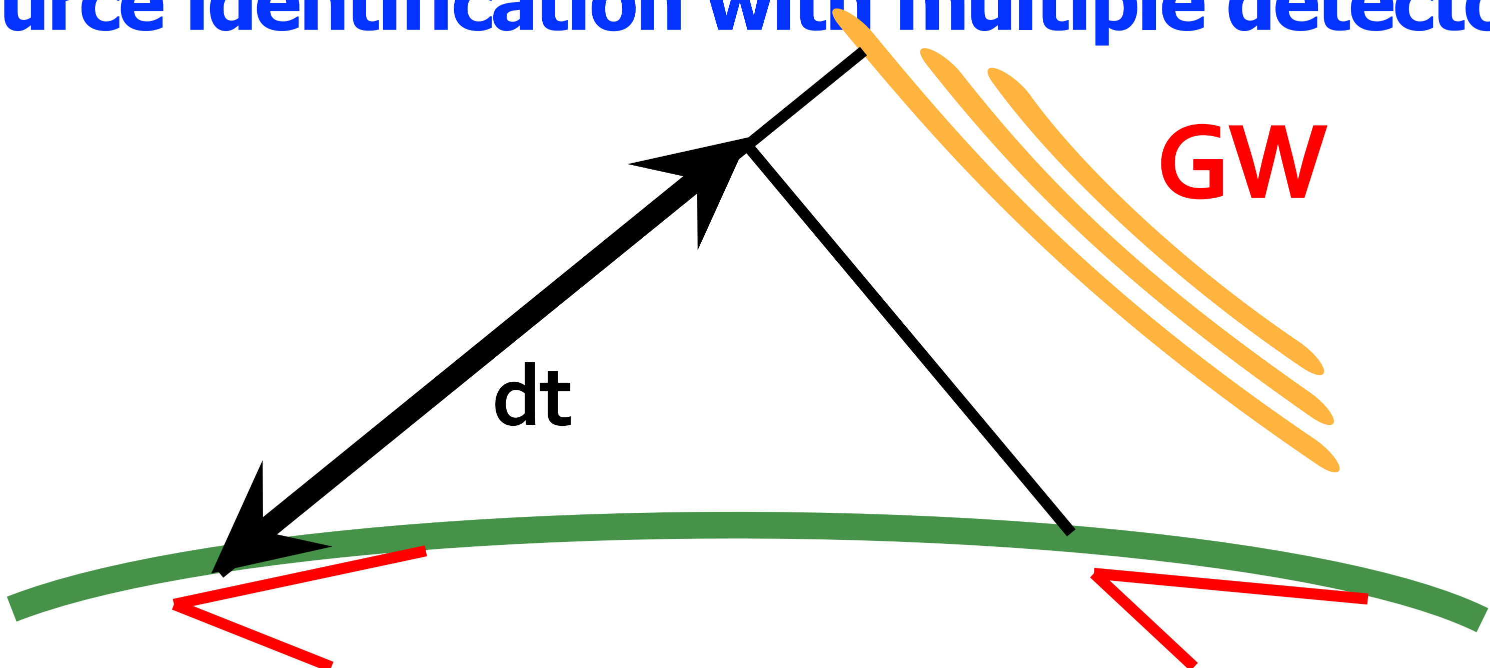
FIG. 2 (color online). Interferometer antenna response for (+) polarization (left), (×) polarization (middle), and unpolarized waves (right).

- **Directivity is very mild**

➔ Need multiple detectors for source identification

Antenna pattern of a Michelson interferometer

- Source identification with multiple detectors

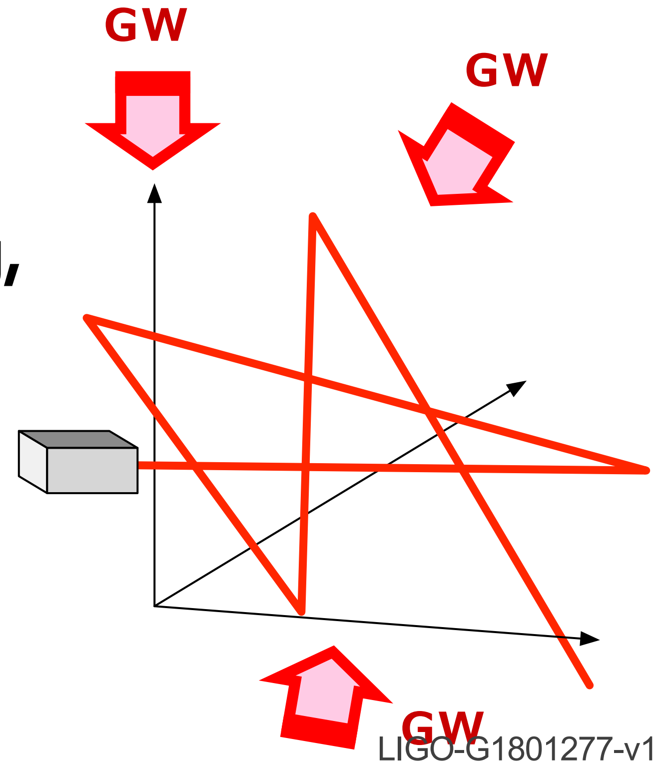


- Use difference of the signal arrival time to triangulate the source direction

- Two detectors are not enough
  Need three or more.

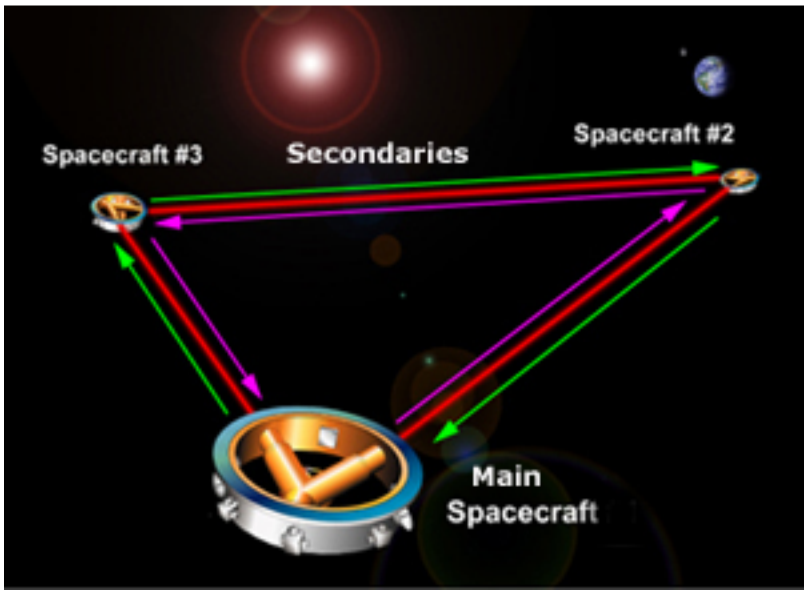
Advanced topics

- Angular & frequency response of an arbitrary optical path
  - Draw an arbitrary optical path.
    What is the angular and frequency response of such a path?
  - Can we use numerical "optimization" for certain criteria?
    e.g.
    better sky coverage, directive beaming, for certain source frequency, etc…



Advanced topics

- **Angular and frequency response of LISA**
  - **R. Schilling, Class. Quantum Grav. 14 (1997) 1513-1519**



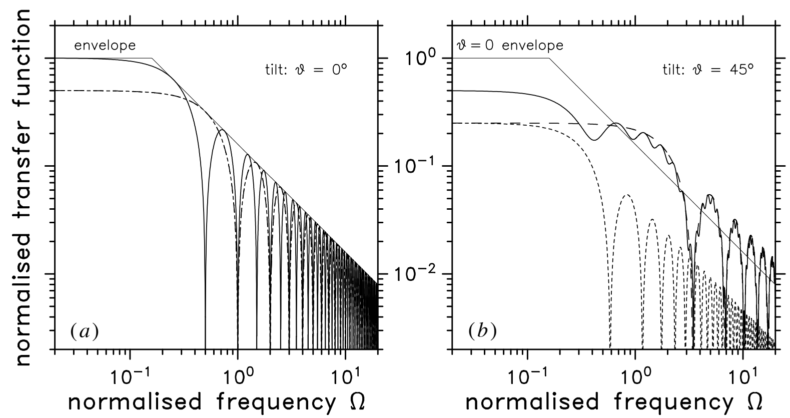
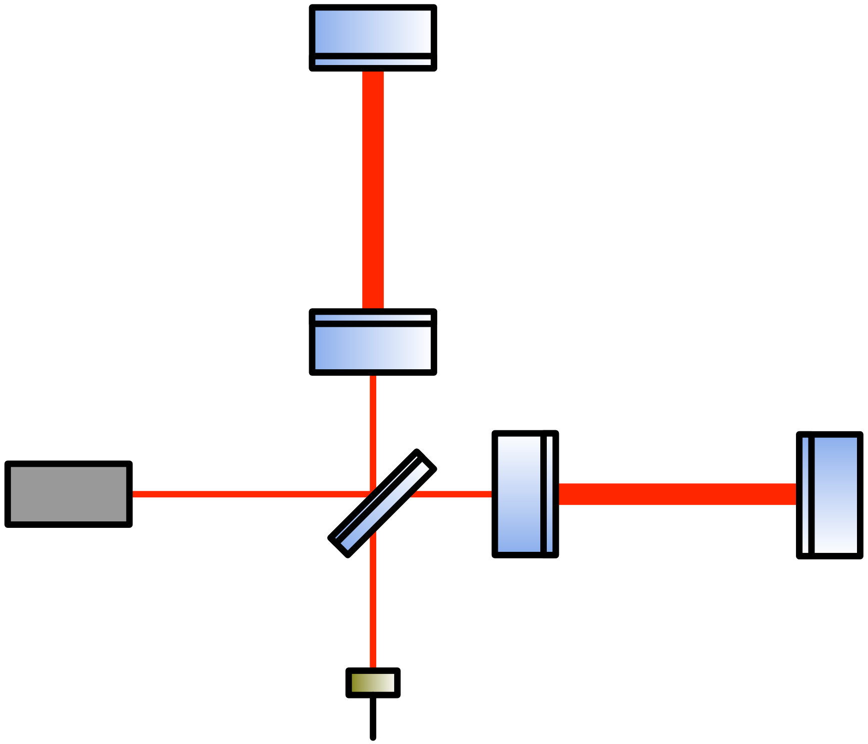


Figure 1. Magnitude of the normalized transfer function for a single round trip in a single arm and a tilt of (*a*) 0° and (*b*) 45°. Full curve, round trip; long broken curve, forward pass; short broken curve, return pass.

Arm length 2.5Gm

Fabry-Perot Michelson Interferometer

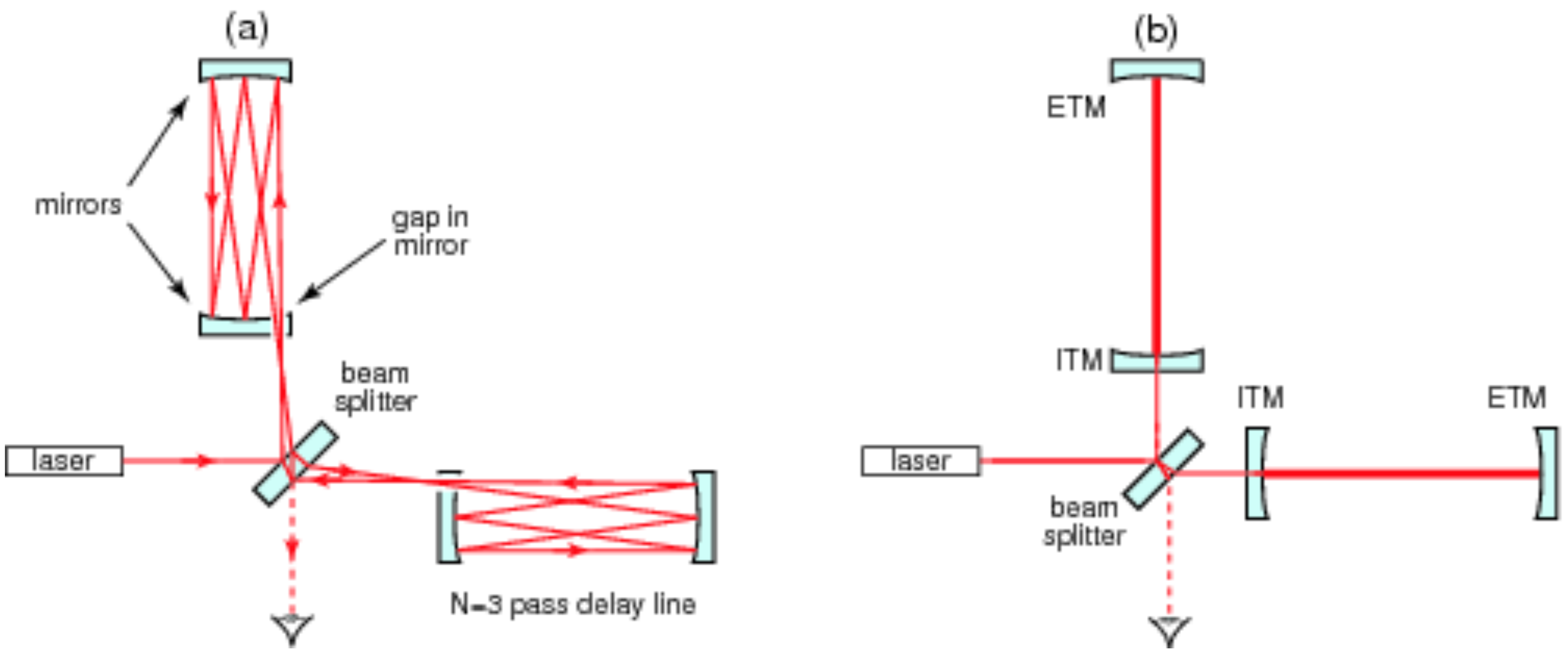
- Arms are too short
- Use Fabry-Perot cavities in the arms to store the light longer



Basic form of the modern interferometer GW detector

Fabry-Perot Michelson Interferometer

- In the past, a delay line arm was considered (easier to understand)
- A FP arm is similar but use a single optical path



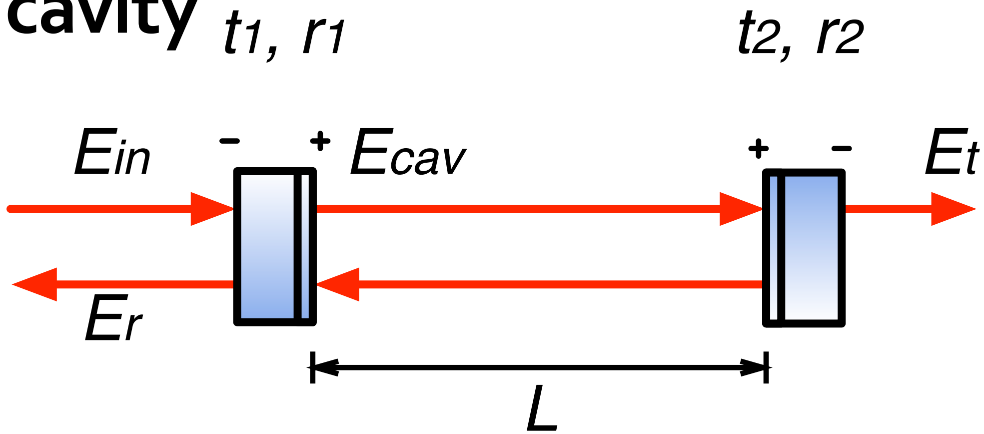
Fabry-Perot optical resonator

- Storing light in an optical cavity
  - Field equations

$$E_{\rm cav} = t_1 E_{\rm in} + r_2 e^{-i\phi} E_{\rm cav}$$

$$E_{\rm t} = t_2 e^{-i\phi/2} E_{\rm cav}$$

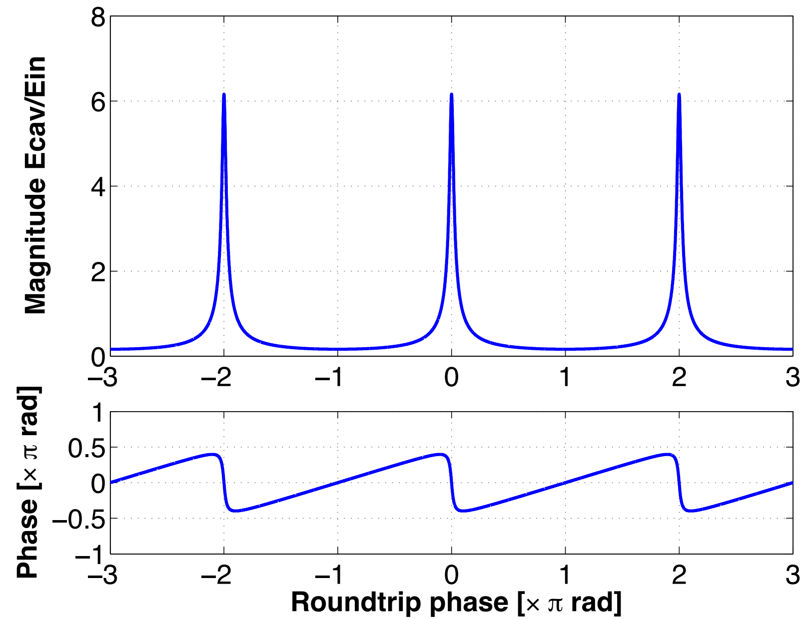
$$E_{\rm r} = -r_1 + t_1 r_2 e^{-i\phi} E_{\rm cav}$$



$$\frac{E_{\rm cav}}{E_{\rm in}} = \frac{t_1}{1 - r_1 r_2 e^{-i\phi}}$$

$$\frac{E_{\rm r}}{E_{\rm in}} = -r_1 + \frac{t_1^2 r_2 e^{-i\phi}}{1 - r_1 r_2 e^{-i\phi}}$$

$$\frac{E_{\rm t}}{E_{\rm in}} = \frac{t_1 t_2 e^{-i\phi/2}}{1 - r_1 r_2 e^{-i\phi}}$$



Very fast phase response

Fabry-Perot optical resonator

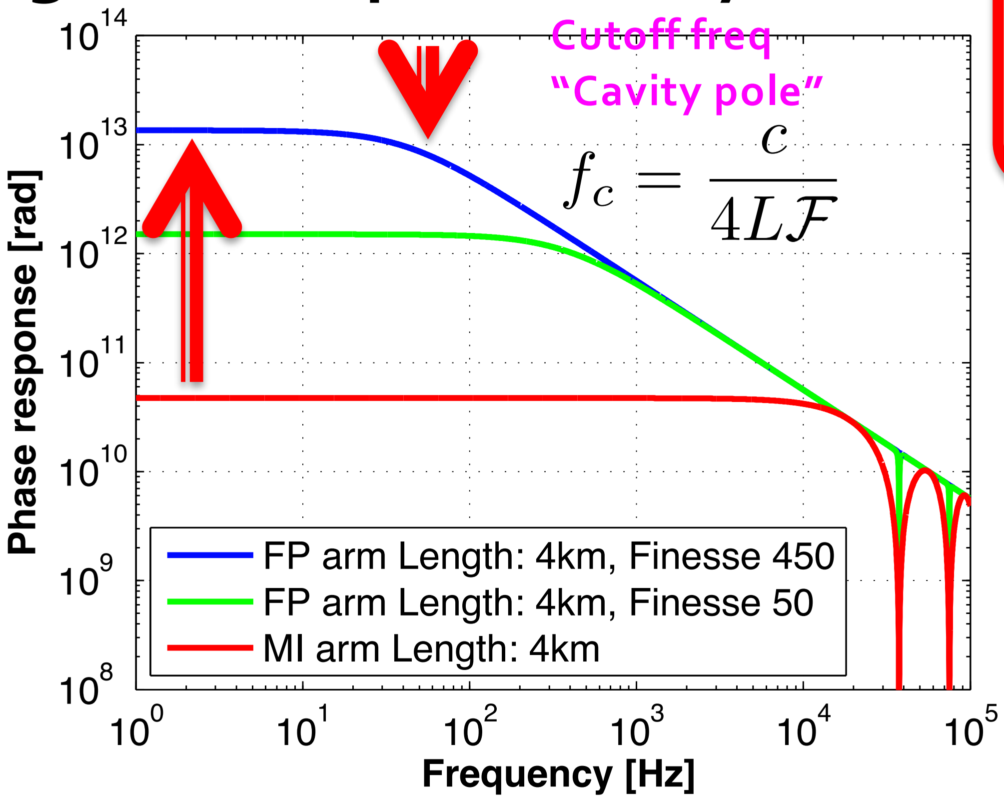
- **Storing light in an optical cavity**

Finesse

$$\mathcal{F} = \frac{\pi\sqrt{r_1 r_2}}{1 - r_1 r_2}$$

DC Response amplification

$$N = 2\mathcal{F}/\pi$$

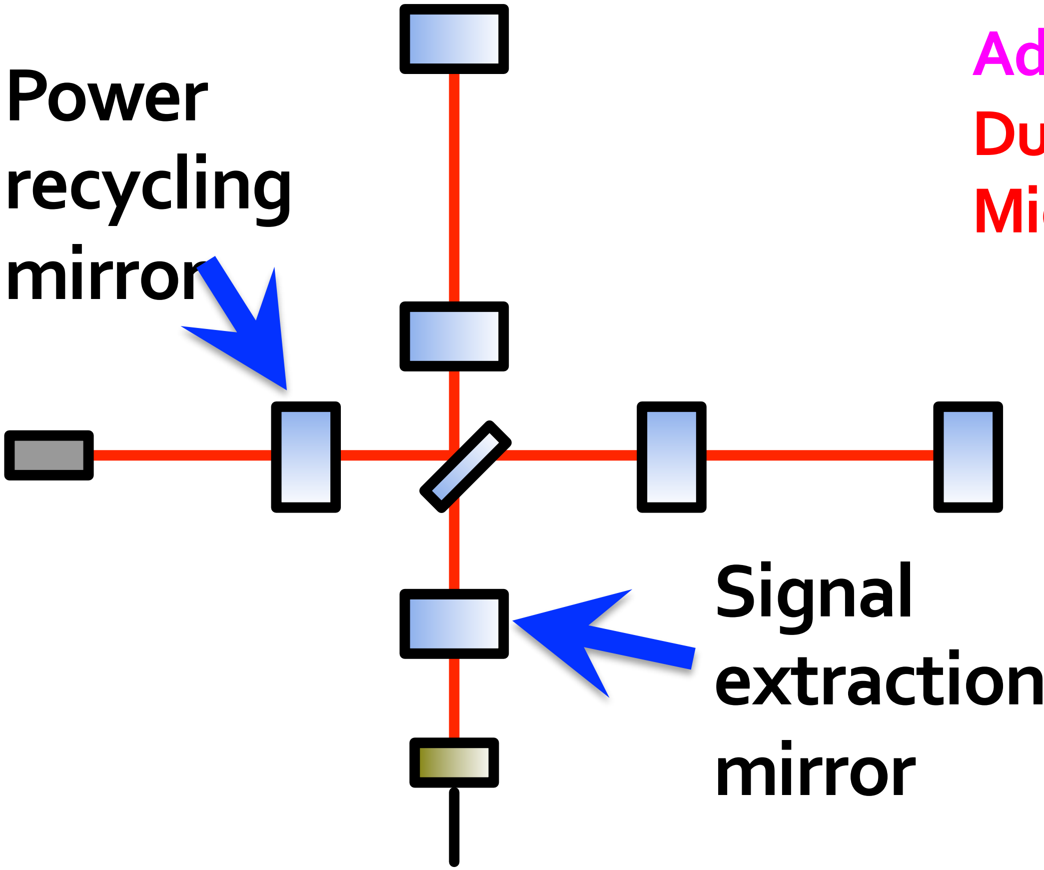


1. FP increases stored power in the arm
2. FP increases accumulation time of the signal

=> Above the roll-off, increasing F does not improve the response

Dual recycling

- Even more mirrors for more light and signals



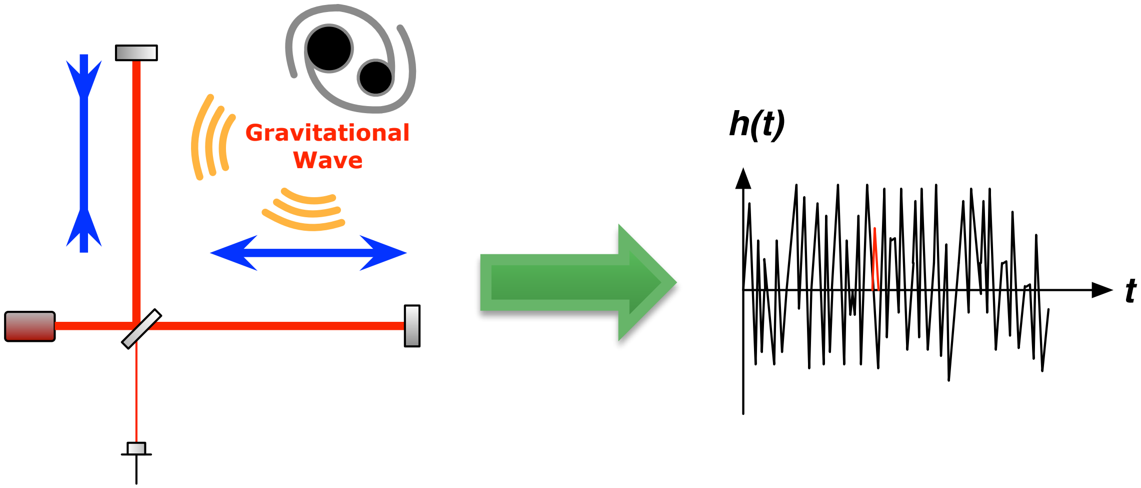
Advanced LIGO

Dual-Recycled Fabry-Perot Michelson Interferometer

These recycling mirrors allows us to control the power increase in the arm and the signal storage time independently

GW telescope?

- A continuous signal stream from an interferometer

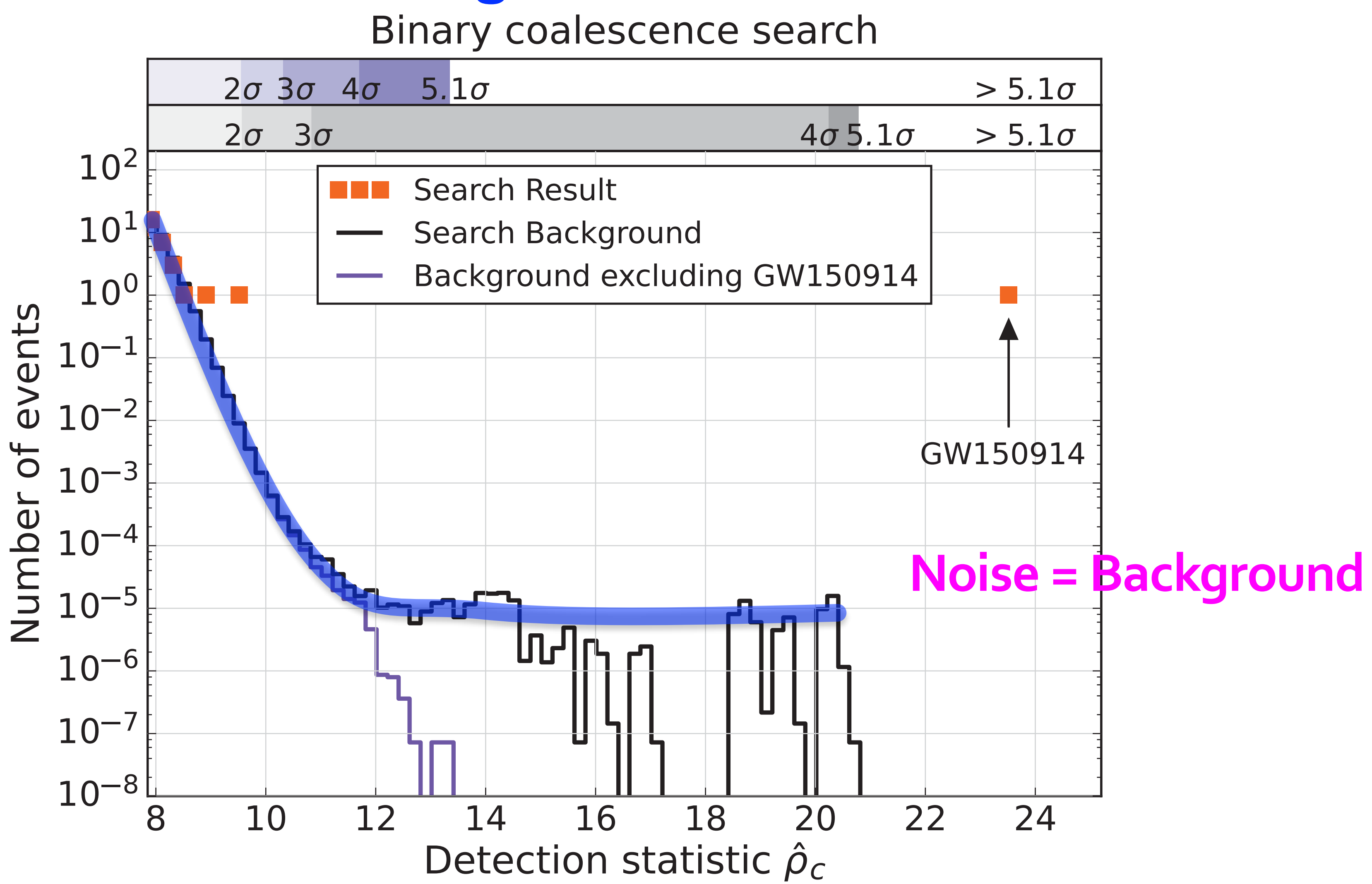


- Find GW signals in the sea of noise
  - Detector noises produce **fake** detections

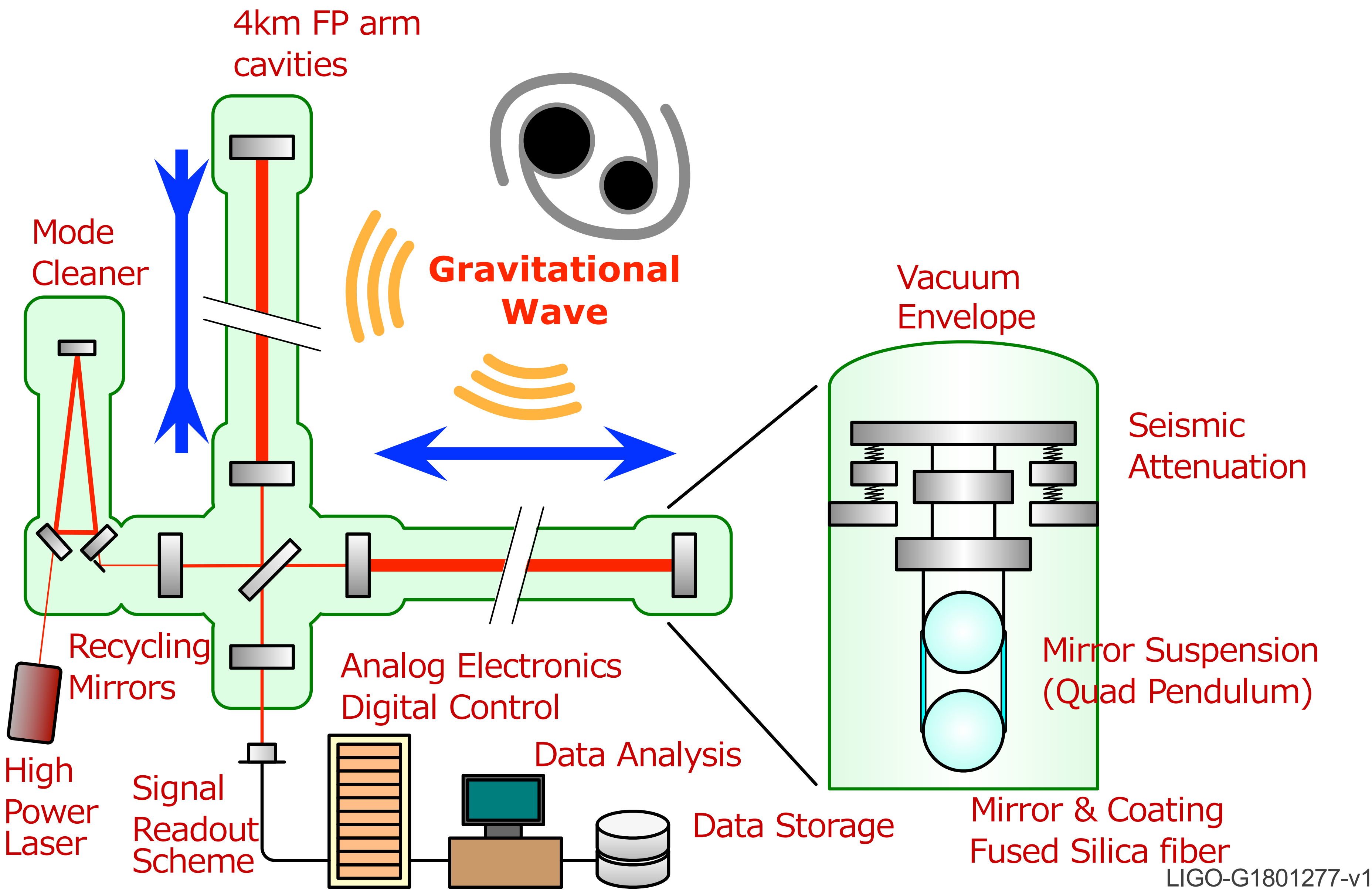
- **Reduce noises!**
  - Obs. distance is inv-proportional to noise level
  - x10 better => x10 farther => x1000 more galaxies

GW telescope?

- ## Noises & the signal



Actual GW detector



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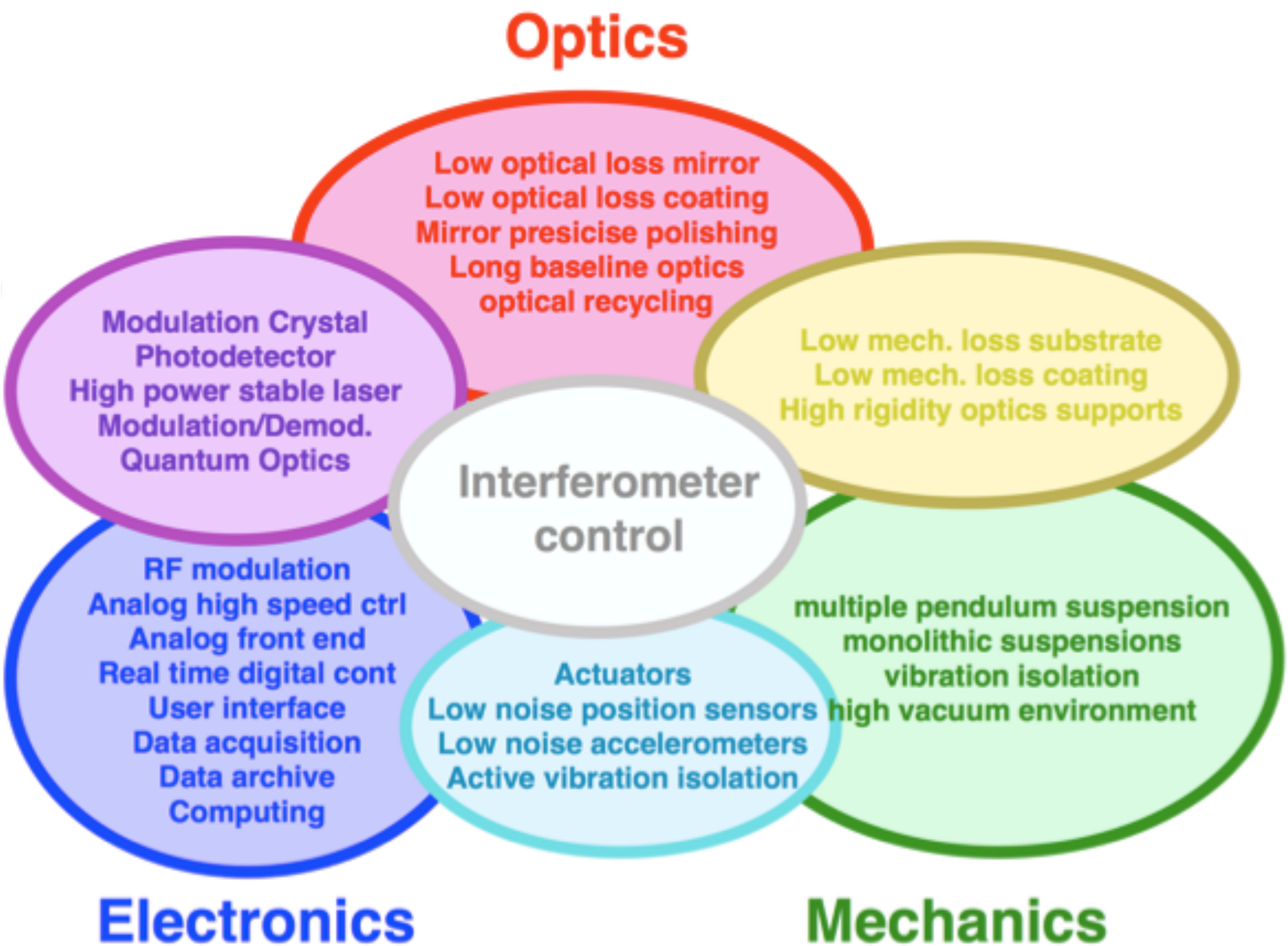
Technologies of the GW detectors

- 3 elements of a laser GW detector
  - Mechanics
  - Optics
  - Electronics

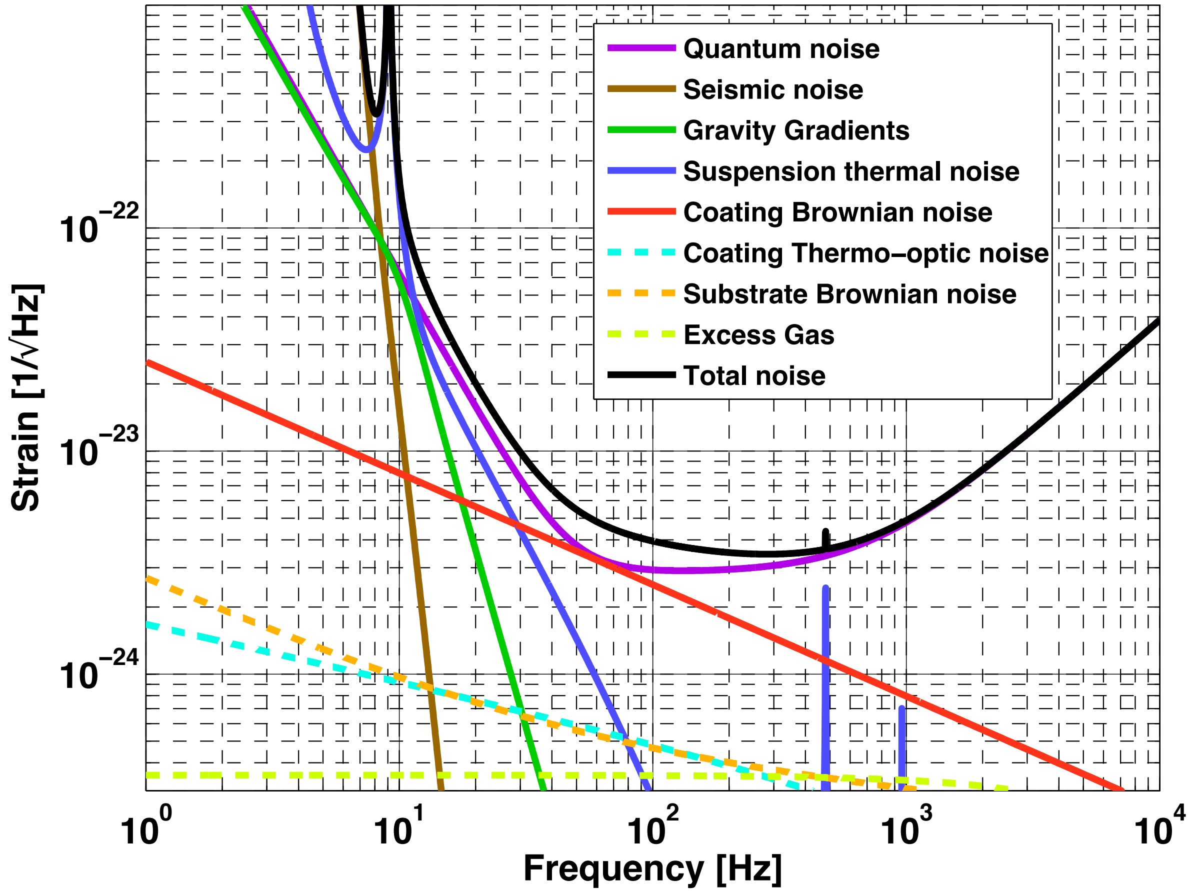
Technologies of the GW detectors

- 3



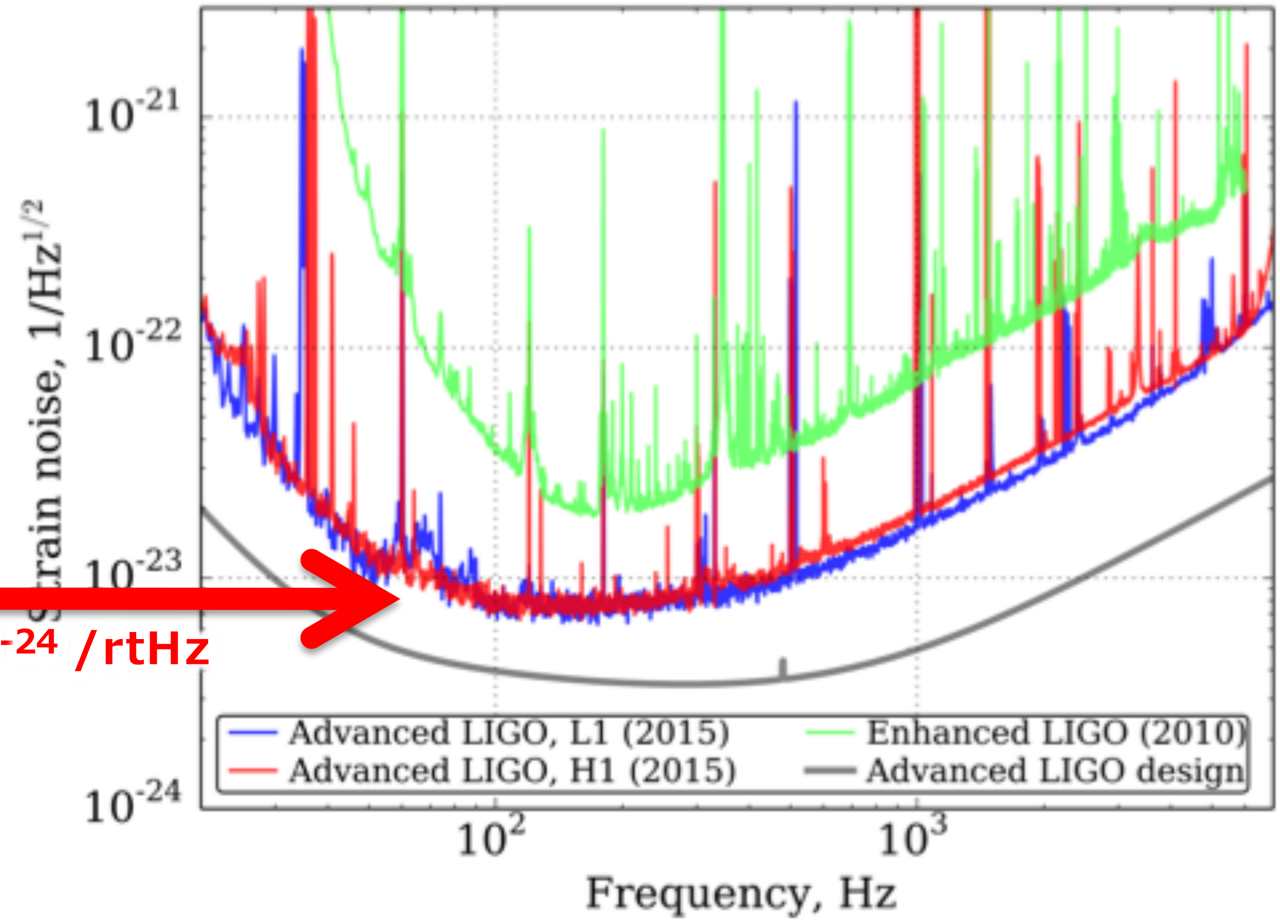
Sensitivity and noise

- Sensitivity (=noise level) of Advanced LIGO
- Design



Sensitivity and noise

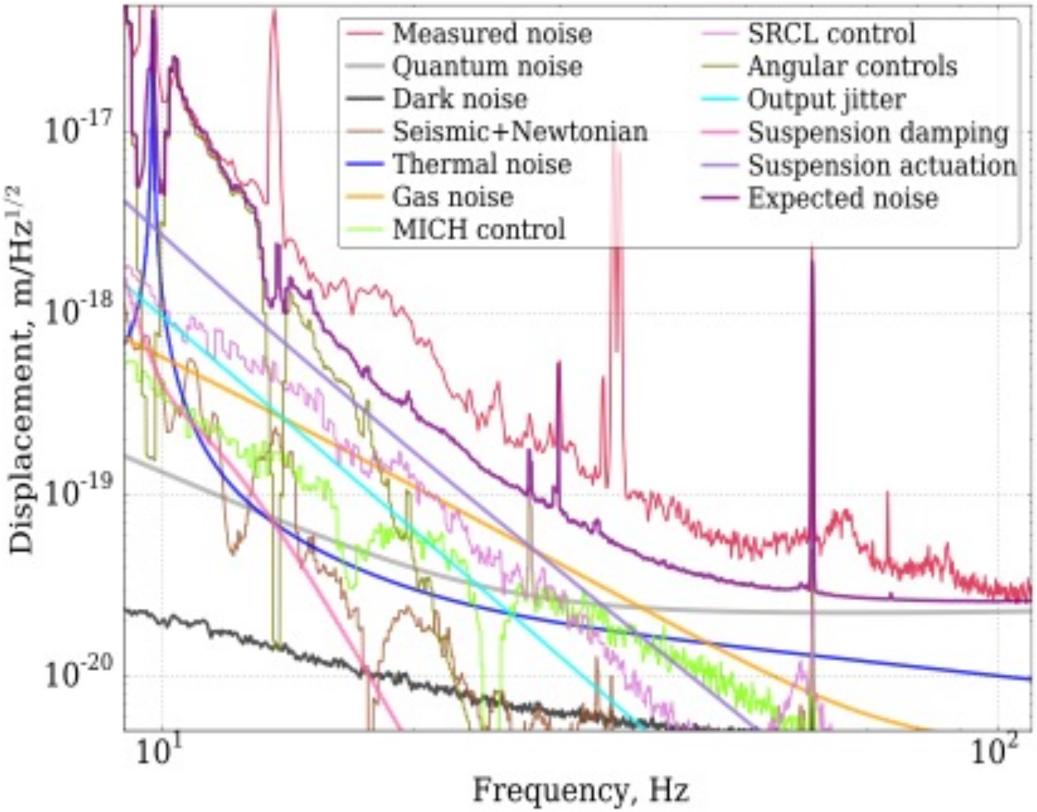
- Sensitivity (=noise level) of Advanced LIGO
- Current sensitivity



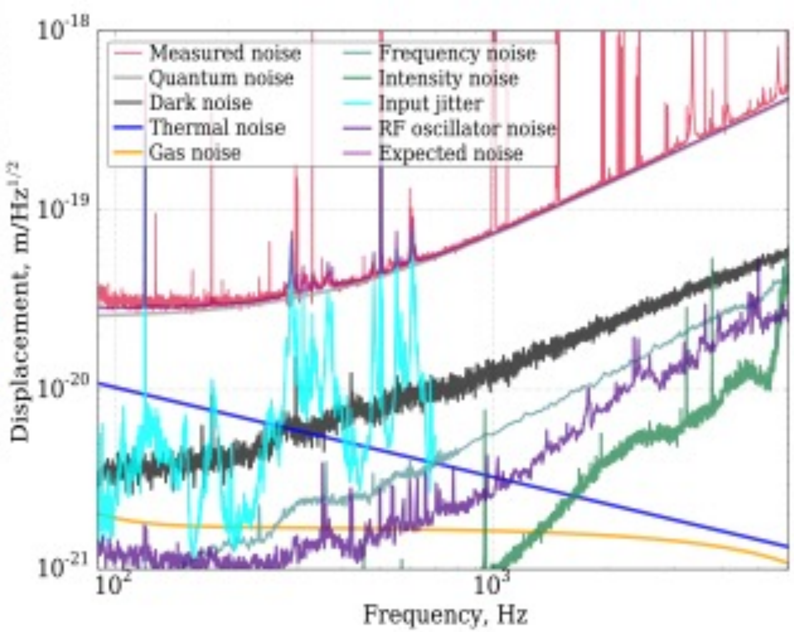
h= $8x10^{-24}$ /rtHz

Sensitivity and noise

- Sensitivity (=noise level) of Advanced LIGO
- Noise budget



(a) LIGO Livingston Observatory



(b) LIGO Hanford Observatory

Summary

- **Interferometric GW detector**
  - GWs: **Spacetime strain** with quadrupole patterns
  - Laser interferometry: **Michelson** type interferometer
  - Multiple detectors: **Triangulation** for source identification
  - Optical resonators: **Fabry-Perot cavity** and recycling
  - Technologies: All for **noise reduction**