# Thermal State of Advanced LIGO Test Masses: Implementation of a Real-Time Mirror Degradation Monitor

Guadalupe Quirarte, Harvey Mudd College

Mentors: Carl Blair and Joseph Betzwieser

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LIGO-G18xxxxx-v1

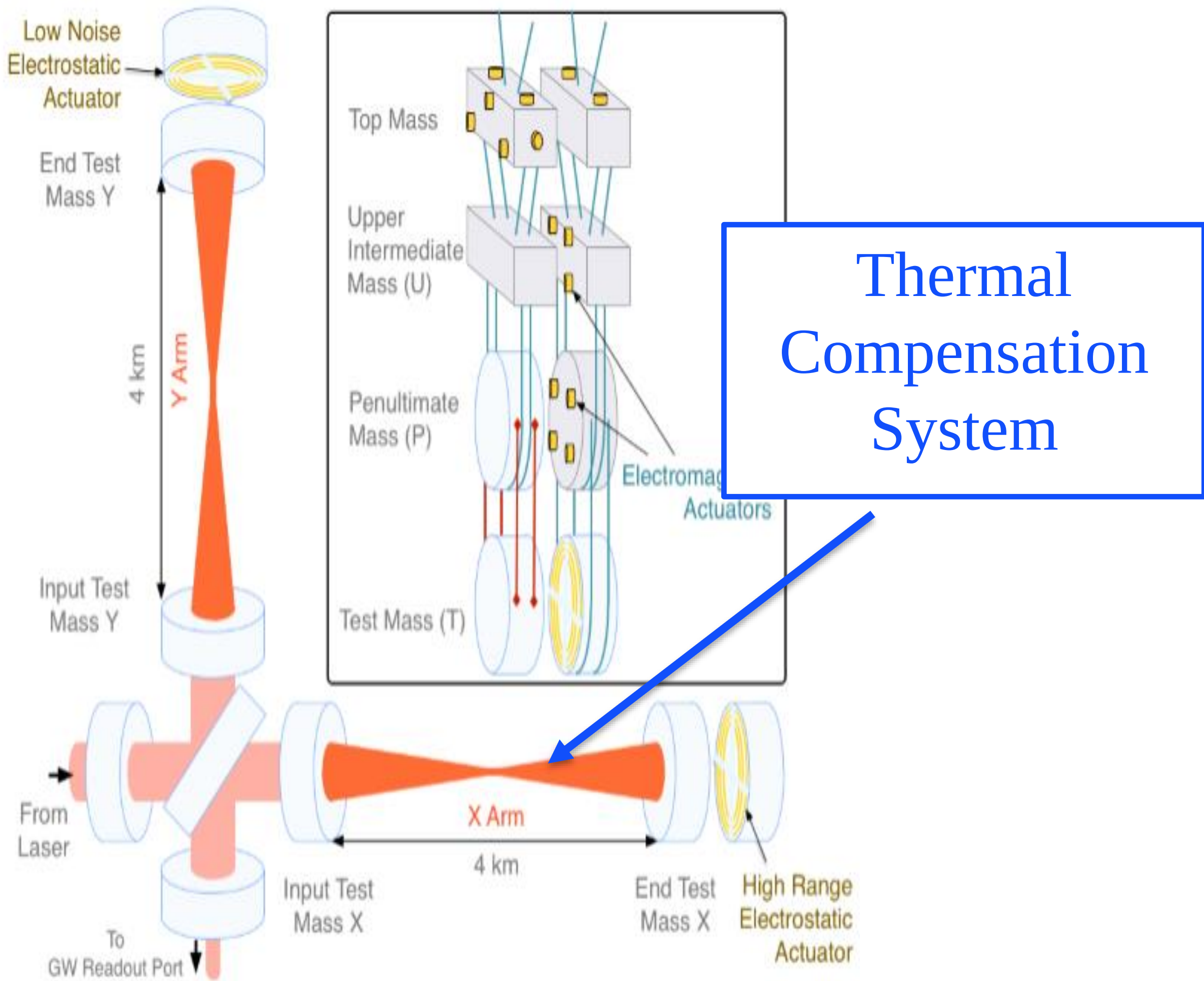
# Objectives:

**I will focus on the following topics:**

- Background – Thermal Compensation System
- Finite Element Modelling
- Parameterization
- Kalman Filter Implementation
- Results
- Future Work

# Advanced LIGO (aLIGO)

- Michelson interferometer with Fabry-Perot optical cavities
- Utilizes high-reflectivity fused silica mirrors
- Optical cavity with 800 kW ultimate optical power
- Apply Heat → Thermal transient forms in the mirrors

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# Thermal Transient Effects

- Two Main Effects:

Thermal Lensing →

- Aberrations in the beam
- Mode matching problems between cavities

→ **Reduces sensitivity of detector**

Thermal Expansion →

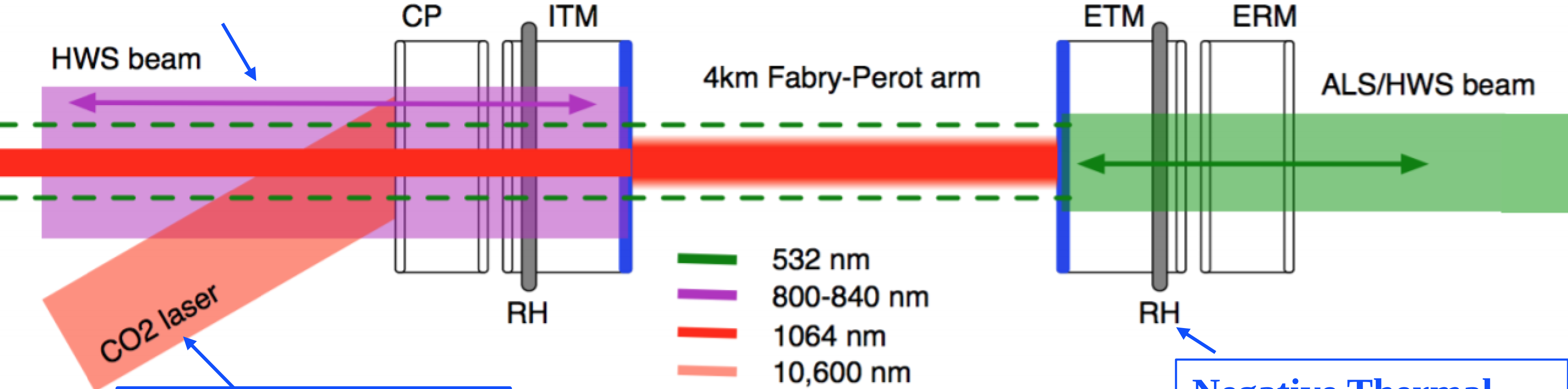
- Change of radius of curvature of mirrors (ROC)
- Shifts frequency of transverse optical modes (TEM)

→ **Parametric instabilities**
-Freq. between TEM & Fundamental mode = MM

Warming shifts mechanical mode (MM) frequencies

# Thermal Compensation System

**HWS-measures wavefront distortion**

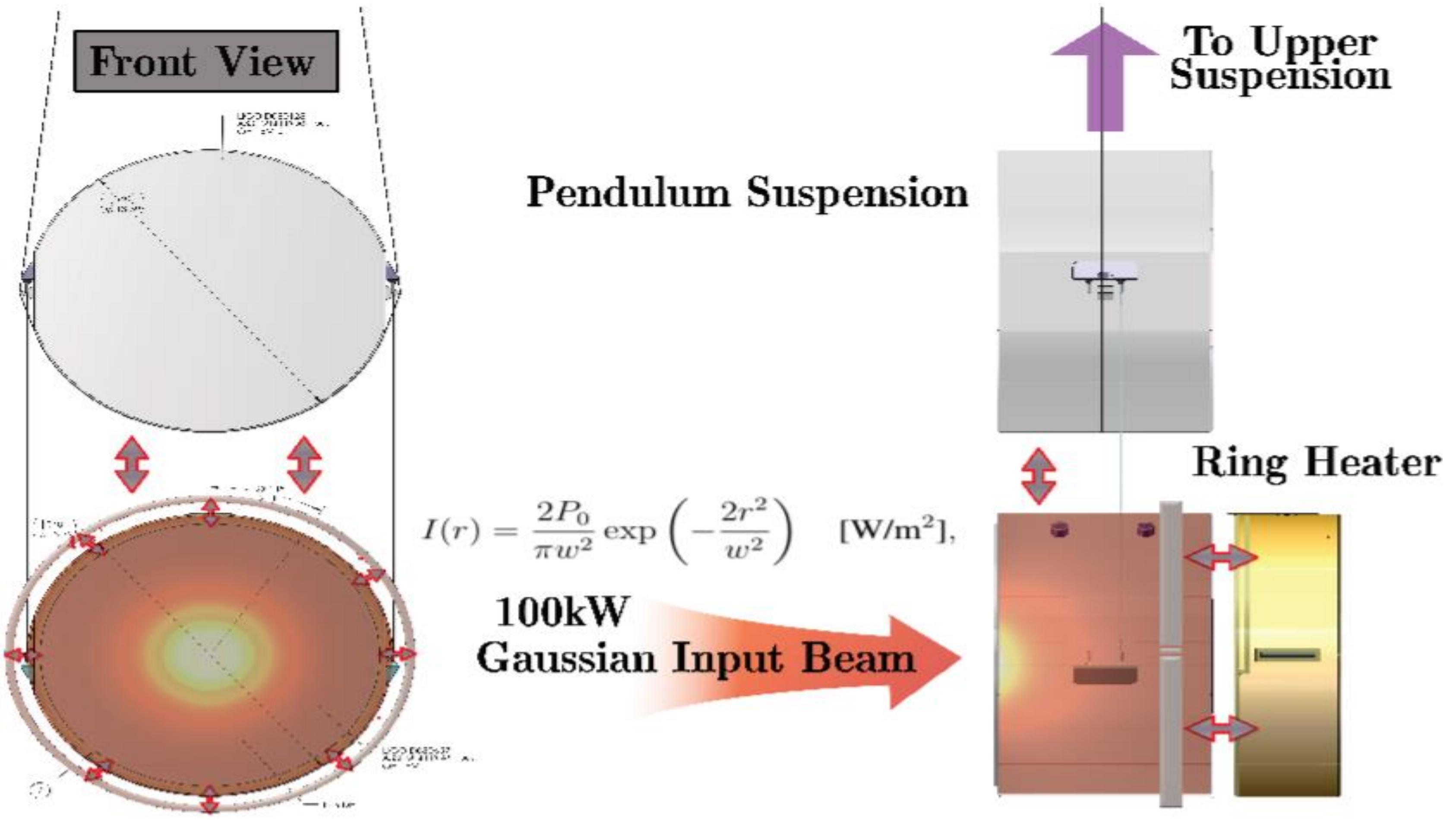



**Positive thermal lens**

**Negative Thermal lens, decreasing ROC**

- Purpose: compensate for laser power absorbed in test masses
- Helps mitigate thermal lensing optical distortion effects

# Shift in Mechanical Mode Frequencies



# Finite Element Model: Method

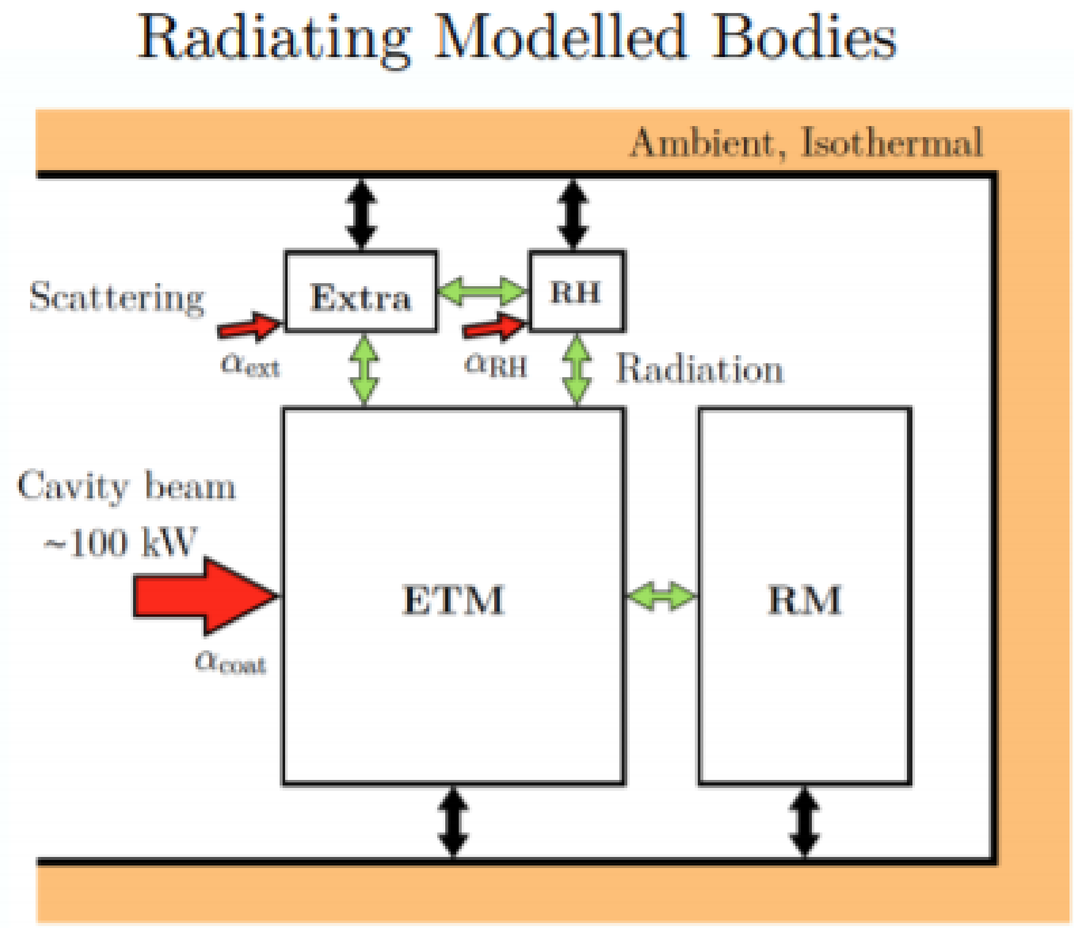
- Mechanical Mode Frequencies = Test Mass Thermometers
- Depend on:
  - Dimensions of the Test Mass (Mirror)
  - Elastic Constants:
    - Young's modulus $Y(T)_{bulk}$- relation between stress and strain- uniaxial deformation
    - Poisson's Ratio $\nu$- ratio between transverse strain to axial strain

$$\omega_m = \beta_m \sqrt{\frac{Y(T)_{bulk}}{\rho(1+\nu)}} \quad [1]$$

- Mechanical Mode Frequency: $\omega_m$
- Constant Dependent on the geometry of the cylinder: $\beta_m$

# Finite Element Model: Thermal Model

- Heat Transfer model between ETM & surrounding elements
- Transfer of heat when arm cavity is locked
- Mechanisms involved
  - RH
  - RM
  - Extra Term: Complex Structures surrounding it

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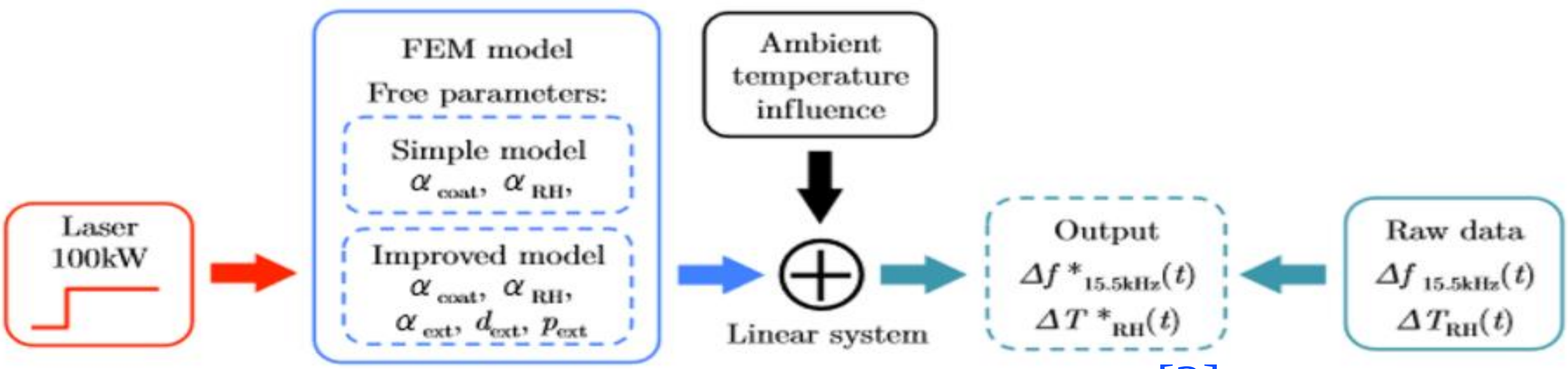
# Finite Element Model: Thermal Model

- aLIGO test mass
  - Cylinder: 170 mm radius, 200 mm thickness
  - Heraeus Suprasil 3001 fused silica
  - 100 kW laser beam
  - Coating absorption of 1 ppm corresponds to total absorbed energy 0.1 W
- Inputs, outputs, and the free parameters involved when modelling a test mass

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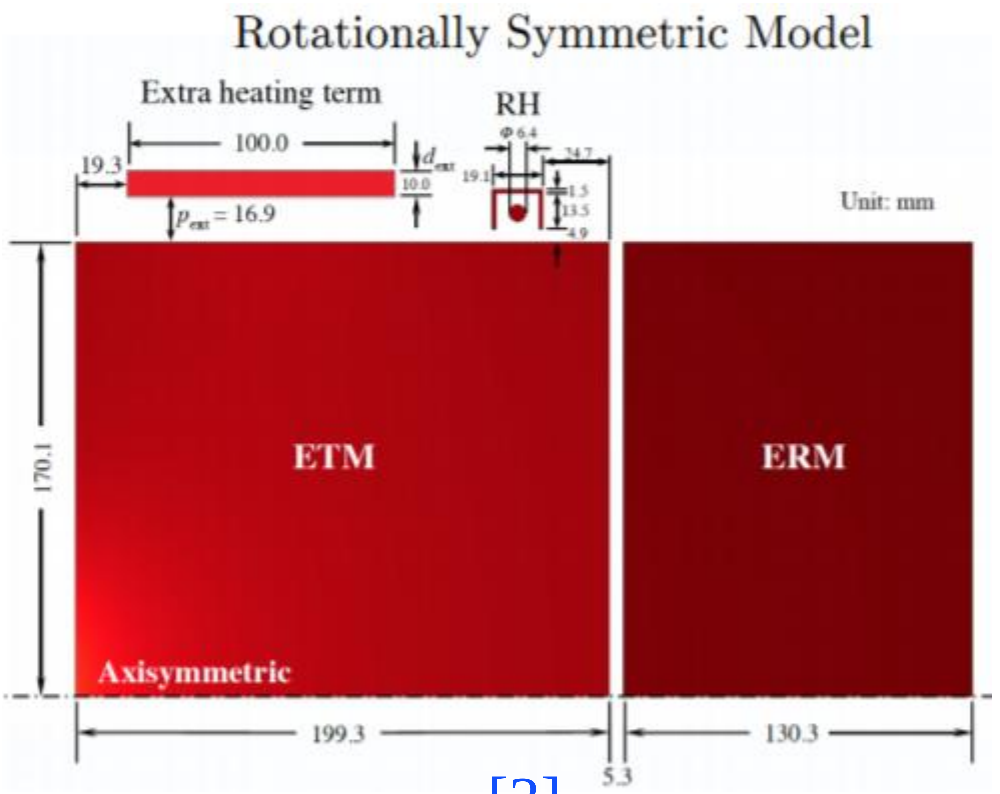
# Finite Element Model: COMSOL

- 2-dimensional Axis-symmetric representation of the system
- Input laser beam heating load
- Restricted to only monitor circularly symmetric mechanical eigenmodes
- LIGO historic data for 5.9, 6.0 and 8 kHz modes
  - Only the 8 kHz mode is axis-symmetric

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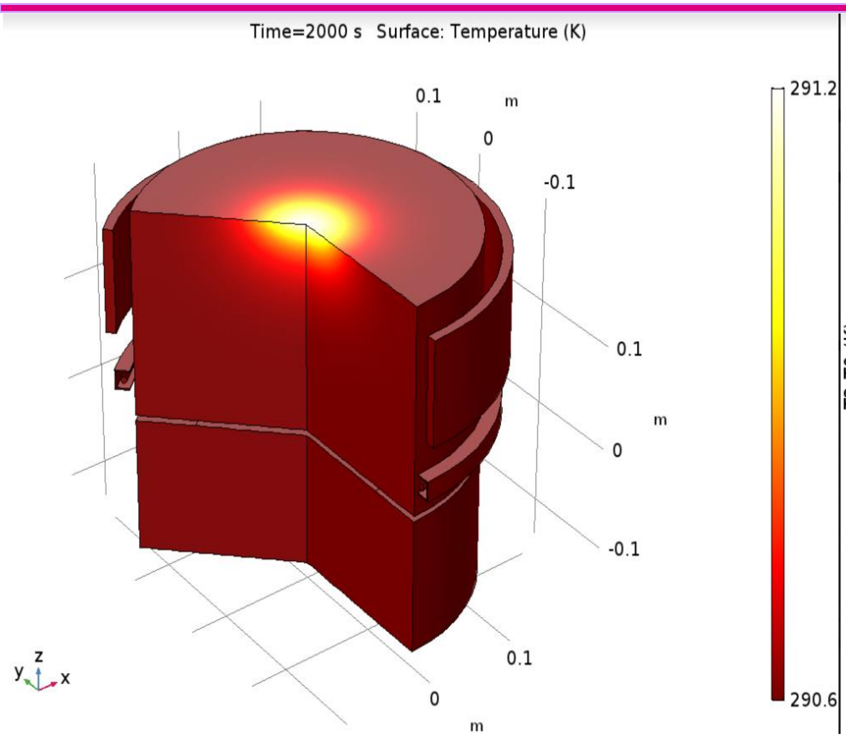
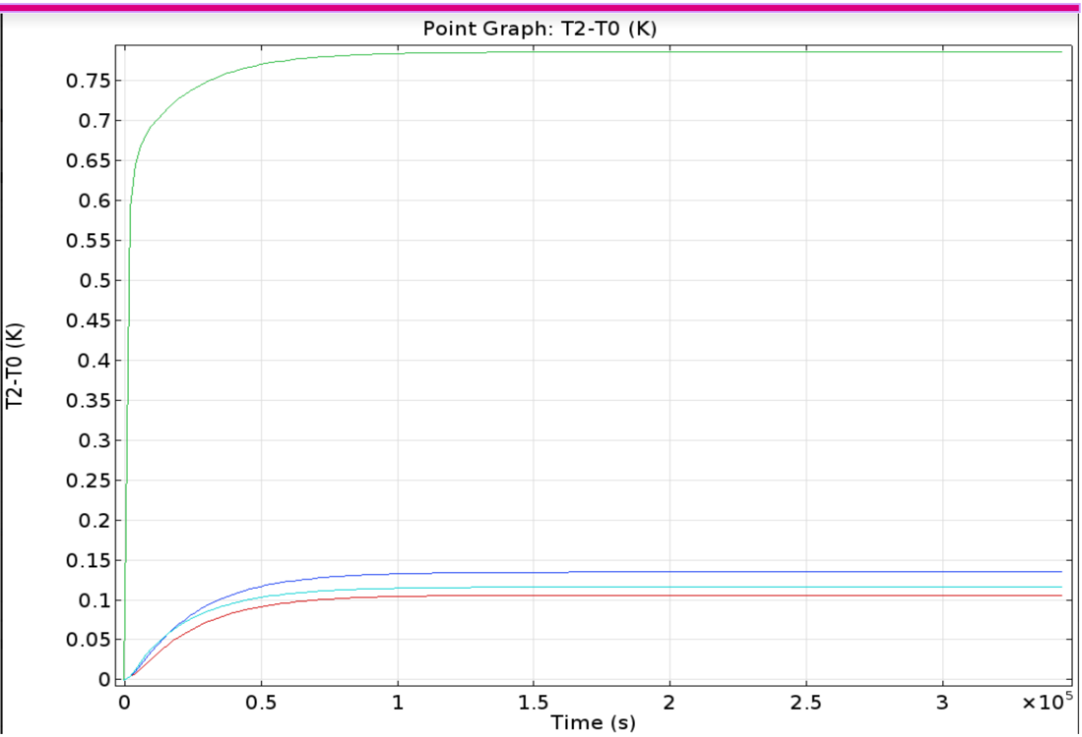
# Finite Element Model: COMSOL



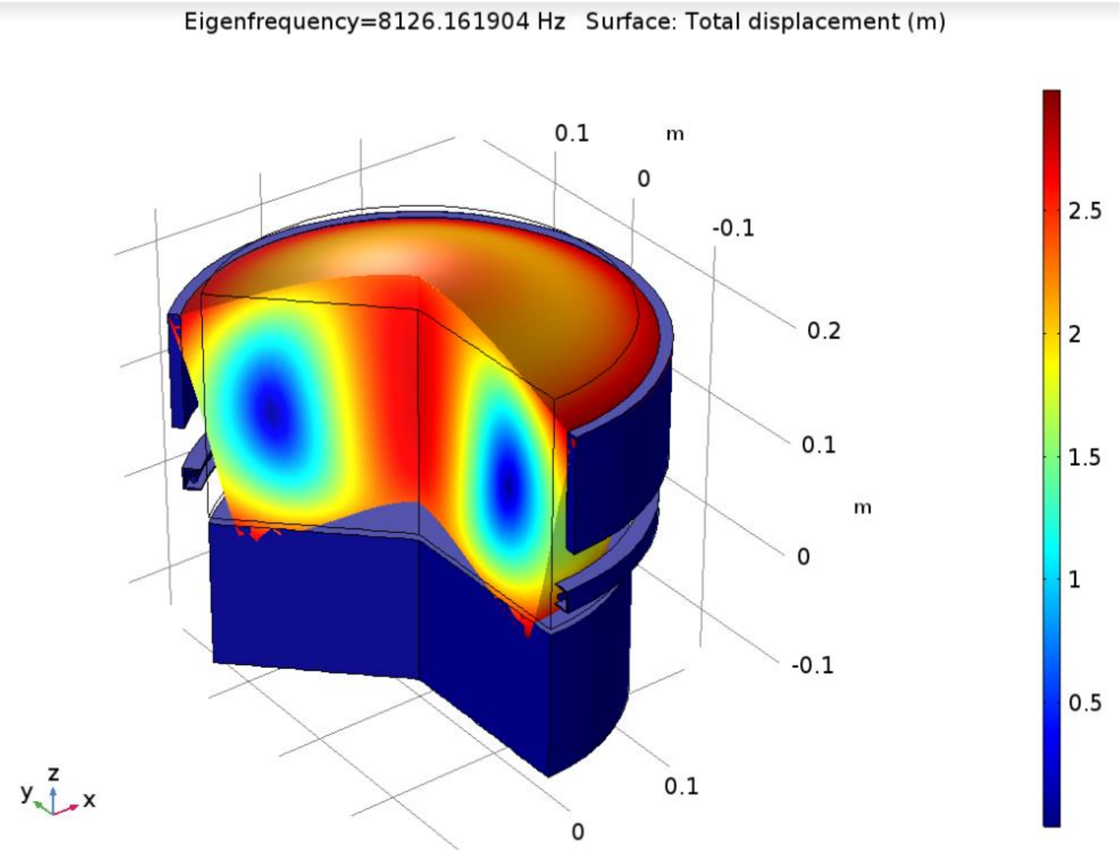

**COMSOL depiction of ETMX**




**Temperature Change with Self Heating**

Applying Heat Equation in a system with a fixed laser beam

# Finite Element Model: COMSOL




- Modelled ETM mode shape for the 8 kHz eigenmode

# Model Parameterization

❖ Take COMSOL numeric simulation model of 8 kHz eigenfrequency shift and fit a model

**First- Order Exponential Model**

$$A\left(1 - e^{-b_1 t}\right) + c_1$$

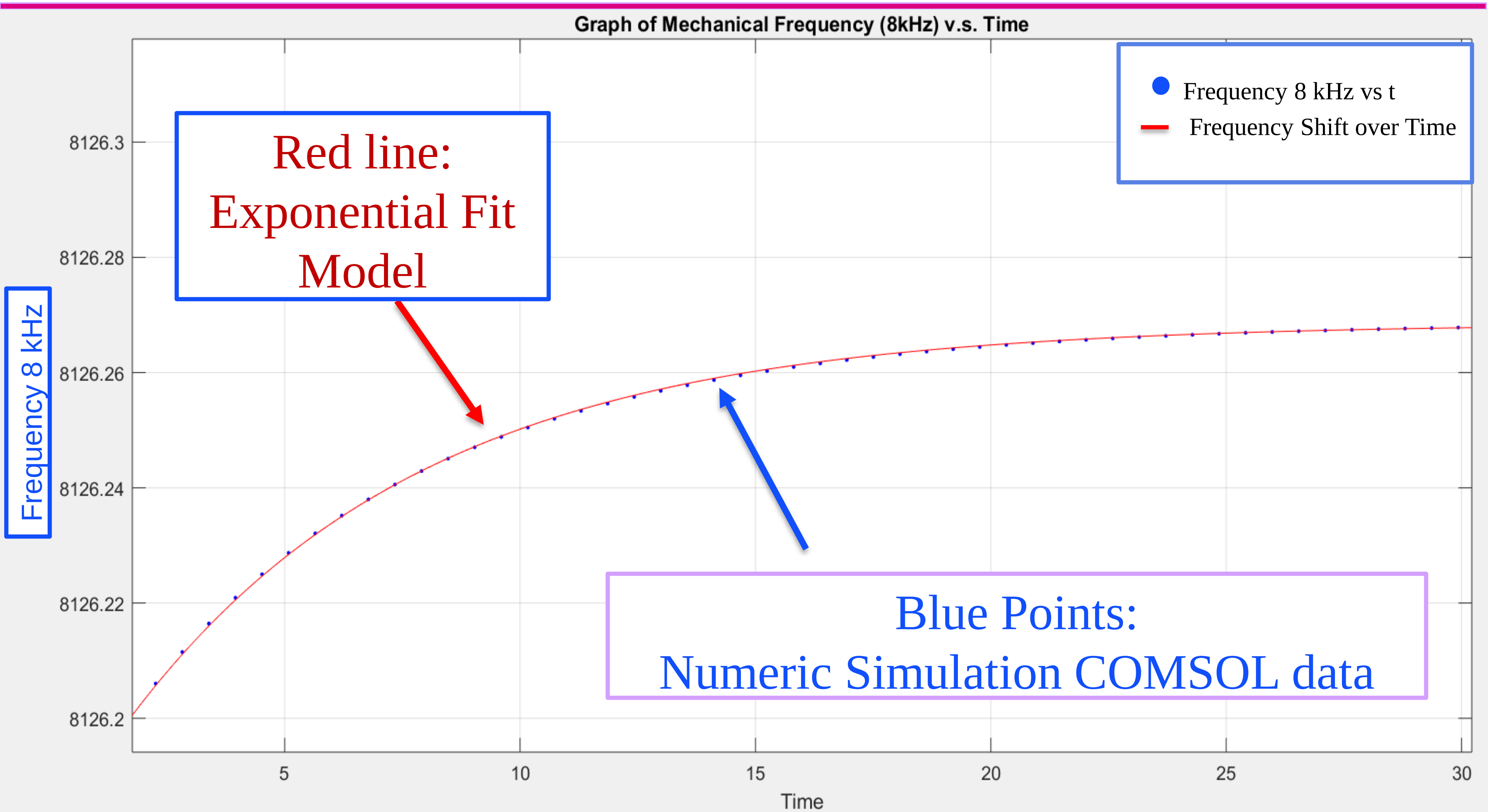
A: Total change in frequency
$b_1$: Model time constant
$c_1$: Frequency at room temperature

**Second-Order Exponential Model**

$$A\left(1 - 2e^{-b_2 t} + e^{-2c_2 t}\right) + d$$

A: Total change in frequency
$b_2$ & $c_2$: Time constants
d: Frequency at room temperature

# Model Parameterization



# Coating Absorption Extraction Techniques

**Simon Tait's Technique**

↓

Applying the exponential model and tracking eigenfrequency shift data

↓

Experimental frequency tracking to extract coating absorption

**This Project's Technique**

↓

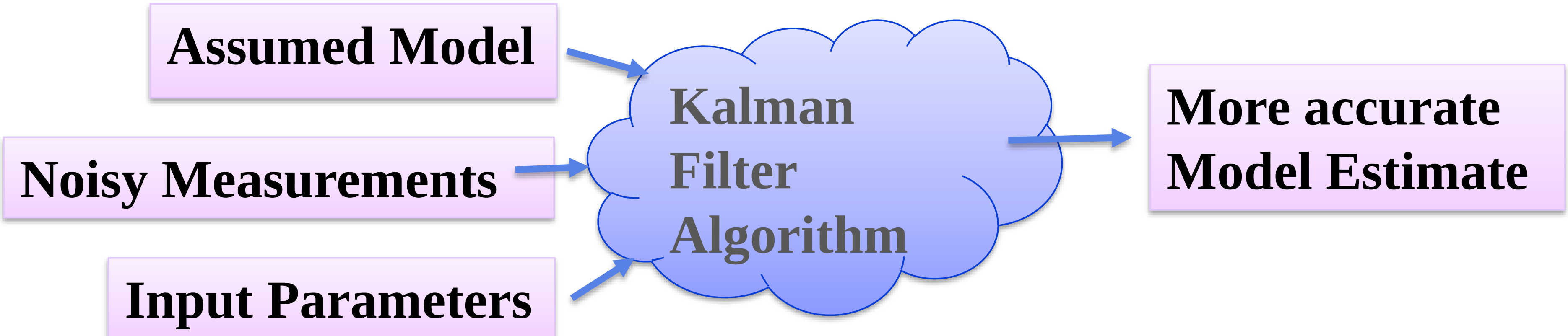
Applying the exponential model, eigenfrequency measurements, control parameters

↓

Implement a **Kalman Filter** to extract coating absorption

# Kalman Filter Theory

- Recursive algorithm
- Input:  A linear model and noisy measurements
- Output: Less noisy and more accurate estimates
- Only requires current state to propagate to next time step
- Error (variances) are used to optimize estimates




- Combines inputs and measurements into model of the system → **minimize uncertainty in the model**

# Kalman Filter Theory

- Requires state space representation of system
- Present state is dependent on the previous state:

$$\boldsymbol{x_k} = \boldsymbol{A_k x_{k-1}} + \boldsymbol{B_k u_k} + \boldsymbol{w_k}$$

$\boldsymbol{A_k}$: State Transition Model

$\boldsymbol{x_{k-1}}$: Previous state

$\boldsymbol{B_k}$: Input Control Model

$\boldsymbol{u_k}$: Control Vector

$\boldsymbol{w_k}$: Process noise with $\boldsymbol{Q_k}$ *covariance*

- Observation is taken representing the true state $x_k$:

$$\boldsymbol{z_k} = \boldsymbol{C_k x_k} + \boldsymbol{v_k}$$

$\boldsymbol{z_k}$: Observation

$\boldsymbol{C_k}$: Observation Model

$\boldsymbol{v_k}$: Measurement noise with $\boldsymbol{R_k}$ *covariance*

# Kalman Filter Theory

## Prediction:

- Predict state ahead

  $\boldsymbol{x_{k|k-1}}$

- Predict the error covariance ahead

  $\boldsymbol{P_{k_k-1}}$

## Correction:

- Calculate Kalman Gain (Minimum Mean Square Error: minimize trace of the state error)

  $\boldsymbol{K_k}$

- Update estimate with measurement

  $\boldsymbol{y_k}$ & $\boldsymbol{x_{k|k}}$

- Update error covariance

  $\boldsymbol{P_{k|k}}$

Initiate with $x_{k-1}$ $and$ $P_{k-1}$

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# Building a Kalman Filter

1. Understand the situation → **Inputs:** Exponential Model, Noisy Eigenfrequency Measurements
   **Input Control Parameter:** Laser Power
2. Model the state process
3. Model the measurement process
4. Model the noise
5. Test the Filter
6. Refine the Filter

| Advantages | Disadvantages |
| --- | --- |
| • Recursive nature<br>  • Does not depend on the history to determine the next state | • Relies on an accurate model<br>• Depends on linearity of system |

# Project Initial Kalman Filter

**Approach:**

Normalized exponential model

$$f(t) = \mu\left(1 - e^{-\frac{t}{\tau}}\right)$$

**State-Space Representation:**

$$f(s) = \mathcal{L}\left\{\mu\left(1 - e^{-\frac{t}{\tau}}\right)\right\} = \frac{N}{s^2 + Ds} \qquad N = \frac{\mu}{\tau} \qquad \mathrm{D} = \frac{1}{\tau}$$

Transfer function of the system: $\frac{f(s)}{P(s)} = sf(s) = \frac{N}{s+D}$

Inverse Laplace Transform → Differential Equation

$\mathcal{L}^{-1}\{f(s)(s + D) = P(s)N\}$ $\qquad \dot{f}(t) + Df(t) = Np(t)$

$$[f_k] = [1 - D\Delta k][f_{k-1}] + [N\Delta k][P_k]$$

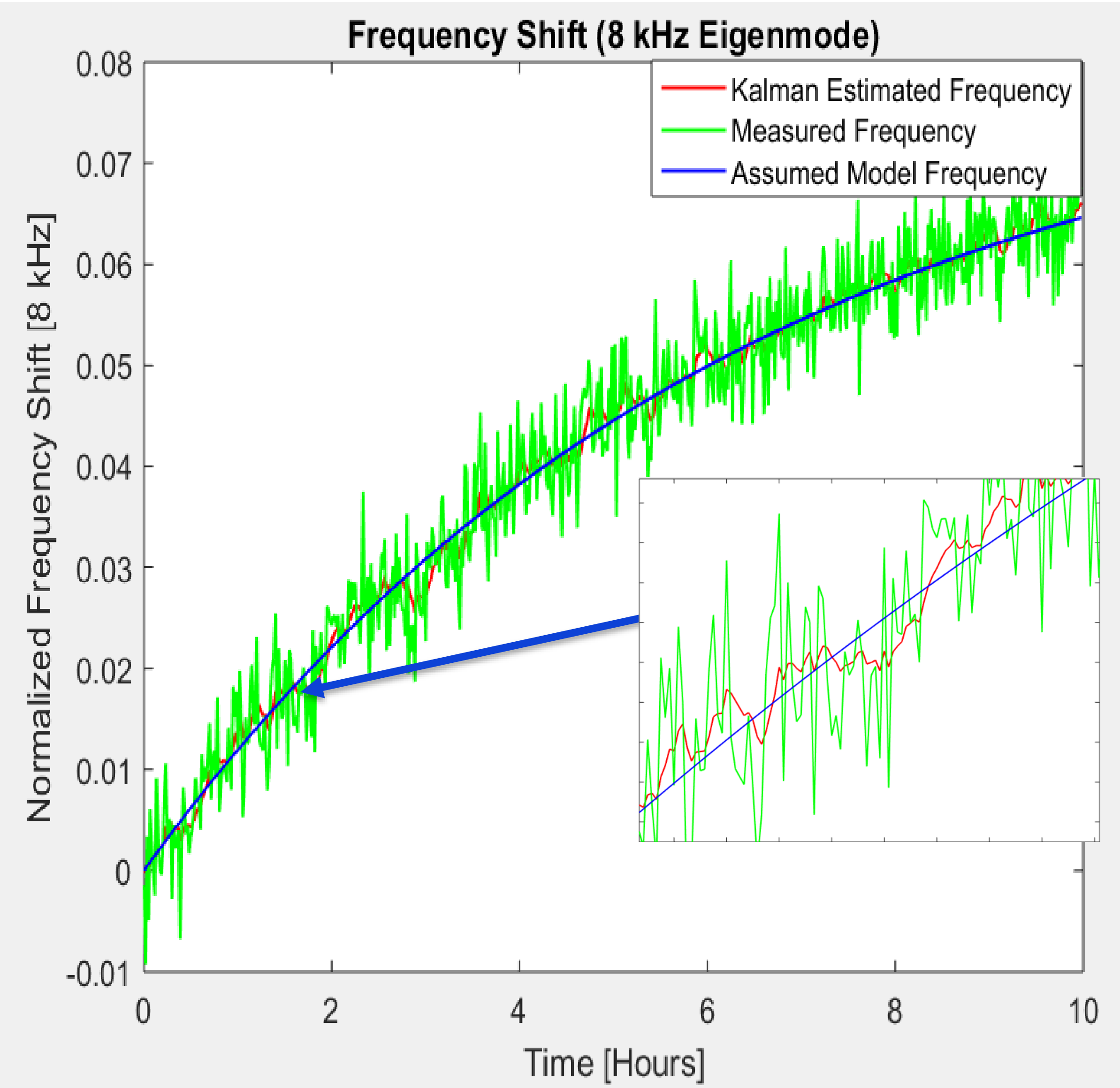
**State-Matrix:** $A = [1 - D\Delta k]$ **Input-Control Matrix**: $B = [N\Delta k]$

**Measurement-Matrix:** C = [1] **Observation Model**: $z_k = f_k$

**Parameters:**

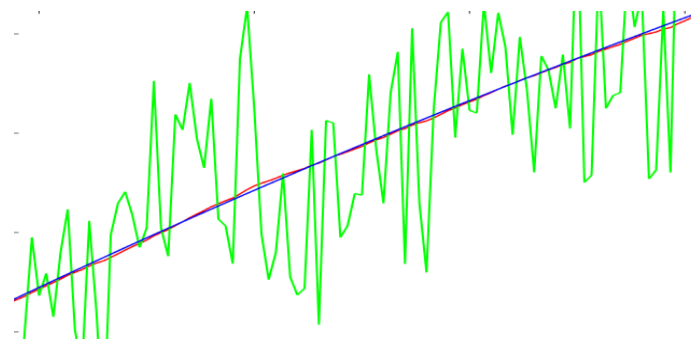
| **Gain ($\mu$)** | **Time Constant ($\tau$)** |
|---|---|
| **0.8114 Hz** | **6.289 Hrs.** |

# Initial Kalman Filter: Simulated Results




## Underfit:

- $R_k >> Q_k$ variance
- Trusts model over measurements



## Overfit:

- $Q_k >> R_k$ variance
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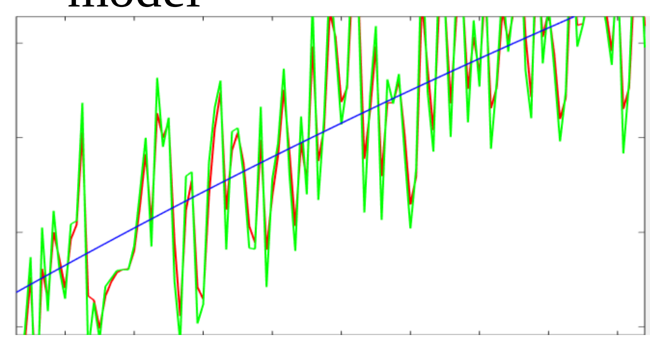


# Problem Extracting Coating Absorption

- Dependent on the change of the gain parameter:

$$P_\alpha = \frac{2(P_{in} k_{PRC} k_{AC})}{\pi \omega^2} e^{\left(-\frac{2r}{\omega^2}\right)} \frac{1}{\alpha_c}$$

$P_\alpha$: Power absorbed by the optic

$\omega$: beam radius of the incident Gaussian light source (6.2 cm)

r: distance from the center of the beam

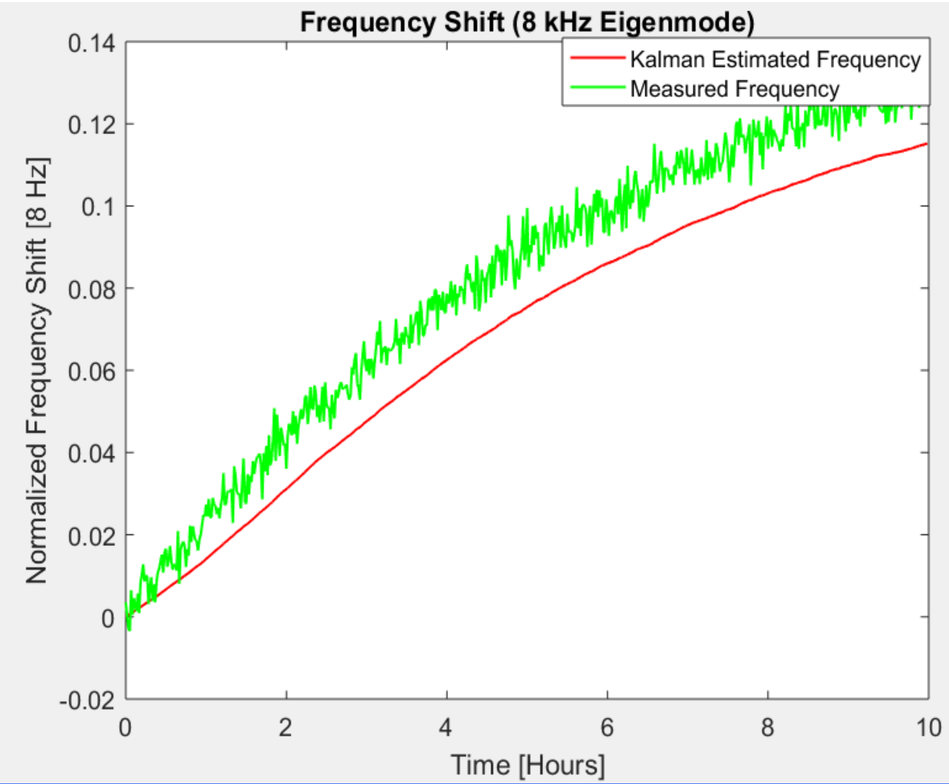
$P_{in}$: power input into the interferometer

$k_{PRC}$: gain of the power recycling cavity

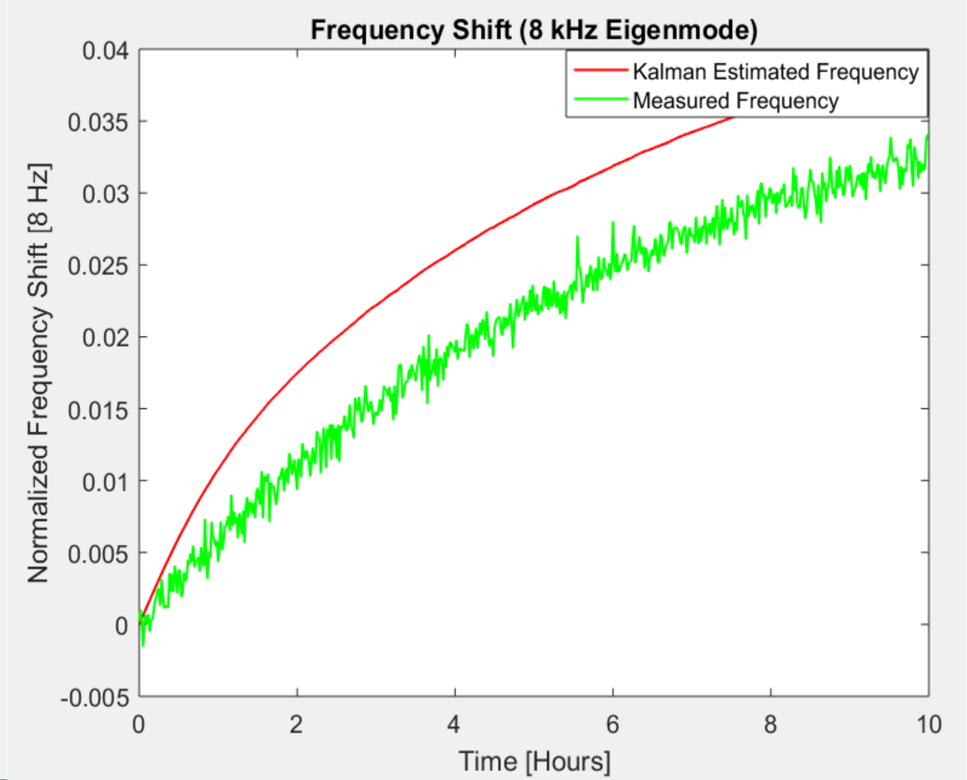
$k_{AC}$: gain from the arm cavity

$\alpha_c$: coating absorption

# Problem Extracting Coating Absorption




Simulated Lock state where the noisy measurements have factor of 2 multiplied to input laser power




Simulated Lock state where the noisy measurements have factor of 0.5 multiplied to input laser power

# Problem Extracting Coating Absorption

- Coating absorption $\alpha_{coating}$ is proportional to the gain parameter in the state space model
- The gain is not linearly related to the system
- Kalman Filters function with linear systems
- Options:
  - Linearize the parameter to the system
  - Create a nested Kalman Filter that updates the change in gain

# Nested Kalman Filter Approach

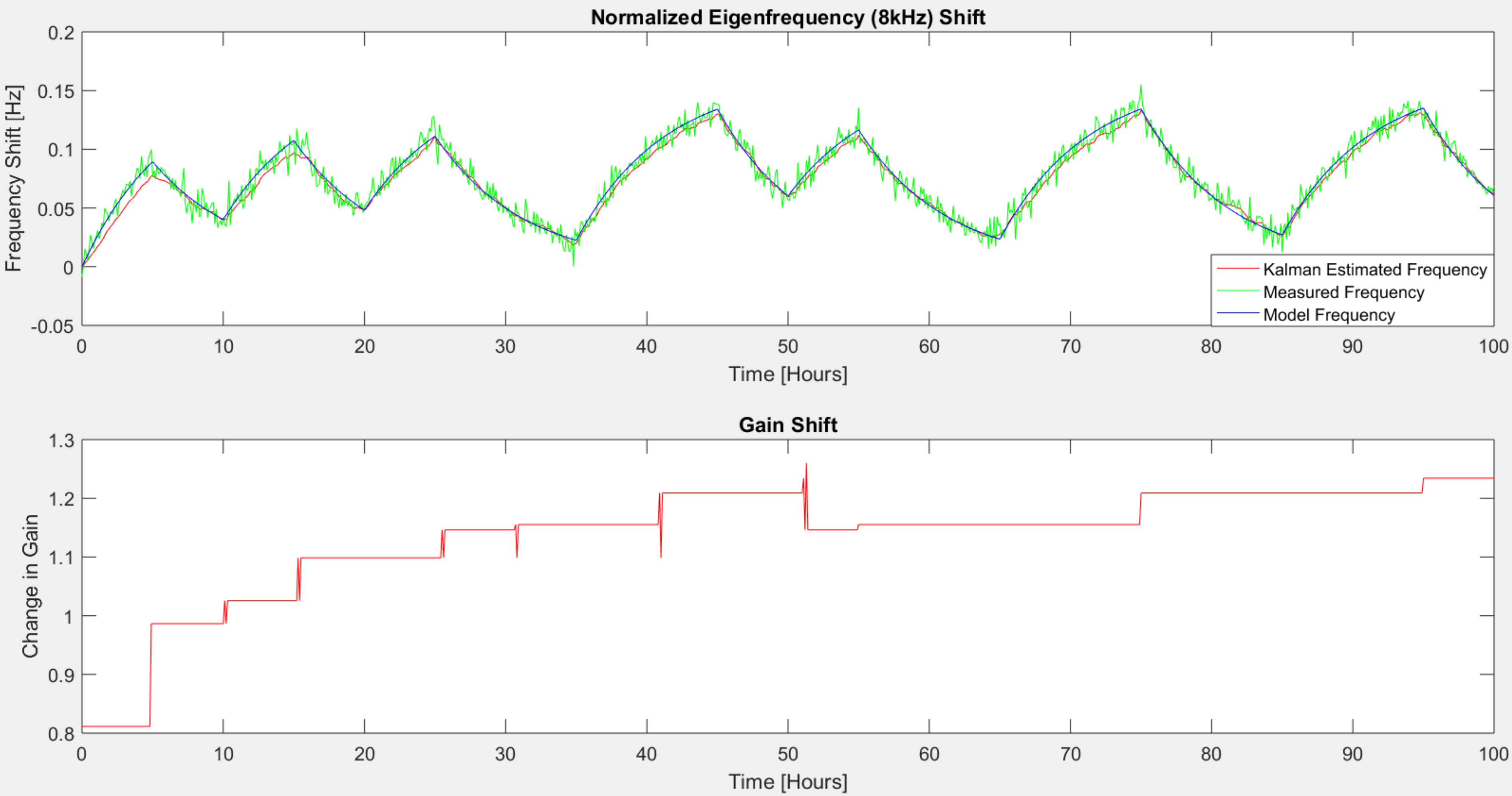
- Gain directly related to the input-control model B
- Update B and the process covariance at the end of each lock state

Process Covariance: $Q_k = Bw^2B'$

- Measure average residuals during lock time frame between measurement data and Kalman Estimate

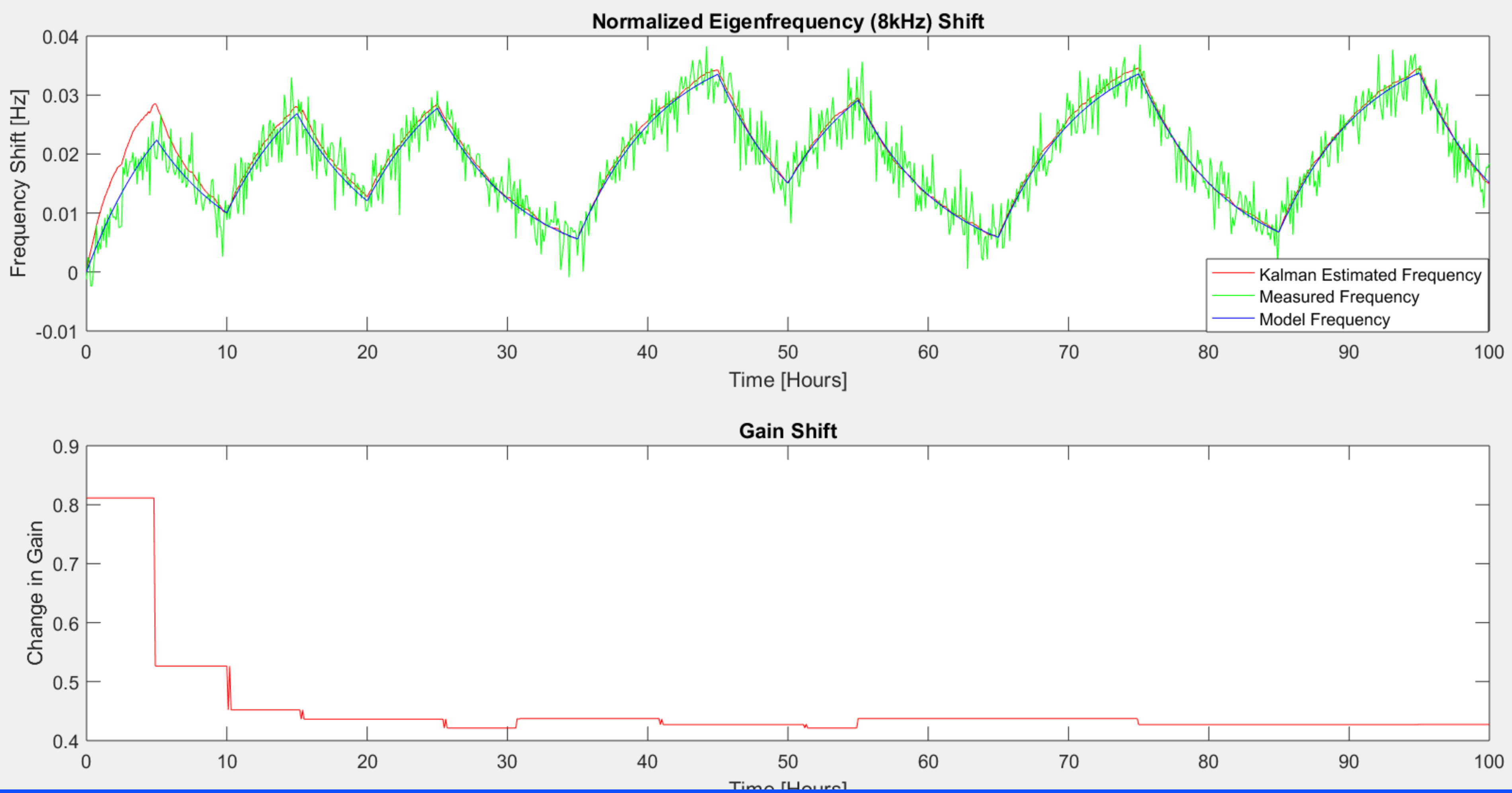
$$B_{updated} = B_{initial} + (average\ residual) * B_{initial}$$

# Nested Kalman Filter: Overfit Simulation



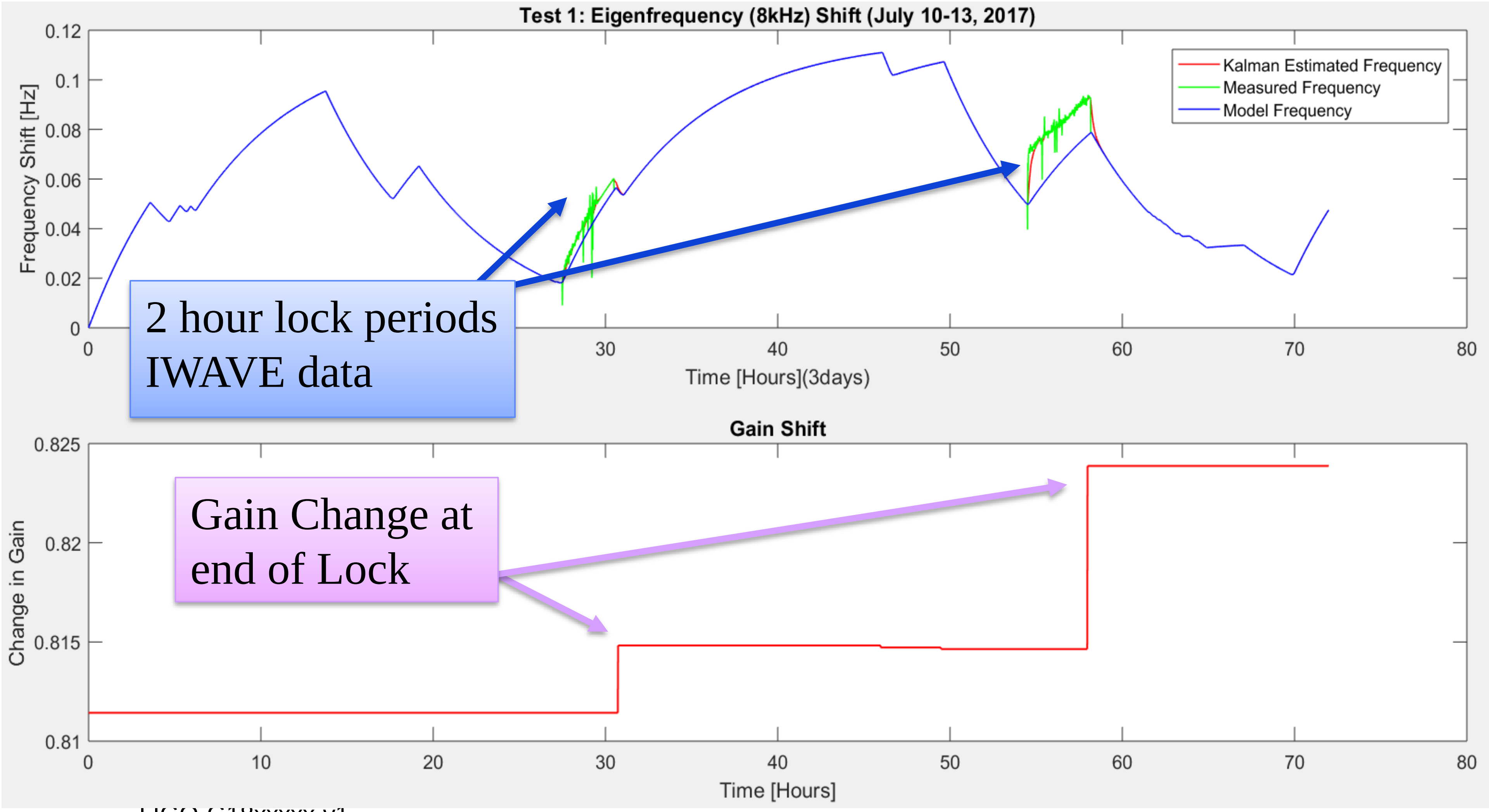

Simulated period of locked and unlocked states where the noisy measurements have factor >1 applied to the input laser power and the nested Kalman Filter updates in a bias towards the noisy behavior

# Nested Kalman Filter: Underfit Simulation

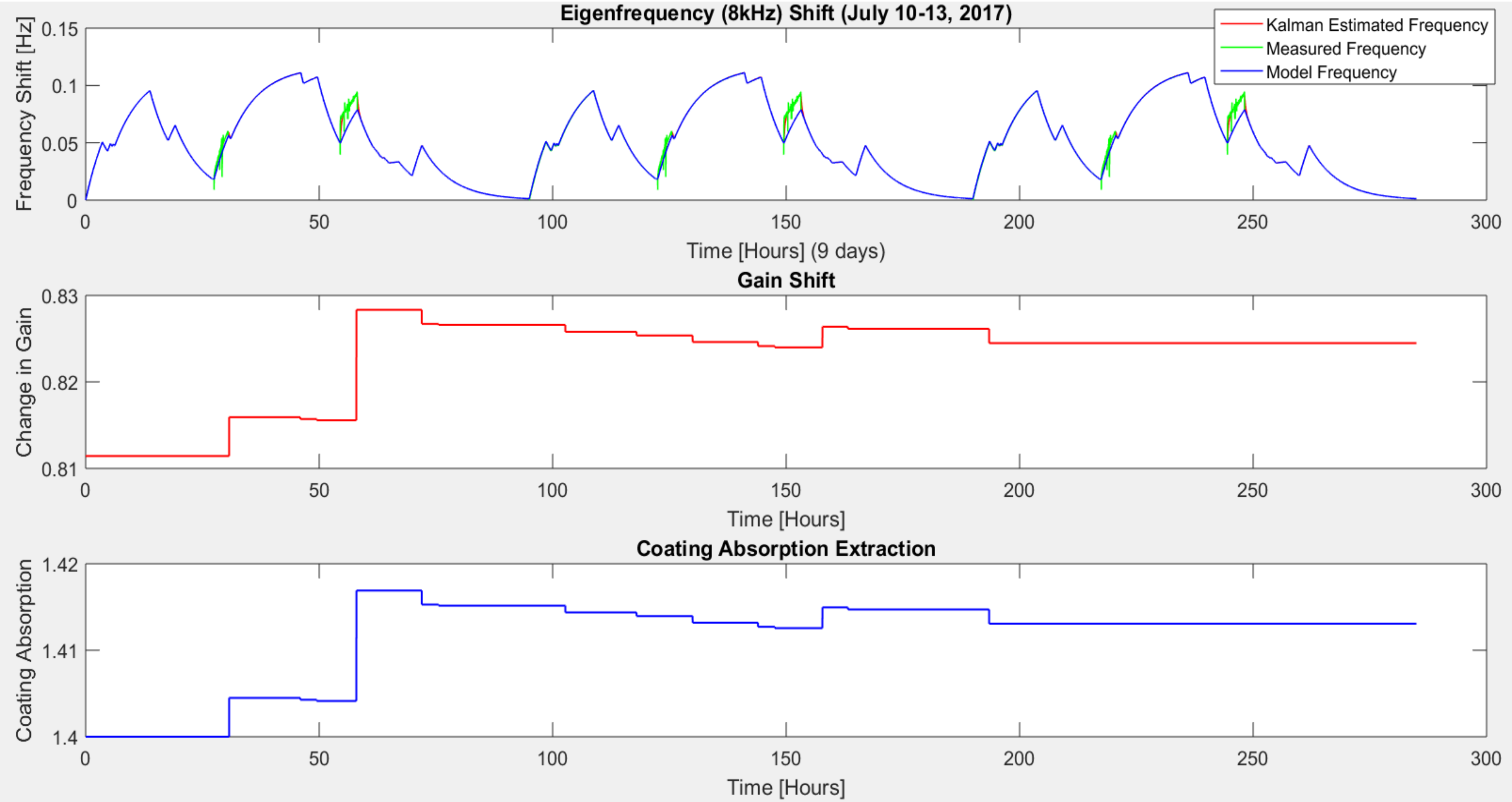



Simulated period of locked and unlocked states where noisy measurements have a factor <1 applied to the input laser power and nested Kalman Filter updates itself in a bias towards the noise's behavior

# Results: Testing Data from July 2017



# Results: Testing Data from July 2017



# Conclusions

## Takeaways:

- Several other factors including ambient temperature need to be incorporated to improve the model of the system
- Kalman Filters provide useful monitors and more accurate models of a system

## Future Work:

- Run the filter over longer periods of time with IWAVE data to improve absorption estimate
- Improve and change the model to incorporate other parameters, remove outliers from noisy IWAVE frequency data
- Implement the Kalman Filter as a real-time LLO monitoring system to further improve absorption estimation and other parameter estimations
- Combine this time evolution ( 1 eigenmode over time) behavior with spatial evolution (several eigenmodes) behavior to create a stronger mirror degradation monitor

# Acknowledgements

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- Alan Weinstein for organizing and administering the LIGO SURF program
- All LLO staff, Caltech SURF, NSF, and LIGO Scientific Collaboration

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[6] A. Brooks, *Seminar on Kalman Filters*. (2014).

# Any Questions?

# In-Depth Kalman Filter Analysis

$\boldsymbol{A_k}$ represents the state transition model used to the previous state $\boldsymbol{x_{k|k-1}}$ and $\boldsymbol{B_k}$ represents the input-control model. The input-control model is applied to the control-vector $\boldsymbol{u_k}$ and the state matrix is applied to the state-vector $\boldsymbol{x_{k|k-1}}$. The process noise is represented by which in this case is a univariate normal distribution with covariance $\boldsymbol{Q_k}$.

$$\boldsymbol{x_{k|k-1} = A_k x_{k|k-1} + B_k u_k}$$

$$\boldsymbol{P_{k_k-1} = A_k P_{k-1|k-1} A_k^T + Q_k}$$

Calculating Kalman Gain:

$$\boldsymbol{S_k = C_k P_{k|k-1} C_k^T + R_k}$$

$$\boldsymbol{K_k = P_{k|k-1} C_k^T S_k^{-1}}$$

Updating Estimate with Measurement and updating error covariance

$$\boldsymbol{y_k = z_k - C_k^T x_{k|k-1}}$$

$$\boldsymbol{x_{k|k} = x_{k|k-1} + K_k y_k}$$

$$\boldsymbol{P_{k|k} = (I - K_k C_k) P_{k|k-1}}$$