

Searching for Lensed Gravitational Wave Signals from Compact Binary Coalescences

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3 Method, 4 Results and 5 Future work

1 Background

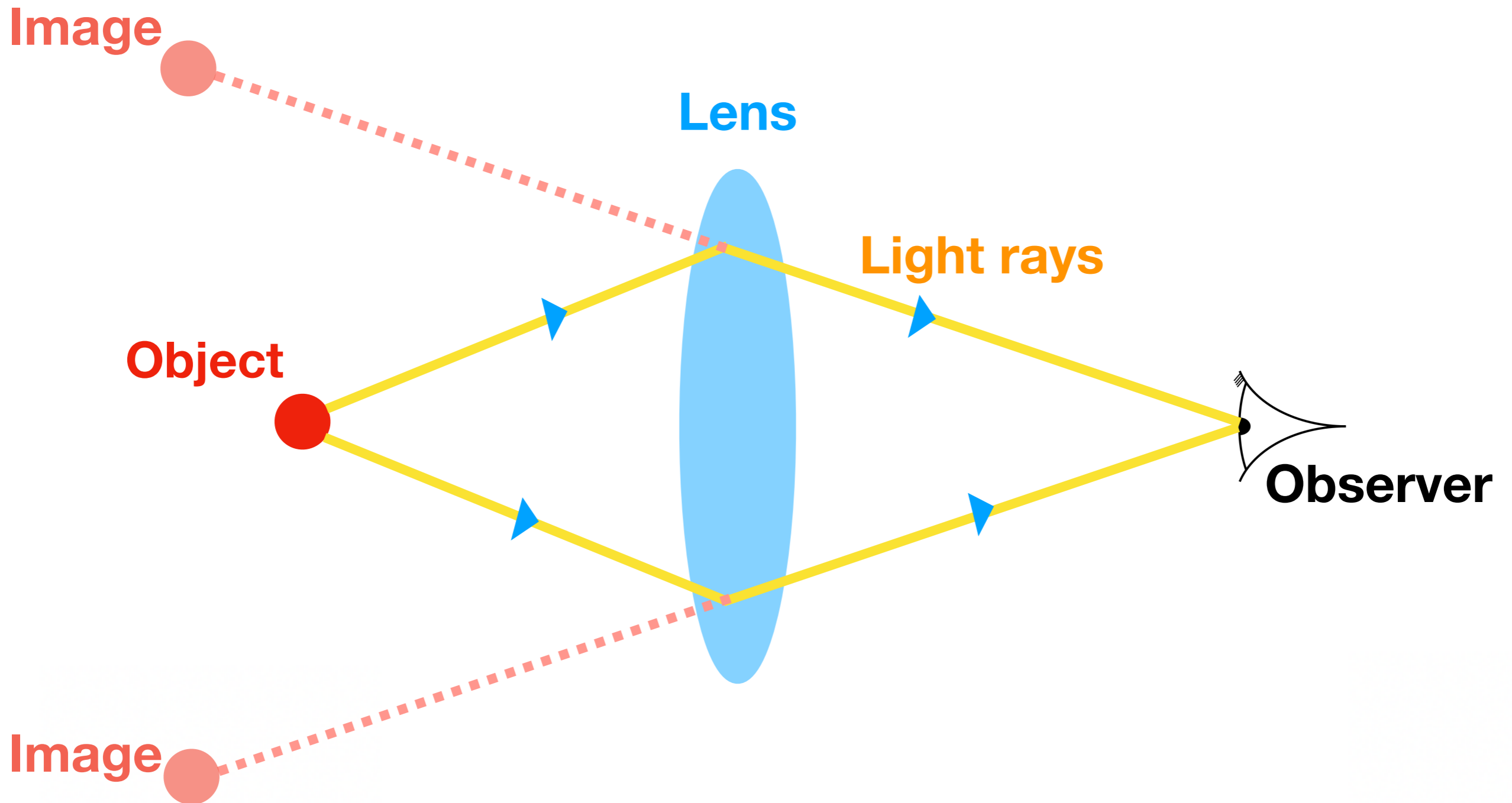
Searching for Lensed

Gravitational Wave Signals from

Compact Binary Coalescences

2 Theory

Gravitational Lensing? What's that?



Gravitational Lensing? What's that?

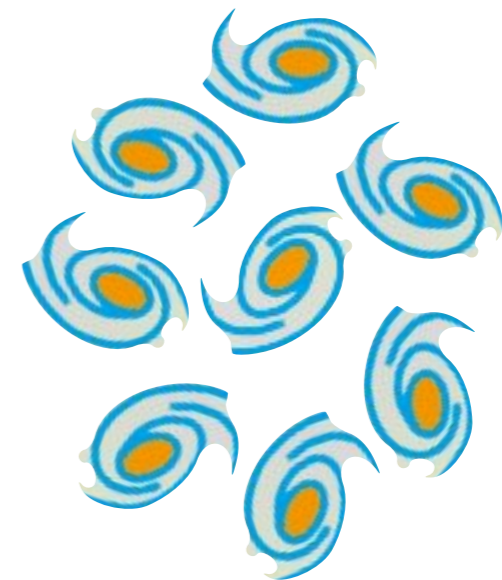
What is this?



A galaxy



What are these?



Many galaxies

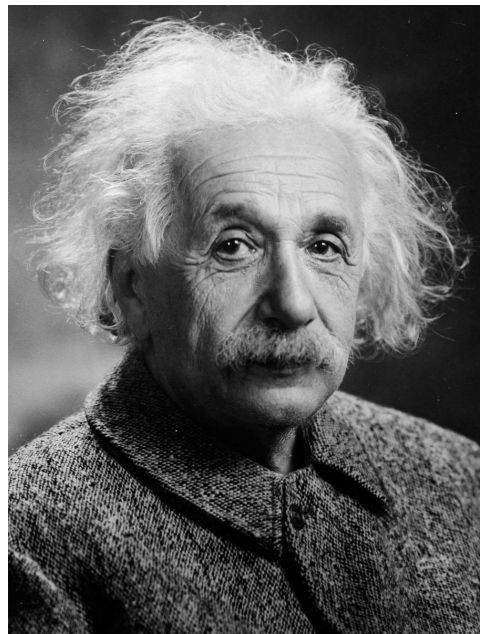


Galaxy cluster

Explain a bit more, please....

- Quoting John Archibald Wheeler :

“Spacetime tells matter how to **move**,
matter tells spacetime how to **curve**.”

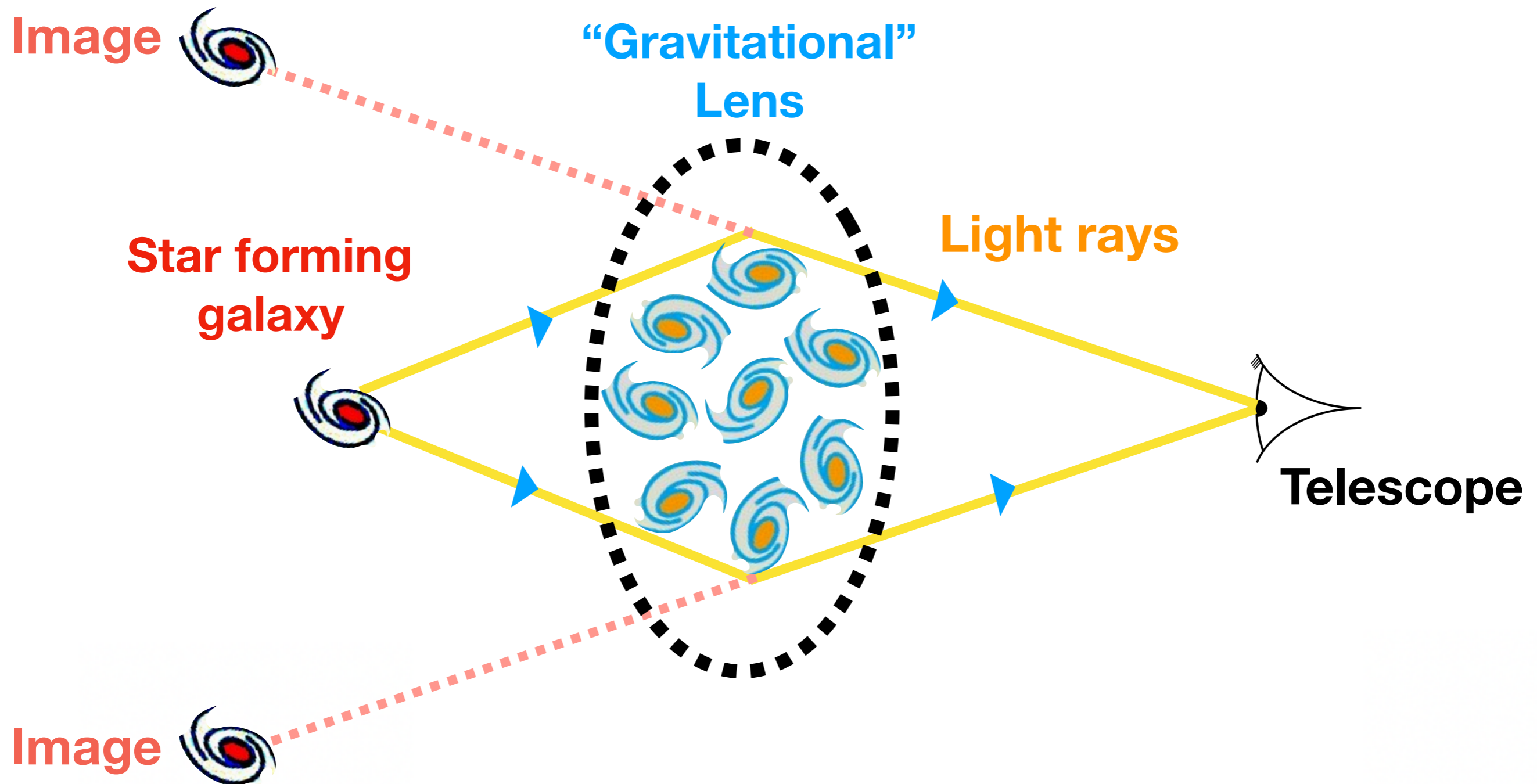


- According to General Relativity :
 - ☑ Masses can **curve spacetime**.
 - ☑ Path of light rays can be **bent and deflected**.

Gravitational Lensing!

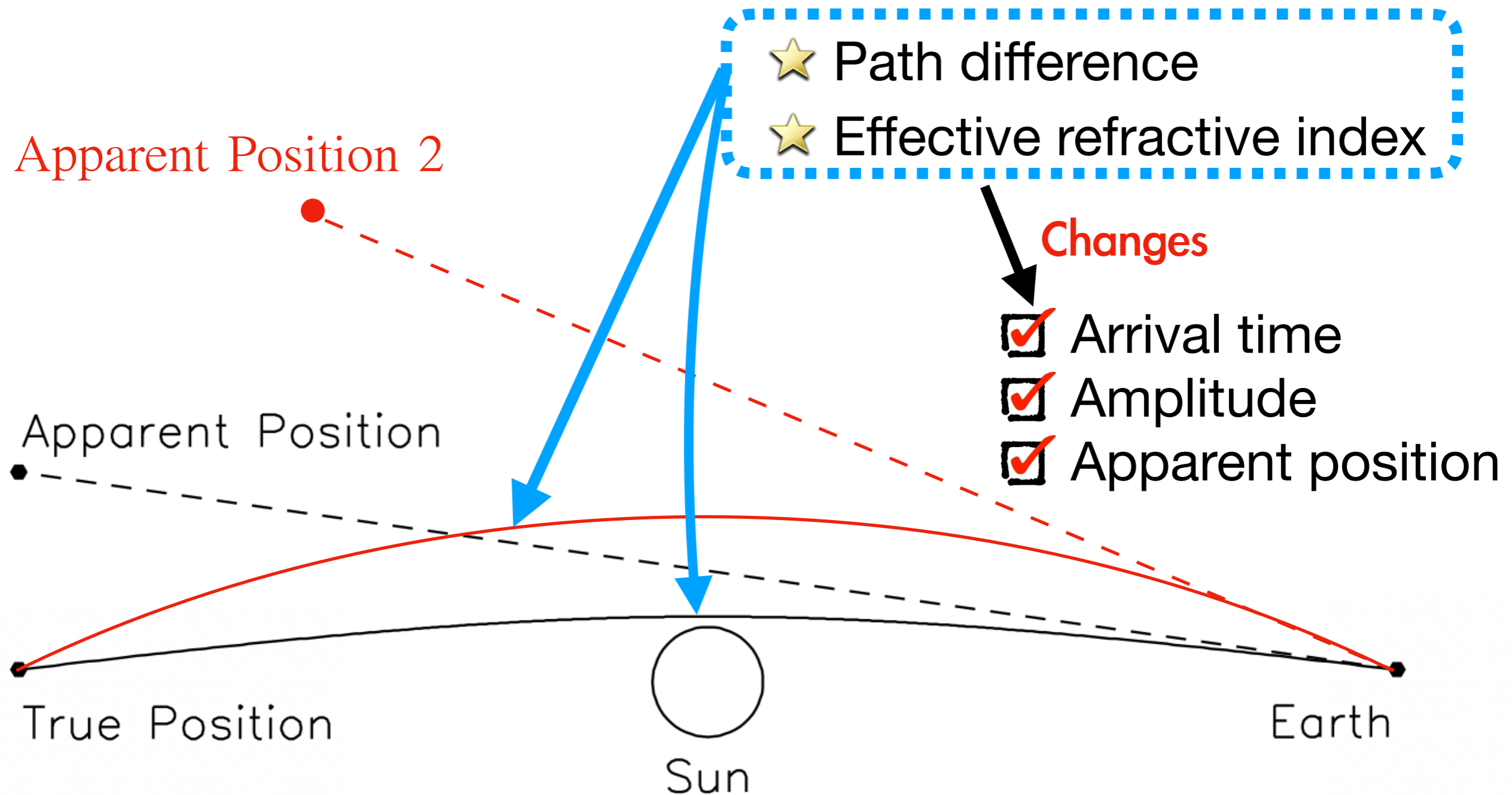


Gravitational Lensing? What's that?



So what happens to the image?

- For TRANSIENT objects...



Wait...That's for light, not GW!

- By the **EQUIVALENCE PRINCIPLE**, all EM waves, as well as **GW**, can be gravitationally lensed in the same way.

So why not study light but GW?

• Obvious reasons :

★ This is **LIGO SURF**, not **ASTRO SURF**

★ Light have already been **extensively studied**.

• Not-so-obvious reasons :

○ Light can be blocked by **dust clouds**.

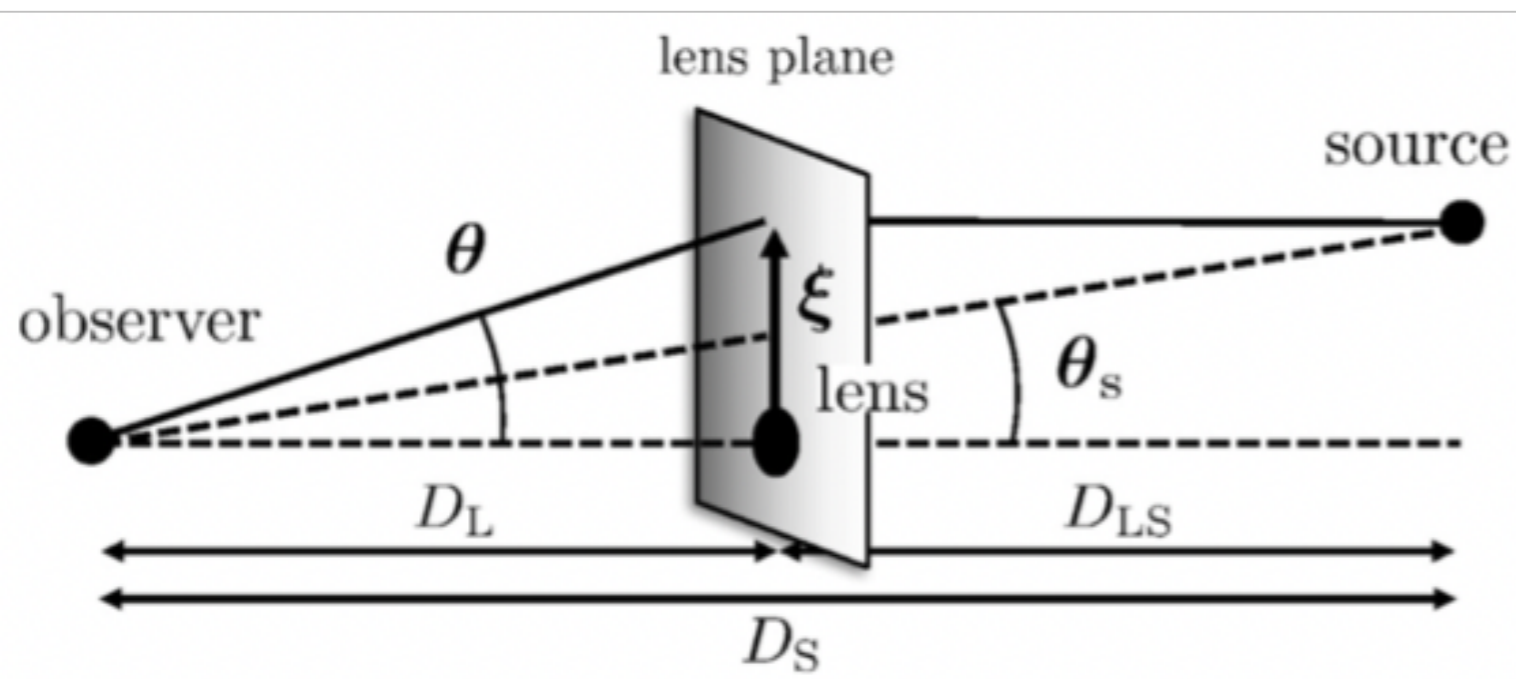
○ Large noise in the Universe screens the light signals.

THAT's BECAUSE...

- GWs have **weak interaction** with matter

So what's the physics and mathematics behind?

- If we know the source and lens position...



- D_L : Lens-observer distance
- D_S : Source-observer distance
- D_{LS} : Source-Lens distance
- $\vec{\theta}$: 2D angle between horizontal and lensing point
- $\vec{\theta}_S$: 2D angle between horizontal and source

- Separation between lensed and unlensed rays at lens :

$$\vec{\xi} = \frac{D_L D_{LS}}{D_S} (\vec{\theta} - \vec{\theta}_S)$$

So what's the physics and mathematics behind?

- Geometrical path difference between lensed and unlensed signals :

$$\Delta\lambda = \frac{\xi(\vec{\theta} - \vec{\theta}_S)}{2}$$

Time delay

$$\Delta t = (1 + z_d) \frac{D_L D_{LS}}{2D_S c} (\vec{\theta} - \vec{\theta}_S)^2$$

Amplitude magnification

$$h_{+,x}^{\text{lensed}}(f) = F(\omega, y) h_{+,x}^{\text{unlensed}}(f)$$

z_d : Gravitational redshift

c : Speed of light in vacuum

$h_{+,x}^{\text{lensed}}(f)$: Amplitude of lensed signal

$h_{+,x}^{\text{unlensed}}(f)$: Amplitude of unlensed signal

$F(\omega, y)$: Amplification function

Before I start searching...

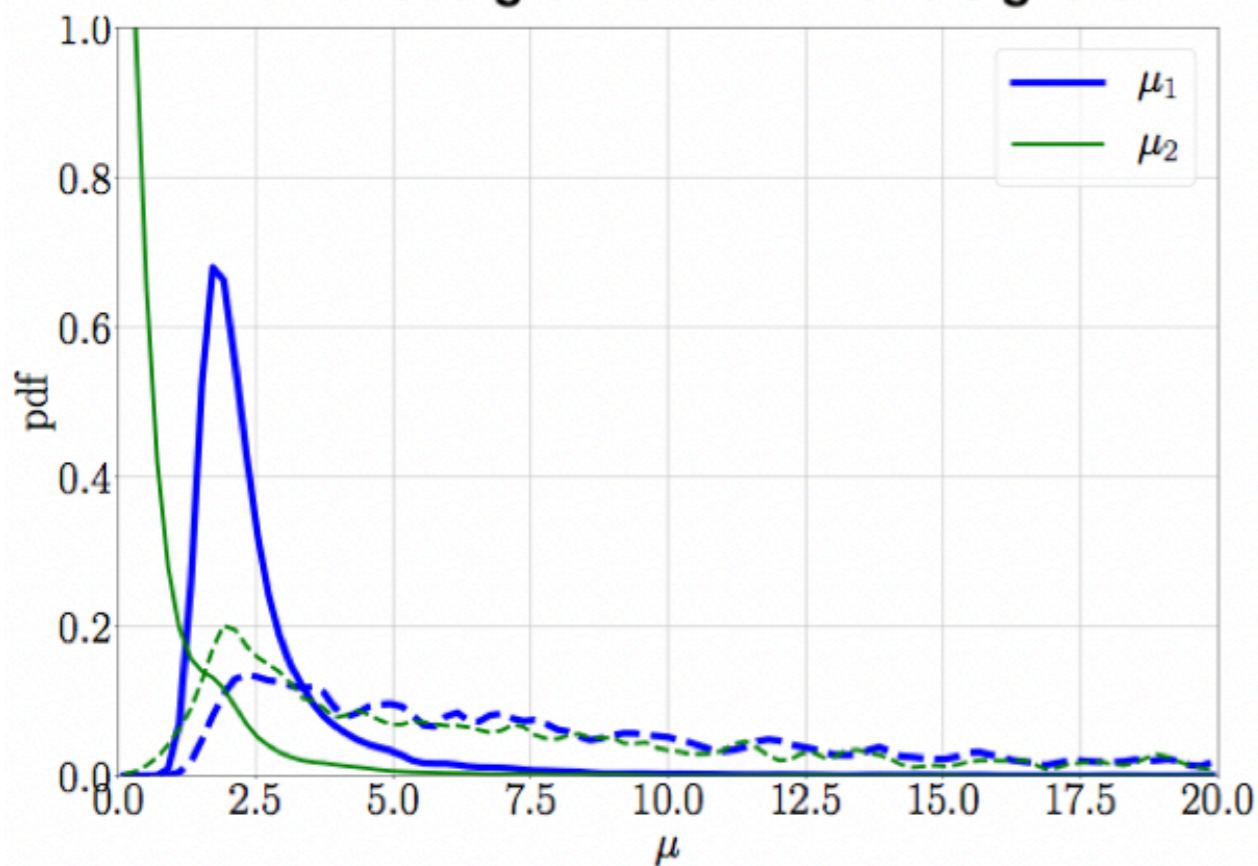
- **Study the magnification and relative time delay probability distribution for lensed GWs**
 - Try to follow available paper[1] to compute the probability distribution of relative time delays and magnification of lensed GWs
 - Type of lens : Singular Isothermal Ellipsoid lens model
 - This model can produce either two or four images

[1] HARIS, K., MEHTA, A. K., KUMAR, S., VENUMADHAV, T., , AND AJITH, P. Identifying strongly lensed gravitational wave signals from binary black hole mergers. - (2018).

Before I start searching...

- Original results

Probability distribution function of magnification of lensed gravitational wave signals



Probability distribution function of relative time delay of lensed gravitational wave signals

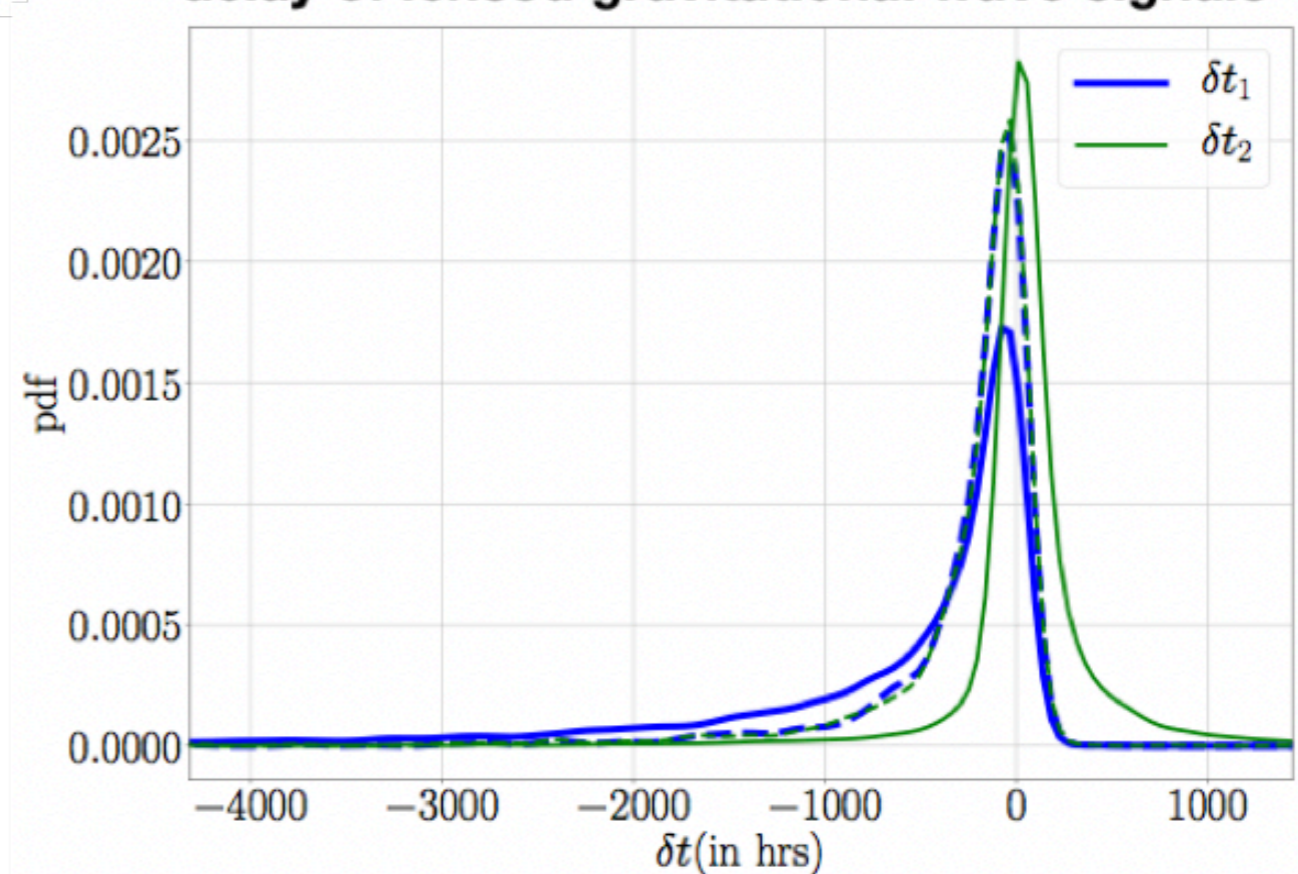


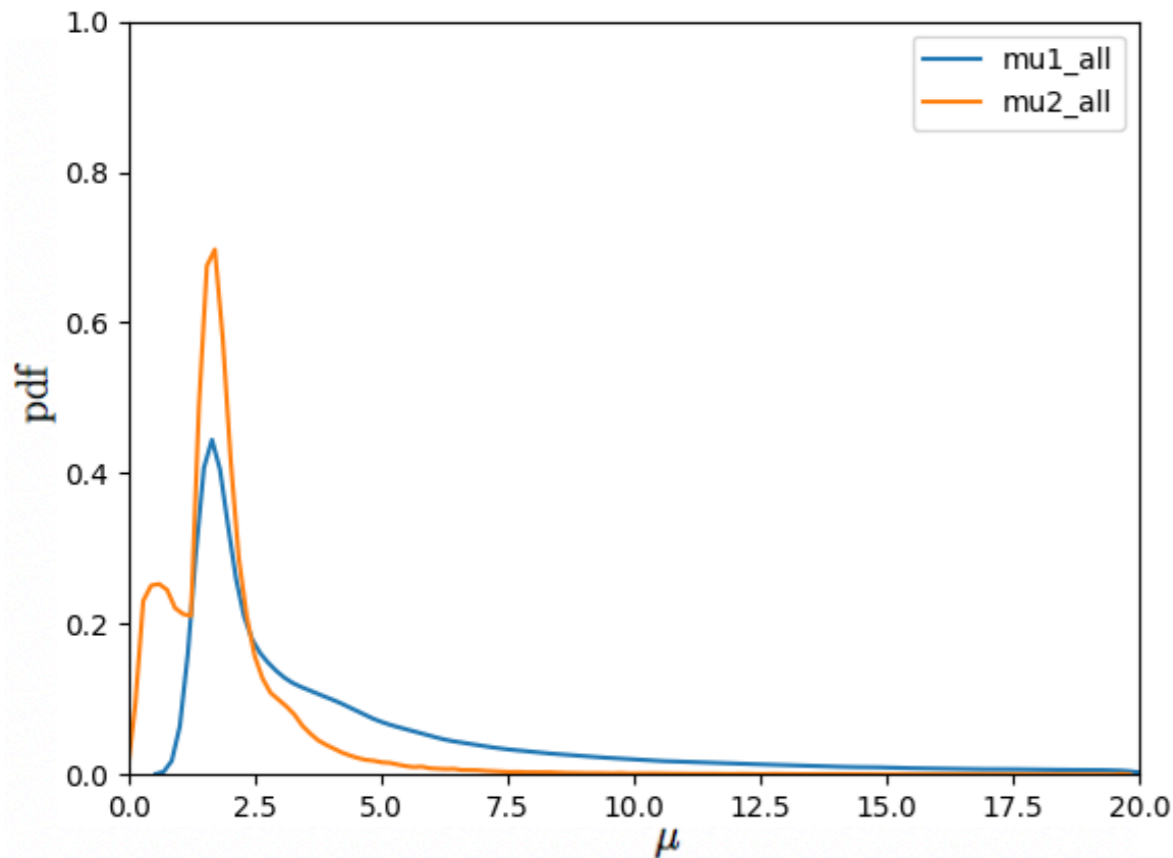
Fig 19. Probability distribution of magnification μ_1 and μ_2 of the two dominant lensed gravitational wave signals. The Solid (dashed) traces show distributions before (after) applying the detection threshold $\text{SNR} > 8$. The component masses of the simulated events are sampled from power law 1 distribution.

Fig 20. Probability distribution function of relative time delay δt_1 and δt_2 of the two dominant lensed gravitational wave signals. The Solid (dashed) traces show distributions before (after) applying the detection threshold $\text{SNR} > 8$. The component masses of the simulated events are sampled from power law 1 distribution.

Before I start searching...

- Reproduced results

Probability distribution function of magnification of lensed gravitational wave signals



Probability distribution function of relative time delay of lensed gravitational wave signals

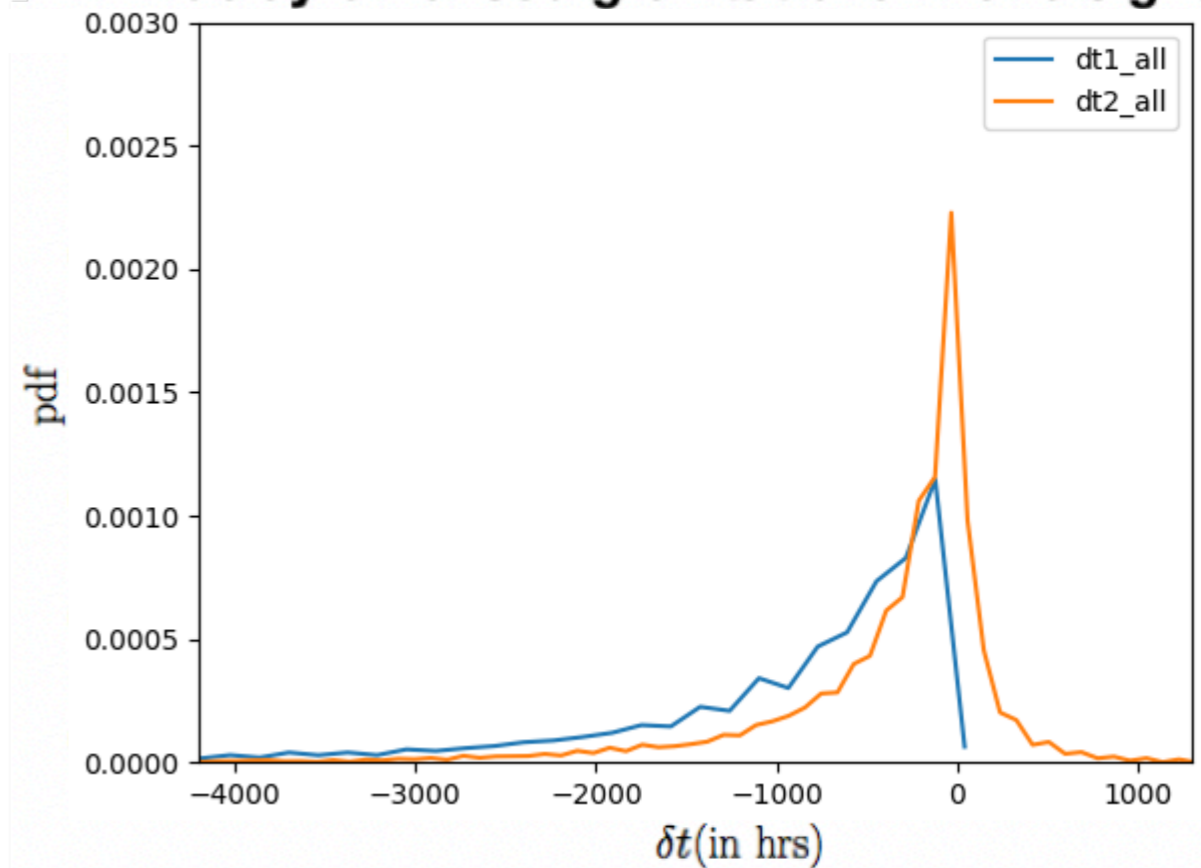


Fig 21. Probability distribution of magnification μ_1 and μ_2 of the two dominant lensed gravitational wave signals. The Solid (dashed) traces show distributions before (after) applying the detection threshold $\text{SNR} > 8$. The component masses of the simulated events are sampled from power law 1 distribution.

Fig 22. Probability distribution function of relative time delay δt_1 and δt_2 of the two dominant lensed gravitational wave signals. The Solid (dashed) traces show distributions before (after) applying the detection threshold $\text{SNR} > 8$. The component masses of the simulated events are sampled from power law 1 distribution.

Before I start searching...

- Some questions...


- Most lensed GWs have their amplitude **magnified**.

- ...Is **GW150914 a lensed GW?**

- If in fact GW150914 is a lensed event, how can we search for the “unlensed” version of it?

- How can we check if there is in fact something between us and the source of GW150914?

Here's the plan...

- ☑ Have we seen any lensed event? **Maybe?**
 - ☑ We want to search for lensed / unlensed version of detected events. These are going to be **weaker**.
 - ☑ But the earlier methods are not **GOOD** enough, so we need to **IMPROVE** the search.
 - ☑ We end up with doing a **TARGETED SEARCH**, which is so far a good method.
- 

How do you look for them? Targeted Search

• Retrieving a detection statistics distribution of lensed GWs

Objectives :

- Figure out the range of parameters to search for lensed GWs so that they will be consistent with the observed events.

How?

- Use a much smaller template bank.
- Much less background.

Why?

- By lowering the background, we can uncover the originally hidden lensed GWs.

How do you look for them?

Simply speaking, we want THIS :

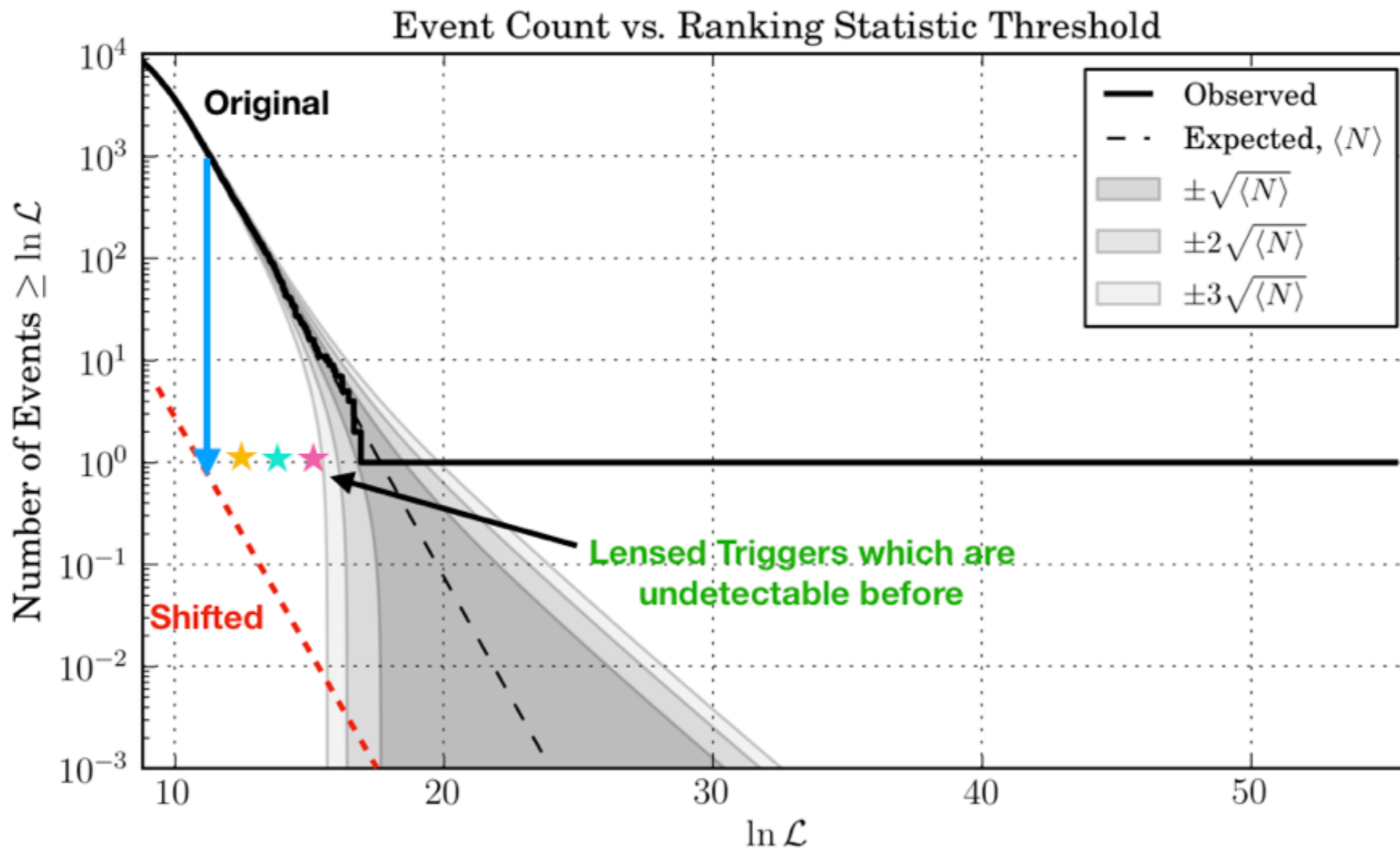


Fig 8. Expected event-count vs ranking statistic threshold curve for lensed GWs, using GW170608 as an example. Note that the red-shifted line and the green lensed triggers are not real data, and is only for illustrative means.

How do you look for them?

- Trust me...It's not that easy!



We tried **MANY METHODS**, but they are not good...



- Magnification and relative time delay probability distribution for lensed GWs

How do you look for them?

- Trust me...It's not that easy!

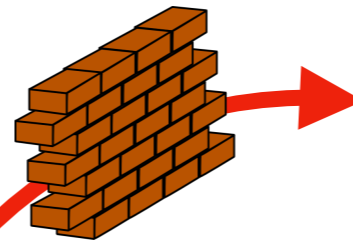


○ Using LALInference posterior data

○ Magnification and relative time delay probability distribution for lensed GWs

How do you look for them?

- Trust me...It's not that easy!



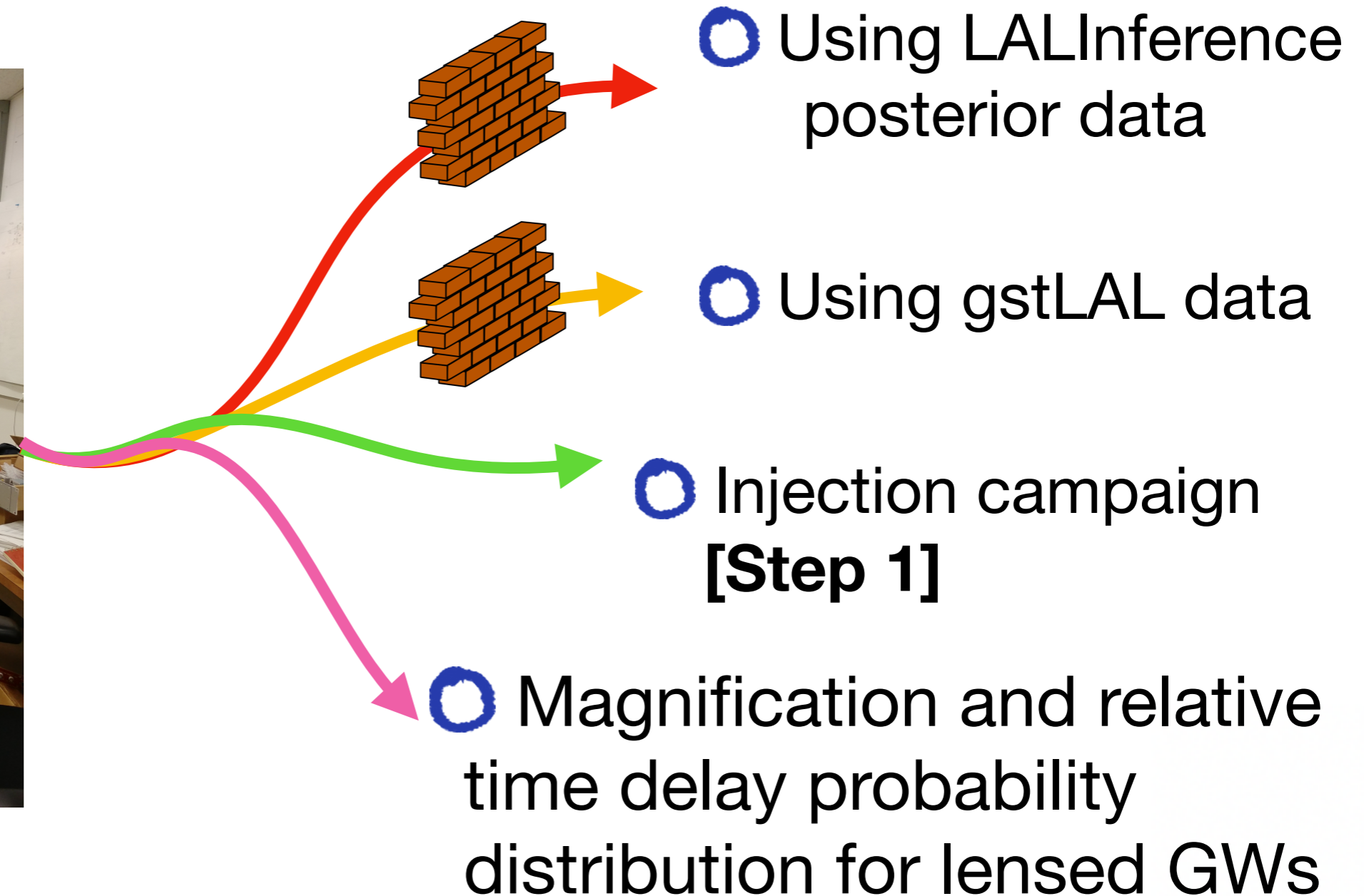
○ Using LALInference posterior data

○ Using gstLAL data

○ Magnification and relative time delay probability distribution for lensed GWs

How do you look for them?

- Trust me...It's not that easy!



How do you look for them?

- Step 1: Rough estimate using unclustered gstLAL data

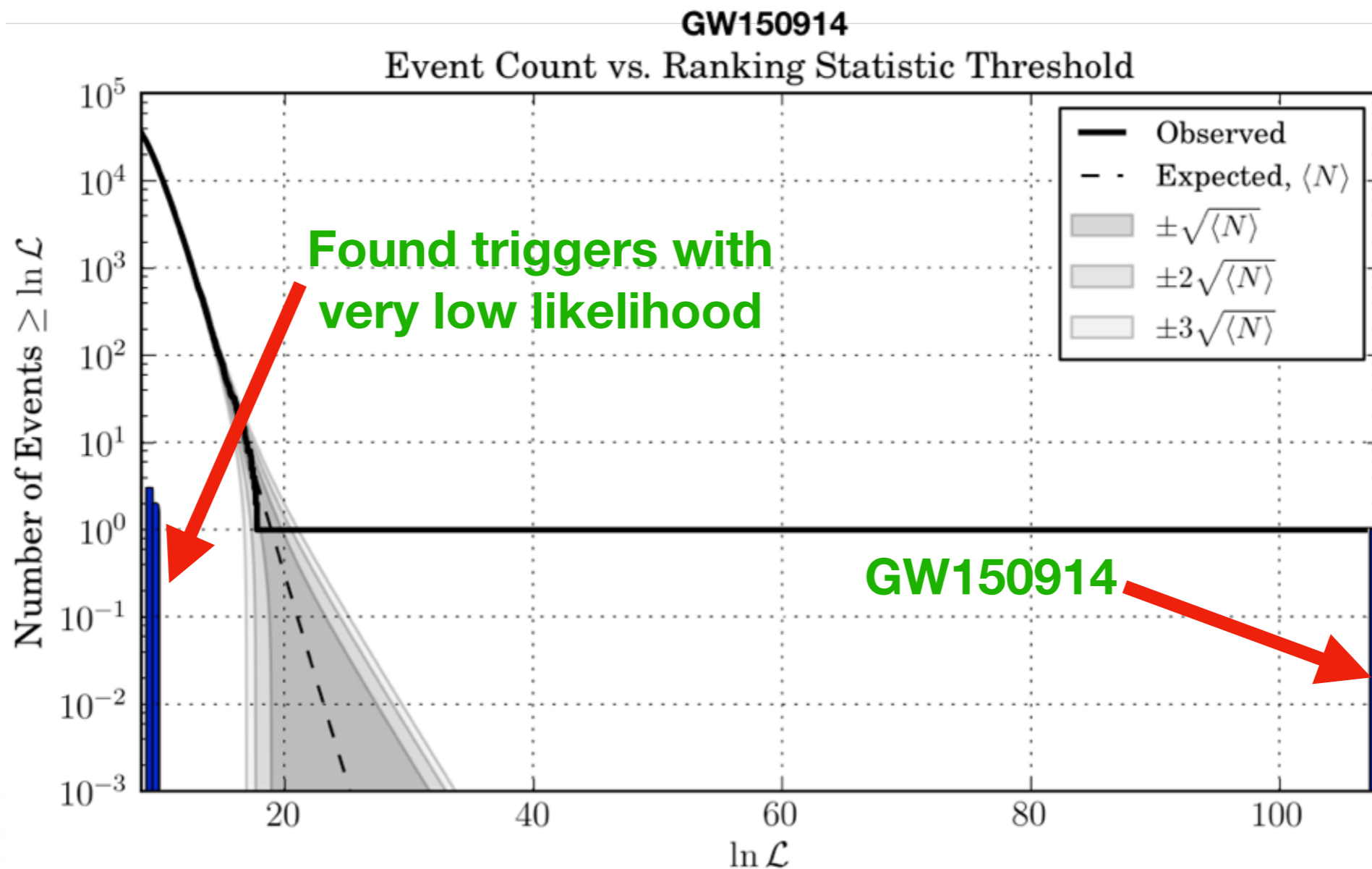


Fig 9. Distribution of likelihood (blue bars) of searched matching triggers in O1-chunk1 using raw data for the event GW150914. Note that the barely visible blue bar on the right boundary corresponds to the detection of the event GW150914.

How do you look for them?

- Step 1: Rough estimate using unclustered gstLAL data

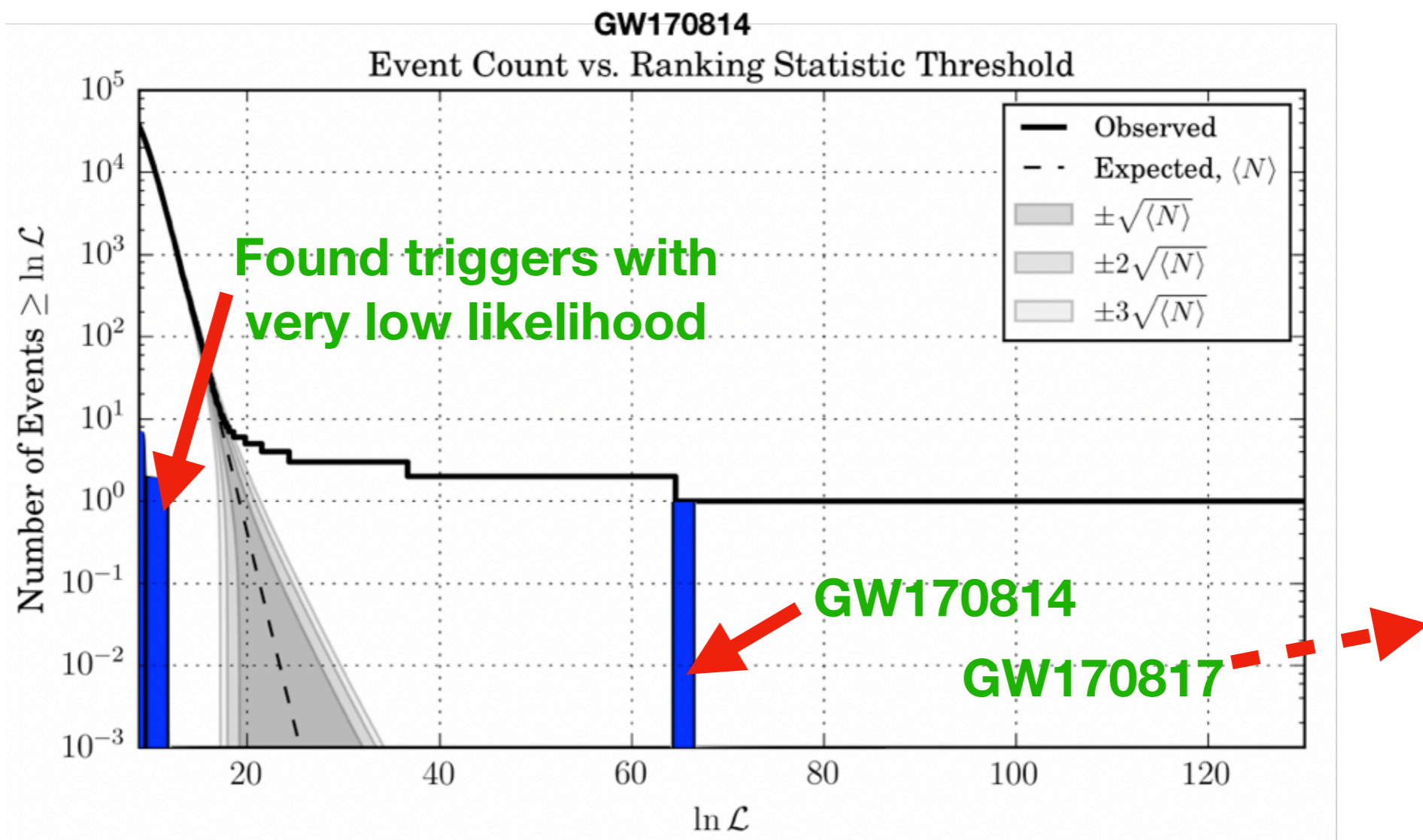


Fig 12. Distribution of likelihood (blue bars) of searched matching triggers in O2-chunk21 using raw data for the event GW170814. The blue bar in the middle refers to the detection of the event GW170814. Note that the solid (observed) event-count versus ranking statistics threshold curve extends beyond the middle blue bar instead of stopping there, since there is another detection, which is GW170817, in the same chunk we are analysing here.

How do you look for them?

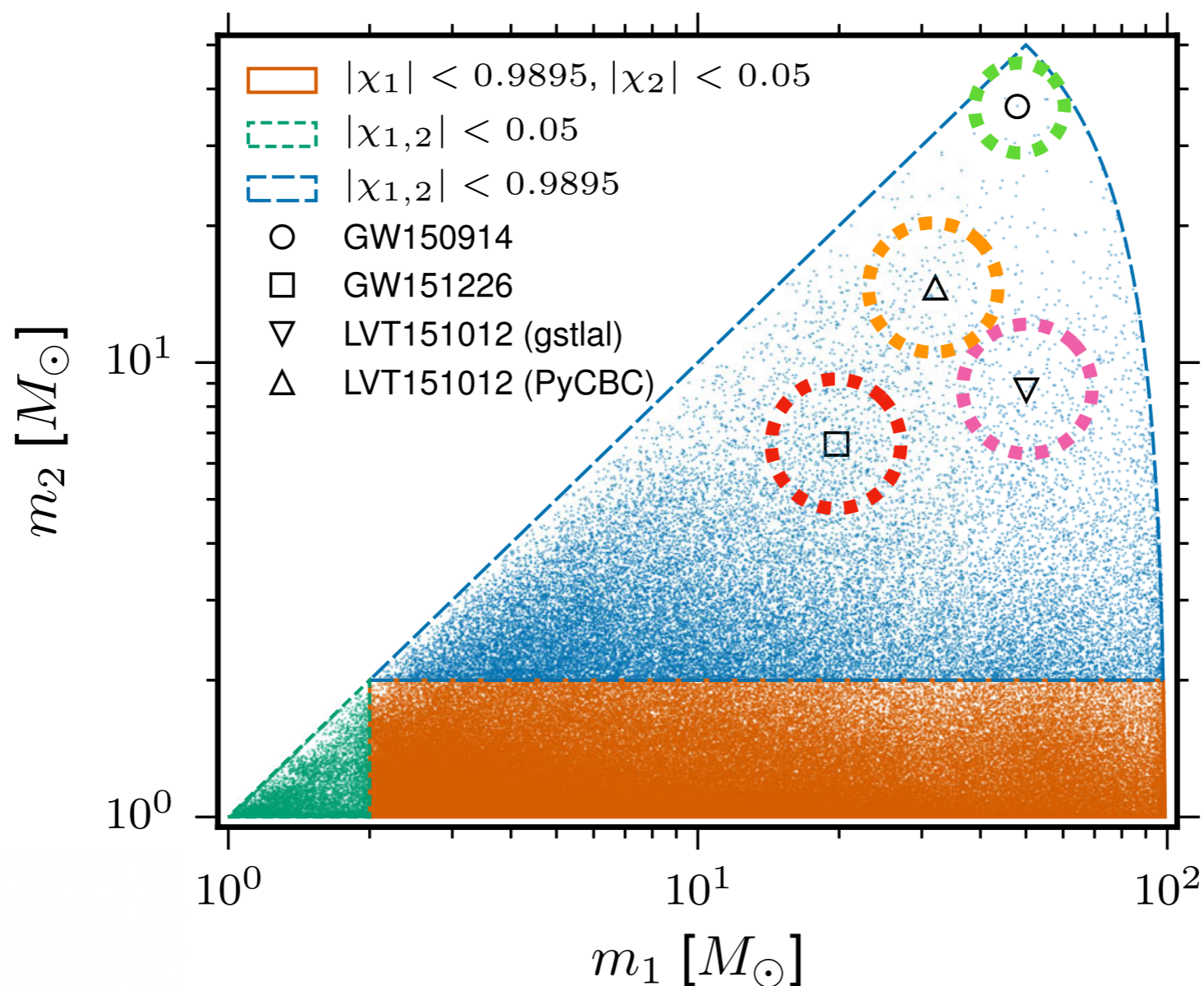
- Step 1: Rough estimate using unclustered gstLAL data

Interim conclusion(s) :

- Get a sense of how the detection statistics distribution of lensed GWs will be.
- But what is the searching range for the masses and spins? **UNKNOWN!**

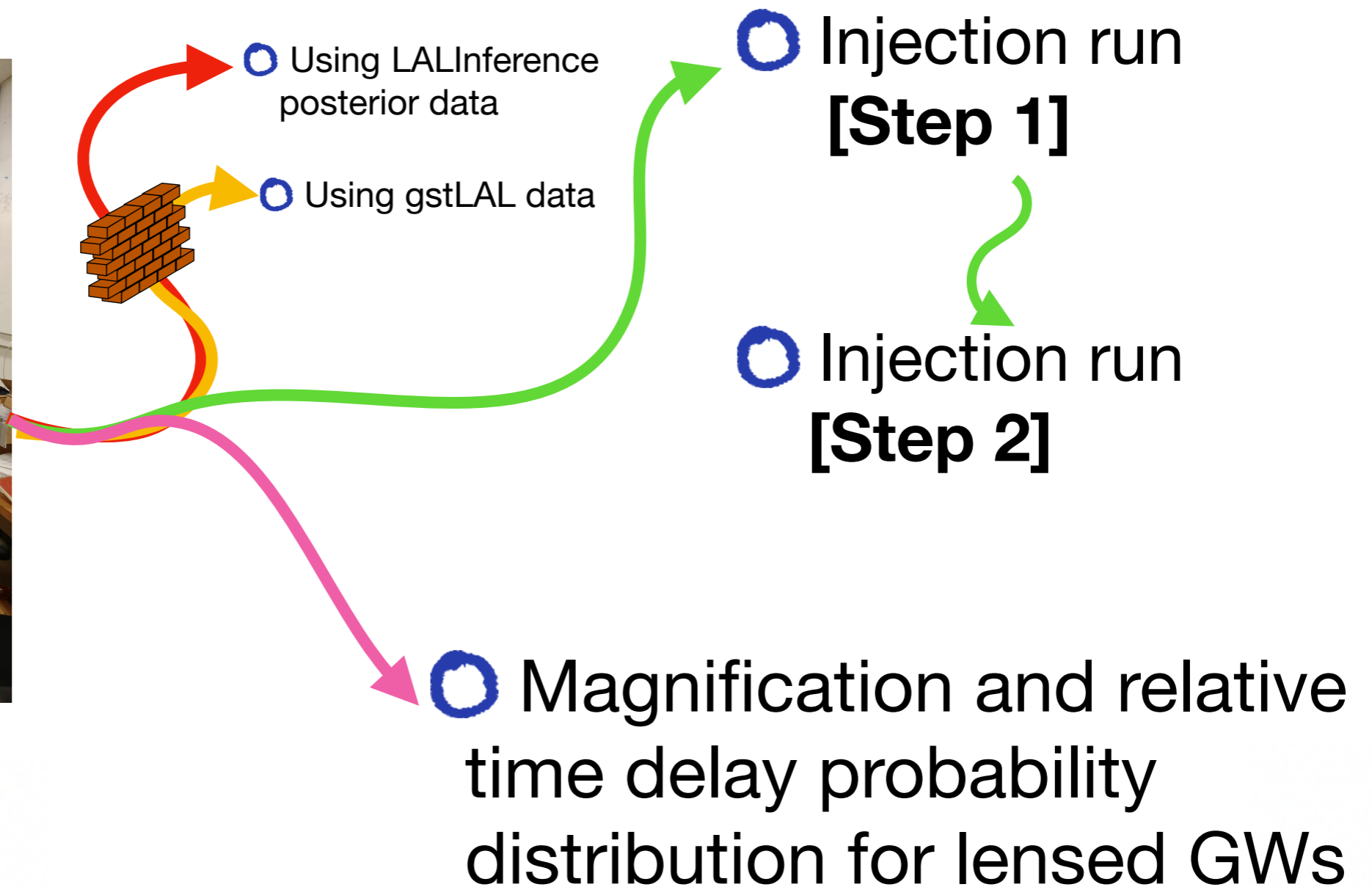
What's next?

- Step 2: An injection run



How do you look for them?

- Trust me...It's not that easy!



How do you look for them?

• Step 2: Injection campaign

- Read in LALInference posterior samples.
- Make injection file containing simulated lensed GWs.
- Run the gstLAL injection run.
- Use recovered triggers as templates for targeted search

It's just a piece of cake, right?

NO!

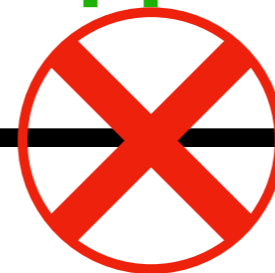
How do you look for them?

• Step 2: Injection campaign

Conversion from posterior samples to sim inspiral table

Item	Content [3]
l1_end_time	Reference time at Livingston site (time of coalescence / peak amplitude)
v1_end_time	Reference time at VIRGO site (time of coalescence / peak amplitude)
h1_end_time	Reference time at Hanford site (time of coalescence / peak amplitude)
time	Reference time at geocentre (time of coalescence / peak amplitude)
m1	Mass of the primary object (detector frame)
m2	Mass of the secondary object (detector frame)
a1z	The z-component of spin of the primary object
a2z	The z-component of spin of the secondary object
mc	Chirp mass (detector frame)
distance	Distance to source
dec	Declination of the gravitational wave source
ra	Right ascension of the gravitational wave source
psi	Polarisation angle (3 rd Euler angle) required to transform the tensor perturbation in the radiation frame to the detector frame
costheta_jn	Cosine of the angle between the total angular momentum and the line of sight vector
theta_jn	Angle between total angular momentum and line of sight
eta	Symmetric mass-ratio
optimal_snr	Optimal Signal-to-Noise Ratio (SNR) of the model
logl	Natural log of the likelihood
lal_amporder	Post Newtonian amplitude order

No direct mapping!



Item	Content and related posterior samples items
h_end_time	Reference time at Hanford site (time of coalescence / peak amplitude) [Integral value] Related item(s) : h1_end_time
h_end_time_ns	Reference time at Hanford site (time of coalescence / peak amplitude) [Nanosecond] Related item(s) : h1_end_time
l_end_time	Reference time at Livingston site (time of coalescence / peak amplitude) [Integral value] Related item(s) : l1_end_time
l_end_time_ns	Reference time at Livingston site (time of coalescence / peak amplitude) [Nanosecond] Related item(s) : l1_end_time
v_end_time	Reference time at Virgo site (time of coalescence / peak amplitude) [Integral value] Related item(s) : h1_end_time
v_end_time_ns	Reference time at Virgo site (time of coalescence / peak amplitude) [Nanosecond] Related item(s) : h1_end_time
geocent_end_time	Reference time at geocentre (time of coalescence / peak amplitude) [Integral value] Related item(s) : time
geocent_end_time_ns	Reference time at geocentre (time of coalescence / peak amplitude) [Nanosecond] Related item(s) : time
mass1	Mass of the primary object (detector frame) Related item(s) : m1
mass2	Mass of the secondary object (detector frame) Related item(s) : m2
mchirp	Chirp mass (detector frame) Related item(s) : mc
spin1z	The z-component of spin of the primary object Related item(s) : a1z
spin2z	The z-component of spin of the secondary object Related item(s) : a2z
distance	Distance to source Related item(s) : distance, ra, dec, optimal_snr
longitude	Right ascension* of the gravitational wave source Related item(s) : ra
latitude	Declination* of the gravitational wave source Related item(s) : dec
eta	Symmetric mass-ratio Related item(s) : eta
inclination	angle between total angular momentum and line of sight Related item(s) : theta_jn
polarization	Polarisation angle (3 rd Euler angle) required to transform the tensor perturbation in the radiation frame to the detector frame Related item(s) : psi
amp_order	Post Newtonian amplitude order Related item(s) : lal_amporder

How do you look for them?

- Step 2: Injection campaign

Generating simulated lensed GWs

- **A biased estimate of effective distance to source:**

$$D_{\text{eff}} = \left(\frac{\sigma}{\rho} \right) Mpc$$

σ : Sensitivity of instrument

ρ : SNR ratio of matched filter

- Generate simulated lensed GWs by altering the effective distance of samples.

But that is not so simple...

- We have to take into account for the **difference in sky location** between the samples and injected signals.

How do you look for them?

• Step 2: Injection campaign

Running the injection campaign

- make -f Makefile
- Submit_condor_dag trigger.dag



How do you look for them?

• Step 2: Injection campaign

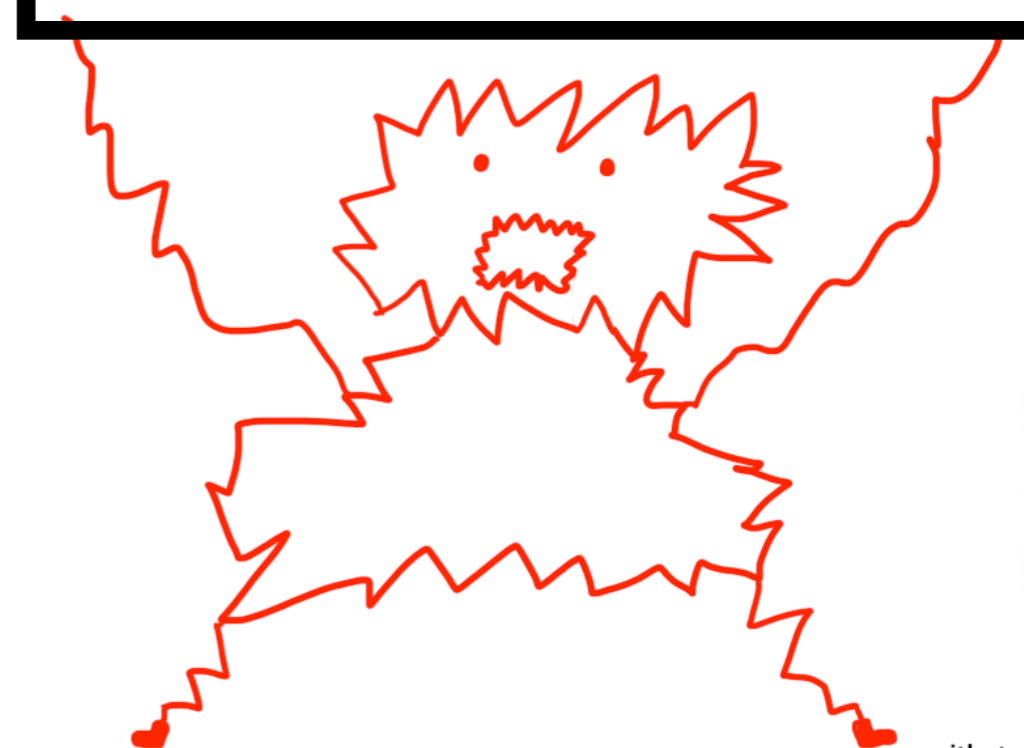
COMPUTING CLUSTERS!

- The computing clusters have been running slow
- The computing clusters have been failing my jobs for loads of reasons :
 1. Memory usage
 2. Sink events
 3. XAL generic error
 - 4.....

**CLUSTER
MONSTER**



**YOU SHALL
NOT PASS!**



How do you look for them?

- Step 2: Injection campaign

Uh oh....what should we do now?



How do you look for them?

- Trust me...It's not that easy!



Using LALInference posterior data

Using gstLAL data

Magnification and relative time delay probability distribution for lensed GWs

Injection run [Step 1]

Injection Run [Step 2]

Injection Run [Step 2 - Shortcut version]

How do you look for them?

- Step 2: Injection campaign [Shortcut Version]

- Reduce the injection time to a week time

- Results may not be **PERFECT**, but for now will still be **SATISFACTORY**.

How do you look for them?

• Step 2: Injection campaign [FULL Version]

GW150914

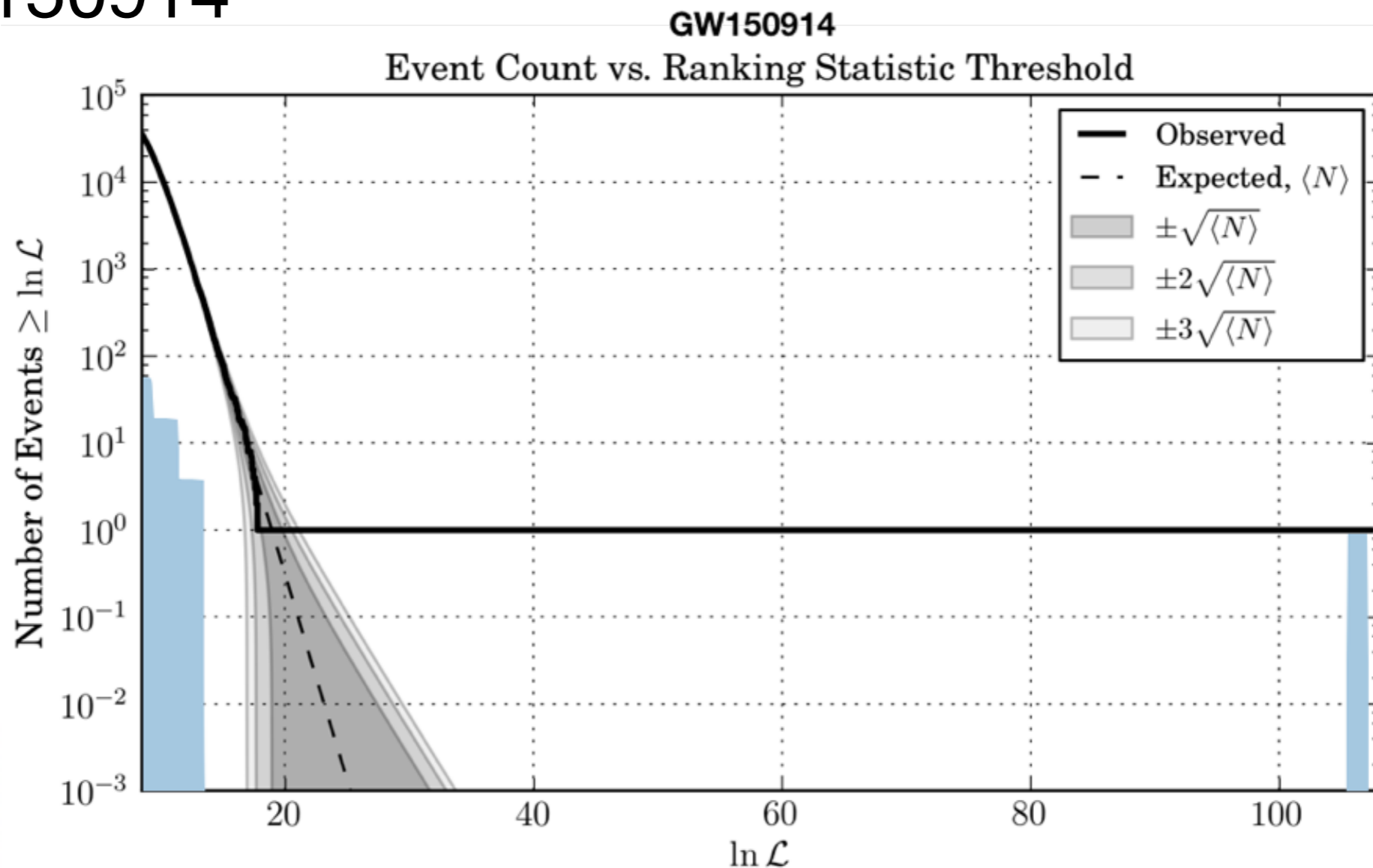


Fig 13. Distribution of likelihood (blue bars) of searched matching triggers in O1-chunk1 using recovered templates from injection run for the event GW150914.

How do you look for them?

• Step 2: Injection campaign [FULL Version]

GW150914

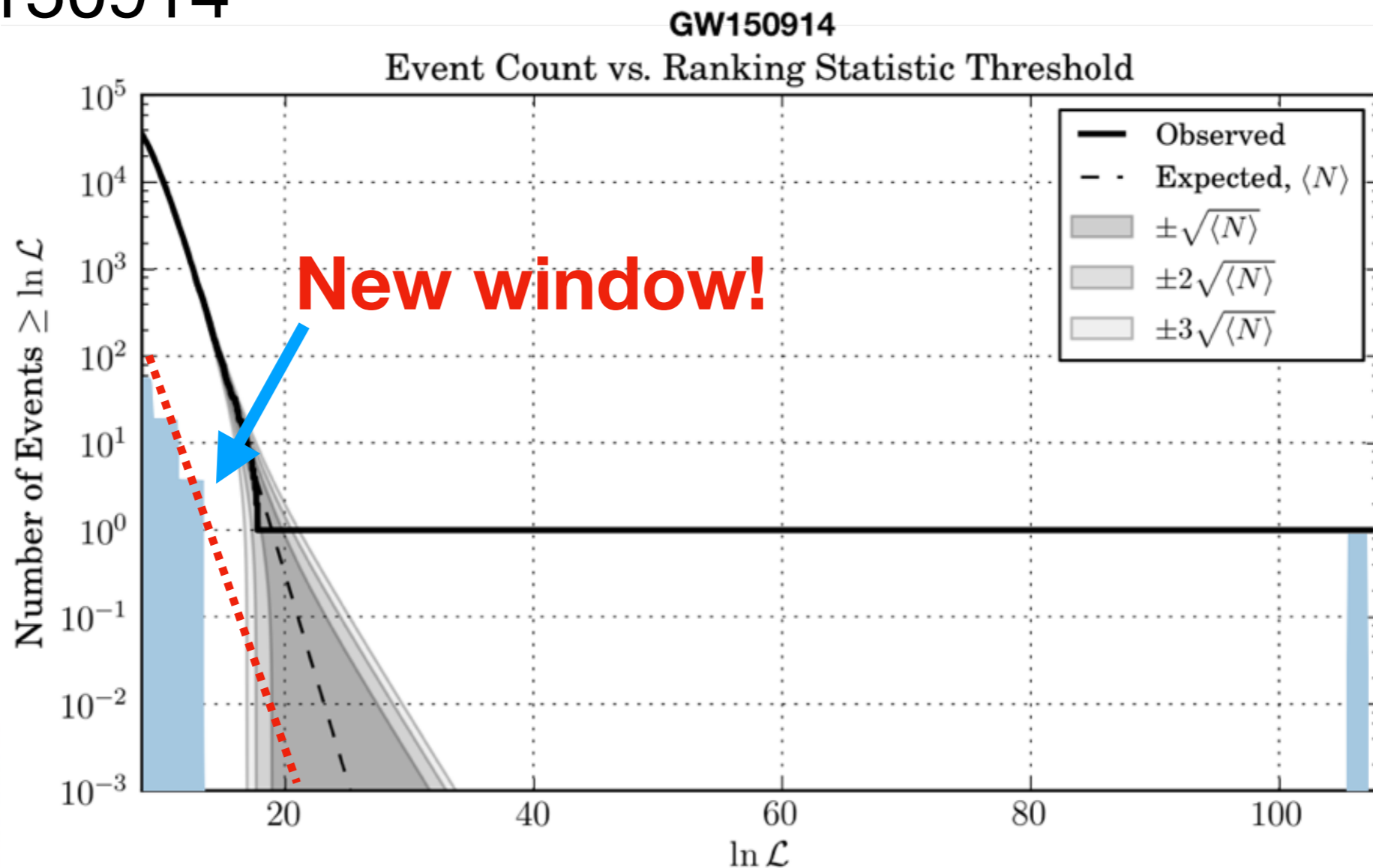


Fig 13. Distribution of likelihood (blue bars) of searched matching triggers in O1-chunk1 using recovered templates from injection run for the event GW150914.

How do you look for them?

• Step 2: Injection campaign [FULL Version]

GW150914

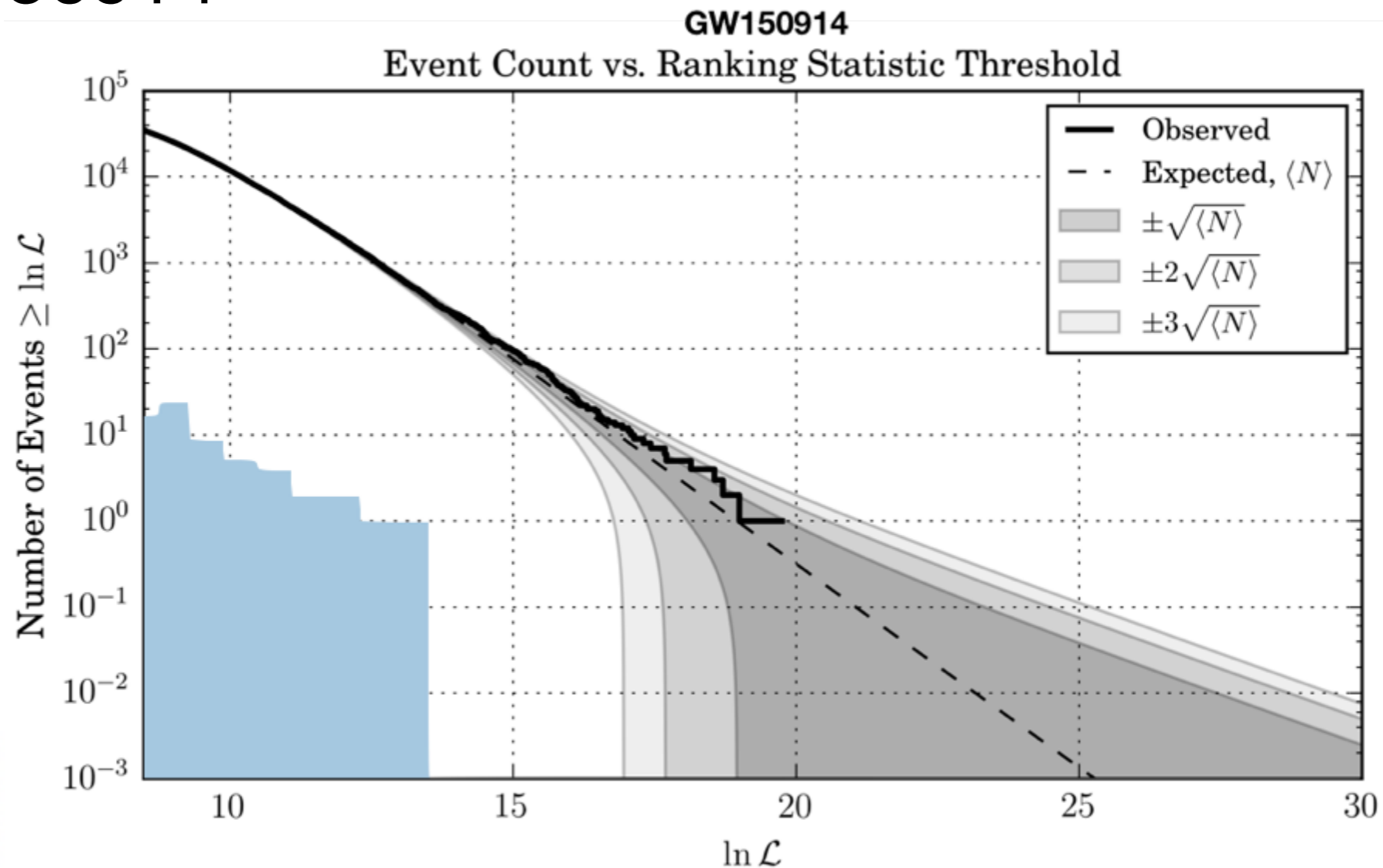


Fig 14. Distribution of likelihood (blue bars) of searched matching triggers in O1-chunk2 using recovered templates from injection run for the event GW150914.

How do you look for them?

- **Step 2: Injection campaign [FULL Version]**

GW150914

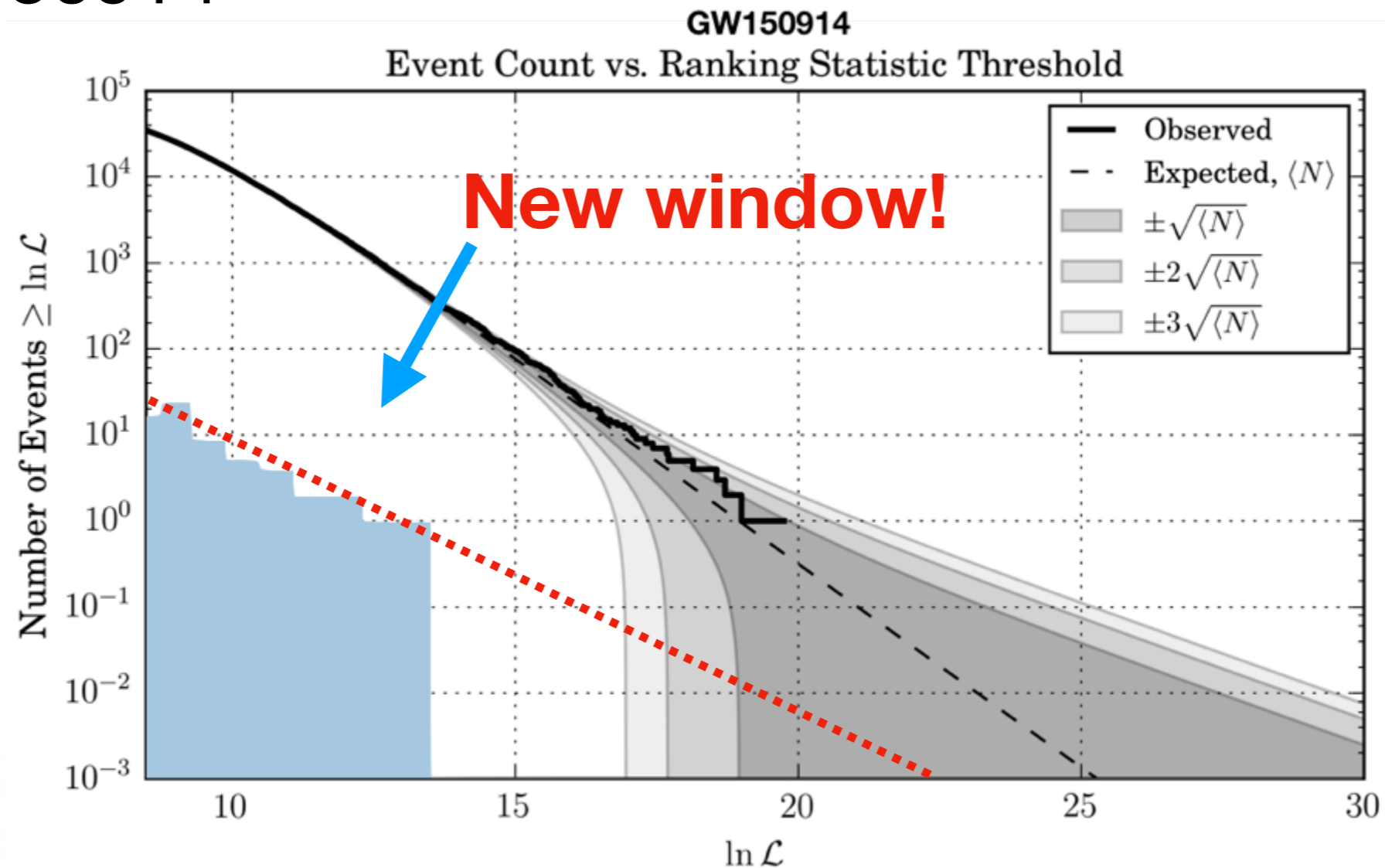
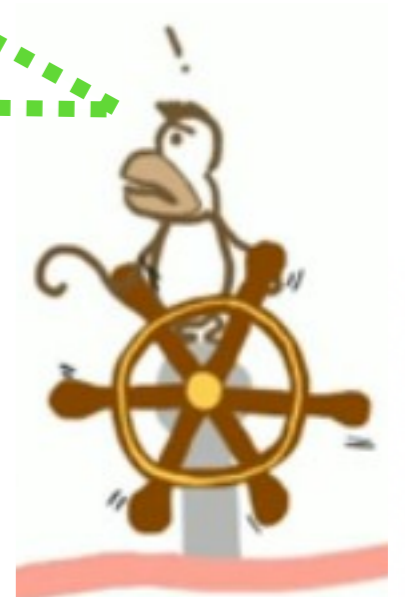


Fig 14. Distribution of likelihood (blue bars) of searched matching triggers in O1-chunk2 using recovered templates from injection run for the event GW150914.

- ☑ We use targeted search to lower the background in order to uncover the lensed / unlensed GWs.
- ☑ We investigated some methods to do so, some are good and some are bad.
- ☑ The injection run results shows that it is a good method of lowering the background

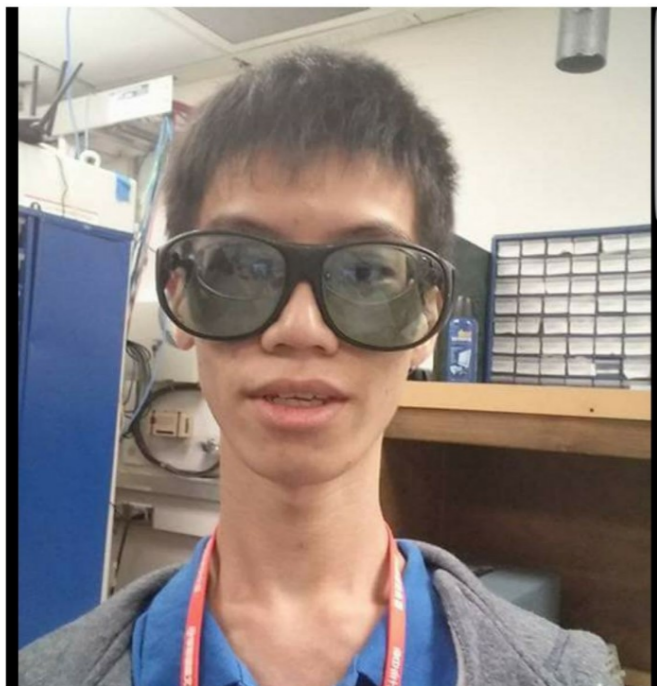
Are these your **conclusions**? Because they do not sound like conclusions to me...

WHAT ON EARTH have you done in this whole summer?



☑ I think it's **WORTHWHILE** to restate that, the end of 2018 LIGO SURF does not mark the end of this project...

I LOVE PHYSICS, BUT MY FUTURE IS AS DARK AS I SEE.



Me visiting the 40m Lab in **2017** as a visitor



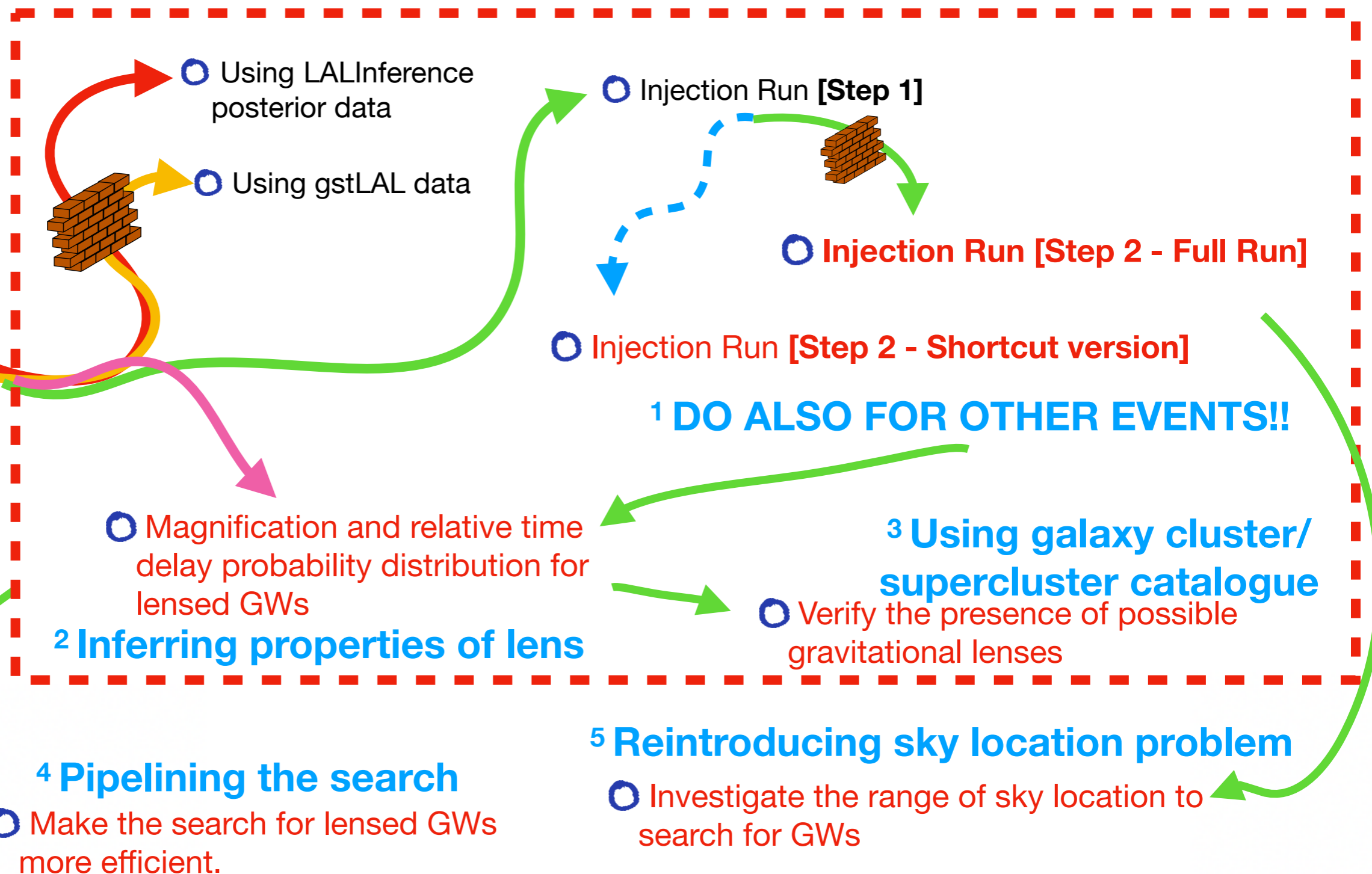
Me visiting the 40m Lab in **2018** as a LIGO SURF



Who knows what will happen in the future?

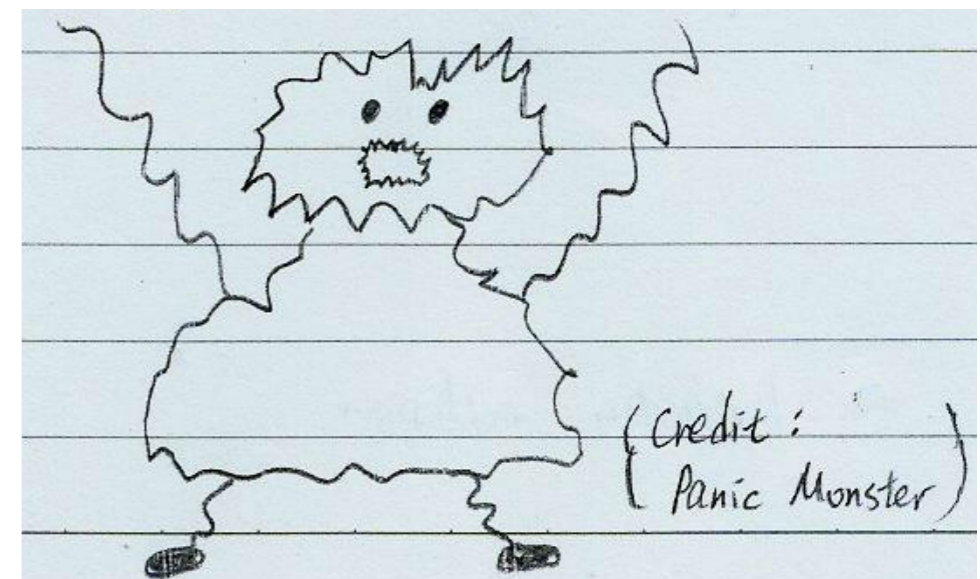
☑ This is **JUST THE BEGINNING...**

It's not yet finished!



Here, I would like to thank...

- Professor Alan Weinstein
- Surabhi Sachdev
- Professor Tjonnie Li Guang Feng
- Rico Lo Ka Lok
- LIGO SURF (staffs, mentors and students)
- Caltech SFP
- Department of Physics, The Chinese University of Hong Kong



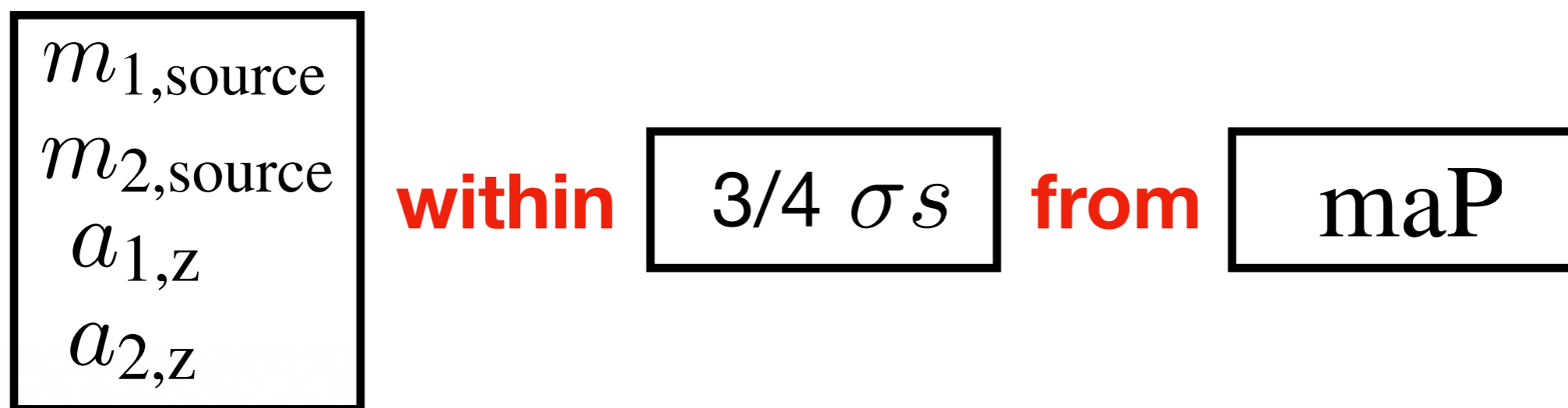
LALInference posterior data

How do you look for them?

- Using LALInference posterior data (GW150914)

Parameter	Maximum Posteriori (maP)	Variation (σ)
$m_{1,\text{source}}$	$32.9M_{\odot}$	$4.9M_{\odot}$
$m_{2,\text{source}}$	$13.7M_{\odot}$	$3.5M_{\odot}$
$a_{1,z}$	-0.618	0.218
$a_{2,z}$	0.083	0.243

- We look for triggers through O1 and O2 with



FOUND triggers are possible lensed candidates of GW150914

How do you look for them?

- For each found candidate, we evaluate its relative time delay and magnification compared to the detected GW150914 event by :

Relative time delay

$$\Delta t = \text{Time of arrival of candidate} \\ - \text{Time of arrival of GW150914}$$

Magnification

$$\mu = \frac{\text{Signal-to-noise ratio of found trigger}}{\text{Signal-to-noise ratio of GW150914}}$$

How do you look for them?

- Everything seems good, so what's wrong?

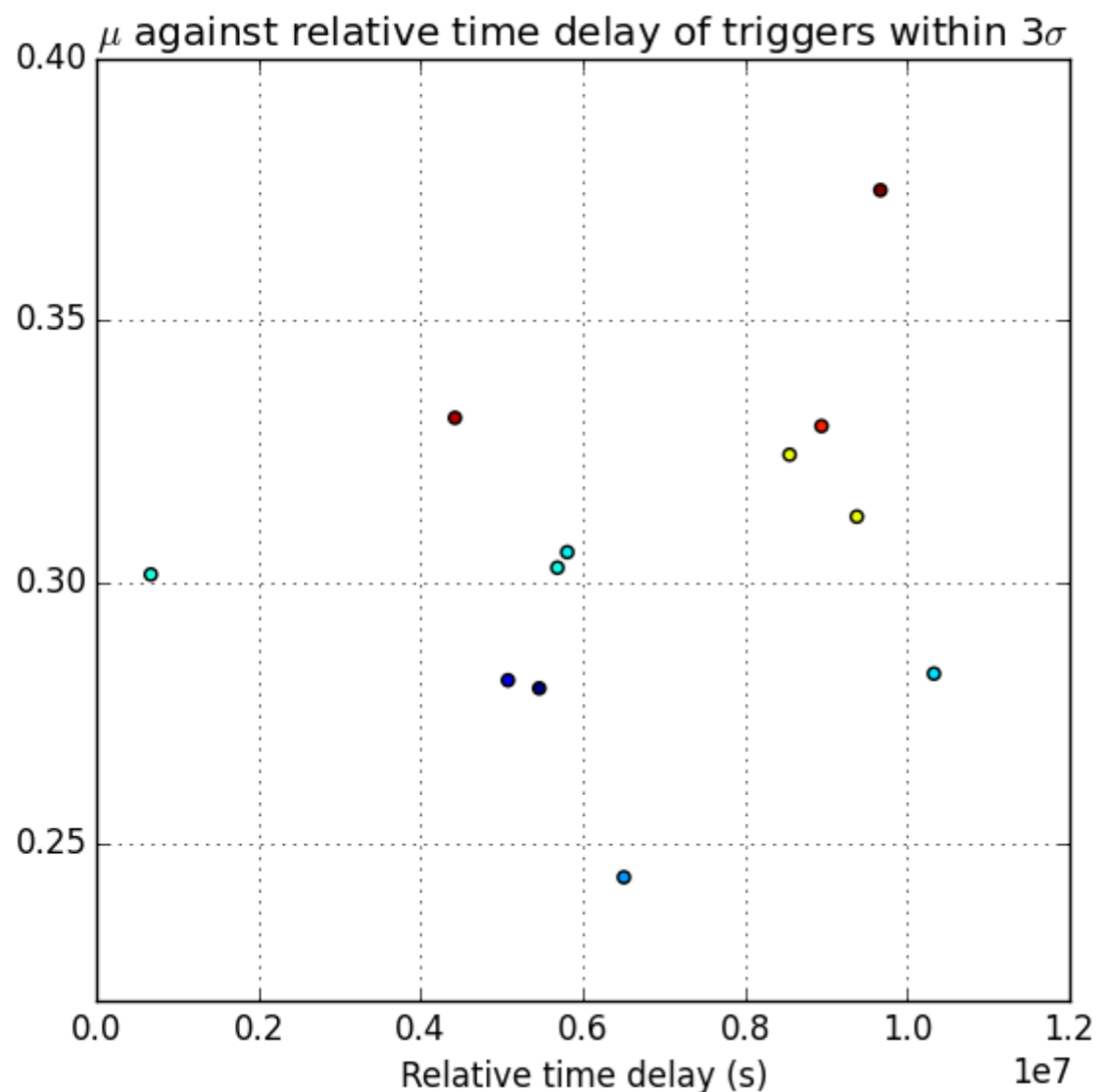


Fig 1. Searched triggers in O1 with parameters within 3 sigma range from GW150914

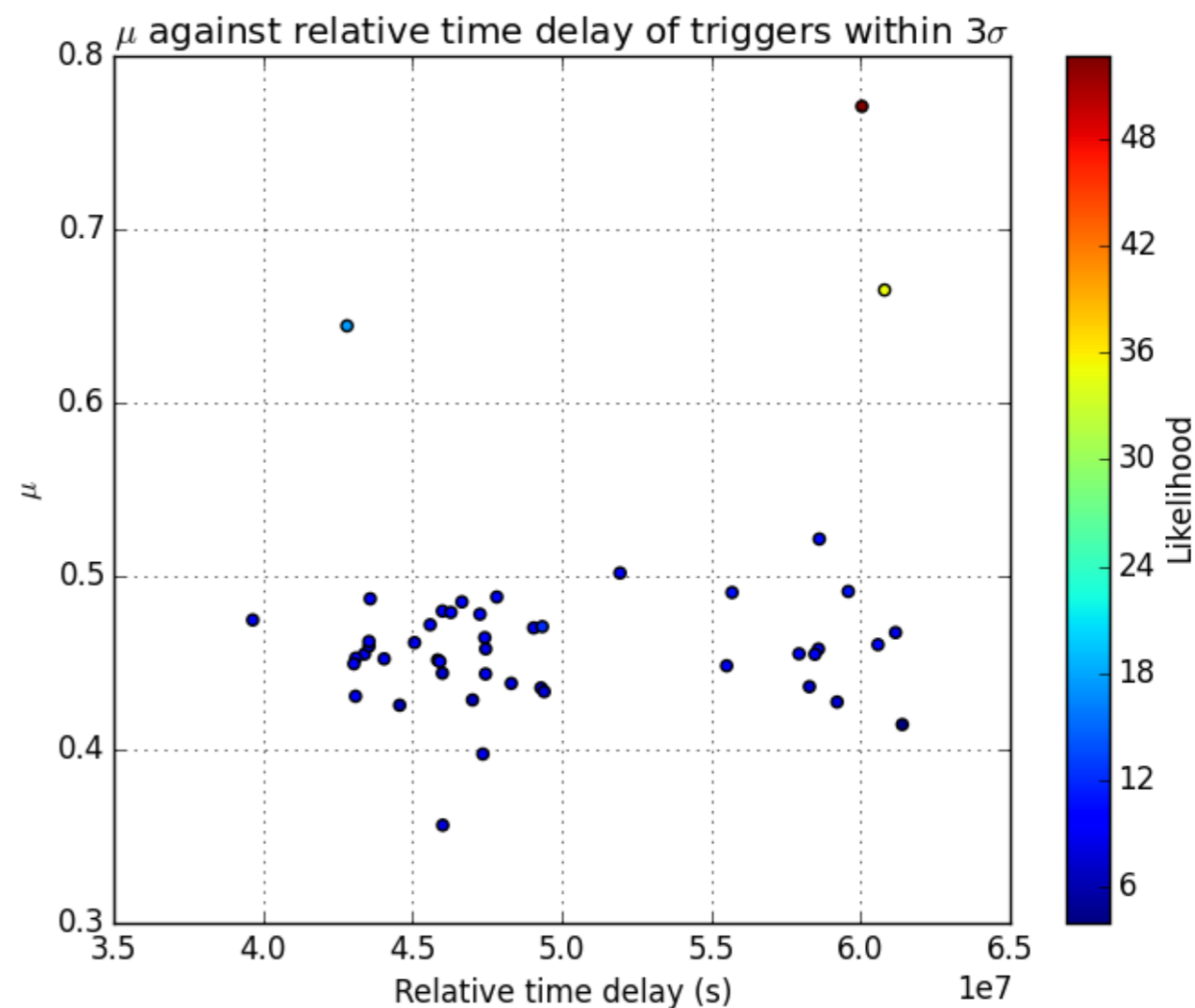


Fig 2. Searched triggers in O2 with parameters within 3 sigma range from GW150914

How do you look for them?

- Everything seems good, so what's wrong?

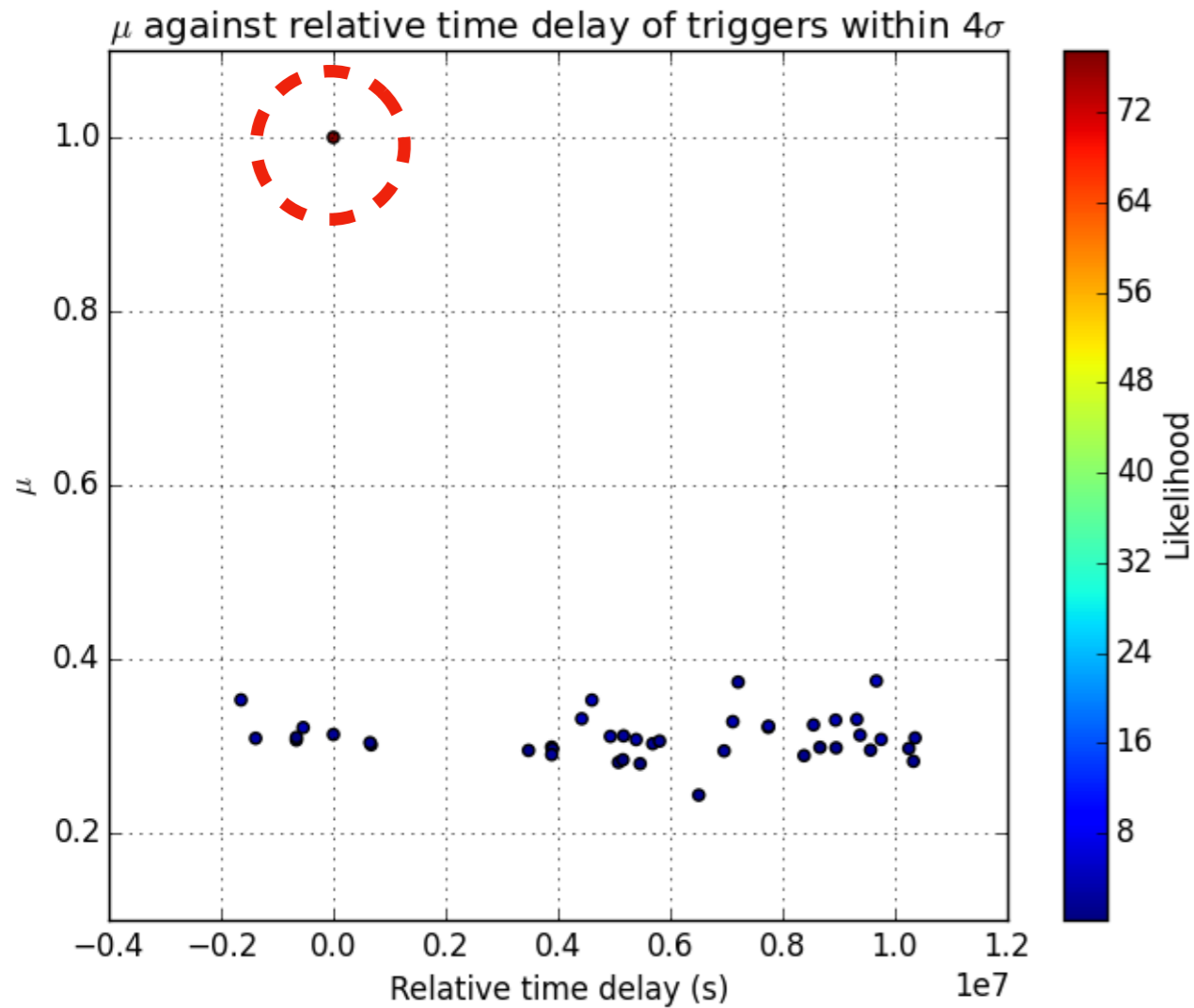


Fig 3. Searched triggers in O1 with parameters within 4 sigma range from GW150914

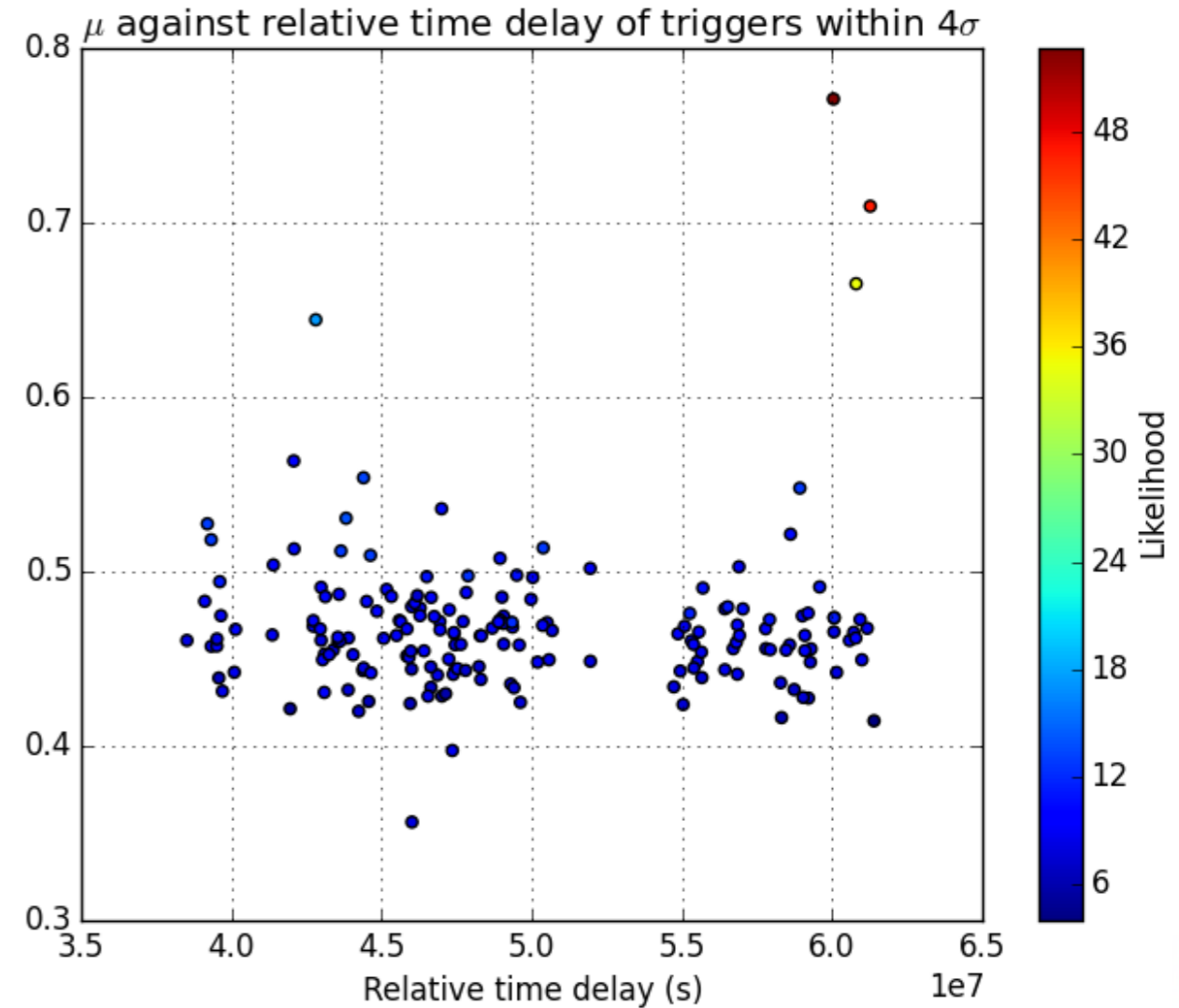


Fig 4. Searched triggers in O2 with parameters within 4 sigma range from GW150914

How do you look for them?

- Everything seems good, so what's wrong?

- GW150914 does not appear in Fig. 1, but only in Fig. 3.

Reason : Inconsistency of design between
LALInference and gstLAL

- SNR ratio for O1 & O2 may have discrepancies.

Reason : Background noise is varying
every moment

CANCELLED

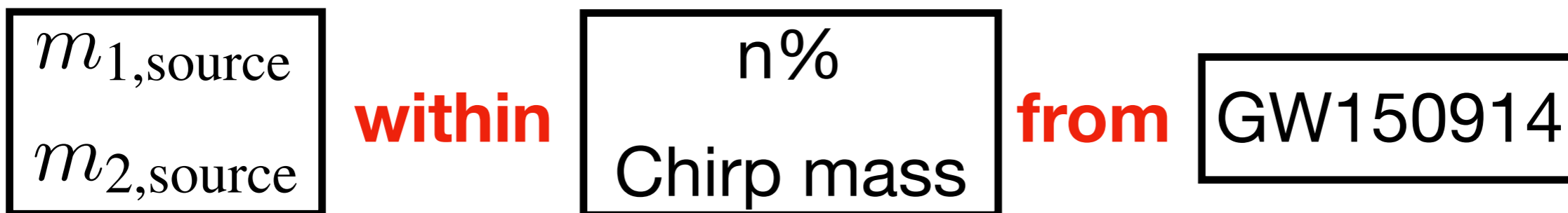
gstLAL
posterior data

How do you look for them?

- Using **gstLAL data (GW150914)**

Parameter	Value
Mass 1	$47.9M_{\odot}$
Mass 2	$36.5M_{\odot}$
Spin 1 (along z -direction)	0.962
Spin 2 (along z -direction)	-0.900
Chirp mass	$33.8M_{\odot}$

- We look for triggers through O1 and O2 with



FOUND triggers are possible lensed candidates of GW150914

How do you look for them?

- For each found candidate, we evaluate its relative time delay and magnification compared to the detected GW150914 event by :

Relative time delay

$$\Delta t = \text{Time of arrival of candidate} \\ - \text{Time of arrival of GW150914}$$

Magnification

$$\mu = \frac{\text{Likelihood of found trigger}}{\text{Likelihood of GW150914}}$$

How do you look for them?

- Everything seems fine, so what's wrong?

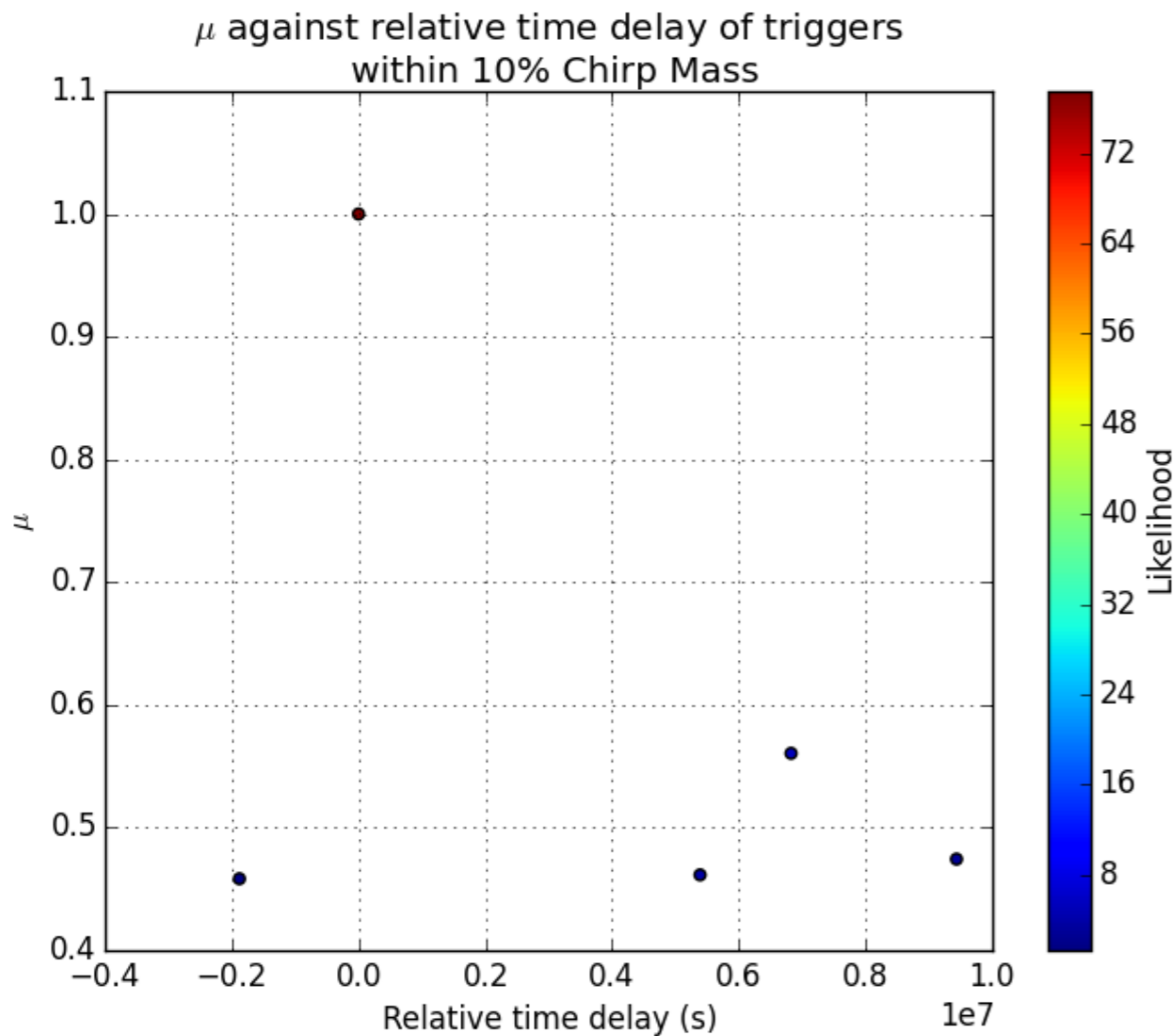


Fig 5. Searched triggers in O1 with parameters within 10% chirp mass from GW150914

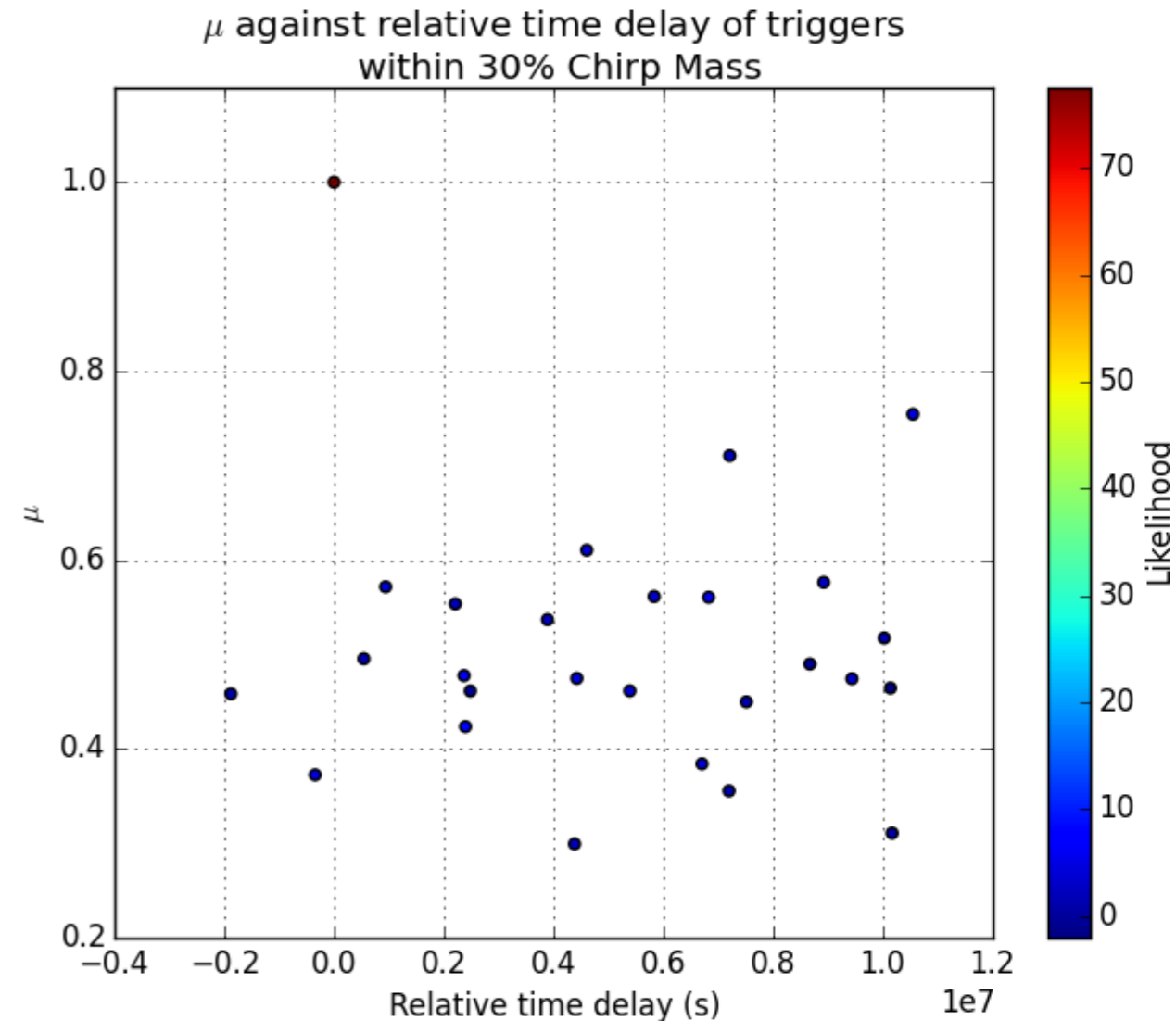
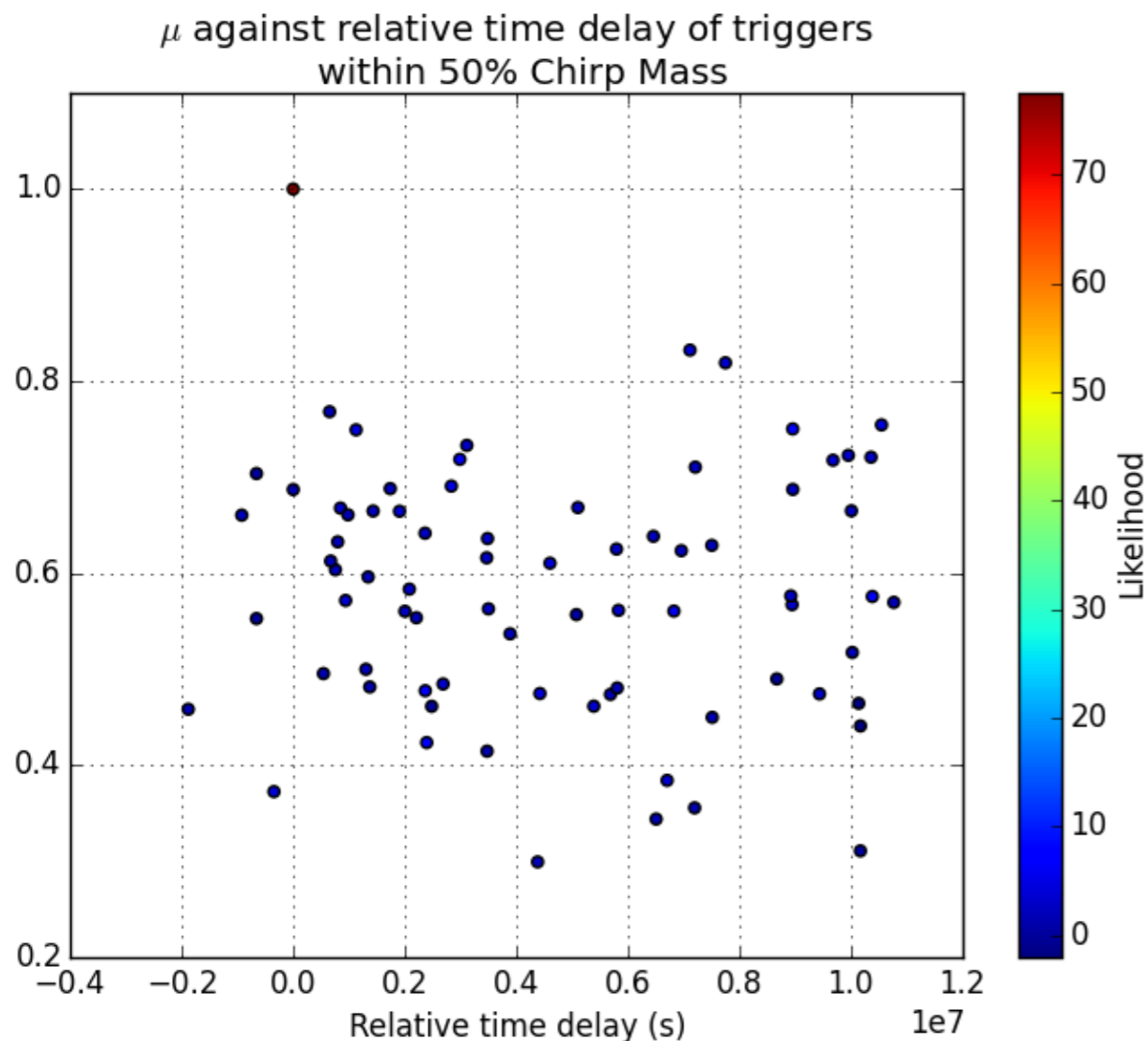


Fig 6. Searched triggers in O2 with parameters within 30% chirp mass from GW150914

How do you look for them?

- Everything seems fine, so what's wrong?



Note :

All of the triggers found in the search has likelihood < 20 , except from GW150914, which has a likelihood > 70

Fig 7. Searched triggers in O1 with parameters within 50% chirp mass from GW150914

How do you look for them?

- Everything seems fine, so what's wrong?

- Magnification of found triggers are unexpectedly high!
- Likelihood of GW150914 is already really high!

Reason : We neglected χ^2 for the detection.

A red, rectangular stamp with a distressed, ink-like texture. The word "CANCELLED" is written in a bold, sans-serif font, slanted upwards from left to right. The stamp has a double-line border.

Unclustered data search

How do you look for them?

- Step 1: Rough estimate using unclustered gstLAL data

- Rerun part of the gstLAL run jobs
- Obtain unclustered data for each focused event
- Select templates around the time of event with $\text{SNR} > 70\%$ of the maximum as lensed GWs templates
- Search through O1 and O2 to find matching triggers.

FOUND triggers are possible lensed candidates of GW events

- Plot likelihood distribution of found triggers.

How do you look for them?

- Step 1: Rough estimate using unclustered gstLAL data

- Just for examples, we are showing the results for GW150914 and GW170814.
- Blue boxes on the left : **Matched lensed candidates**
- Blue boxes on the middle / right : **Detected event(s)**

Note : We also did similar work for GW170608 and GW170823, see **final report** for details.

How do you look for them?

- Step 1: Rough estimate using unclustered gstLAL data

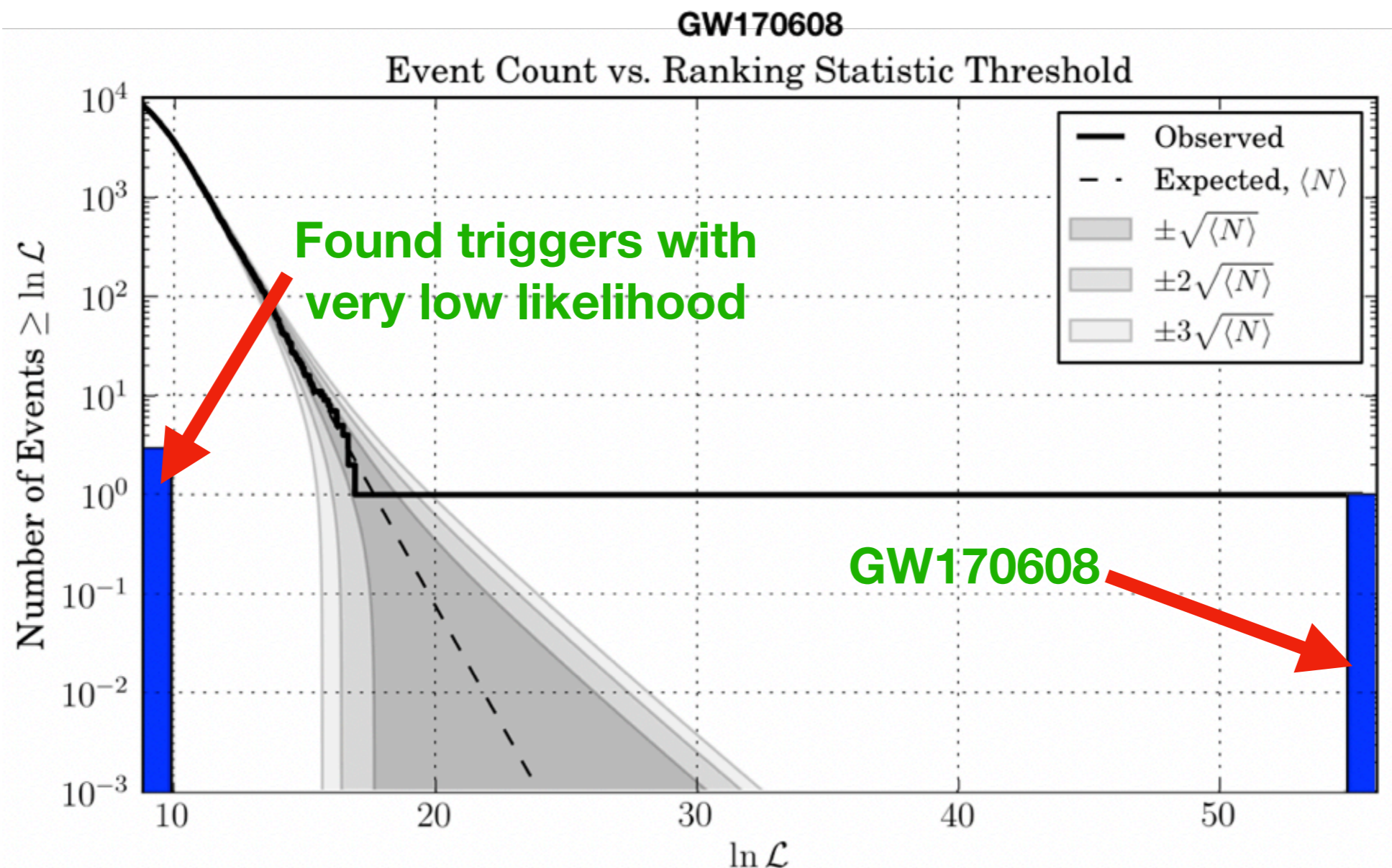


Fig 10. Distribution of likelihood (blue bars) of searched matching triggers in O2-chunk-GW170608 using raw data for the event GW170608. The blue bar on the right boundary corresponds to the detection of the event GW170608.

How do you look for them?

- Step 1: Rough estimate using unclustered gstLAL data

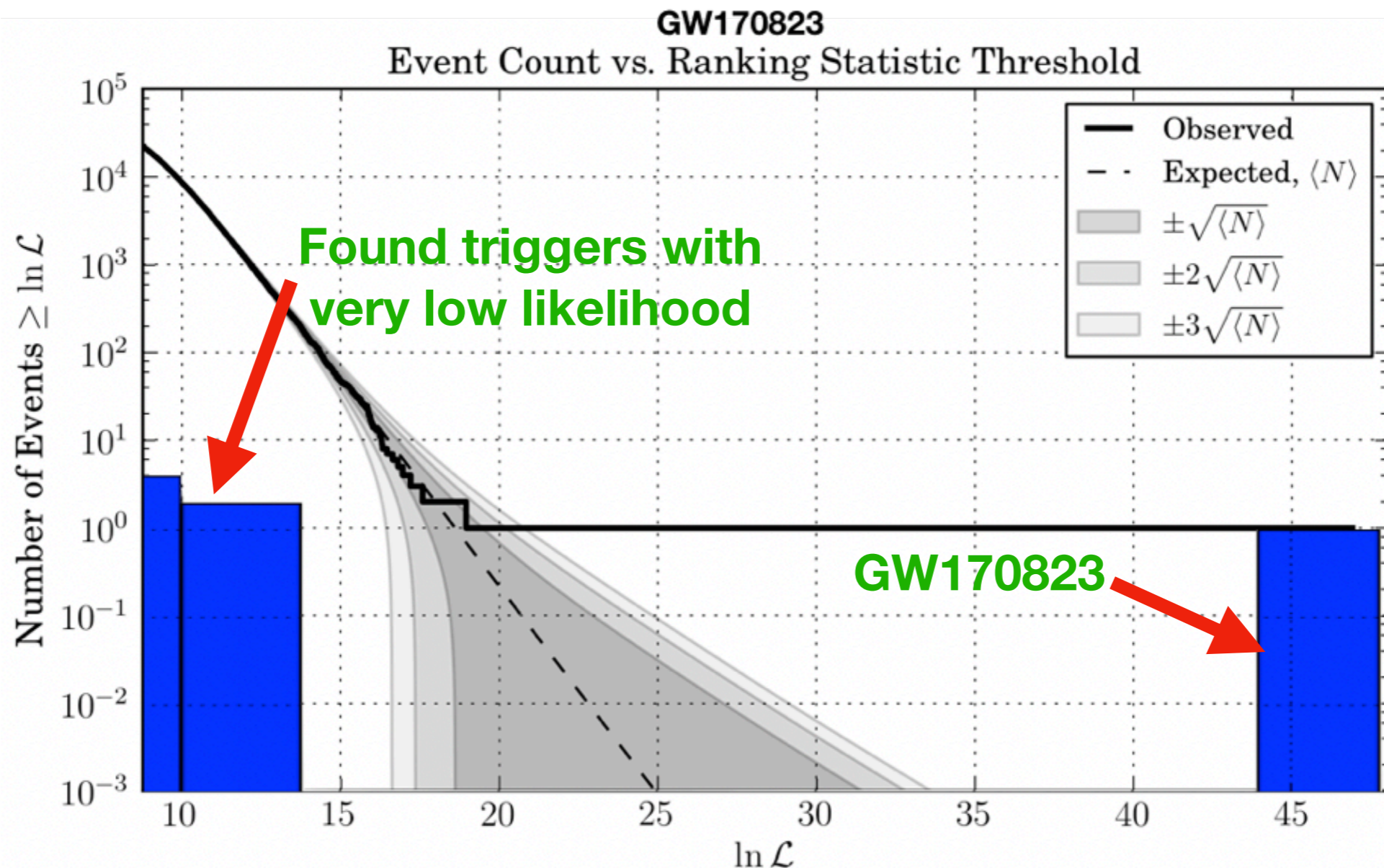


Fig 11. Distribution of likelihood (blue bars) of searched matching triggers in O2-chunk22 using raw data for the event GW170823. The blue bar on the right boundary corresponds to the detection of the event GW170823.

**More details on
generating simulated
lensed signals**

How do you look for them?

- **Step 2: Injection campaign**

Generating simulated lensed GWs

- **SNR ratio of matched filter:**

$$\rho(t) = \frac{|z(t)|}{\sigma}$$

- **Sensitivity of instrument:**

$$\sigma^2 = 4 \int_0^{\infty} \frac{|\tilde{h}(f)|^2}{S(f)} df$$

\tilde{h}_1 : GW Signal amplitude

$S(f)$: Power spectral density

- **Modulus of complex filter output:**

$$z(t) = 4 \int_0^{\infty} \frac{\tilde{s}(f) [\tilde{h}^*(f)]}{S(f)} e^{2\pi i f t} df$$

How do you look for them?

- Step 2: Injection campaign

Generating simulated lensed GWs

- The samples only store “distance” D instead of “effective distance” D_{eff}
- Both depends on sky location!
- Particularly...

$$D_{\text{eff}} = D \left[F_+^2 \left(\frac{1 + \cos^2 \iota}{2} \right)^2 + F_\times^2 \left(\cos^2 \iota \right) \right]^{-\frac{1}{2}}$$

F_+, F_\times : Antenna response function for the GW signal

Solve by using the code ***ComputeDetAMResponse***

**Injection Run
GW150914
O1 - Chunk 3**

How do you look for them?

• Step 2: Injection campaign [FULL Version]

GW150914

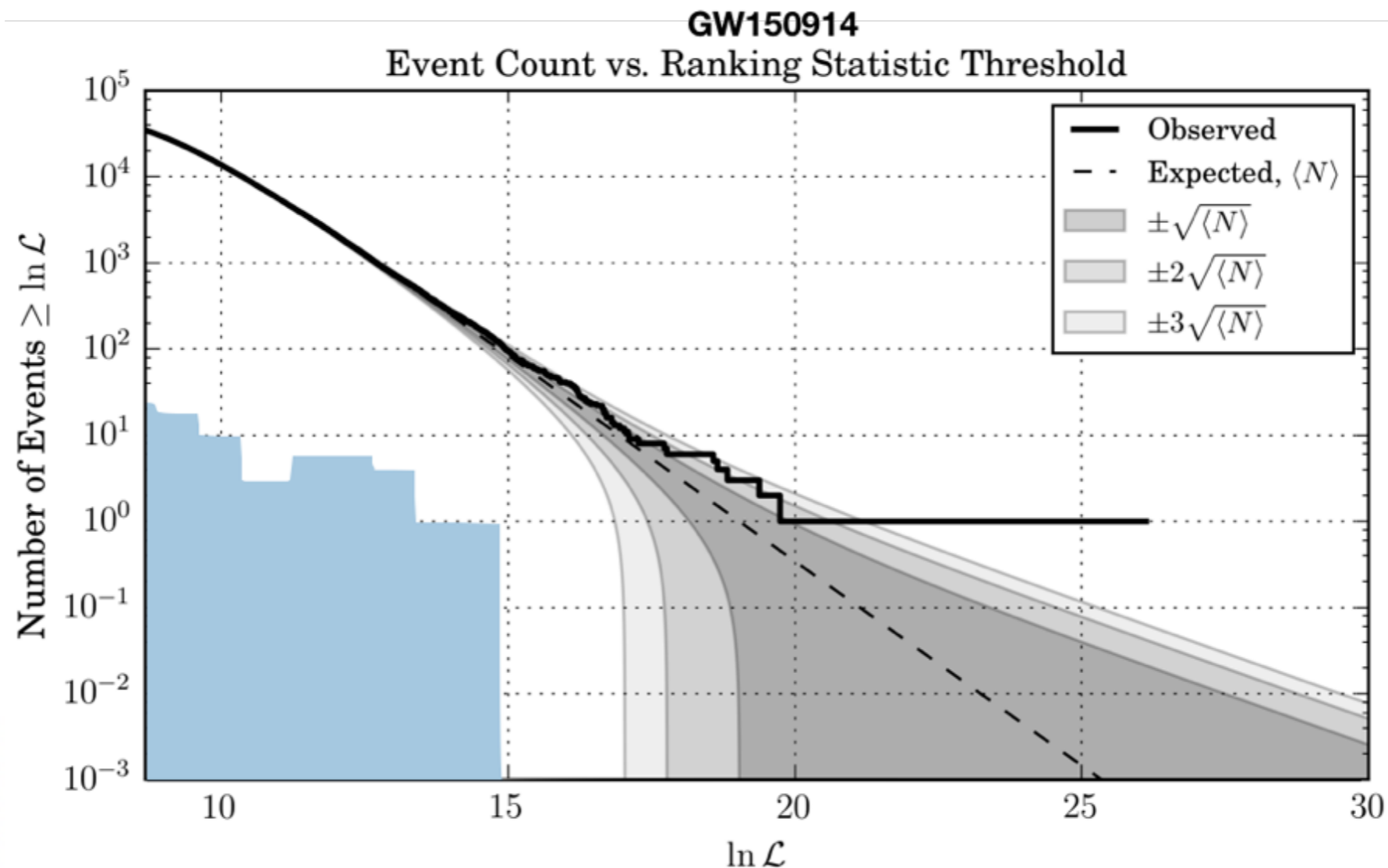


Fig 15. Distribution of likelihood (blue bars) of searched matching triggers in O1-chunk3 using recovered templates from injection run for the event GW150914.

Injection Run GW151226

How do you look for them?

• Step 2: Injection campaign [Shortcut Version]

GW151226

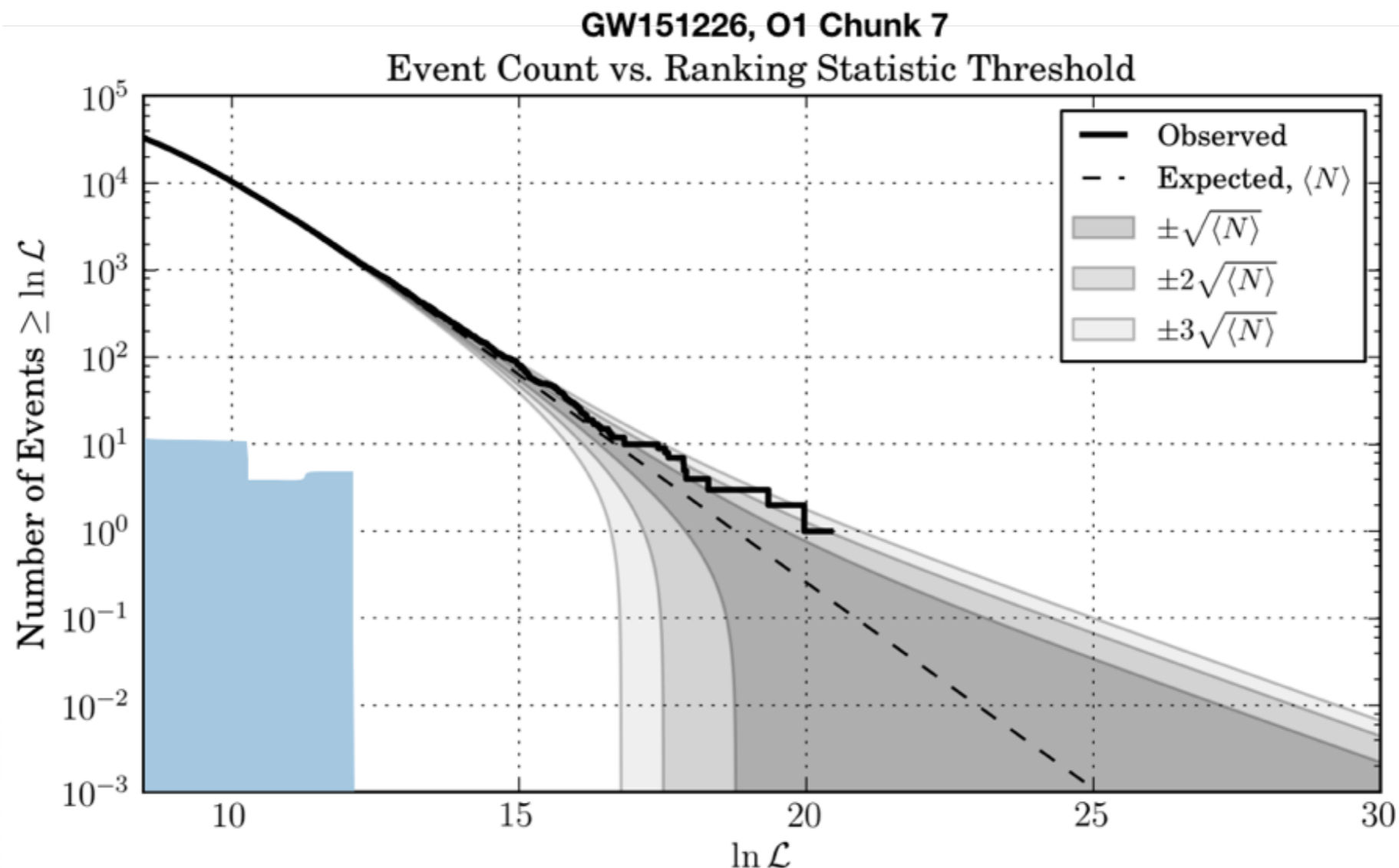


Fig 16. Distribution of likelihood (blue bars) of searched matching triggers in O1-chunk7 using recovered templates from injection run for the event GW151226.

How do you look for them?

- Step 2: Injection campaign [Shortcut Version]

GW151226

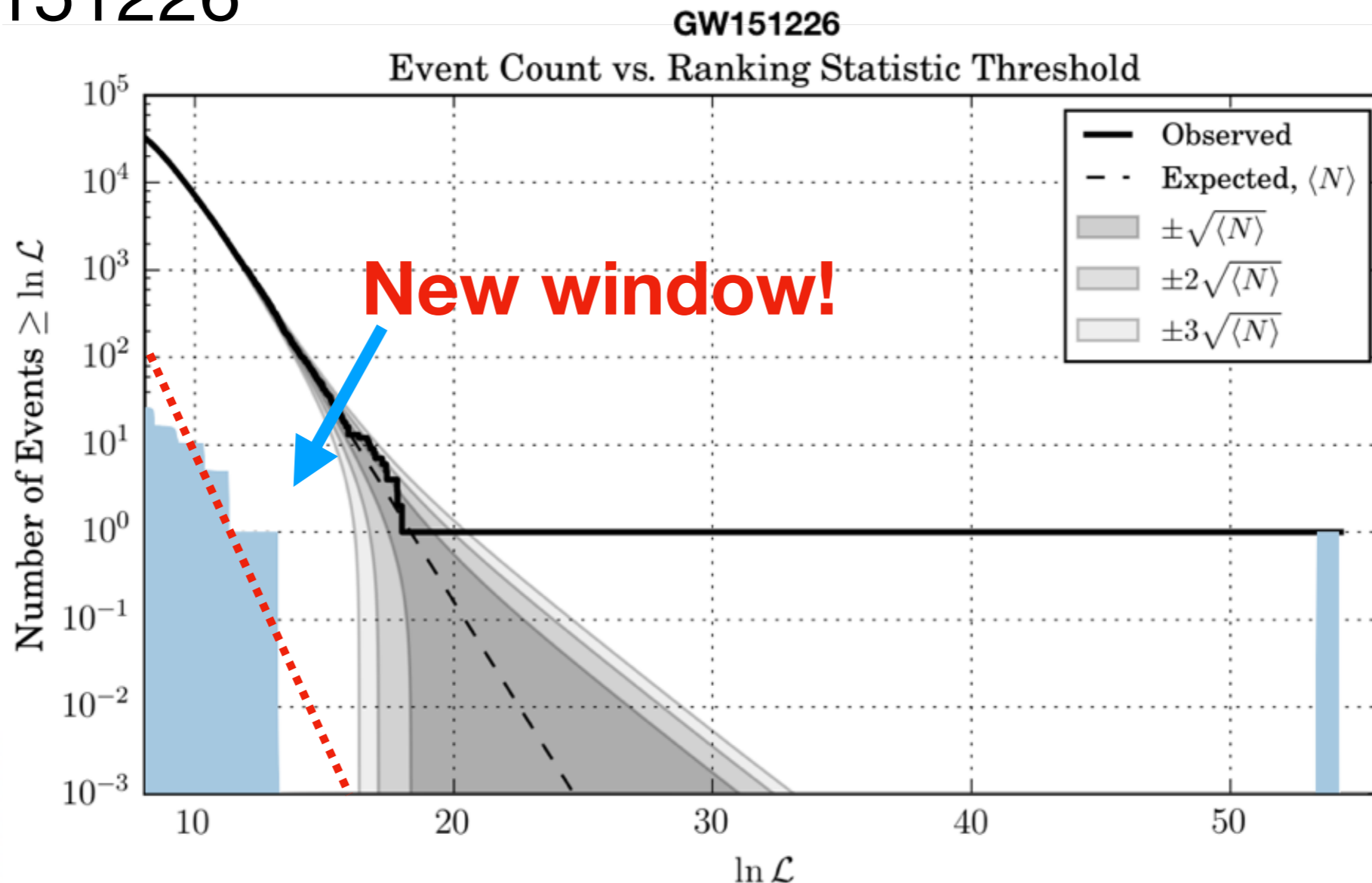


Fig 17. Distribution of likelihood (blue bars) of searched matching triggers in O1-chunk8 using recovered templates from injection run for the event GW151226.

How do you look for them?

• Step 2: Injection campaign [Shortcut Version]

GW151226

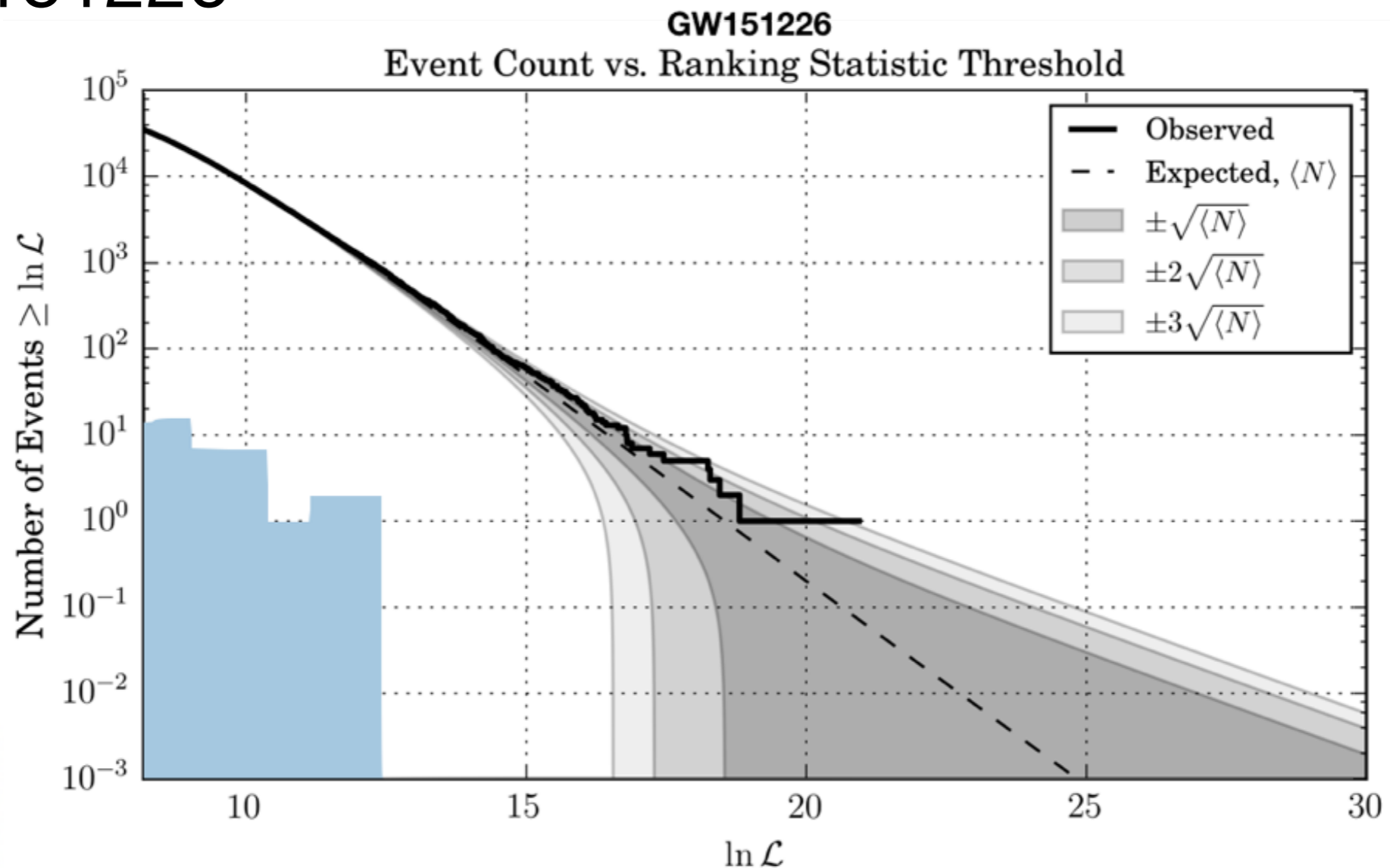
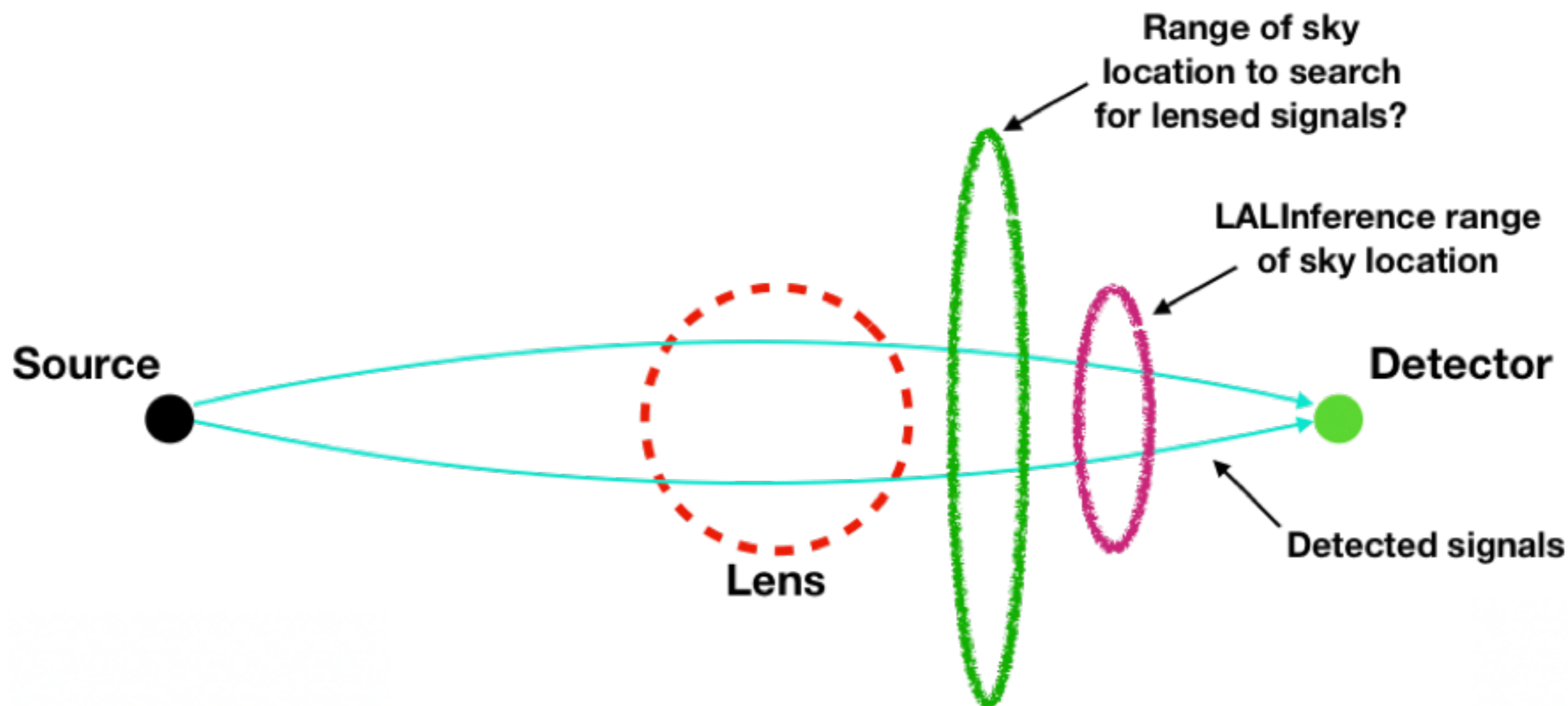


Fig 18. Distribution of likelihood (blue bars) of searched matching triggers in O1-chunk9 using recovered templates from injection run for the event GW151226.

What is the sky location problem

It's not yet finished!

5 Reintroducing sky location problem



Mischief Managed!