Searching for Lensed Gravitational Waves from Compact Binary Coalescences
Interim Report 1

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Einstein's general relativity predicts the radiation of gravitational waves when masses accelerates, for instance, as the components of a binary black hole system orbit each other. This was confirmed when LIGO (The Laser-Interferometry Gravitational-wave Observatory) made the first detection of gravitational waves from a binary black hole merger on 14 September 2015. The success of gravitational waves detection opens a new window for scientists to study the Universe. In Einstein's general theory of relativity, it is also predicted that light rays bend when passing by masses in spacetime, a phenomenon known as gravitational lensing. As a manifestation of Einstein's equivalence principle [1], everything in motion, independent of their nature, is gravitationally lensed in the same way. In such sense, gravitational waves will also be lensed, resulting in multiple signals which differ in arrival times and amplitudes. Since the amplitudes of such signals may differ, there are cases that they are not identified as signals. In this research, we aim to search for lensed signals of the binary black hole signals detected by LIGO. We generate templates of possible lensed gravitational wave signals for detected events by simulating gravitational wave signals as observed by LIGO. Our major objective is to make use of those templates to re-identify possible lensed signals which may have insufficiently high signal-to-noise ratio to be distinguishable from detector noise. We will further attempt to infer the intrinsic properties of the gravitational lenses from the lensed gravitational wave signals identified.

I. Introduction and Motivation

With the successful detections of gravitational waves [2]–[7] over the past few years, we have already verified the existence of gravitational waves predicted by Albert Einstein's general theory of relativity in 1915 [8]. Therefore, it is now the right time to test the other properties of gravitational waves as General Relativity predicts, and in this project, our main focus is gravitational lensing. In particular, we aim to search for lensed gravitational wave signals of confirmed LIGO events from compact binary coalescences. Our major goal is to set up and test a methodology for re-identify possible lensed candidates which are initially indistinguishable from the background. We will then try to infer the intrinsic properties of the gravitational lenses by making use of the identified lensed gravitational wave signals.

In this report, Section II introduces the motivation of the project. Section III provides background information on the research. Section IV discusses the work progress and the problems and challenges encountered so far. Section V describes the upcoming plans for the research and the anticipated challenges. Finally, Section VI illustrates the work plan for the research.

II. Background

A. Properties of Gravitational Waves

According to Albert Einstein's general theory of relativity in 1915 [8], the Universe can be perceived as a fabric of spacetime. Masses like black holes and neutron stars on this fabric produce spacetime curvature [9]. When masses move in spacetime, they cause ripples like water waves generated when one throws a stone into water. Such ripples are known as **gravitational waves**. In this project we focus on four fundamental properties for gravitational waves as predicted by General Relativity, namely their speed, polarization, weak interaction with matter and ability to be lensed gravitationally.

The speed of gravitational waves is predicted by General Relativity to be the same as the speed of light c in vacuum [10]. This has been experimentally confirmed by the detection of gravitational wave from the neutron star inspiral GW170817 in 2017, which constrained the difference between the speed of light and the speed of gravitational wave to between -3×10^{-15} and $+7 \times 10^{-16}$ [11] times the speed of light c.

For the polarization of gravitational waves, as discussed in Ref. [12], we imagine placing a circular ring of test masses on the $x - y$ plane with its center coinciding with the origin. If we assume there exists a transverse - traceless (TT) gravitational wave propagating in the z-direction, then the effect of such gravitational wave is constrained to be on the $x-y$ plane only. Under the influence of the wave, the test masses ring can exhibit two orthogonal deformation modes.

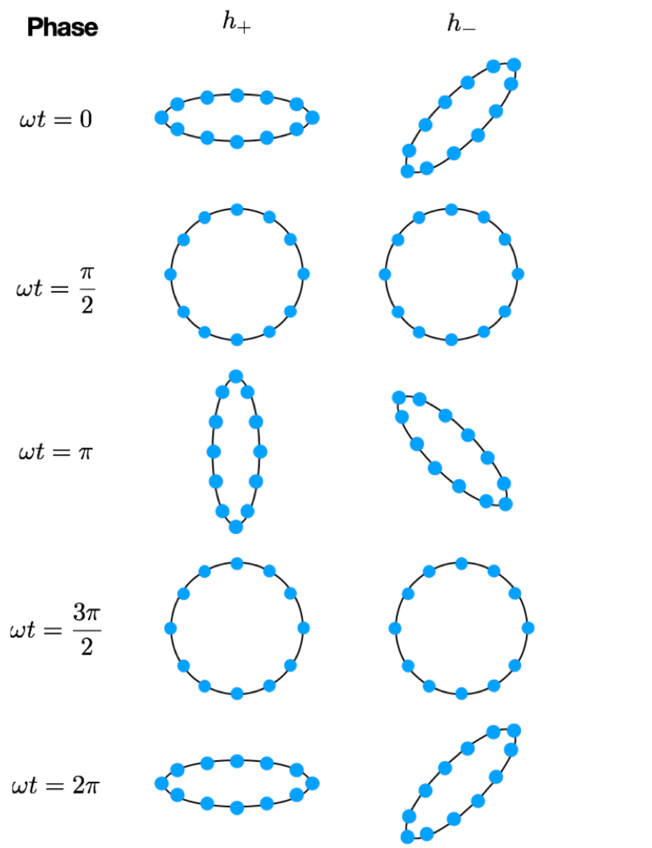

Fig. 1. Two orthogonal deformation modes within one period of the test mass ring in response to a TT-gravitational wave. The upper row refers to the plus polarization (denoted by +) and the lower row refers to the cross polarization (denoted by $\times$) of the gravitational wave. Image reproduced from [12].

As shown in Figure 1, when a **plus (+) polarized** gravitational wave passes through our ring of test masses, the ring is stretched along the y-direction and then along the x-direction into an ellipse of the same area as the original circle throughout one period. On the other hand, if the gravitational wave passing through is **cross** ($\times$) **polarized** instead, the ring will be stretched along the $y = x$ and $y = -x$ line in a similar way as for plus (+) polarized gravitational wave. We can see that gravitational waves can be polarized in two particular modes, namely the plus (+) polarization, and cross ($\times$) polarization. The effect of stretching and shrinking of proper lengths between test masses in the ring by polarized gravitational waves is applied to the detection of gravitational waves. In particular, detectors including LIGO and VIRGO detect gravitational waves using interferometry.

B. Detection of Gravitational Waves

A schematic overview of the gravitational wave detector used by LIGO is shown in Figure 2 [13]. It is a Michelson interferometer consisting of two arms, each of 4 km long. A laser beam is incident on a beam splitter, which splits the incident laser beam into two beams propagating along the two arms of the interferometer. At the end of the arm, a mirror reflects the beams which then rejoin at the beam splitter and is finally collected by a photodetector to observe the interference pattern.

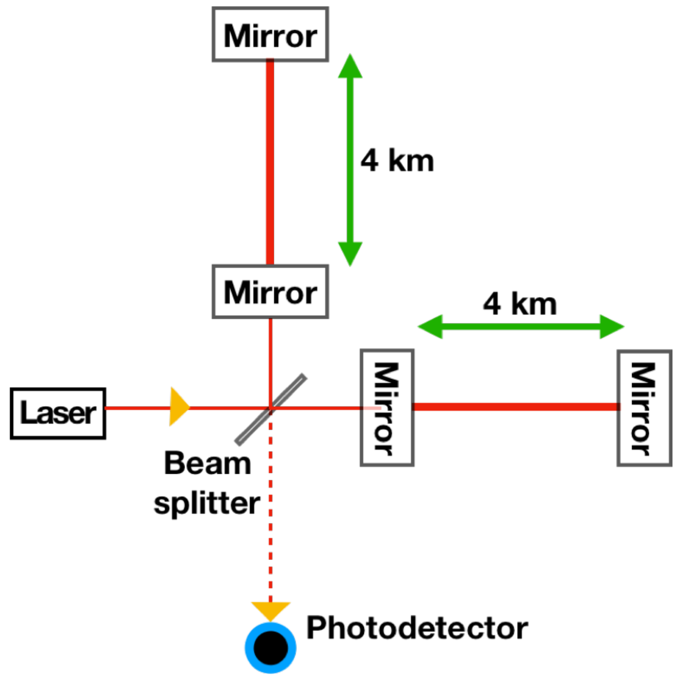

Fig. 2. A schematic overview of the gravitational wave detector used by LIGO. Image reproduced from [13].

The interference of the two laser beams is set to be destructive at the photodetector. Alternatively, when there are alterations to the lengths of the arms which cause a path difference between the two laser beams, a constructive interference pattern will be observed. The change in arm lengths is not necessarily caused by gravitational waves because there is also noise which can cause such effect. These noises include seismic noises, thermal noises, gravity-gradient noises and quantum noises [14].

In order to detect gravitational waves, we must constrain ourselves to those which have a sufficiently large perturbation to spacetime. Typically, we focus on four types of gravitational waves, namely Continuous Waves, Stochastic Waves, Bursts and Compact Binary Coalescenes, the last one is the focus of this research. When two compact objects, for instance, neutron stars and/or black holes, orbit about their common center-of-mass, they will **inspiral** due to loss of orbital energy by means of gravitational radiation, then **merge** into a single object which then **rings down** . This sequence, "inspiral-merger-ringdown", is referred to as "coalescence". Among the four mentioned types of gravitational waves, Compact Binary Coalescences are sources of gravitational waves with well modelled waveforms compared to other kinds of gravitational wave sources, and hence one can use a technique called **matched filtering** to search for such signals.

We now outline the major steps in analyzing gravitational-wave data [12]. Currently, matched filtering is a method to distinguish the weak gravitational wave signals from the detector noise fluctuations. The principle of matched filtering is to slide templates of an expected waveform from an astrophysical event across the received data and look for a strong cross-correlation between the two.

We denote $s(t)$ as the signal received from a detector, $n(t)$ as the background noise and $h(t)$ as the gravitational wave signal (if it exists). $s(t)$ is the sum of $n(t)$ and $h(t)$, that is

$$s(t) = n(t) + h(t). \tag{1}$$

If we have a filter $P(t)$, we may define

$$\hat{s} = \int s(t)P(t)dt. \tag{2}$$

We denote $\langle S \rangle$ and N as the expectation value and root mean square value of $\hat{s}$ if a gravitational wave signal is included in the data respectively. Then we have

$$\begin{aligned} \langle S \rangle &= \int \langle s(t)P(t) \rangle dt \\ &= \int \langle (n(t) + h(t))P(t) \rangle dt \\ &= \int \langle h(t)P(t) \rangle dt \\ &= \int \tilde{h(f)}\tilde{P}^*(f)df, \end{aligned} \tag{3}$$

where we have taken $\langle n(t) \rangle = 0$ (since the noise is assumed to be random and gaussian) and the tilde ($\tilde{A}$) denotes the Fourier-transformed function of A.

Also, if $h(t) = 0$, we have

$$\begin{aligned} N^2 &= \langle \hat{s}^2 \rangle - \langle \hat{s} \rangle^2 \\ &= \langle \hat{s}^2 \rangle dt \\ &= \int \int P(t)P(t') \langle n(t)n(t') \rangle dt dt' \\ &= \frac{1}{2} \int S_n(f) |\tilde{P}(f)|^2 df, \end{aligned} \tag{4}$$

where $S_n(f)$ denotes the power spectral density. We can therefore define **signal-to-noise ratio (SNR)** as

$$\begin{aligned} \rho &= \frac{\langle S \rangle}{N} \\ &= \frac{\int \tilde{h}(f)\tilde{P}^*(f)df}{\sqrt{\int \frac{1}{2} S_n(f) |\tilde{P}(f)|^2 df}}. \end{aligned} \tag{5}$$

Furthermore, we define the inner product between two functions $x(t)$ and $y(t)$ to be:

$$\langle x, y \rangle = \Re \left(\int_{-\infty}^{\infty} \frac{\tilde{x}^*(f)\tilde{y}(f)}{\frac{1}{2} S_n(f)} \right) df. \tag{6}$$

Consequently, we can have equation (5) simplified as

$$\rho = \frac{\langle d, h \rangle}{\sqrt{\langle h, h \rangle}}, \tag{7}$$

where d is the data received in the detector. With an optimal matched filter $P(t)$, and provided that the identical gravitational wave signal can be seen in coincidence between two or more detectors, LIGO detectors can detect inspiral signals with a network SNR $\rho_{\text{net}} > 8$ [15]. The network SNR for two or more detectors is simply calculated by adding the SNR of individual detectors in quadrature, that is

$$\rho_{\text{net}}^2 = \Sigma_i \rho_i^2, \tag{8}$$

where the index i runs over the individual detectors [12].

In reality, the rate of gravitational wave events occurring is expected to as low as about a few per year [14]. To avoid mistaking large and infrequent detector noise fluctuations mimicking events as signals, we need to find out the **false alarm rate (FAR)**, which is how often an abnormal noise signal mimicking event can be measured. The smaller the FAR is, the more plausible the candidate is a real astrophysical event. The FAR for any signal is estimated by [16]

$$\text{FAR} = \frac{N}{\Sigma_i T_i}, \tag{9}$$

where N is the total number of background triggers similar to the one which we consider as a real signal, and T_i is the analyzed time interval in the i^{th} background trial.

C. Gravitational Lensing

As predicted by General Relativity, since masses can curve spacetime, the path of a light ray from a source can be bent and deflected before reaching the observer (See Figure 3). Such effect is known as gravitational lensing, in the sense that it is similar to light rays being bent by optical lenses, but in this case the "lenses" are masses instead. In particular, since a source emits light rays in all direction, light rays propagating along different directions are bent differently and may, therefore, form multiple images. The images can vary in arrival time and amplitudes.

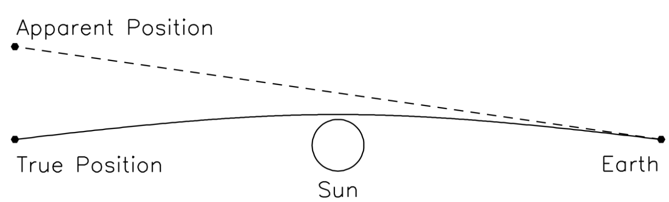

Fig. 3. Light rays from a source are bent because of a gravitational lens in between the source and the observer. Image from [17].

In fact, all electromagnetic waves, as well as all gravitational waves, can be gravitationally lensed in the same way, and this phenomenon has been observed on astronomical scales for light of all wavelengths. However, the study of gravitational lensing of light encounters difficulties from the blocking of light by dust clouds in the Universe, as well as the large noise which screened the light signals [18]. General relativity also predicts that gravitational waves, having a similar nature as light, can also be lensed gravitationally, producing multiple signals, and same as light, is achromatic. In contrast to light, gravitational waves are not disturbed by the dust clouds between the source and observing point.

Over the past two years, more than six gravitational wave detections have been successfully made [2]–[7], which have confirmed the prediction of the existence of gravitational waves. Among the four predicted fundamental properties of gravitational waves, which are their speed, polarization, weak interaction with matter and ability to be lensed gravitationally, that we mentioned at the very beginning of Section II, we are only left with the last one - ability to be gravitational lensed untested. Therefore, it is now the right time for us to start searching for lensed gravitational wave signals so as to test the final property of gravitational wave predicted from general relativity.

Due to lensing, there are time delays among the waves of images. In the discussion for electromagnetic waves, there are two major contribution to the delay, namely refraction and gravitational time delay. Gravitational lensing occurs when light rays pass through spacetime perturbed by masses. This will form multiple signals which differ in amplitude and time of arrival. The difference in arrival time is due to 1) The path lengths travelled from the images to the observer vary, and 2) The effective speed of light can be different under the influence of a refractive index larger than one, resulting in arrival time delay.

The same thing happens with gravitational waves, except that their weak interaction with matter means that the refractive index is negligibly different from one. Therefore, the only crucial effect to account for is the geometric effect, which causes both magnification and time delay of lensed signals. An important point to note here is that there is no dispersion, and hence the geometric lensing is achromatic. That is to say, it affects all frequency components of the wave in exactly the same way.

In the derivations below, for cosmological distances, they are referred to as the angular diameter distances. As shown in Figure 4, the angular diameter distances from the observer to the lens and the source are given by D_L and D_S respectively, and that from the source to the lens is D_{LS}. When we compare the path difference between the unperturbed ray (dotted line between the observer and the source in figure 4), that is when the lens is absent, and the lensed ray (solid lines between the observer and the source), we have [19]

$$\vec{\xi} = \frac{D_L D_{LS}}{D_S}(\vec{\theta} - \vec{\theta_S}), \tag{10}$$

where ξ is the separation between the two rays at the lens, $\vec{\theta}$ is the two-dimensional angle between the horizontal of the observer and the point where the gravitational waves strike the lens, and $\vec{\theta}_S$ is the angle between the horizontal of the observer and the source.

With this, we have the geometrical path difference $\Delta\lambda$ between the unperturbed ray and lensed ray is given by

$$\Delta\lambda = \frac{\vec{\xi}(\vec{\theta} - \vec{\theta_S})}{2}. \tag{11}$$

Finally, the geometrical time delay Δt due to gravitational lensing is given by

$$\Delta t = (1 + z_d)\frac{D_L D_{LS}}{2 D_{S} c}(\vec{\theta} - \vec{\theta_S})^2, \tag{12}$$

where z_d denotes the gravitational redshift. From the calculation of the time delay, we are able to infer the distance of the lens from the observer.

For gravitationally lensed gravitational wave signals, the lensed waveform has an amplitude $h^{\text{lensed}}_{+,\times}(f)$ given by [20], [21]

$$h^{\text{lensed}}_{+,\times}(f) = F(\omega, y) h^{\text{unlensed}}_{+,\times}(f), \tag{13}$$

where $h^{\text{unlensed}}_{+,\times}(f)$ denotes the amplitude of the unlensed gravitational waves, and $F(\omega, y)$ is the amplification function given by

$$F(\omega, y) = \exp\left[\frac{\pi\omega}{4} + i\frac{\omega}{2}\left(\ln\left(\frac{\omega}{2}\right) - \frac{\sqrt{y^2+4} - y}{4}^2 + \ln\left(\frac{\sqrt{y^2+4}+y}{2}\right)\right)\right]\Gamma\left(1 - \frac{i}{2}\omega\right) \times {}_1F_1\left(\frac{i}{2}\omega, 1; \frac{i}{2}\omega y^2\right), \tag{14}$$

where $h^{\text{lensed}}_{+,\times}(f)$ is the waveform without lensing, Γ is the complex gamma function, ${}_1F_1$ is the confluent hypergeometric function of the first kind, $\omega = 8\pi M_{\text{Lz}} f$; $M_{\text{Lz}} = M_{\text{L}}(1 + z_L)$ is the redshifted lens mass, $y = \frac{D_L S}{\Xi_0 D_S}$ is the source position, $\Xi_0 = (\frac{4 M_L D_L D_{\text{LS}}}{D_S})^{\frac{1}{2}}$ is a normalisation constant, and M_L and z_L are the lens mass and redshift respectively. From finding the amplitude of the lensed gravitational waves, we can infer both the mass M_L and the position of the lens.

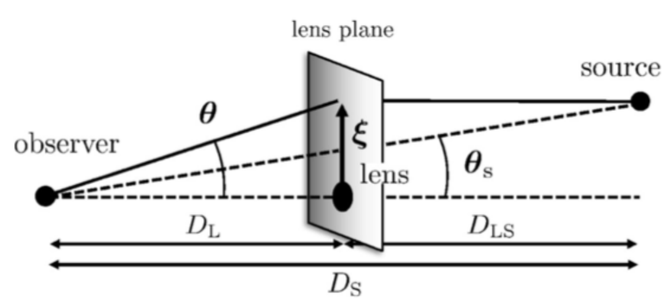

Fig. 4. In this figure, D_L denotes the distance between the lens and the observer, D_S denotes the distance between the observer and the image, D_{LS} denotes the distance between the lens and the image, $\vec{\theta}$ denotes the two dimensional angle between the observer and lensing point, and $\vec{\theta_s}$ denotes the two dimensional angle between the source and the observer. Note that θ and $\theta_{\mathbf{s}}$ are both two-dimensional angles. Image from [19].

Consider a point mass lens, particularly for compact objects like black holes or stars. In the geometrical optics limit ($f >> M_{\rm Lz}^{-1}$) from the equation above, we have [21]

$$F(\omega, y) = |\mu_+|^{1/2} - i|\mu_-|^{1/2} e^{2\pi i f \Delta t_d}, \qquad (15)$$

where the magnification of each image is

$$\mu_{\pm} = \frac{1}{2} \pm \frac{(y^2+2)}{2y\sqrt{y^2+4}}, \qquad (16)$$

and the time delay between the double images is

$$\Delta t_d = 4M_{\rm Lz}\left[\frac{y\sqrt{y^2+4}}{2} + ln\left(\frac{\sqrt{y^2+4}+4}{\sqrt{y^2+4}-y}\right)\right]. \qquad (17)$$

The typical time delay for the point mass lens is therefore $2 \times 10^3 s \times \left(\frac{M_L}{10^8 M_\odot}\right)$. Furthermore, for gravitational waves from coalescence of super massive black holes of mass $10^4 - 10^7 M_\odot$ under the lensing effect of a point mass lens of mass in the range $10^6 - 10^9 M_\odot$, then the typical time delay will be $10 - 14s$ [21]. Therefore, for gravitational waves from blackholes of masses lower than $10^4 M_\odot$, we would expect a time delay in the range $10^1 - 10^3$s.

III. Work Progress

A. GstLAL search pipeline

This research is based on the use of GstLAL search pipeline. Figure 5 shows the schematic flow of the pipeline. Before the start of SURF period, the working scheme of the pipeline has been studied and GstLAL search practice has been run.

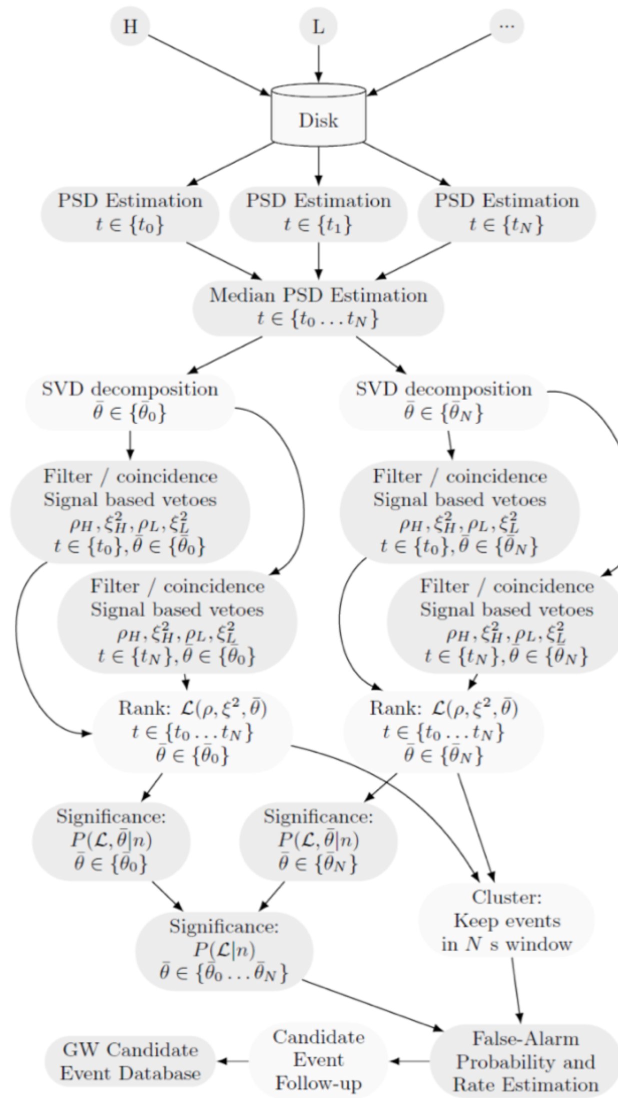

Fig. 5. A schematic flow of the GstLAL search pipeline. Image from [22].

B. Searching for possible lensed candidates for GW150914 in O1 and O2 using LALInference posterior data

We make use of LALInference software library [23] posterior data analysis of the event GW150914. The following table shows the posterior estimation of the parameters of the two black holes involved in GW150914:

Parameter	Maximum Posteriori (maP)	Variation (σ)
$m_{1,\rm source}$	$32.9M_\odot$	$4.9M_\odot$
$m_{2,\rm source}$	$13.7M_\odot$	$3.5M_\odot$
$a_{1,\rm z}$	-0.618	0.218
$a_{2,\rm z}$	0.083	0.243

where $m_{1,\rm source}$, $m_{2,\rm source}$, $a_{1,\rm z}$ and $a_{2,\rm z}$ are the respective masses and components of spins aligned with the orbital angular momentum of the binary blackhole system of the two black holes in GW150914 evaluated by the LALInference library. Using the information, we search for triggers throughout O1 and O2 with masses and spins within 3 and 4 sigmas from the maP of GW150914 which are regarded as possible lensed candidates for the event. Figure 6 to 9 show the search results for O1 and O2 within 3 sigma range and 4 sigma range. Note that μ on the y-axis refers to the magnification of the triggers comparing to GW150914, which is evaluated by:

$$\mu = \frac{\text{Signal-to-noise ratio of found trigger}}{\text{Signal-to-noise ratio of GW150914}}, \qquad (18)$$

and the relative time delay on the x-axis refers to the time delay of the found triggers relative to the geocentric arrival time of GW150914, which is $1126259462s$ [2] (The corresponding UTC time is 2015-09-14 09:50:45). The colours of the dots indicate the likelihood, a measure of the distinguishability of the event from the detector noise, of the triggers.

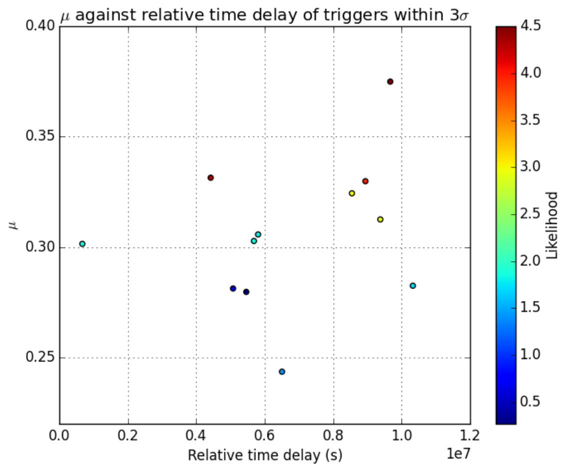

Fig. 6. Searched triggers in O1 with parameters within 3 sigma range from GW150914.

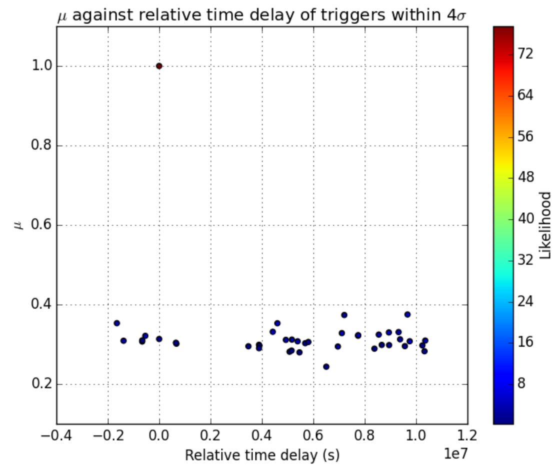

Fig. 8. Searched triggers in O1 with parameters within 4 sigma range from GW150914. Note that the detected GW150914 event is visible at relative time delay = 0 and μ = 1

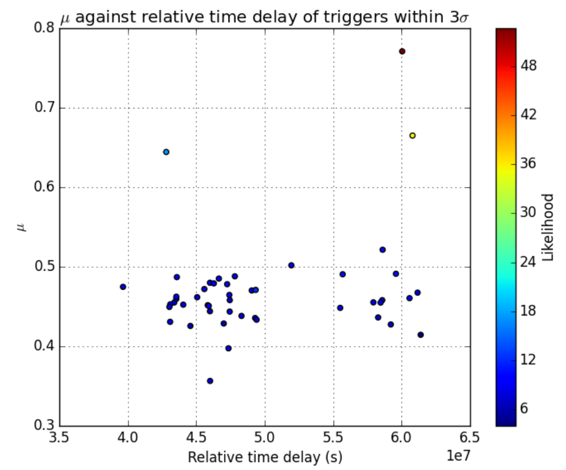

Fig. 7. Searched triggers in O2 with parameters within 3 sigma range from GW150914.

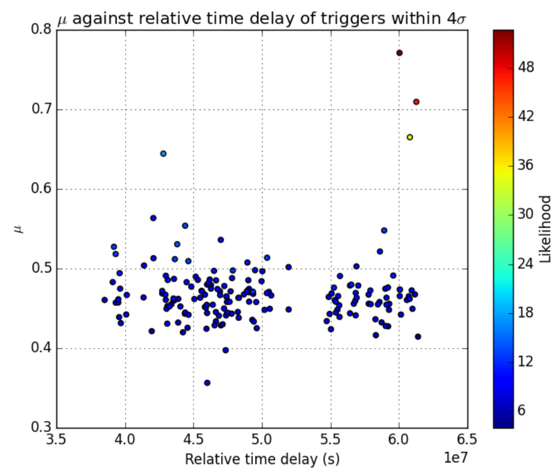

Fig. 9. Searched triggers in O2 with parameters within 4 sigma range from GW150914.

Problems

We note that the trigger, which is a candidate event where the SNR $\rho(t)$ peaks in time above a certain threshold, corresponding to GW150914, which should have $\mu = 1$ and Relative time delay = 0, does not show up in the 3 sigma range plot in O1, and it only shows up when we loosen the range to 4-sigma. This is due to the inconsistency of data used between LALInference and GstLAL. In fact, LALInference is designed to accurately infer the parameters of the source, while GstLAL is not. Therefore, their results are not completely agreeing with each other, leading to the absence of GW150914 in the 3-sigma plot.

Also, we are aware that the signal-to-noise ratio (SNR) evaluated in both O1 and O2 may have discrepancies since the background noise is varying every moment. Initially, we proposed looking into the power spectral density in O1 and O2 to link the SNRs, but we decided to do better and hence this method is called off.

C. Searching for possible lensed candidates for GW150914 in O1 and O2 using GstLAL data

Regarding the problems in the previous method, we rerun the search by using GstLAL data. The following table shows the GstLAL parameter data regarding the event GW150914 [2]:

Parameter	Value
Mass 1	$47.9M_{\odot}$
Mass 2	$36.5M_{\odot}$
Spin 1 (along z-direction)	0.962
Spin 2 (along z-direction)	-0.900
Chirp mass	$33.8M_{\odot}$

Using similar techniques from the last method, we search through O1 to look for triggers with mass 1 and mass 2 within a certain percentage range of the chirp mass. The objective is to find a distinctive feature for separating possible lensed triggers from the background. Figure 10 - 14 show the results for 10%, 30% and 50% chirp mass range.

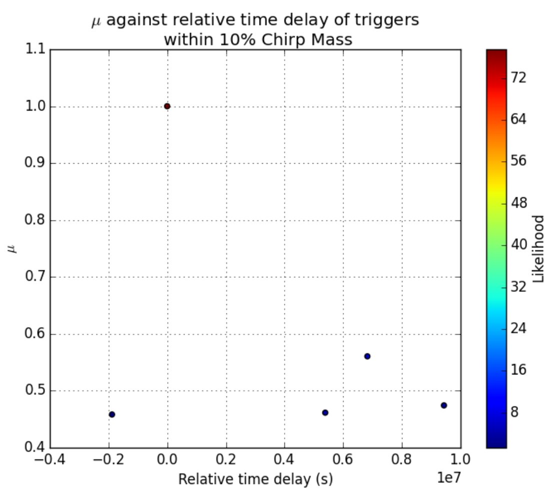

Fig. 10. Searched triggers in O1 with mass 1 and mass 2 within 10% chirp mass range from GW150914.

We note that all of the triggers in the search have likelihood smaller than 20, except for the detected GW150914 event which has a likelihood above 70.

Problems

Although some triggers appear to be distinguishable from the background cluster in the 30% and 50% plots, the magnification of those triggers is unexpectedly high (up to 0.7) considering the exceptionally high SNR of GW150914. A possible reason behind is our neglecting of χ^2 for the detection. We decided to shelve this method and to obtain a distribution of the likelihood of possible lensed triggers as our next step.

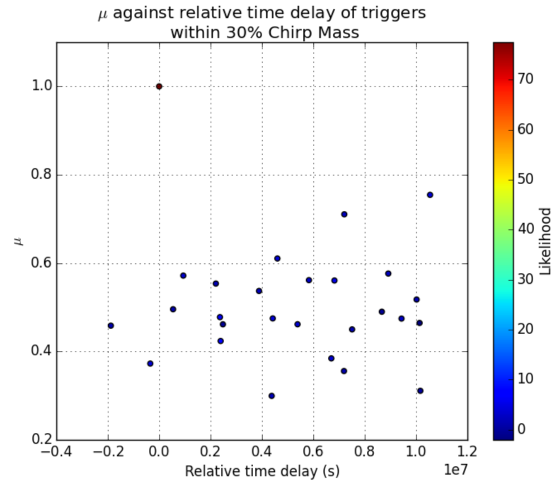

Fig. 11. Searched triggers in O1 with mass 1 and mass 2 within 30% chirp mass range from GW150914.

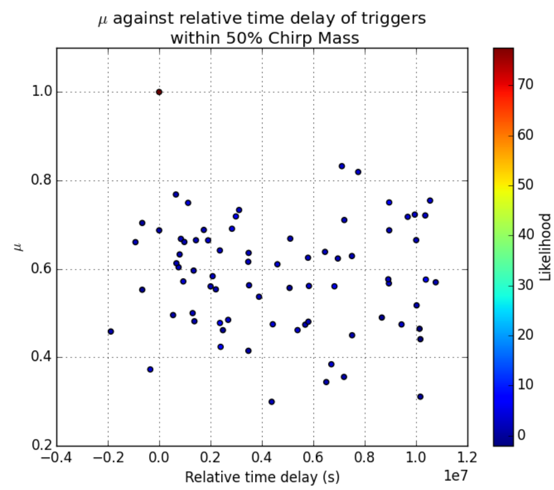

Fig. 12. Searched triggers in O1 with mass 1 and mass 2 within 50% chirp mass range from GW150914.

D. Searching for possible lensed candidates for GW150914 and other events in O1 and O2 using unclustered GstLAL data

We aim to retrieve a likelihood distribution of lensed gravitational wave signals at this stage. The objective of doing so is to figure out the range of search for them. It is expected that the event-count vs ranking statistic threshold curve will be shifted downward for lensed triggers, as shown in Figure 13, since in our GstLAL run, the background noise distribution is not from the entire template bank, but instead from a much smaller template bank, and therefore produces much less background, which allows us to do a targeted search for the lensed gravitational wave signals. This is the major reason why we are launching the injection campaign in the later stage of our project.

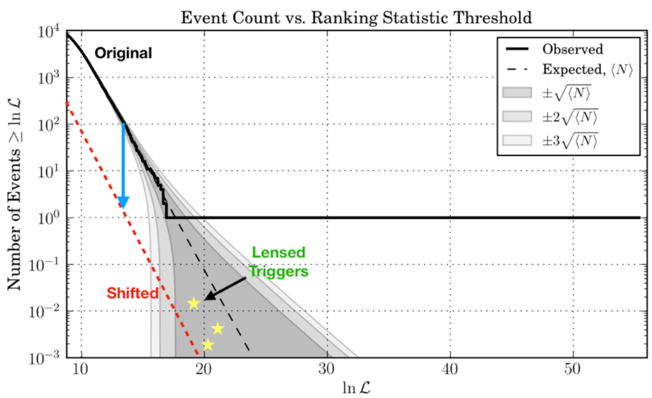

Fig. 13. Expected event-count vs ranking statistic threshold curve for lensed gravitational wave signals, using GW170608 as an example. Note that the shifted red event-count vs ranking statistic threshold curve, and also the yellow stars denoting the lensed triggers we expect to find, are only for illustrative means. In other words, they are not actual data.

We rerun part of the GstLAL run jobs and obtain the unclustered data for each focused event. We then obtain templates around the time of the event and select those with SNR higher than 70% of the maximum. We search through the chunk in which the event happened to find triggers which match the parameters (mass 1, mass2, spin1z, spin2z) in our template bank exactly and regard them as possible lensed triggers. Finally, we plot the distribution of the likelihood of the triggers and compare it with the event-count vs ranking statistic threshold graph. Figure 13 - 16 show the results for GW150914, GW170608 and GW170814. In each of the figure, one sees the solid black line (observed) and the dashed line (expected) indicating the event-count vs ranking statistics threshold curve with the background from the entire template bank. The curve with background from the much smaller template banks we used for our search is not yet to be known. The blue bar(s) in the middle and/or right side of each graph corresponds to the detected event, while those on the left refer to some found triggers with very low likelihood from our search. We hope to get a sense of how the likelihood distribution for lensed gravitational wave signals would look like from

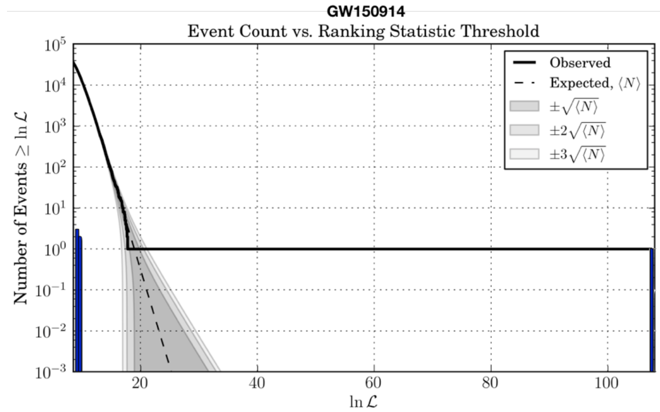

Fig. 14. Distribution of likelihood (blue bars) of searched matching triggers in O1-chunk1 using raw data for the event GW150914. Note that the barely visible blue bar on the right boundary corresponds to the detection of the event GW150914.

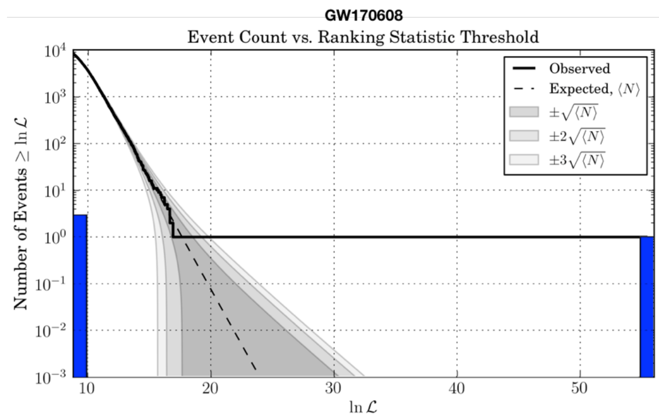

Fig. 15. Distribution of likelihood (blue bars) of searched matching triggers in O2-chunk-GW170608 using raw data for the event GW170608. The blue bar on the right boundary corresponds to the detection of the event GW170608.

these searches.

Problems

From the plots, we can already get a sense of how the distribution of the likelihood of possible lensed triggers will be. However, we are still uncertain about the searching range for possible lensed triggers. Until now, we are still varying the SNR percentage threshold to get a satisfactory result. Therefore, a more systematic way to actually obtain the likelihood distribution of lensed candidates will be to run an injection campaign, which is done in Week 4 - 6.

IV. Plans and foreseeable challenges

A. Inferring the properties of the gravitational lens

Once we identify lensed gravitational wave signals, we will go on and try to infer some properties of the gravitational lens, including its mass, the lens-observer distance and the lens-source distance.

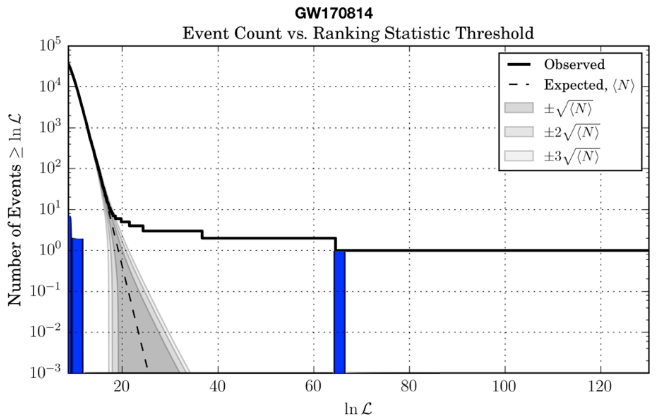

Fig. 16. Distribution of likelihood (blue bars) of searched matching triggers in O2-chunk21 using raw data for the event GW170814. The blue bar in the middle refers to the detection of the event GW170814. Note that the solid (observed) event-count versus ranking statistics threshold curve extends beyond the middle blue bar instead of stopping there, since there is another detection, which is GW170817, in the same chunk we are analysing here.

Foreseeable challenge(s)

The parameters inference step requires further literature review on gravitational lensing and involves more calculations. Alternatively, we may try to use available lens models, like in Section V we follow [24] to reproduce the results for probability distribution of relative time delay and magnification of lensed gravitational wave signals under the influence of a singular isothermal ellipsoid, to check how likely the identified triggers are under the effect of the proposed models. We may also consider making use of the package LENSTOOLS [25]. The package is being widely used for lens optimisation in studying gravitational lensing but requires further literature review before it can be applied to our research.

B. Running injection campaign for all other detected events

If the injection campaigns for GW150914 and GW151226 succeed, we will go on to run injection campaign for other detected events such as GW170104 and GW170608.

Foreseeable challenge(s)

The clusters are still unstable and slow. Therefore, the injection campaign may take more time than expected.

C. Using galaxy cluster / supercluster catalogue

We may try to make use of the available galaxy cluster / supercluster catalogue to verify the presence of possible gravitational lens between the detectors and the source once lensed gravitational wave signals are identified.

Foreseeable challenge(s)

The catalogues available in the present only covers a small region of the sky. It is possible that the gravitational wave source is not in the covered region. We have to check the coverage of catalogues to see if they are applicable to our research.

D. Pipelining the search for lensed gravitational wave signals

To make the search for gravitational wave signals efficient, we may try to construct a pipeline (makefile) for the search. This can make the process more time-efficient, and it will also make error-shooting easier.

Foreseeable challenge(s)

Regarding my unfamiliarity towards pipeline and makefile construction, more literature review has to be conducted before making the pipeline for the lensed gravitational wave signals search.

E. Re-introducing the sky location problem to the project

At this stage of our project, we have neglected the alteration in sky location of the source when we inject simulated lensed signals using LALInference posterior samples. In particular, we need to investigate the range of sky location to search for lensed gravitational wave signal, instead of using the range suggested by LALInference. Figure 24 illustrates the problem.

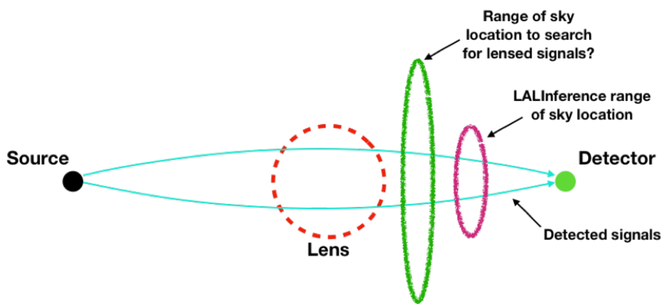

Fig. 17. An illustration of the sky-location problem on the project.

We may also try to find the correlation between the relative time delay δt of lensed gravitational wave signals and the corresponding sky-location of the source (i.e. right ascension α and declination δ), or even trace the lensed gravitational wave signals.

Foreseeable challenge(s)

This part is rather time-consuming and may not be finished in the coming few weeks. It will be carried on after the SURF period.

V. Work Plan

1) Pre-Surf Period:

- Literature review on gravitational lensing, gravitational wave data analysis and related mathematics
- Construct models for lensed gravitational wave signals
- Complete the mathematical formulation for the SNR, FAR and related data analysis for this research
- Learn how to run a GstLAL search
- Run template searches on the drafted models to observe the effect, and modify them in accordance.

2) SURF Period:

- 19 June, 2018 : Arrive at Caltech
- 19 June - 9 July, 2018 [Week 1-3] :
  (i) Complete the follow-up work during the Pre-Surf Period
  (ii) Rerun searches using the modified models
  (iii) Analyse the results and suggest further modifications on the models
- 10 July, 2018 : First Interim Report to be submitted
- 11 July - 20 July, 2018 [Week 4-5] :
  (i) Run injection campaign
  (ii) Analyse the results
- 21 July - 2 August, 2018 [Week 5-7] :
  (i) Run injection campaign
  (ii) Analyse the results
  (iii) Start preparing the second interim report and the final paper
- 3 August 2018 : Second Interim Report to be submitted
- 4 August - 17 August, 2018 [Week 7-9] :
  (i) Attempt to infer the properties of the lens
  (ii) Pipeline the search process
  (iii) Preparation of final presentation and final paper
- 23 - 24 August 2018 : Summer Seminar Days

3) Post Surf Period:

- 24 August 2018 : Returning to Hong Kong
- Complete final paper
- Follow-up work on the summer research
- Continue related research in gravitational wave science

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