Exploring the Detection Process of Gravitational Wave Memory

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Abstract

From studies of general relativity it has been established that two types of gravitational wave memory exist: linear and nonlinear, also called Cristodoulou memory [27]. These effects are predicted to be detectable in strong-field, highly dynamical regime collisions which until recently have been some of the least explored events in the universe. The linear memory is very small, whereas the nonlinear memory is believed to be large enough to be detected by Advanced LIGO and 3rd generation detectors. The current complications of detecting memory involve the fact that it is extremely small and can easily be overlooked in the gravitational wave signal. Also, memory is not detectable with just one collision; multiple events must be stacked in order to produce a resolution decent enough to detect the memory. The final obstacle is the low-frequency detector noise. While some forms of this noise are manageable, other forms such as quantum and instrument noise, which can look very similar to the memory signal, is difficult to control. As the sensitivity of Advanced LIGO and 3rd generation detectors improves, the goal is they will be responsive enough to pick up nonlinear memory signals. Scientists have attempted to detect memory with a range of techniques including integrating up the signal to $t = 1/f_{opt}$ where f_{opt} is the optimal frequency of the detector, or using the "effective-one-body" (EOB) approach which will eventually allow one to develop an analytical function for gravitational wave memory. The technique of focus for my project will be stacking multiple events, and we will estimate parameters relevant to this technique in order to infer how well the memory parameter can be measured for future events detected with Advanced LIGO.

Introduction: LIGO, Gravitational Waves, and Memory

Gravitational wave detection by LIGO is a developing phenomenon that began in 2015 with the detection of a gravitational wave from a binary black hole merger that occurred about

1.3 billion years ago [11]. Since this first occurrence, there have been 6 other cases of gravitational wave detections by LIGO [14]. With each detection providing new information, there is always a push to analyze the data in hopes of further understanding the source that produced the wave. Along with gaining information concerning some of the universe's most extreme events, the properties of the waves themselves can also be further examined in order to increase our understanding of the effect they have on matter they pass through and our general relativistic predictions.

Motivation for Memory Detection

For a binary inspiral there is a non-oscillatory component to the "+" polarization (similar to polarization of light except gravitation wave polarizations are 45 degrees apart) which makes the amplitude of the gravitational waves dampen to a non-zero value [7]. This non-zero amplitude represents the gravitational wave memory, a weak stretching that permanently alters spacetime [4]. There are numerous reasons scientists want to detect memory. One reason is that exploring the strong-field, highly-dynamical regime in connection to general relativity is a relatively new and unexplored area of physics. For systems under gravitational influence with unbound components, a linear memory effect is applicable. Linear memory, discovered in the 1970s, arises from near-zero-frequency changes in the time derivatives of the source's multipole moments. Multipole moments are a combination of the mass moment, the extent to which an object resists rotational acceleration about a particular axis, and the mass-current moment which corresponds to the star's spin angular momentum (the star's moment of inertia about its spin axis multiplied by its spin angular frequency Ω) [20, 9, 15]. Linear memory also appears in systems that experience kick such as a rogue black hole, or systems that eject particles such as neutrinos from supernovae [7]. Non-linear memory grows slowly and is also a non-oscillatory contribution to to the gravitational; wave's amplitude. It originates from gravitational waves that are sourced by the previously emitted waves [9]. It is believed that all gravitational waves carry a component of nonlinear memory which means it should be included in LIGO waveform models [25].

Since linear and nonlinear memory depend on the form of Einstein's field equation, a set of 10 general relativistic that describe gravity as a result of spacetime being curved by mass and energy, it is then possible that different forms of memory could be uncovered if general relativity were to be modified [6]. Studying these two fields side by side is highly intriguing as together they could help us undercover unexplored areas of physics. Since memory is difficult for LIGO to detect, it has been mostly disregarded by scientists studying gravitational waves. However, the most studied gravitational wave source is currently compact binaries, for example black holes. The memory scales linearly with the black hole's mass, and it is estimated that the memory effect in these cases will have a detectable contribution to the calculated waveform amplitude of the resulting gravitational waves [12]. This memory effect is thought to be so large that the order it enters the waveform is equivalent to the leading-order term, in this case the quadrupole. From this arises the conjecture that the memory effect should not be impossible to detect [7]. Finally, there is little information known about how the memory signal grows and fluctuates throughout the inspiral and ringdown phase, and also post-merger. For these reasons one can understand the importance of exploring methods to detect memory.

Background

While there are numerous methods one can utilize to begin to understand gravitational radiation, one helpful analogy is electromagnetic radiation. As electric charges move they create electric and magnetic waves that propagate outward from their source at the speed of light. The waves carry energy and their strength falls off as $1/r^2$where r is the distance away from the source. They can be detected by the forces they apply to charged particles, or by the amount of energy the source loses from the wave propagation. In a similar fashion, gravitational waves arise when moving masses send out waves that cause the curvature of spacetime to fluctuate. The amplitude of the waves also falls off as 1/r over long distances and they can be detected either by the gravitational strain they apply to groups of massive objects in free fall, or the amount of energy that is lost by the source. While there are strong similarities between gravitational and electromagnetic radiation, the differences become apparent when the strength of the two forces are compared. Due to the weakness of gravity, only very powerful astrophysical interactions are capable of producing gravitational waves that are detectable on earth. Some of these interactions include mergers of neutron stars, black holes, or a combination of both [5]. An additional component that differs between electromagnetism and gravitation is gravitational waves have a large nonlinearity [23].

While the nonlinear component of gravitational radiation is very important, it becomes easier to understand the background physics when only the linear portion is considered at first. In this case, the relevant Einstein field equation reduces to a form similar to that of one of Maxwell's equations. After taking the time derivatives of the sources multipole moments the resulting equation becomes $h(r,t) = \frac{2G}{rc^4}\frac{d^2 I_{ij}}{dt^2}(t - \frac{r}{c})$[19] where I_{ij} represents the mass quadrupole moment. This is known as the quadrupole formula of general relativity which is used to calculate energy loss. While this equation is only used in situations without the presence of gravity, it can be altered using a "post-Newtonian approximation" to include gravity's influence. The equation for this quadrupole moment gives rise to an indirect way to detect gravitational waves, that is, by considering a system where motion is measured very accurately. An example of this is a binary neutron star system with one pulsar and one neutron star, where the rate general relativity predicts the system will lose energy at can be calculated. This loss of energy will cause the orbit of the neutron star and the pulsar to shrink. The shift in orbital features can be tracked through doppler shift of the arrival time of the pulses. From the formula for energy loss one can then predict what the orbital period of the binary system will be at a particular moment in time. This can be checked with an astronomical measurement of the changing period. These two values were shown to match which means gravitational waves were indirectly detected [9].

In order for one to directly detect gravitational waves, the concept of geodesics must be used. Geodesics refer to the shortest path an object in free fall can follow on a curved surface. The distance between two geodesics changes as a non-uniform gravitational field embedded in the curvature of spacetime [10]. If one considers the geodesic deviation equation and focuses on the "weak field slow motion" case, one can integrate twice and obtain the equation $\Delta S^i = \frac{1}{r} S^j P[\Delta Q_{ij}]$[1] The delta shows that there is a difference between the original and final distance of the geodesic. This is due to gravitational wave memory that has left the wave field at a final non-zero value. The specific component of memory as described above is only the linear aspect, the nonlinear effect makes up the primary portion of memory. Scientists discovered that linear memory arises in situations with unbound components such as ejected neutrinos from a supernova or gamma-ray burst jets, while Christodoulou established that this 'nonlinear' memory is from energy radiated in gravitational radiation [7]. Additionally, while linear memory is a result of fields that do not reach null infinity, nonlinear memory is due to fields that do reach null infinity. Christodoulou then went on to prove that the linear memory effect is relatively small which is why non-linear memory is usually the focus.

Limitations of Memory Detection

While we have good reason to believe gravitational memory exists, it is the detection process that has limited us thus far. In understanding why gravitational memory has not yet been directly detected, it is helpful to first examine the details that make finding it difficult. The first reason is due to the extremely small size of the memory effect. The size of the memory is expected to about one-tenth to one-hundredth of that of the gravitational waves. This is why this effect is predicted to mostly be detectable in the most violent collisions in the universe. For example, the first gravitational wave signal, GW150914, caused the LIGO arms to stretch and shrink by about one-five-hundredth of a femtometer. It was predicted this memory effect would only be about one-twentieth of the size of the gravitational waves, which is about one-ten-thousandth of a femtometer [4].

Another way to interpret this difficulty of memory detection is by considering the presence of low-frequency detector noise. While seismic noise occurs at low frequencies, scientist have learned how to cancel its effects. Quantum noise and instrument noise however are much harder to suppress. Quantum noise arises from the statistical uncertainty of photon arrival time and radiation pressure from the random motion of the mirrors [13]. Instrument noise is a concern because it can overwhelm or mimic the strain pattern that is being looked for. The instrument noise is smallest around a few hundred hertz but increases sharply at low and high frequencies. At all frequencies there are narrow spikes due to vibrating fibers that suspend the mirrors and test masses in the interferometers [18].

Approach: Potential Methods of Detection

Even though there are still numerous challenges to overcome before scientists obtain a clear memory signal, there are still strategies that have allowed us to begin this hunt. By analyzing a binary neutron star system one can use the calculated metric perturbation, or disturbance, to find the strain, or the difference in displacement due to the passing of a gravitational wave. The strain is small, on the order of magnitude of 10^{-21}. Measuring this type of strain means that a length of 1cm is required to be measured to length of 10^{-21} cm [3].

However, the memory strain is even smaller than the oscillatory strain. This memory strain monotonically increases with the length of the signal. For signals with relatively short burst lengths, (time shorter than the inverse frequency of the detectors sensitivity), the memory can be approximated by a step function which is proportional to 1/f [16] (Figure 1). To estimate the step function a sine-Gaussian waveform is used.

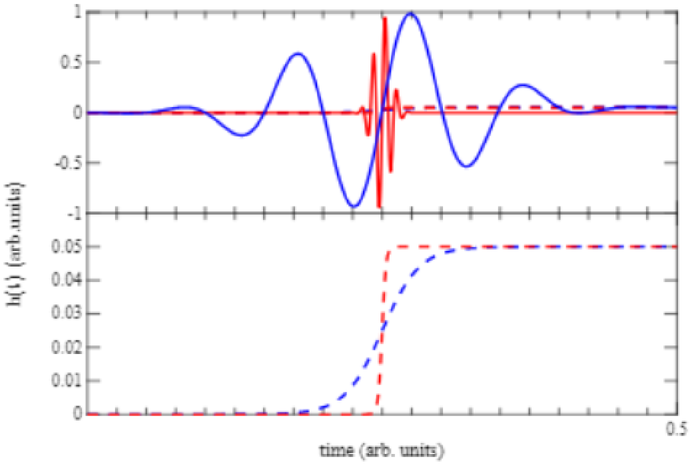

Figure 1 [26]: The top figure shows the sine-Gaussian burst (solid curve) and memory (dashed curve) strains for two bursts with an equal amplitude. The high-frequency burst (red) has a frequency ten times greater than the low-frequency burst (blue). The bottom figure is a close up of the memory time series'. The rise time approaches zero as the frequency increases, and the step function becomes an acceptable approximation of the memory.

From here scientists have concluded that the memory from high frequency waves has a low-frequency component whose value is below 1/t where t is the length of the gravitational wave signal. This means that if the source of the burst, also called the parent burst, is of a frequency that is above that of LIGO's observing band, then there may be a detectable orphan memory.

While orphan memory has not been directly detected yet, its theoretical maximum value can still be calculated. The maximum gravitational wave frequency is $f_{max} \sim 1/r_s \sim 1/M_s$ where $r_s$is the Schwarzschild radius and M_s is the mass. To find an estimate of the possible maximum memory, we have to assume the mass of the black hole completely radiates away. It follows that the maximum occurs at a frequency equal to $1/E_{gw}$which leads to a possible maximum memory of ${E_{gw}}^{1/2}/f^{1/2}d$ [16] which scales in a similar fashion to the oscillatory strain. Also, the maximum memory for edge-on binaries has a scaling relationship proportional to $sin^2\theta(17 + cos^2\theta)$ [12] (Figure 2)

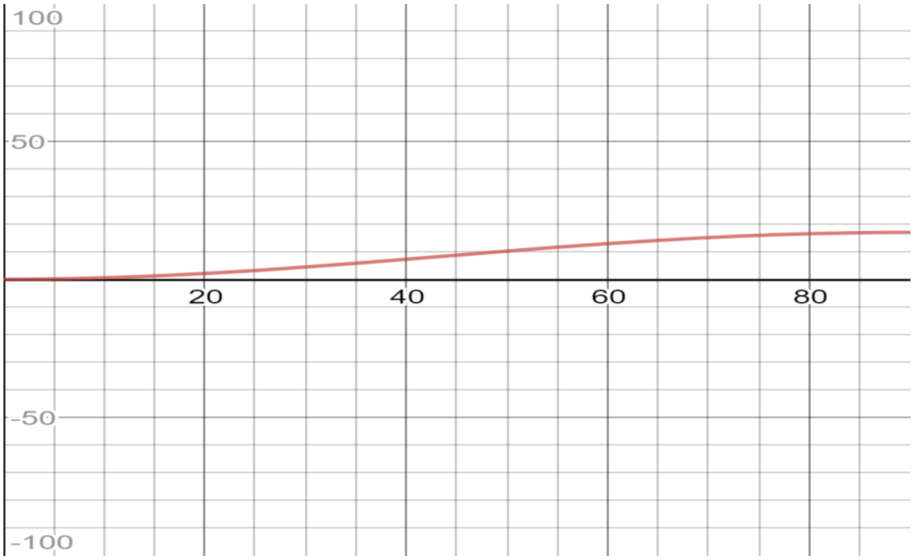

Figure 2: graph of $sin^2\theta(17 + cos^2\theta)$ from 0° to 90°

While scientists are not confident that orphan memory is detectable with Advanced LIGO, there is still potential for its discovery with future detectors.

Another method that could be useful in detecting gravitational memory is to first predict a function that represents it. This model will likely help to check results if gravitational memory is discovered. The first step in finding this representative memory equation is to first use a combination of Bayesian statistics techniques. The next step is to implement the "effective-one-body" (EOB) approach which can model the multipole moments, or the coefficients of a multipole expansion in the inspiral and ringdown phases [7]. A multipole expansion is composed of a group of functions known as spin-weighted spherical harmonics that depend on the angles (usually two) on a curved surface. They are useful for gravitational field study where the fields at distant locations are described in terms of the sources located in a small region of space. The expansion is expressed as a sum of the angles as they approach an increasingly finer degree with the zeroth term named the monopole, the next term the dipole moment (which only varies once around a sphere), and eventually the quadrupole which changes quickly with the angles [17]. Once the angles used for the expansion are combined with a certain distance, then the result is a function that describes three-dimensional space. Next, the minimal-waveform model can be implemented in order to sum the quasi-normal modes. [12]. These quasinormal modes can help us understand the energy loss of dynamical systems such as a black hole mergers. Quasinormal modes of black holes describe the exponential decrease of the asymmetry of the hole in time as it evolves to a spherical shape in the ringdown phase. The result will be an analytical function for gravitational wave memory.

A more general approach to compute Christodoulou memory is to treat the oscillatory component of the wave as a parameter. This means one must look at both the cross and plus polarization of the wave. It can then be integrated over a specific angle and time to calculate the memory. Another benefit of this approach is one can use a series of waveforms from compact binary coalescences instead of a single estimated one. This approach also yields potential for scientists to compute high-order memories, or memories produced by memories [22].

A fourth method to detect gravitational wave memory is to integrate along the signal to $t = 1/f_{opt}$where f_{opt}(optimal frequency) is the frequency at which the detector is the most sensitive to ordinary gravitational wave bursts. If the length of the burst with memory (BMW) is smaller than $1/f_{opt}$, the detector's sensitivity to BMW is practically equivalent to that of bursts without memory that are one cycle long and whose frequency is f_{opt}. A benefit of this method is it has the potential to be implemented despite the type of detector used for the study [2].

A final method of detection is the stacking of events. It is believed that combining information from the mergers could, overtime, boost the detectability of memory enough for scientists to obtain a clearer picture. Paul Lasky, an astrophysicist at Monash University in Australia, has predicted that 35 to 90 black hole mergers similar in mass and distance as G150194 may be enough for LIGO to detect memory [4]. Also, since LIGO is going through advancements until 2021 which will make it more sensitive, there is potential that fewer mergers than predicted will be needed to detect memory [24].

Objectives for LIGO SURF 2018 Project

For my project I plan to first learn background information relevant to gravitational wave memory. This will include learning how memory is encoded in the models of the waveform and how to create parameters for memory as part of this model. I will need to study Bayesian analysis in order to establish a procedure that estimates a Bayesian parameter. I will also explore the techniques that have been developed to search for memory including the stacking of events. Memory slowly builds overtime which is why data from a number of extreme events will be needed in order to view memory in a decent resolution. I also plan to examine the LIGO data with the stacking method and explore how memory could manifest itself in the events we have already detected. The stacking procedure will also be used to estimate the accuracy of the memory parameter in order to determine how well it can be measured for future events from Advanced LIGO and 3rd generation detectors. Necessary tools for these procedures include python and previously determined waveform models from the LIGO Algorithm Library [23].

LIGO SURF 2018 Project Schedule

The timeline for this project, while subject to change, will first involve me learning the relevant background information as listed above. I will also focus on understanding the necessary Bayesian analysis in order to estimate parameters and reviewing the waveform models from the LIGO Algorithm Library. I will also study how python can be used to perform necessary scientific computations. After a few weeks I will likely begin to explore stacking as a technique for gravitational wave memory detection. This is likely the portion of the project my first interim report will be focused on. I will then continue to examine the stacking technique and begin using data from LIGO to explore the ways memory manifests itself in the discovered events. The goal will be to create the most foolproof detection approach possible. By looking at the data that has already been collected, I will then examine the parameters for memory we have developed and determine if they are likely to fit future LIGO data. This will allow me to write my second interim report and eventually final report. The last 3 weeks of the program I will begin working on my poster or presentation which will be presented August 24th.

Conclusion

There is a continual effort of analyzing gravitational wave signals in hopes of gaining a deeper understanding of their sources. While its effects are not entirely known, memory could serve as a complementary factor that helps us reach this goal. Nonlinear memory continues to intrigue scientists due to the interesting information it has revealed thus far, including the way it affects the wave form at a leading order at a level equivalent to that of the quadrupole. This along with other implications are being analyzed so scientists can continue to probe the strong-field, highly dynamical regime. LIGO in combination with other ground-based gravitational wave interferometers will potentially be able to detect memory after the discovery of dozens of extreme collision events. Additional gravitational wave detectors such as Virgo, KAGRA or LIGO-India will further reduce detection time [12]. While it has not yet been directly detected, the capability of our technology is one of the many reasons why some scientists are optimistic that we will detect gravitational memory at some point in the future.

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