A Study of Gravitational Wave Memory and Its Detectability With LIGO Using Bayesian Inference

Interim Report #2

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Update For Interim Report #2

I have made significant progress since my first interim report. I have accomplished the next steps I outlined in my previous interim report and I am also on track with the project schedule I made at the beginning of the year. So far I have met my expectations for the summer, and I am continuing to think about the next best step for my project. I have run into a few additional challenges since my last report. I have continued to have some issues with getting my code to work so I have had to take a substantial amount of time to debug. From this however I have built my intuition in regards to programming with Python which will be very beneficial for future projects and research. I have continued to use the same resources, mainly my mentors, tutorials, and the internet to answer questions and understand key concepts.

The next step I am currently working on is creating a loop that will allow me to input a set of distance and mass parameters and plot them on the same graph. From here I will calculate the 90% confidence interval so I will be able to communicate how confident we are that lambda is equal to one. This calculation will also hopefully show that zero is not included at all in the peak which means that there is a very low probability that memory does not exist. The data analysis techniques I am currently working on is increasing my understanding of functions and for loops in order to produce desired graphs. I will also likely use Python in order to run certain calculations involving the rate density of binaries. This calculation will be important as I will then be able to predict how often we will have binaries where memory would be detectable depending on the binaries mass, distance away, and the specific observatory being used for direction.

A Study of Gravitational Wave Memory and Its Detectability With LIGO Using Bayesian Inference

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Gravitational waves are produced by accelerating masses, but in most cases they are too weak to detect. In 2015 LIGO announced its first gravitational wave detection which was produced by the merging of two black holes 1.3 billions years ago. The detectable component of gravitational waves, known as the oscillatory waveform, is predicted to have a smaller, lower frequency counterpart called the memory: the permanent warping of spacetime. In addition to memory’s small amplitude compared to the oscillatory waveform, low frequency noise sources on earth make it difficult for ground based detectors to reach the SNR (signal to noise ratio) needed to detect this component. While memory is likely not currently detectable due to LIGO limitations, it is of interest to characterize future detector sensitivities to know where and when to look for this phenomenon. Here we implement Bayesian parameter estimation to calculate the likelihood of a simulated set of LIGO data with a template, both of which include memory. Next we explore binary systems of varying masses and distances along with the noise curves of various observatories in order to establish the SNR needed to detect gravitational wave memory. Our final goal is to find a ballpark SNR value for when memory will be detectable.

Introduction: LIGO, Gravitational Waves, and Memory

In 1916 Albert Einstein proposed a theory that unifies gravity, spacetime, and energy which he called The General Theory of Relativity. In his theory he predicted that massive accelerating objects would emit waves that physically distort spacetime; these objects were given the name gravitational waves. In hopes of directly testing for the existence of gravitational waves gravitational a detector called LIGO (Laser Interferometer Gravitational Wave Observatory) was built in 1995. Twenty years later in 2015 the first detection of a gravitational wave was announced: a merger of two black holes that occured approximately 1.3 billion years ago [11]. This merging process is formally known as a binary black hole system where the black holes undergo three main phases. The first phase of this process is the inspiral which consists of the black holes orbit around each other with a shrinking orbit. In this phase gravitational waves are weakly emitted. Next is the merger where the black holes combine to make one black hole; gravitational wave emission peaks at this time. Finally is the ringdown stage where the final black hole oscillates between an elongated spheroid, and a flattened spheroid through gravitational wave emission. Since this first occurence, there have been a handful of gravitational waves detected by LIGO [14]. With each detection providing new information, there is constantly a push to analyze the data in hopes of further understanding the source that produced the wave. Along with gaining information concerning some of the universe's most extreme events, LIGO observations provide unique tests of General Relativity in the strong-field, highly dynamical regime. While we have begun to probe general relativity with these detections, there are other predictions that we have yet to test. Here we begin to characterize the detectability of one of these predictions, gravitational wave memory, through the study of binary black hole mergers.

Motivation for Memory Detection

General Relativity predicts that gravitational waves will have an oscillatory component as well as a memory component. This oscillatory and memory component are polarized in the plus and cross direction. These polarizations are similar to that of light except they are related by a 45 degree rotation compared to the 90 degree rotation for EM (electromagnetic) radiation. These polarizations also have an oscillatory and also a non-oscillatory component. For a binary inspiral there is a non-oscillatory component to the "+" polarization which makes the amplitude of the gravitational wave end with a non-zero value [7]. This non-zero amplitude represents the gravitational wave memory, a weak stretching that permanently alters spacetime [4].

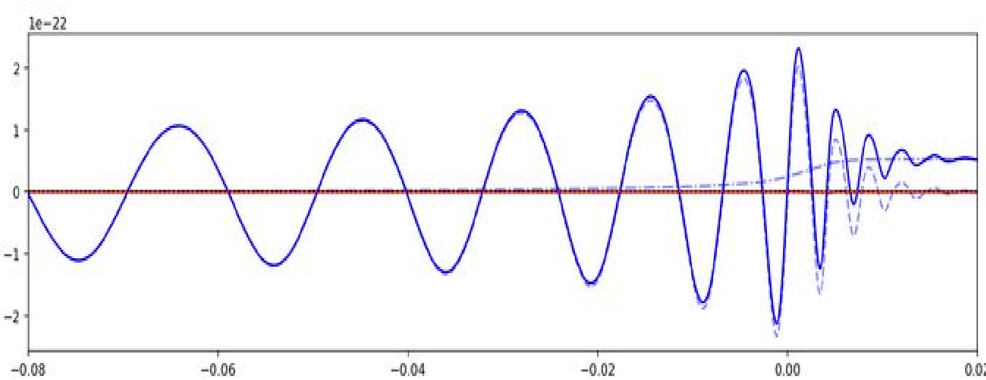

Figure 1: Waveform showing oscillatory binary black hole merger waveform with memory (solid blue line) and without memory (--line). Made with package GWMemory from "Gravitational-wave Memory: Waveforms and Phenomenology" [28]

Linear memory, discovered in the 1970s, arises from near-zero-frequency changes in the time derivatives of the source's multipole moments. Multipole moments are a combination of the mass moment, the extent to which an object resists rotational acceleration about a particular axis, and the mass-current moment which corresponds to the star's spin angular momentum (the star's moment of inertia about its magnetic poles multiplied by its spin angular frequency Ω) [20, 9, 15]. Linear memory also appears in systems that experience kicks such as a rogue black hole, or systems that eject particles such as neutrinos from supernovae [7]. Non-linear memory grows slowly and is also a non-oscillatory contribution to to the gravitational wave's amplitude. It originates from gravitational waves that are sourced by the previously emitted waves [9]. It is believed that all gravitational waves carry a component of nonlinear memory which means it should be included in LIGO waveform models [25].

Since linear and nonlinear memory depend on the form of General Theory field equations, a set of ten coupled non-linear differential equations equations that describe gravity as a result of spacetime being curved by mass and energy, it is then possible that different forms of memory could be uncovered if general relativity were to be modified [6]. Since memory is difficult for LIGO to detect, it has mostly been disregarded by scientists studying gravitational waves. However, the memory scales linearly with the black hole's mass which means there will likely be a detectable contribution to the calculated waveform amplitude of the resulting gravitational waves [12]. This memory effect is computed to be non-negligible as the order it enters the waveform at approximately the same order as the quadrupole. From this one can conclude that the memory effect should not be impossible to detect with the proper equipment and analysis techniques[7]. Now that LIGO has published seven CBC (compact binary coalescence) events, there is more data to explore and a greater potential to detect memory.

Background - Gravitational Wave Memory

While there are numerous methods one can utilize to begin to understand gravitational radiation, one helpful analogy is electromagnetic radiation. As electric charges move they create electromagnetic waves that propagate outward from their source at the speed of light. The waves carry energy and their energy flux falls off as $1/r^2$ where r is the distance away from the source while the amplitude falls off as 1/r ,They can be detected by the forces they apply to electrons, or by the amount of energy the source loses from the wave propagation. In a similar fashion, gravitational waves arise when moving masses send out waves that are the fluctuating curvature of spacetime. The amplitude of the waves also falls off as 1/r over long distances and they can be detected either by the gravitational strain they apply to groups of massive objects in free fall, or the amount of energy that is lost by the source. While there are strong similarities between gravitational and electromagnetic radiation, the differences become apparent when the strength of the two forces are compared. Due to the weakness of gravity, only very powerful astrophysical interactions are capable of producing gravitational waves that are detectable on earth. Some of these interactions include mergers of neutron stars, black holes, or a combination of both [5].

An additional factor that differs between electromagnetism and gravitation is gravitational waves have a large nonlinearity [23]. This nonlinearity is intriguing to study because it will help us further understand the fundamental nature of gravitational waves. Gravitational waves are sourced by energy and mass which allows the particles that carry gravity called gravitons, which carry energy, to source gravitational waves and emit more gravitions. These gravitons interact or couple with one another which gives rise to the residual warping of spacetime; this is what we refer to as gravitational wave memory. By better understanding this nonlinear memory we may also reach a stronger comprehension of other objects in our universe such as black holes. While black holes are produced by collapsed massive stars, the theoretical point at its center, the singularity, is thought to have infinite spacetime curvature. For this reason black holes can be considered a physical representation of memory. However, since quantum mechanics is not currently able to account for this and we do not have a clear picture of strong field quantum gravity, there are certainly flaws in the idea of infinite spacetime curvature. Studying nonlinear memory will allow us to test whether this firm prediction of General Relativity holds up to our quantitative predictions.

While the nonlinear component of gravitational radiation is very important, it becomes easier to understand the background physics when only the linear portion is considered at first. In this case, the relevant Einstein field equation reduces to a form similar to that of one of Maxwell's equations. After taking the time derivatives of the sources multipole moments the resulting equation becomes h(r,t) = $\frac{2G}{c^4 r} \frac{d^2 I_{ij}}{dt^2}(t - r/c)$ [19] where I_{ij} represents the mass quadrupole moment. This is known as the the quadrupole formula of general relativity which is used to calculate energy loss. The equation for this quadrupole moment gives rise to an indirect way to detect gravitational waves, that is, by considering a system where motion is measured very accurately. An example of this is a binary neutron star system with one pulsar and one neutron star, where the rate general relativity predicts the system will lose energy can be calculated. The work of Hulse-Taylor [9] concluded that the loss of energy will cause the orbit of the neutron star and the pulsar to shrink. The shift in orbital features can be tracked through doppler shift of the arrival time of the radio pulses. From the formula for gravitational wave energy loss one can then predict what the orbital period of the binary system will be at a particular moment in time. These two values, observed orbital shrinking from the radio pulse delays and the prediction that energy is being lost through gravitational wave emissions. were shown to match which means gravitational waves were indirectly detected [9].

<u>Limitations of Memory Detection</u>

While we have good reason to believe gravitational memory exists, it is the detection process that has limited us thus far. In understanding why gravitational memory has not yet been directly detected, it is helpful to first examine the details that make finding it difficult. The first reason is due to the extremely small size of the memory effect. The size of the memory is one order of magnitude smaller than that of the oscillatory waveform, making it significantly more difficult to detect. For example, the first gravitational wave signal, GW150914, caused the LIGO arms to stretch and shrink by about one-five-hundredth of a femtometer. It was predicted this memory effect would only be about one-twentieth of the size of the gravitational waves, which is about one-ten-thousandth of a femtometer [4].

Another reason why the detection of gravitational wave memory is difficult with LIGO is from the presence of low-frequency detector noise. While some noise sources are relatively well understood, quantum noise and instrument noise are difficult to suppress. Quantum noise arises from the statistical uncertainty of photon arrival time and radiation pressure from the random motion of the mirrors [13]. Instrument noise is a concern because it can overwhelm or mimic the strain pattern that is being looked for. The instrument noise is smallest around a few hundred hertz, but increases sharply at low and high frequencies. Throughout the frequency band there are narrow spikes due to vibrating fibers that suspend the mirrors and test masses in the interferometers [18]. In summary, the memory effect is dominant at low frequencies but the noise that limits LIGO's sensitivity to gravitational wave strain is orders of magnitudes larger at frequencies below ten to twenty hertz. This low frequency noise makes it much more difficult to detect the low-frequency strain signal from gravitational wave memory.

Approach: Potential Methods of Detection

Even though there are still numerous challenges to overcome before a clear memory signal is obtained, there are strategies that have allowed the search to commence. One approach to compute Christodoulou memory is to treat the oscillatory component of the wave as a parameter. The posterior probability distribution function of the parameter can be calculated, with the help of numerical relativity models, which can then communicate the likelihood of the parameter taking specific values. After taking the confidence interval one can conclude whether the potential value of lambda is likely to take on one.

Another potential method of detection is to integrate along the signal to $t = 1/f_{opt}$ where f_{opt} (optimal frequency) is the frequency at which the detector is the most sensitive to ordinary gravitational wave bursts. If the length of the burst with memory (BWM) is smaller than $1/f_{opt}$, the detector's sensitivity to BMW is practically equivalent to that of bursts without memory that are one cycle long and whose frequency is f_{opt}. A benefit of this method is it has the potential to be implemented for any type of detector used for the study [2].

Additionally, there is the method of stacking events. By combining information from the mergers could, over time as more compact binary coalescence events are detected, boost the detectability of memory enough to obtain a clearer picture. Paul Lasky, an astrophysicist at Monash University in Australia, has predicted that 35 to 90 black hole mergers similar in mass and distance as G150194 may be enough for LIGO to detect memory [4]. Also since LIGO is going through advancements until 2021 which will make it more sensitive, there is potential that fewer mergers than predicted will be needed to detect memory [24].

Specific Approach to Memory Detection with LIGO

As a prerequisite to attempting the stacking method mentioned above, we have been working on general data analysis techniques with an oscillatory LIGO waveform in preparation to work with waveforms with memory. The first steps involve creating simple waveforms with black hole masses of an arbitrary size. With this preliminary step also comesm creating a PSD (power spectral density) with real LIGO data. A power spectral density is a measure of the strain-equivalent noise in a detector.

One can then begin to analyze the memory waveform by computing the likelihood of a template and a set of data. In order to perform our analysis we used a "surrogate" model of the oscillatory waveform built from a large group of pre-existing numerical relativity simulations that combines a large number of pre-existing numerical relativity simulations. From the surrogate we create a waveform composed of the oscillatory and memory component, and a template which contained the oscillatory component and a memory waveform multiplied by a parameter lambda. This memory component is computed directly from the equation {Cite}. From here we use Bayes theorem to calculate the posterior. P(lambda) is the prior which is the prediction of the range of values the parameter will likely take. For our purposes we set the prior from -10 to 10. The P(lambda slash data) is the probability of the data given the prior that was just set. The denominator is the normalization and the (slash) is the noise weighted inner product calculated by. Figure X shows the posterior graphed against lambda using the O1 GW15914 PSD for the noise weighted inner product. The graphs fixed the mass but allowed the distance away from earth of the merger to vary. This confirms the prediction that the farther away the signal, the harder the memory component will be to detect as the peak will then cover a wider range of lambda values. The results also match our original prediction as the graph is distributed across a range of lambda values but peaks at one. This peak communicates that the most likely value for lambda is one, however these results are not significant without performing a confidence interval calculation.

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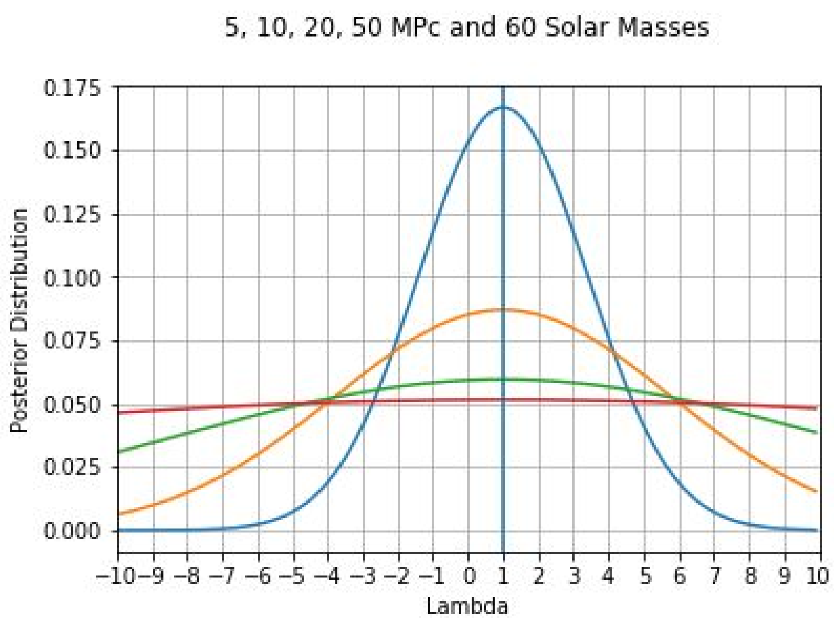

Figure 2: Graph of Likelihood versus lambda for a binary black hole system with a total of 60 solar masses at distances of 5, 10, 20, 50 megaparsecs.

In order to determine if our results were significant we took the 90% confidence interval of the posterior. Figure X shows the 90 percent confidence interval of a 60 solar mass merger at 5 megaparsecs away. Here the confidence interval includes 0 which means lambda could take on the value of one, but it could also just as easily take on the value of zero. However when we bring in the merger to a distance of 0.9 megaparsecs, we can see that 0 is not in the range which means we can much more confidently state that .

The next step is to attempt to characterize the detectability of the memory component using PSD curves from current and future detectors. Our first attempt is by making use of the O1 GW150914 noise curve graphed along with the oscillatory waveform and memory component in the frequency domain. After viewing the memory and oscillatory component graphed with the O1 data, one can understand why memory is not detected, it lies well below the noise curve for a binary black hole merger that is at a distance of 300 megaparsecs away.

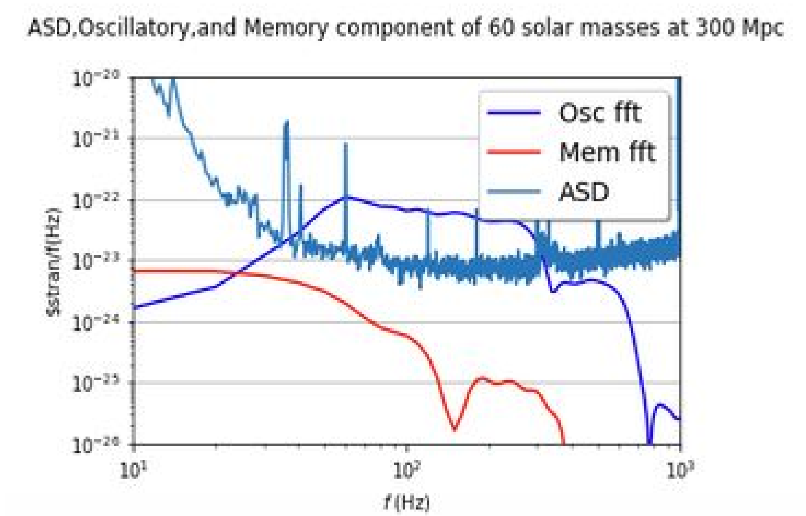

Figure 5: Graph of oscillatory and memory component graph with the ASD for GW091415.

<u>Conclusion and Future Work</u>

There is a continual effort of analyzing gravitational wave signals in hopes of gaining a deeper understanding of their sources. While its effects have not been theoretically tested yet, detecting memory would be an important scientific feat that would allow us to both have a deeper understanding of CBCs and confirm a firm prediction of General Relativity. Nonlinear memory continues to be an intriguing research topic due to the interesting information it has revealed thus far, including the way it affects the wave form at a leading order at a level equivalent to that of the quadrupole. This along with other implications are being analyzed so the strong-field, highly dynamical regime can continue to be probed. LIGO in combination with other ground-based laser detectors will potentially be able to detect memory after the discovery of dozens of extreme merger events. Additional gravitational wave detectors such as Virgo, KAGRA or LIGO-India will further increase the number and types of detections. More events we have more accurately we can measure lambda. Additional gravitational wave detectors will give us more events. [12]. There is also great potential that LISA (to be launched in 2030) will be able to greatly enhance the detectability of gravitational wave memory due to its capability to measure much lower frequencies. While it has not yet been directly detected, the capability of our technology is one of the many reasons why there is optimism surrounding the potential for directly observing gravitational wave memory.

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