Technical Note	LIGO-T1800200–vX	2018/09/19

In Situ Laser Mode Spectroscopy for Mirror Phase Mapping

Interim Report 1

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1 Introduction

1.1 Gravitational Waves

Einstein predicted gravitational waves in his theory of General Relativity, which states that gravity is the distortion of spacetime due to the presence of mass. Mass creates a curvature in the geometry of spacetime. Gravitational waves are ripples in the fabric of spacetime, and these waves distort spacetime geometry as they propagate at the speed of light. Gravitational waves interact with matter by stretching in one direction and compressing space in the perpendicular direction. They originate from the acceleration of large masses, specifically from cataclysmic cosmic events such as the collision of black holes. Gravitational waves are important because they carry information about their origin, the universe, and the nature of gravity itself.

1.2 LIGO

LIGO (Laser Interferometer Gravitational Wave Observatory) is a gravitational wave detector in Livingston, Louisiana, and in Hanford, Washington. LIGO is a Michelson based interferometer with Fabry Perot cavities. The interferometer creates an interference pattern from the constructive or deconstructive interference of waves. The interferometer has two perpendicular arms of equal length. The laser beam will take the same time to travel the same distance, which will produce deconstructive interference, where the light waves cancel each other out. Thus, the photodetector will not detect a light pattern. However, when a gravitational wave passes, the length of the arms changes. One arm is stretched, and the other one is compressed. The photodetector will detect an interference pattern due to the changes in relative length between the masses. Gravitational waves will alter the time the light will travel in the interferometer. The difference in time will produce an interference pattern in which astronomical information can be extracted.

The arms of LIGO are 4 km long. Incorporating Fabry Perot cavities into each arm increases the sensitivity by increasing the interaction time of the gravitational wave with the interferometer. This can be done by introducing additional mirrors near the beam splitter, causing the each part of the laser beam to reflect 280 times before being merged into an interference pattern [1].

1.3 Optical Losses

The success of Advanced LIGO is largely due to precise optical measurements which result in the sensitivity to detect gravitational waves. This project is focused on improving the optics in LIGO by characterizing mirror figure error that contribute to optical losses. Optical losses are a result of light scattering. Defects, absorption, microroughness, transmission by the end test mass, and mirror figure error cause light scattering. Mirror figure error has a low spatial frequency, which causes a low scattering angle of light [2]. Scattered light alters the amplitude and phase of the laser used for gravitational wave detection.

The project is aimed to reduce optical power losses due to mirror figure error. The requirements of Advanced LIGO are that the round trip cavity loss must be less than 75 parts per
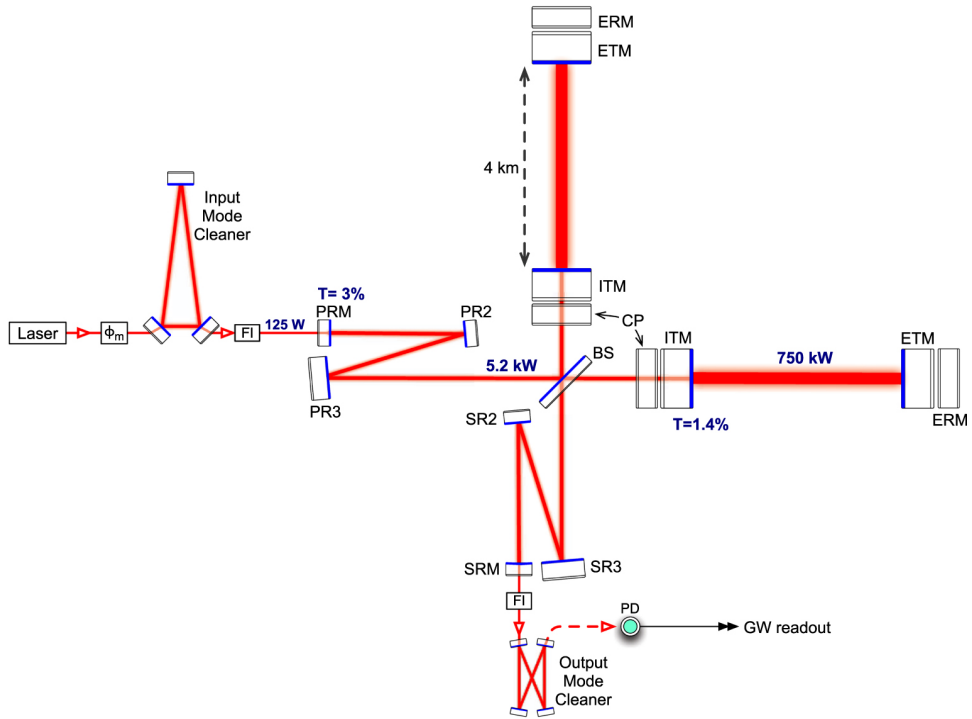
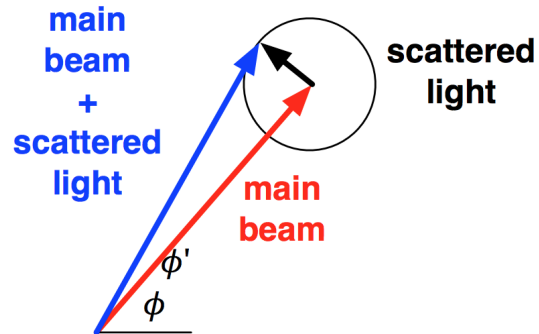Figure 1: Schematic diagram of Advanced LIGO

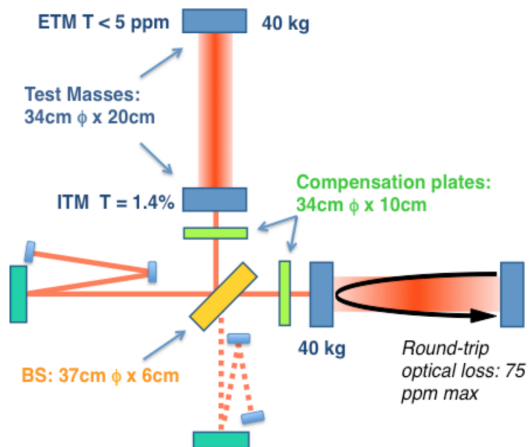Figure 2: The Scattered Light on LIGO's beam used for interferometry

million [3].

Figure 3: The Optical Loss Round Trip in LIGO's interferometric beam

Mirror figure error is the second largest contributor to optical losses, with a budget of 24 ppm for the combination of both test masses [3]. Optical losses destroy the squeezed state of light, reduce the circulating power in the cavities, which increases shot noise in the data. Shot noise is considered quantum noise in LIGO's noise budget. Quantum noise is one of the highest noise contributors. Thus, it is highly important for LIGO to reduce optical losses in order to achieve the sensitivity to detect gravitational waves.

We will develop an in-situ technique to characterize mirror figure error using an optical cavity, which will be demonstrated on the Y arm of the 40m prototype at Caltech. Ultimately, we will measure the transverse mode spacings and create mirror phase maps.

1.4 Optical Resonators

1.4.1 Fabry Perot Cavity

An optical resonator is a system that consists of two mirrors (ITM and ETM) that form a standing wave. A cavity eigenmode describes the standing wave pattern, which is the light pattern that is continuously reflected. Only certain modes are allowed to resonate inside of the cavity.

The Fabry Perot Cavity used in Advanced LIGO is an optical resonator with spherical mirrors. There are two types of modes: longitudinal modes and transverse modes. Longitudinal modes describe the laser frequency and determine the certain frequencies that are allowed to resonant inside of the cavity. Longitudinal modes are determined by the absolute length of the cavity. A transverse mode differs in frequency and amplitude, and it depends on the radius of the curvature of the mirrors.

A system with perfectly spherical mirrors has Gaussian modes. A Gaussian mode is a transverse mode, and the intensity of light can be described in a Gaussian distribution function. The fundamental Gaussian mode has repetitive resonances with the laser frequency of the
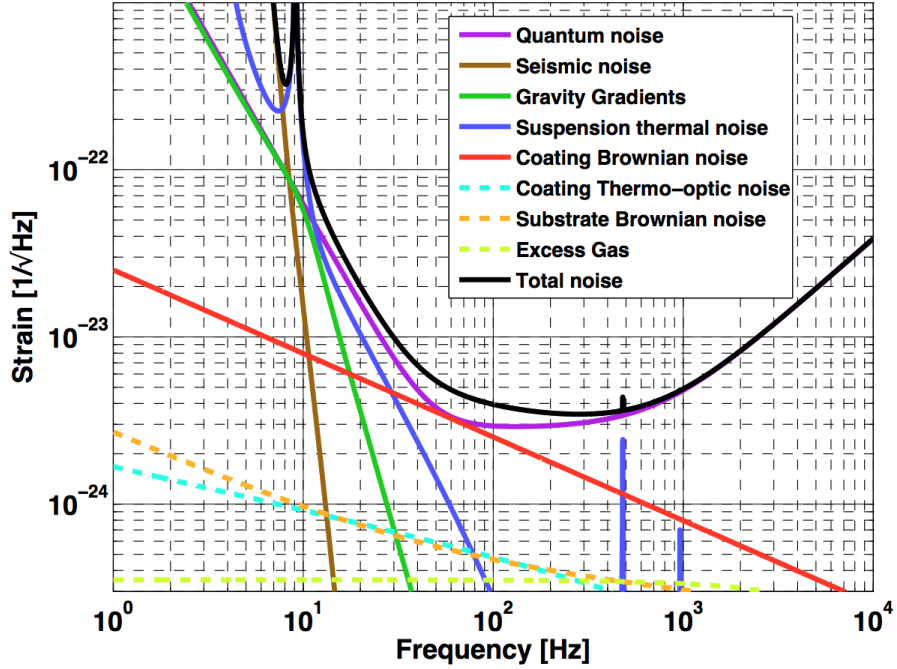
Figure 4: The Noise Budget of Advanced LIGO

Parameter	Value
Cavity length (L)	40 m
ITM Reflectivity (R)	0.98616
ITM Transmissivity (T)	0.01384
ETM Reflectivity (R)	0.9999863
ETM Transmissivity (T)	$1.37\text{x}10^{-6}$
ITM Radius of curvature (R_1)	∞
ETM Radius of curvature (R_2)	60 m

Table 1: Parameters of the 40m prototype LIGO interferometer.

Free Spectral Range (FSR), which describes the spaces between two resonant longitudinal modes. The FSR is defined as,

$$\nu_{FSR} = \frac{c}{2L} = 3.7474 \times 10^6 \text{ Hz}$$

The FSR is determined by the absolute length of the cavity. The FSR is known as the fundamental mode of resonance or TEM(0,0).

1.4.2 Hermite Gaussian Modes

A Hermite Gaussian mode is a higher order mode that has an intensity distribution that differs from the fundamental Gaussian mode. Hermite Gaussian modes are described by spacial indices (n,m) for TEM(n,m).
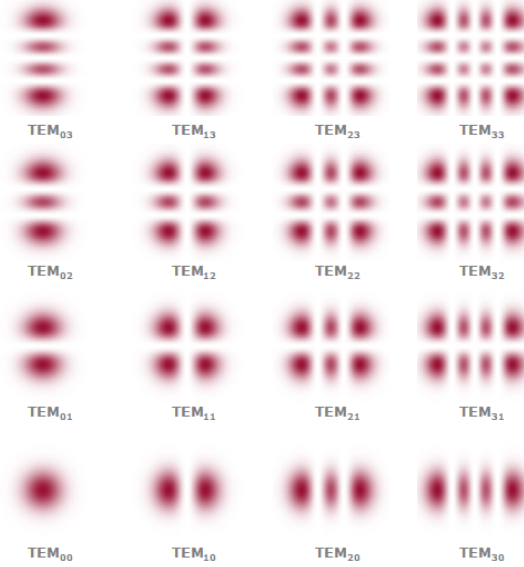

Figure 5: Hermite Gaussian Modes

The Transverse Mode Spacing (TMS) is the distance between transverse modes. The higher order modes have a frequency separation of multiples of the transverse mode spacing from the fundamental mode of resonance.

$$\nu_{TMS} = \nu_{FSR} \times \left(\frac{n+m}{\pi}\right) \cos^{-1} \sqrt{g_1 g_2}$$

Mirror figure error causes a shift in the frequency spacings. Thus, information about mirror figure error will be extracted by precisely measuring the shift of the transverse mode frequencies.

The g parameters depend on the absolute length of the cavity and the radius of curvature of the mirrors.

$$g_1 = 1 - \frac{L}{R_1} \approx 1$$

$$g_2 = 1 - \frac{L}{R_2} \approx 0.298$$

The Gouy Phase is $\Delta\varphi$,

$$\Delta\varphi = \varphi(z_2) - \varphi(z_1) = \cos^{-1} \pm\sqrt{g_1 g_2}. \tag{1}$$

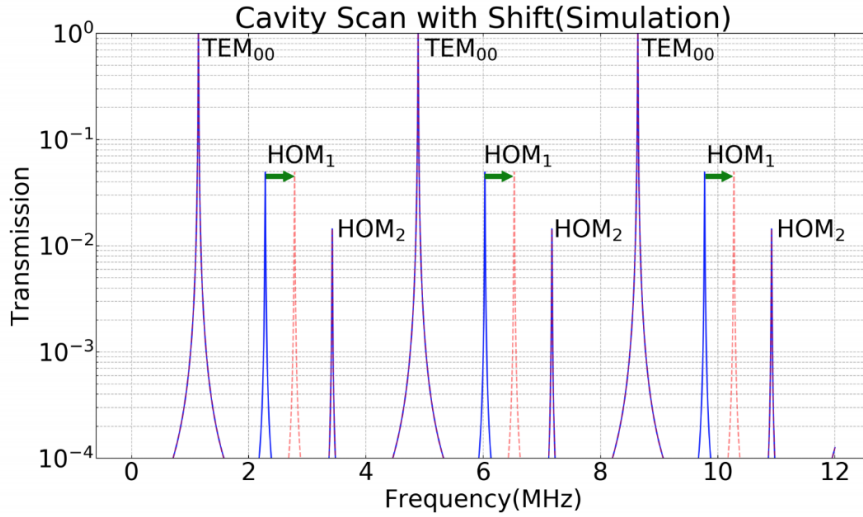The Gouy Phase is stronger for HOMs, which can cause the resonant frequencies to be higher.

Figure 6: Exaggerated Shift in Resonant Frequencies of Higher Order Modes due to Figure Error

2   Objectives

The goal of this project is to create a mirror map that characterizes the figure error on the test masses causing optical losses. A mirror map visually depicts the imperfections on the surface of the mirror. Mirror maps are conventionally created using Fizeau interferometry. However, Fizeau interferometry would effect the gravitational wave detector's output. Thus, we will use an in-situ technique through mode spectroscopy because this research will be applied to measure the mirror figure error in LIGO's gravitational wave detectors.

A cavity scan will be performed by injecting an auxiliary laser into the 40m LIGO prototype interferometer to measure its transverse mode spacings. The transverse mode spacings obtained experimentally will be compared to an ideal model simulated in Finesse in order to analyze the shift in resonant frequencies. A heater will be installed in the ETM of the Y arm to create a known perturbation in the mirror. This will be done in order to check our Bayesian inference and the Monte Carlo Chain Method will be used to characterize the

most probable perturbations of the cavity mirrors and create a high resolution mirror map for the input test mass (ITM) and end test mass (ETM). A mirror map will be a successful outcome of the project because we will be able to identify the mirror figure error causing the optical power losses.

3   Approach

3.1   Experimental Methods

3.1.1   Mode Spectroscopy

Mode spectroscopy is a technique to measure the frequency of a laser and its higher order modes. This will be performed with an in-situ technique using an auxiliary laser (AUX)in 40m prototype interferometer in order to measure mode spacings. Therefore the interferometer will consist of the two beams: the pre stabilized laser (PSL), which is the beam that is used for the detection of gravitational waves, and the auxiliary laser. It is important to note that the auxiliary laser will not disturb the output of the interferometer, and therefore the mode spectroscopy can be performed while the interferometer is observing. Mode spectroscopy will be performed to measure the FSR and TMS. The FSR and TMS are determined by the frequency and the absolute length of the cavity. Thus, we rely on very accurate and precise measurements of the frequency and length of the cavity in order to determine the modes. However, the laser frequency and the arm cavity length are not stable enough to sustain the resonance of the cavity. Thus, feedback control will be used on either the frequency and the arm length using a Phase Lock Loop (PLL) and the Pound Drever Hall technique (PDH). Feedback control ensures that the PSL remains locked in the cavity.

The goal of the PLL is to reduce frequency and arm length fluctuations in order to obtain precise and accurate frequency scans, as well as to ensure that the PSL will resonate within the cavity and that the phase difference between the PSL and AUX is set. The PSL is locked using the Pound Drever Hall Locking, which stabilizes the laser by measuring the frequency with a Fabry Perot and feeding it back into the laser, ultimately suppressing frequency fluctuations [8]. The auxiliary laser will be injected from the optical port, and the will be locked to one of the arms. A beam splitter sends a small amount of the PSL and AUX beam to the PLL. The Phase Lock Loop ensures that the frequency difference between the PSL and AUX is controlled and set with a local oscillator set by the marcorni. The frequency difference between the PSL and the AUX in the PLL is set by RF generator when the PLL is locked.

The coupler sends part of the beat to the control room in order to monitor the beat frequency. The Phase Lock Loop will not lock if the beat frequency is not within a close range of the frequency set by the LO. The frequency of the beat can be altered if needed by changing the temperature of the laser. This will ensure that the PLL is in lock.

Currently, we have developped multiple in-situ techniques to scan the frequency of the AUX laser:

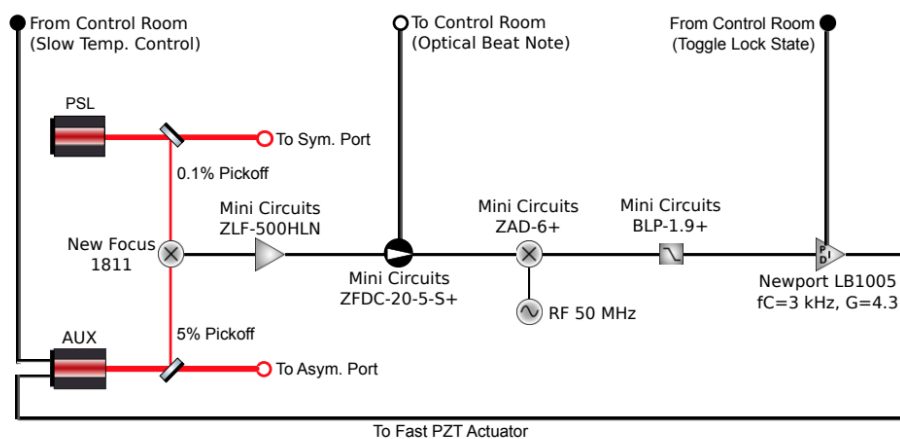- RF offset

AUX-PSL Phase-Locked Loop

Figure 7: Schematic diagram of the AUX-PSL Phase locked loop

- AOM sidebands

- Agilent Drive Signal

The RF offset produces cavity scans by sweeping the frequency of the AUX carrier and its sidebands. The RF offset changes carrier frequency (PSL). The RF frequency goes into the AOM. The AOM created two sidebands. The sidebands resonated with the FSR and the carrier, which resulted in too much power. The sideband frequency spacings were approximately 11 mHz. It can be inferred that the repeated lines were higher order modes (HOM) due to the equal spacing. However, the frequency scan using the RF offset was unsuccessful because it was an incoherent measurement. The sidebands resonated with the FSR and the carrier, which resulted in too much power.

The goal of scanning the frequency using the AOM sidebands was to create a coherent and sentinel measurement. A transfer function will be created in order to measure the input power of the beam relative to the output of the beam at a specific frequency. This creates a pure measurement of the spectral power density. Therefore we are only looking at specific frequencies. The PSL is set to resonate within the cavity. The AOM creates two sidebands and sweeps outworks. However, the AOM sidebands technique was inconclusive because HOM's were unclear. Thus, we inserted a razor blade to add more power to the HOMs, but it was also unsuccessful. The photodetector (PD) measures the light going into the interferometer and the light after the AOM.

The Agilent Drive Signal swept the AUX carrier frequency. The previous methods relied on the frequency of the AM sidebands being swept, and the carrier was locked at a fixed offset from the PSL carrier.

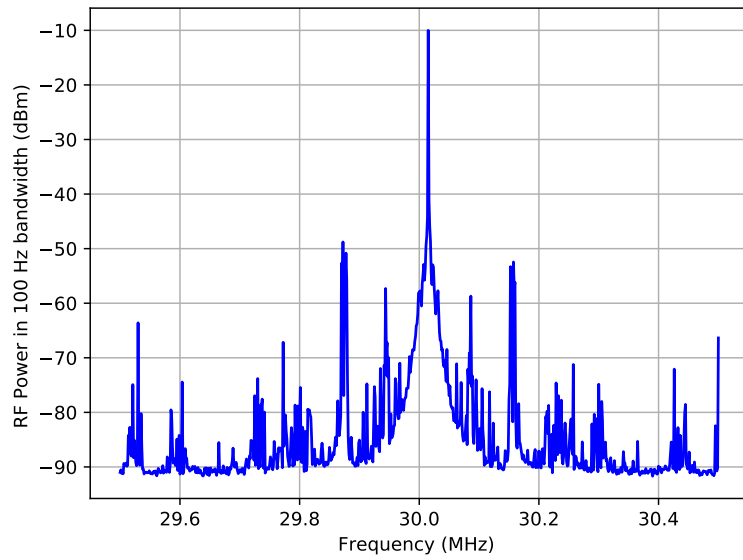We used this method with a double demodulation and simplified it further to a single demodulation.

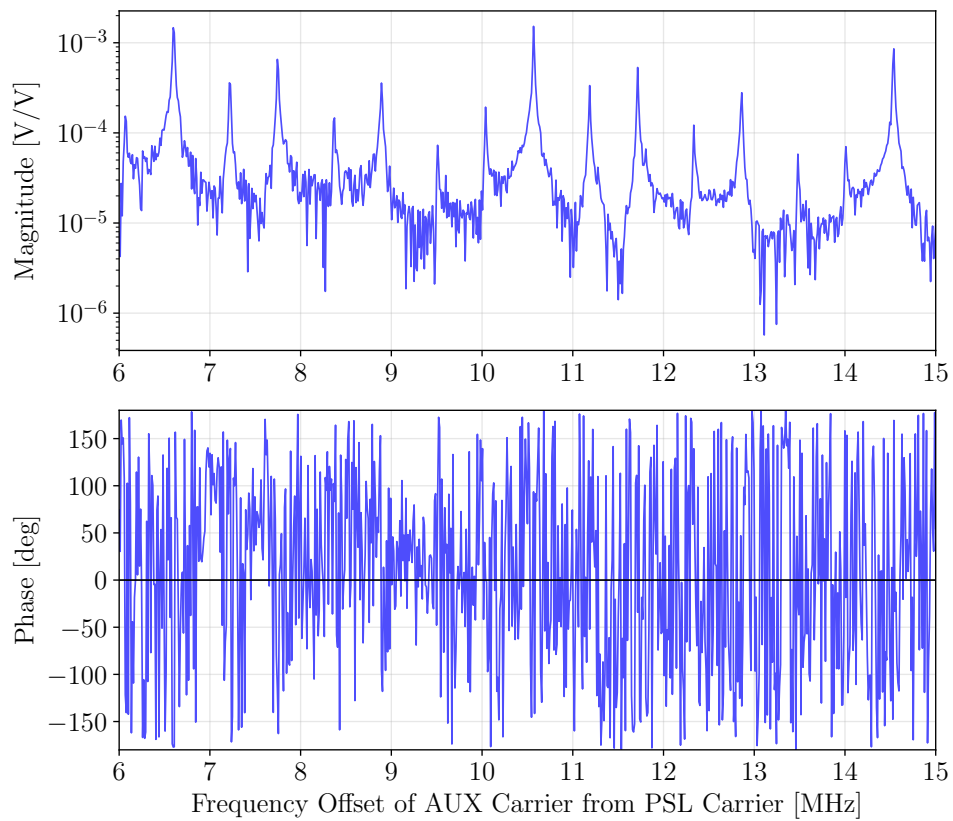Figure 8: Power Spectral Density using the AOM sidebands

Figure 9: The Power Spectral Density using the Agilent Drive
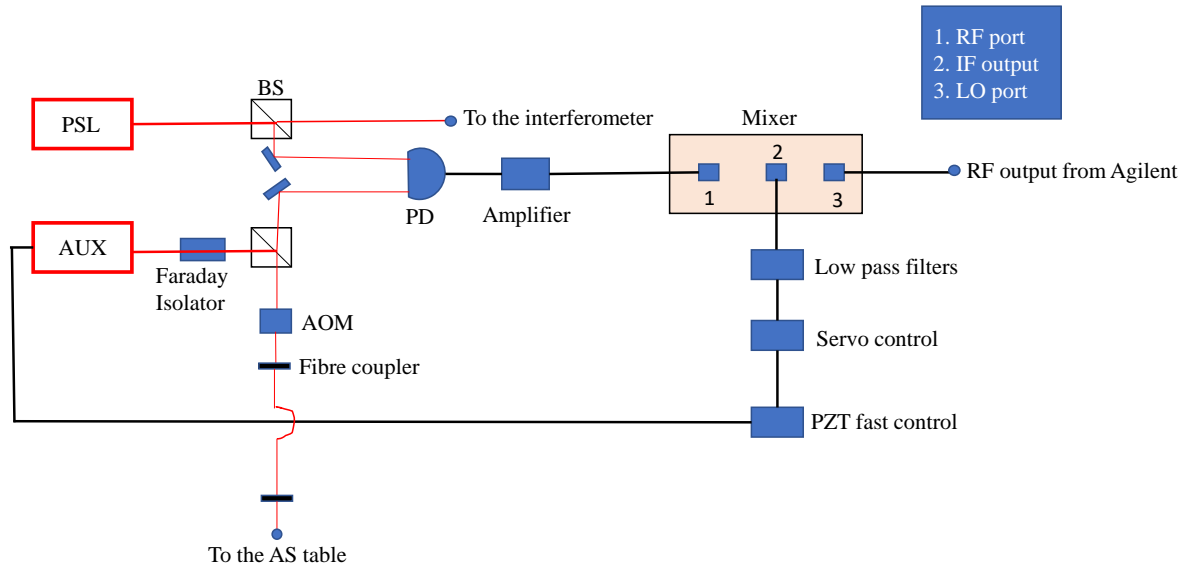
Phase Locked Loop – Single demodulation

Figure 10: The Agilent Drive technique with single demodulation
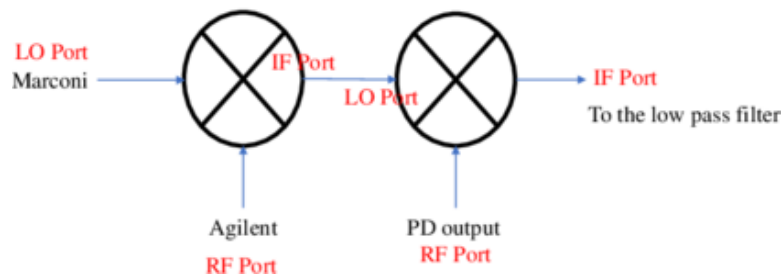

Figure 11: The Agilent Drive technique with double demodulation

3.1.2 Heater

A heater will be installed to radiate electromagnetic radiation in the ETM of the Y arm in order to create a perturbation in the mirror. We will be purposefully inserting one known perturbation in order to use this known reference to make sure our own measurements are correct. The heater will be inserted into an elliptical reflector. However, the reflector will be half an ellipse to ensure that it does not block the laser beam path. The reflector is an ellipse because an ellipse has two focal points. The heater will be set at one focal point, so that the heat will be radiated towards the other focal point. The focal points will be a specific point on the ETM of the Y arm and this point will create a perturbation on the mirror. This is to focus all of the infared radiation and power into one point as precisely as possible.
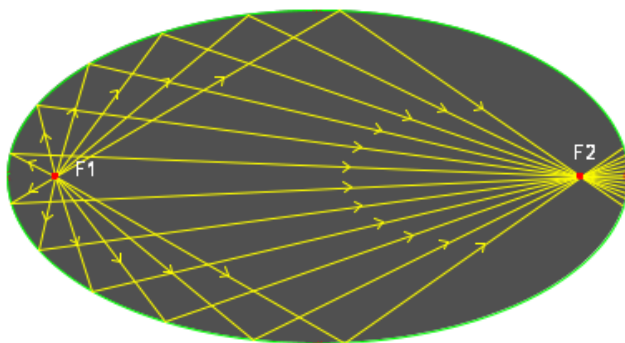

Figure 12: Ellipse, where the heater will be installed in focal point 1 and the perturbation in the ETM mirror is focal point 2.

3.2 Finesse Simulations

Finesse is an optical simulation software. An ideal Fabry Perot cavity will be simulated in Finesse. The ideal model will have no figure error and the same parameters as the 40m prototype interferometer. Its transverse mode spacings will be studied and will be compared to the modes obtained experimentally to analyze the shift in the higher order modes.

Thousands of potential perturbations will be simulated in Finesse, and its effect on the optical power will be observed. Perturbations will be introduced using Zernike Polynomials coefficients. However, information about the mirror figure error cannot fully be extracted from the use of Zernike polynomials, and a mirror phase map cannot be reconstructed from the use of Zernike polynomials. Mirror phase maps can be reconstructed from the potential perturbations by using Bayesian inference.

3.3 Bayesian Inference

Bayesian inference will be used to produce the most probable mirror phase maps. The probability of perturbations will be continually updated as more evidence is available. What will be most important when using Bayesian inference will be understanding what causes the

shift in resonant frequencies, which can tell us information about the surface defects on the mirrors. We will use Bayes' theorem of inverse problems, which will be done by analyzing the cavity transmission, then finding the most probable phase map of the cavity mirror for the given cavity scan. This relies on the Markov Chain Monte Carlo method (MCMC), which states that the future values do not depend on its previous values [4].

The most difficult aspect of this project will be to produce a mirror phase map using Bayesian inference techniques.

4   Project Schedule

4.1   Experimental Methods

- Perform cavity scans to obtain resonant frequencies of HOM.

- (If needed) Add power to HOMs by inserting a razor blade in the experimental set up.

- Install heater to create known perturbation in the ETM of the Y arm.

- Repeat process to obtain cavity scans after inserting heater in experimental set up.

4.2   Finesse Simulations

- Create an ideal model of a Fabry Perot with no figure error using the same parameters in the 40m prototype.

    - Plot the power spectral density.
    - Use the ideal model to analyze shift in frequencies.

- Introduce mode-mismatched and misalignment into Finesse to produce a fitting model.

- Introduce figure error using Zernike polynomials and thousands of possible perturbations in mirror.

    - Observe how transmitted power is effected.

4.3   Creating mirror phase maps

- Evaluate optical losses as perturbations in mirrors

    - Use Monte Carlo Method to fit experimental data as a linear combination of the simulated perturbations.

- Use Bayesian Inference to investigate the most probable perturbations in the cavity mirror contributing to power losses in the interferometer. This will produce a mirror phase map.

References

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