In-Situ Laser Mode Spectroscopy for Mirror Phase Mapping

Sandrine Ferrans

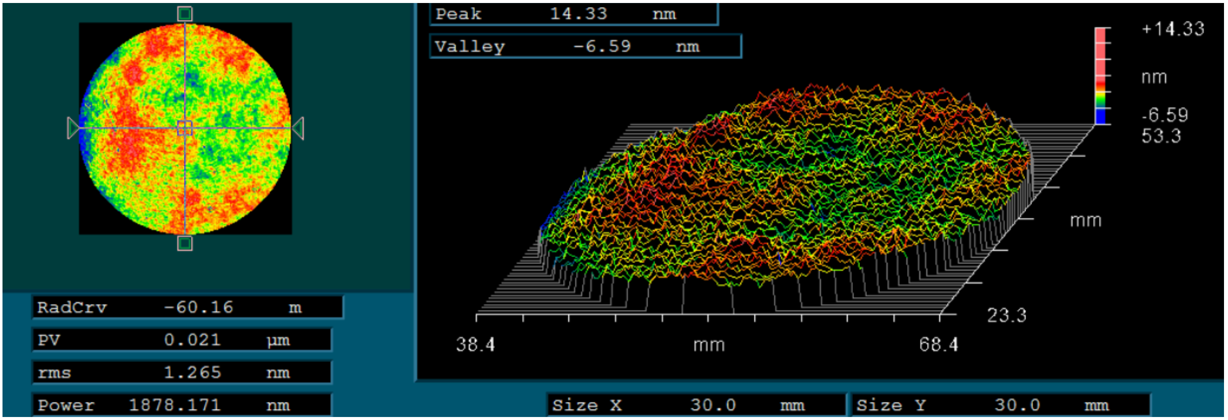
Rana Adhikari, Koji Arai, Gautam Venugopalan

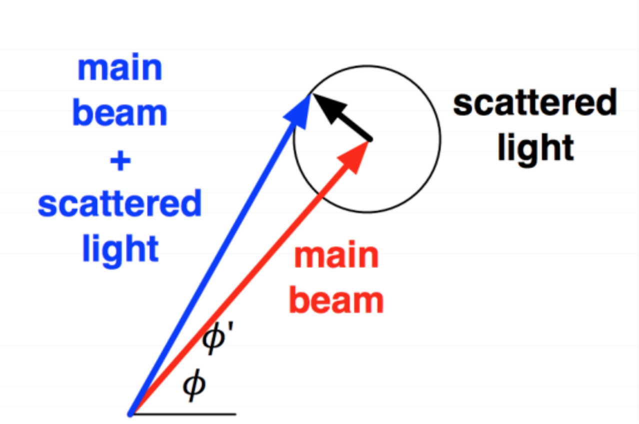
Objectives

- Improving the sensitivity in LIGO by characterizing mirror figure error that contributes to optical losses in the laser
- Measure Transverse Mode Spacings (TMS) using Mode Spectroscopy to extract information about the cavity mirrors
- Analyze shift in resonant frequencies of Higher Order Modes (HOMs)
- Use Bayesian Inference to create the most probable mirror phase maps of the ETMY in the 40m prototype

Mirror Figure Error

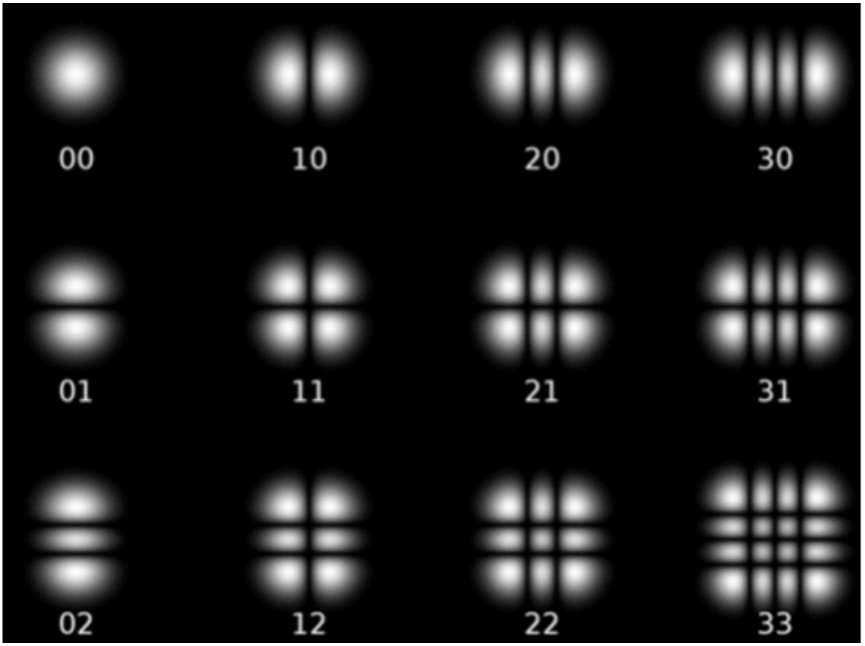
- Mirror figure error has a low spatial frequency, which causes a low scattering angle of light
- LIGO relies on a phase based measurement. Scattered light alters the amplitude and phase of the laser used for gravitational wave detection.
- Mirror figure error can be evaluated with **Phase Maps**

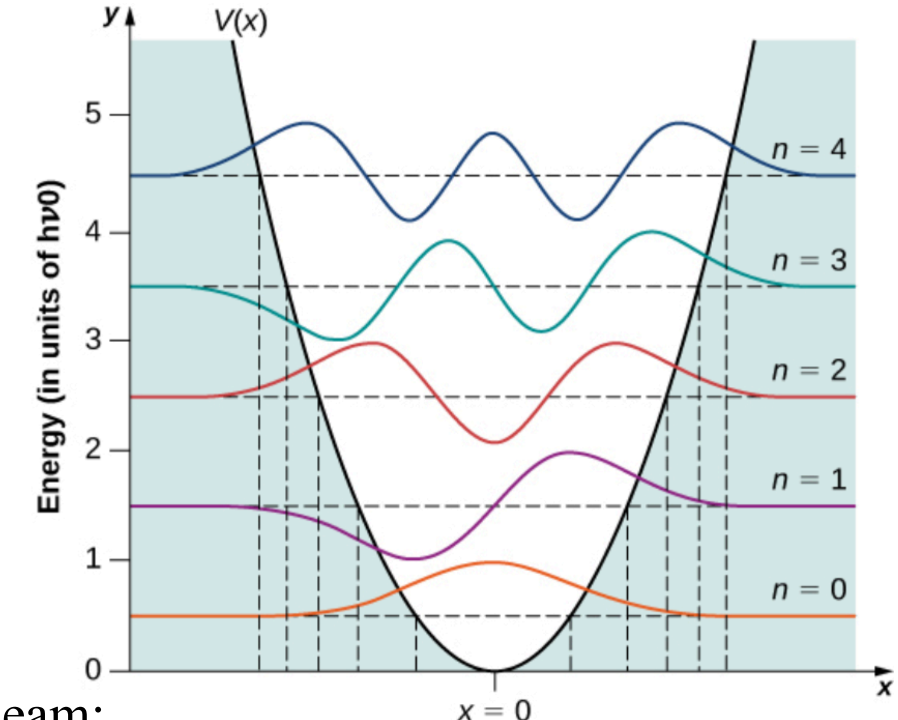




measured phase Map of ETM in 40m (not in-situ)

Hermite-Gaussian Modes

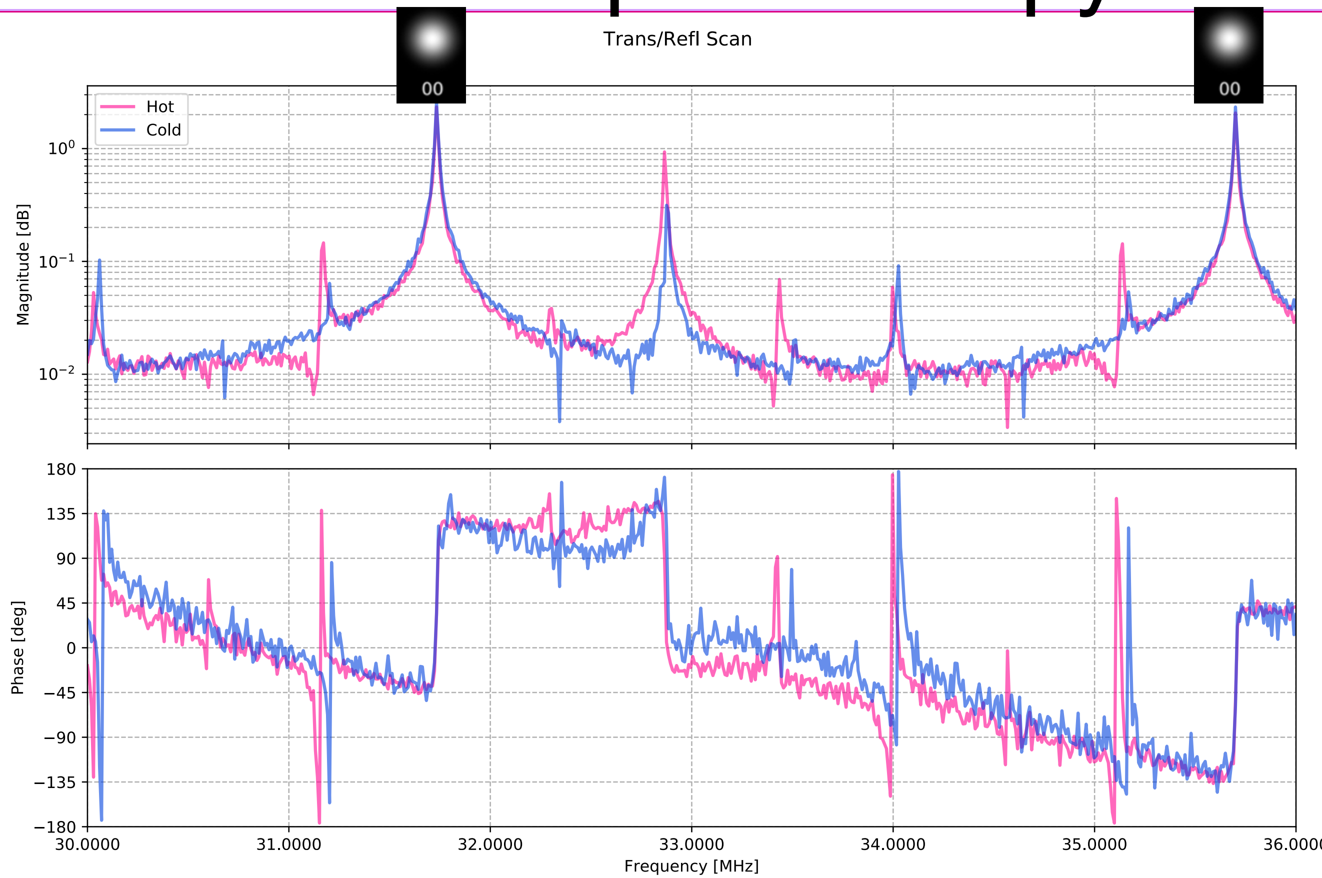




- Electric field intensity distributions of a beam:

$$E_{nm}(x,y,z) = E_0 \frac{w_0}{w(z)} \cdot H_n\left(\sqrt{2}\frac{x}{w(z)}\right)\exp\left(-\frac{x^2}{w(z)^2}\right)\cdot H_m\left(\sqrt{2}\frac{y}{w(z)}\right)\exp\left(-\frac{y^2}{w(z)^2}\right) \cdot \exp\left(-i\left[kz-(1+n+m)\arctan\frac{z}{z_R}+\frac{k\left(x^2+y^2\right)}{2R(z)}\right]\right)$$

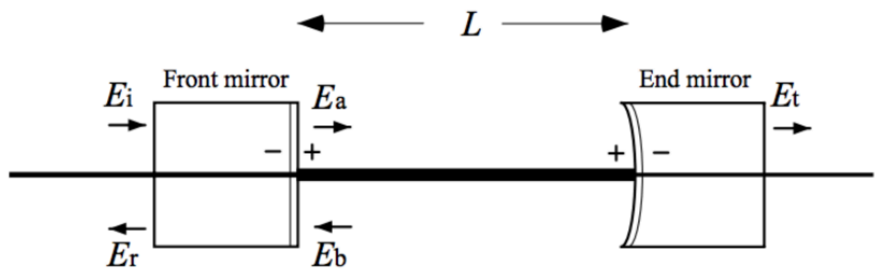
Mode Spectroscopy



Free Spectral Range: $\nu_{FSR} = \frac{c}{2L}$

Transverse Mode Spacings: $\nu_{TMS} = \nu_{FSR} \times \left(\frac{n+m}{\pi}\right) \cos^{-1}\sqrt{\left(1 - \frac{L}{R_1}\right)\left(1 - \frac{L}{R_2}\right)}$

Fabry Perot Cavity



- g parameters:

$$g = 1 - \frac{L}{R}$$

- Guoy Phase:

$$\Delta\varphi = \varphi(z_2) - \varphi(z_1) = \cos^{-1} \pm\sqrt{g_1 g_2}.$$

- Transmitted Power:

$$P_t = \left|\frac{E_t}{E_i}\right|^2 = \left|\frac{t_1 t_2}{1 - r_1 r_2^{-i\Phi}}\right|^2$$

LIGO

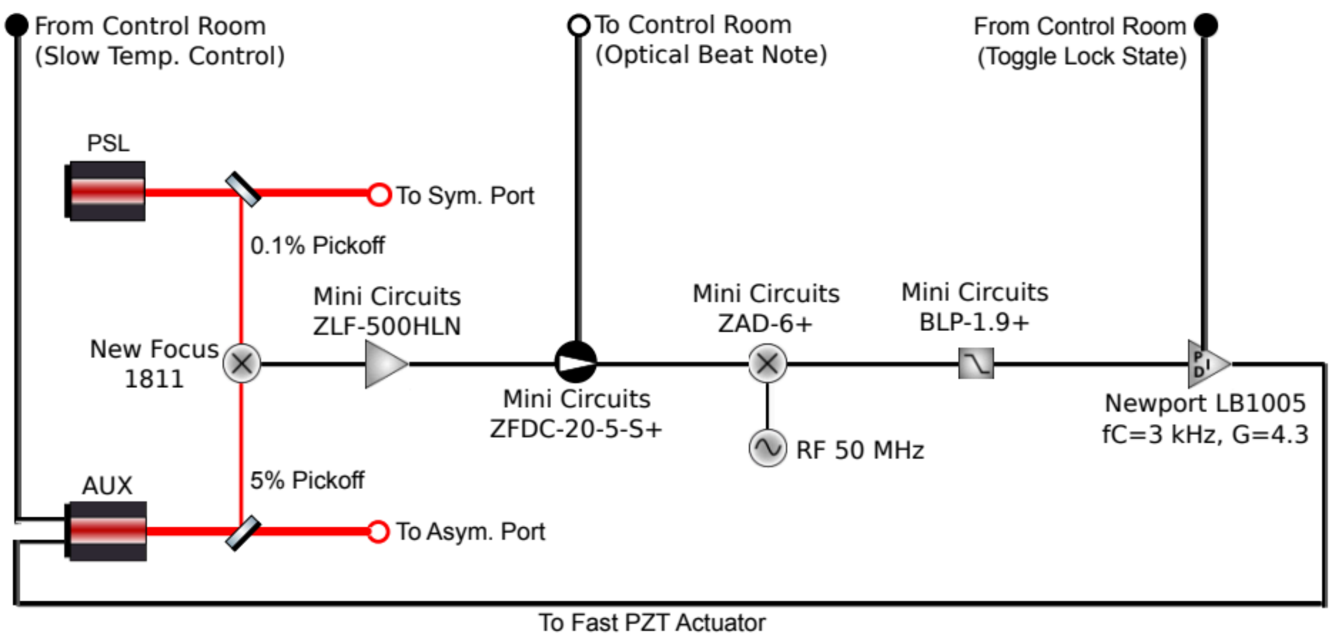
Mode Spectroscopy

- Using resonant frequencies of HOMs, we can extract information about the cavity mirrors
- Performed on 40m prototype - insert auxiliary (AUX) laser from antisymmetric dark port
- ***In-Situ*** - allows for extremely precise measurements
  - does not disturb pre-stabilized laser (PSL) use for gravitational wave detection
  - observe what the interferometric beam sees

Phase Lock Loop

- Uses phase lock loop (PLL) to set a frequency difference between the lasers and ensures that the PSL is always locked to the cavity.

AUX-PSL Phase-Locked Loop

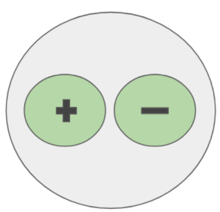


LIGO

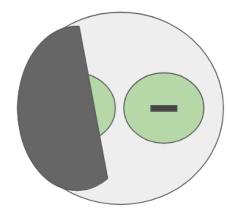
Set up

- Razor blade inserted on the output of the photodiode to **clip** the ~1/3 of laser beam

- Disrupts symmetry of beam so phase does not cancel each other out.



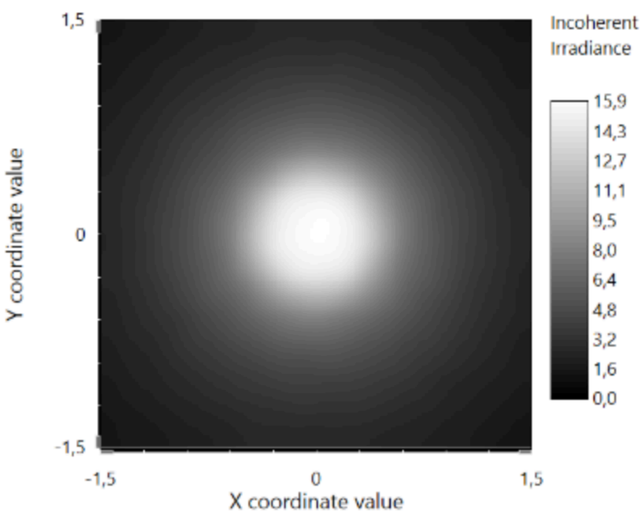
Unclipped TEM_{10}



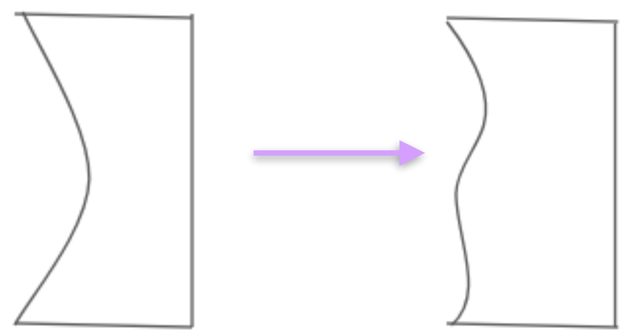
Clipped with Razorblade

Heater

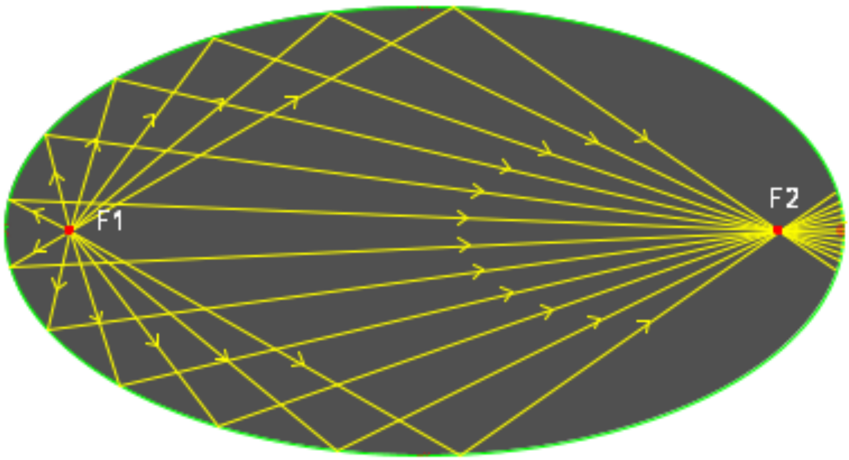
- Insert temporary perturbation into ETMY to induce a shift in HOMs.
- The heater is an **elliptical reflector**.
- Increases the RoC and decreases TMS.



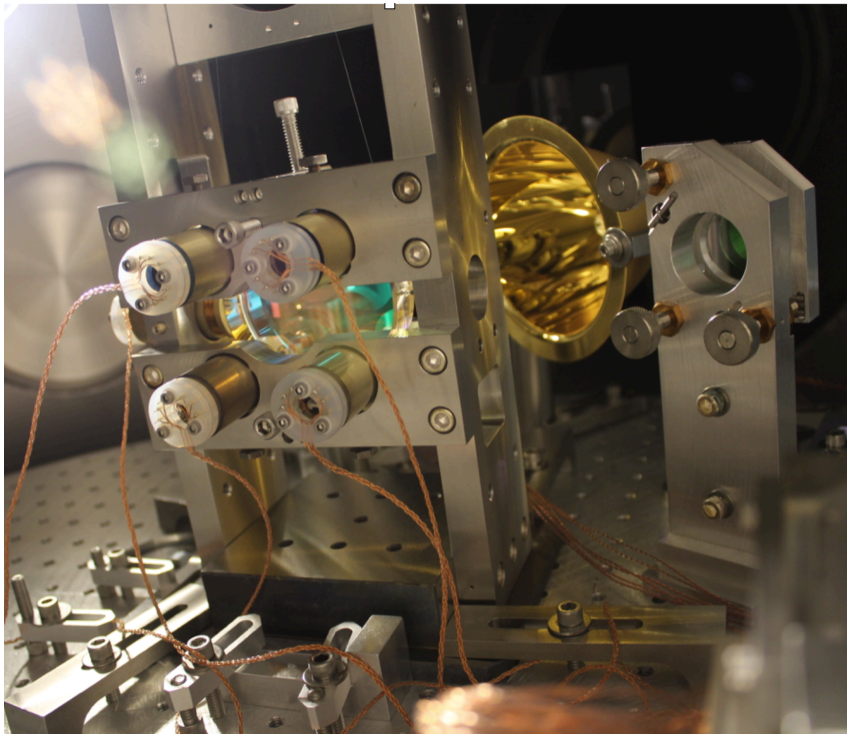
Simulation of Heating Pattern


ETM Y

Heater

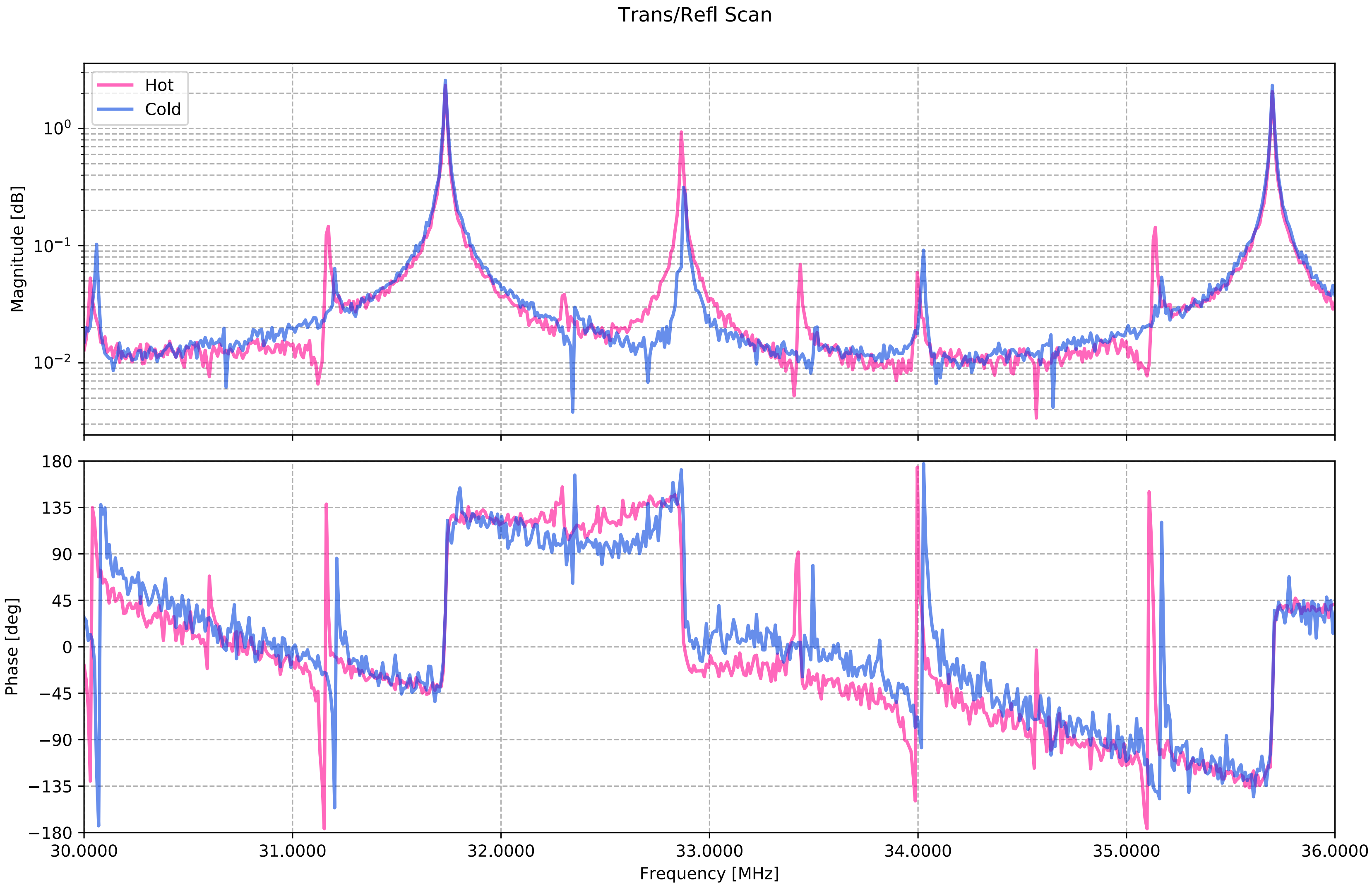


Ellipse

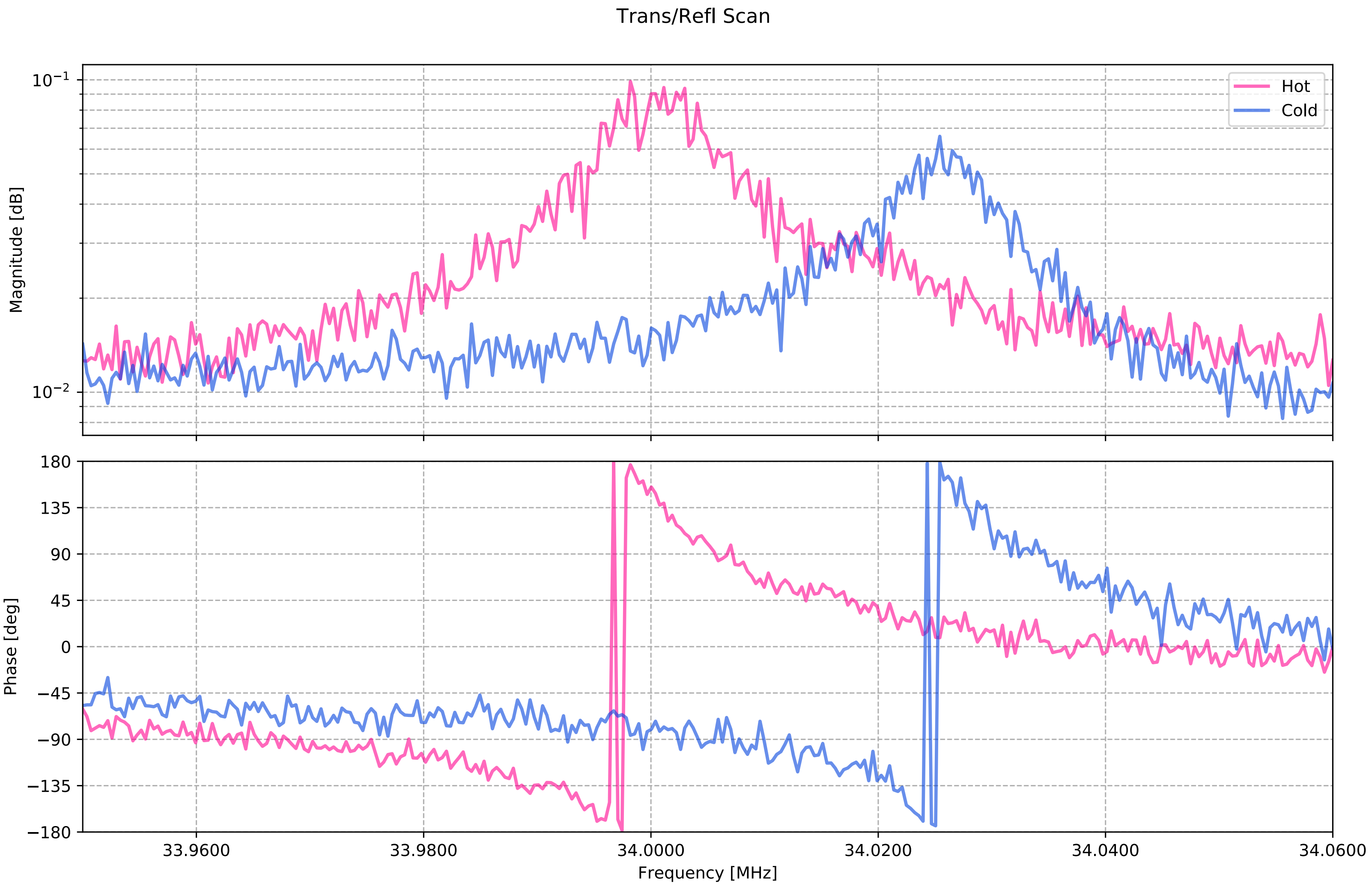


ETMY Chamber

Cavity Scan Data



Cavity Scan Data



LIGO

Magnitude Cavity Scan Fitting

- Curve fitting calculates the values of the parameters obtained experimentally for a given equation. This makes the function ‘fit’ the data as close as possible.

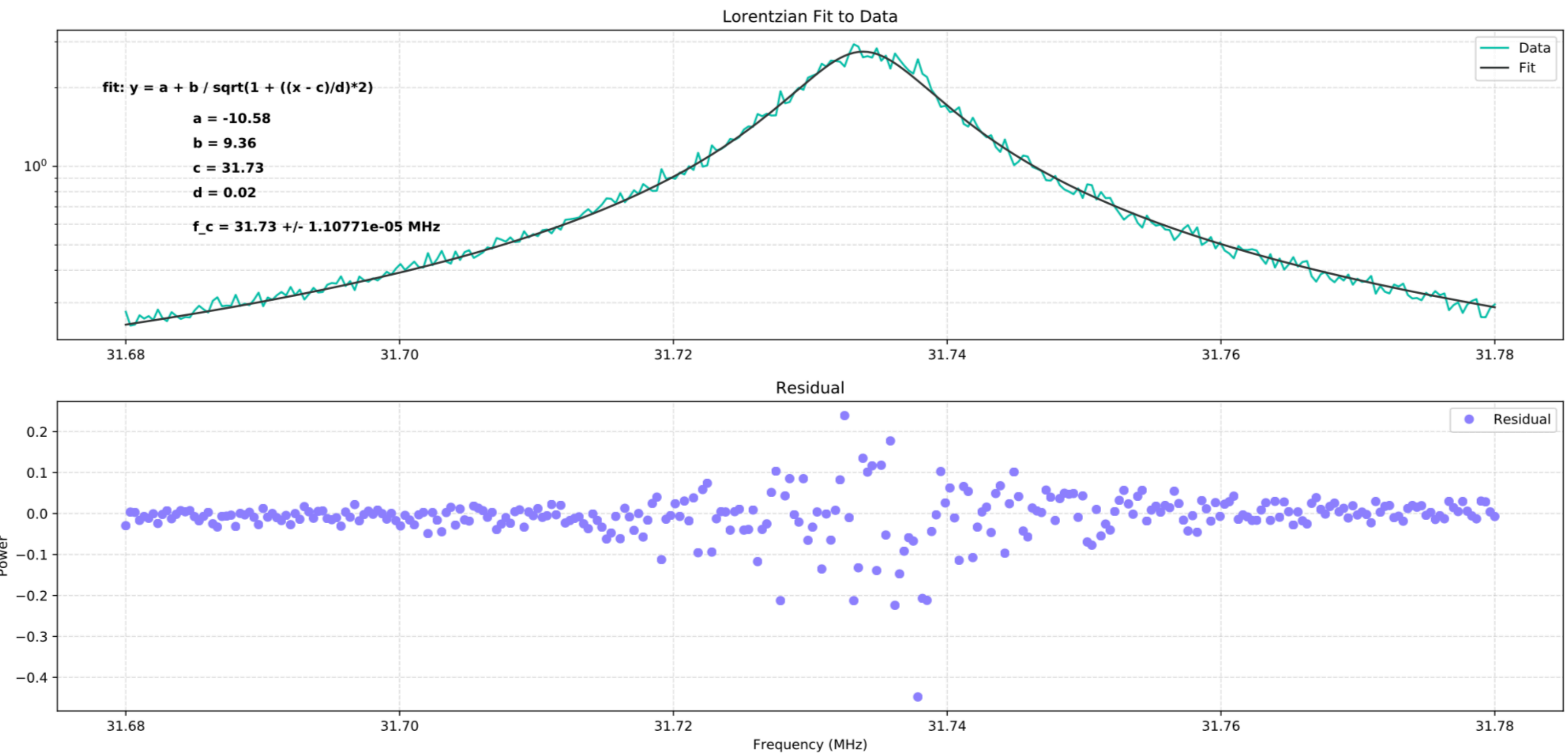
- Data was fitted with equation:

$$A + \frac{B}{\sqrt{1 - (\frac{X-C}{D})^2}}$$

- A - Constant background
- B - Magnitude of resonant peak
- C - Central position of the peak,
- D - Full width at half maximum

- Tells us how precise and accurate our measurements are.

Magnitude Fit



Complex Cavity Scan Fitting

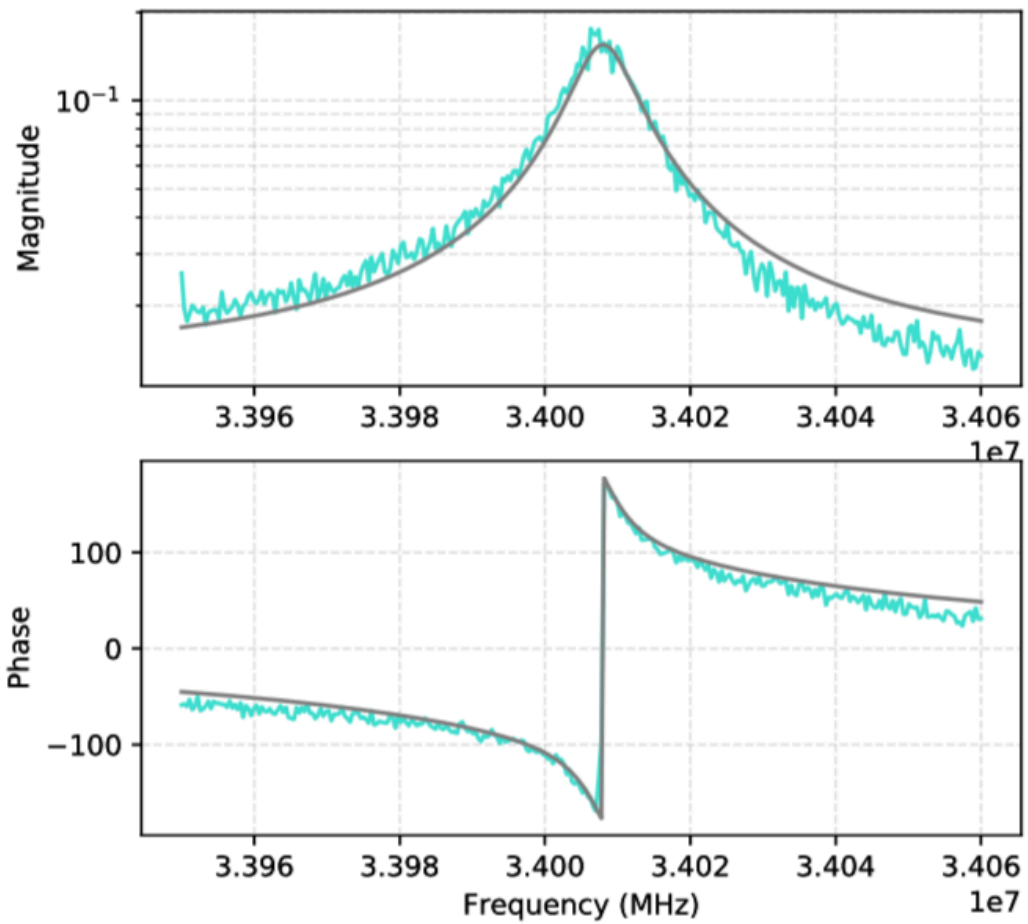
- Magnitude and Phase were combined to make a complex output
- Data was fitted with equation:

$$\frac{kjw^2}{\frac{w_0^2 + jw_0}{Q - w^2}}$$

- w_0 = resonant frequency
- Q = $\frac{w_0}{FWHM}$

- Tells us how precise and accurate our measurements are.

Complex Cavity Scan Fitting



f = 34006287.63 +/- 1.37 Hz

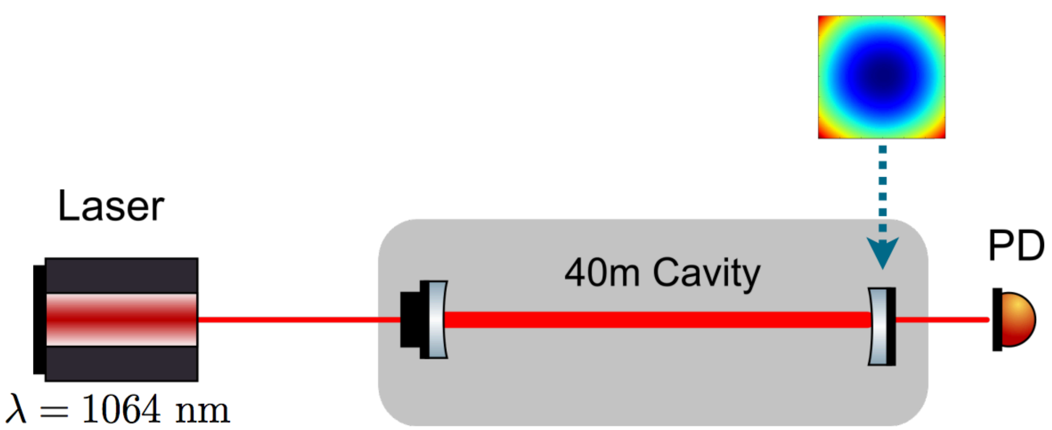
chi square = 0.03

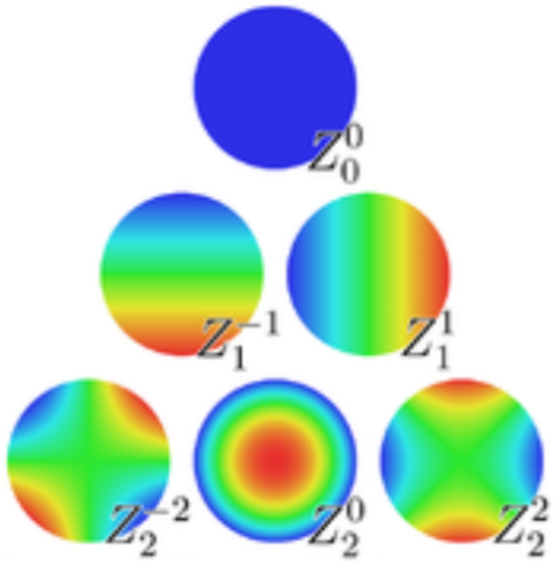
Finesse Model

- Finesse - interferometer simulation software
- Used to analyze shift in TMS and to reconstruct phase map of ETM

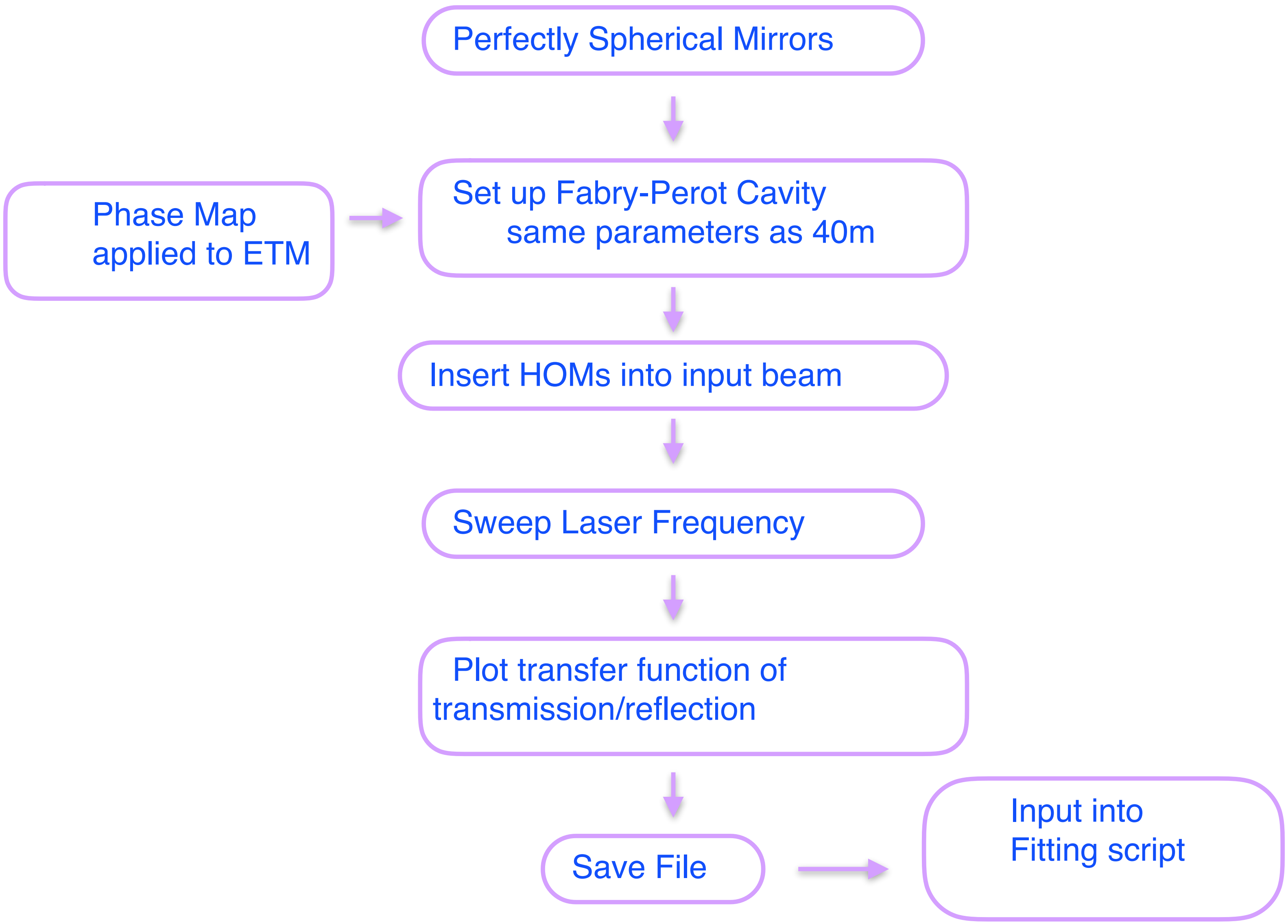
- How are phase maps created?
  - **Zernike Polynomials** insert Figure Error into ETM
  - Each polynomial inserts optical aberrations onto ETM surface

- Analyze shift in TMS

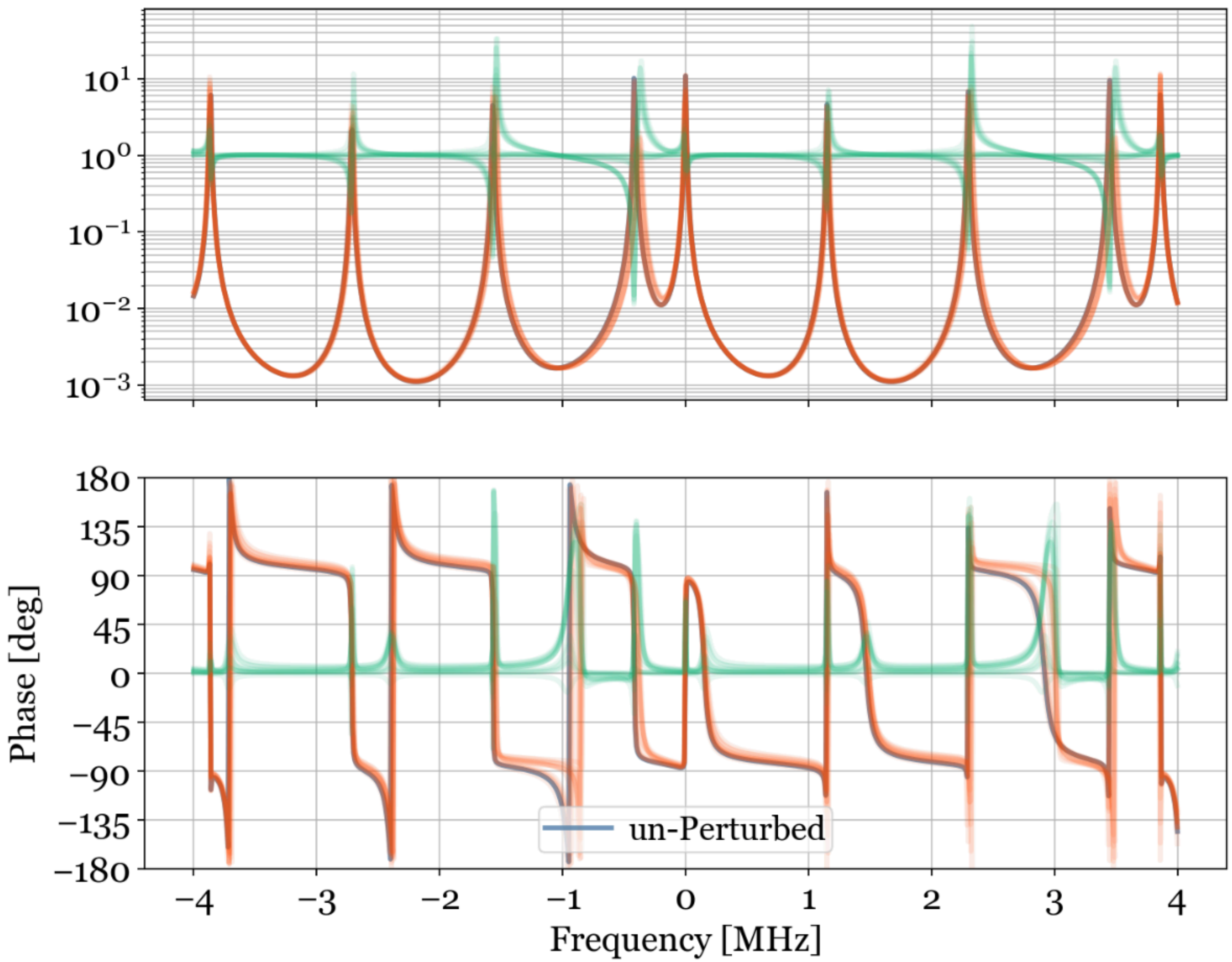




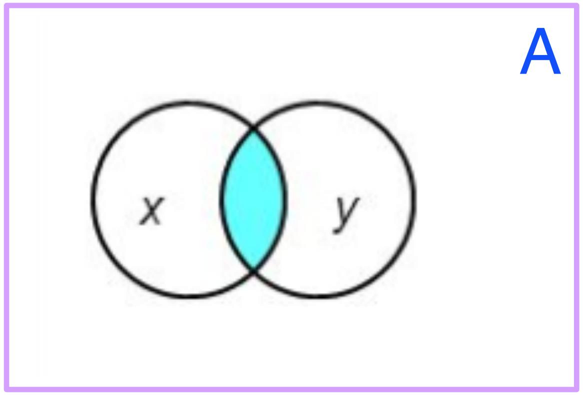
Finesse Model



Finesse Model



Bayesian Inference



- Bayes' theorem:

$$P(X|Y) = \frac{P(Y|X)P(X)}{P(Y)}$$

$$P(Z_{nm}|D) = \frac{P(D|Z_{nm})P(Z_{nm})}{(P|D)}$$

- Finesse returns: Zernikes - $S(f, Z_{nm})$
- Cavity scan returns: Data - $D_i(f)$ from the i_{th} cavity scan
- Both are a function of frequency

Posterior Probability

- Posterior Probability - conditional probability distribution of an unknown variable given some evidence.

- Assumes that noise is Gaussian with variance σ^2

$$p(Z_{nm}|\{D(f)\}) \propto \exp\left[-\frac{1}{2\sigma^2}\sum_i \int [S(f, Z_{nm}) - D_i(f)]^2 \, \mathrm{d}f\right] p(Z_{nm})$$

- $p(Z_{nm})$ is the prior on the amplitudes.

Future Work

- Bayesian Inference
  - Run Markov Chain Monte Carlo Script to get most probable phase maps
- What is the deformation needed to create a phase map?
- Calculate uncertainty in length of cavity from fitting uncertainty
- Improve fits

Thank you!

- Rana Adhikari
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- Alex Urban
- Jon Richardson
- Ladies of LIGO!