# Optimal and robust control of the arm alignment with mu-synthesis

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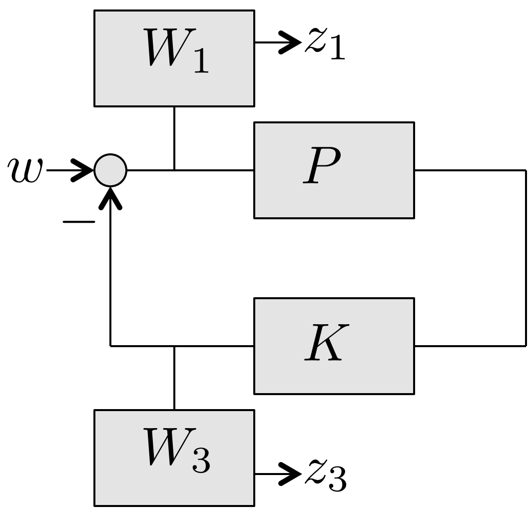
# OPTIMAL CONTROL

- H_2 optimization:
  - Closely related to LQG; min rms
- ***H_inf optimization*** (Doyle et al. 1989; Doyle 1984):
  - Minimizes pk mag of CL transfer functions.
  - Well-suited to freq resp shaping.
  - Handles robustness constraints explicitly

    (main advantageous over H_2 opt)

# H-INF/MU OPTIMIZATION




- Given plant $P$, find controller $K$ s.t.
  - Guarantees loop stability.
  - Minimizes $||H(s)||_\infty$.

$$\begin{bmatrix} z_1 \\ z_3 \end{bmatrix} = Hw = \begin{bmatrix} W_1 S \\ W_3 T \end{bmatrix} w$$

$$S = \frac{1}{1+KP} \qquad T = \frac{KP}{1+KP}$$

- All we need to do is to play with the weights!
- If $P$ is uncertain => \mu-synthesis.

# FREQUENCY RESPONSE SHAPING

- $|W_1 S|^2 + |W_3 T|^2 = \gamma^2$ (i.e., equalizing property) $\gamma = \mathcal{O}(1)$
- Low freq,

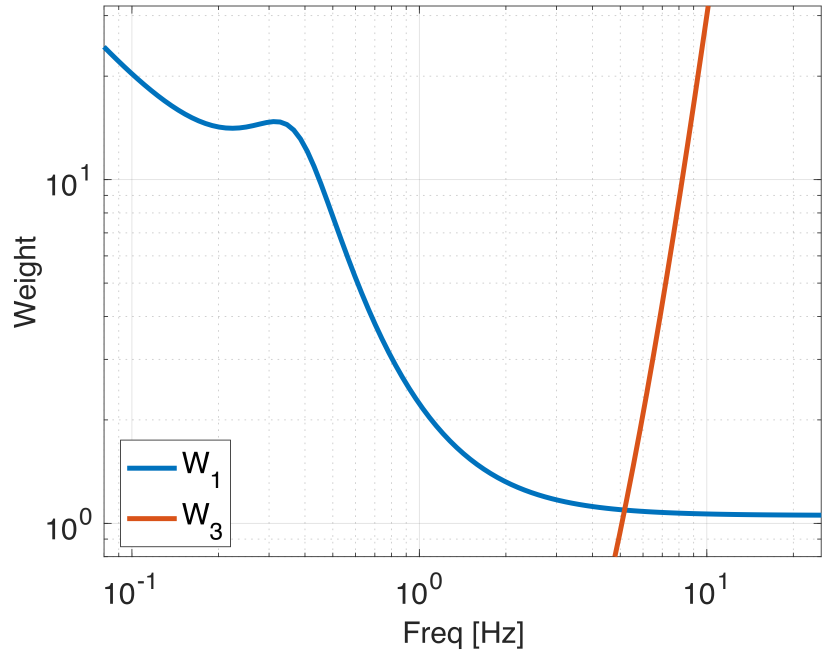
  $|W_1 S| \simeq \gamma;\ S \simeq 1/KP,$

  $\implies \boxed{|W_1| \simeq |KP|.}$

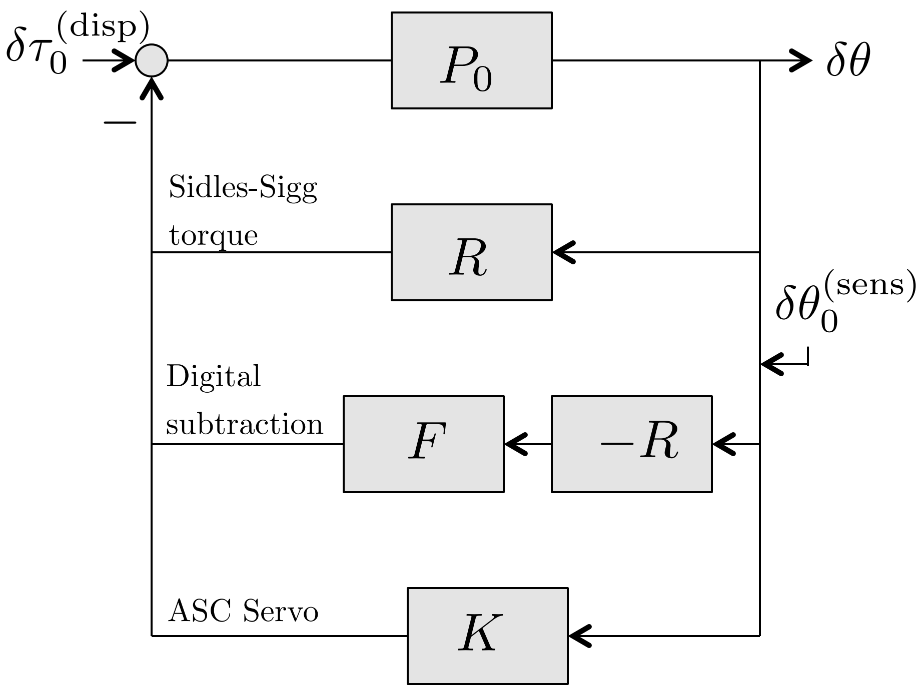
- High freq,

  $|W_3 T| \simeq \gamma;\ T \simeq KP,$

  $\implies \boxed{|W_3| \simeq 1/|KP|}$



# ARM ALIGNMENT CONTRL




- $P_0$ = free pendulum torque to angle TF,

  $$R = -\frac{2P_{\mathrm{arm}}}{c}\frac{dy}{d\theta},$$

  $F$ = switch to turn on/off digital sub.,

  $K$ = control servo from $\mu$-synthesis.

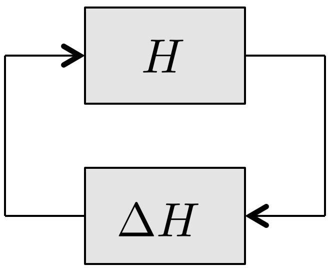
- $OL = [(1-F)\,R + K]\,P_0,$

  $$\delta\theta^{(\mathrm{disp})} = \frac{P_0}{1+OL}\delta\tau_0^{(\mathrm{disp})},$$

  $$\delta\theta^{(\mathrm{sens})} = \frac{-(K-FR)}{1+OL}\delta\theta_0^{(\mathrm{sens})}$$

# ROBUSTNESS

- Small gain theorem:

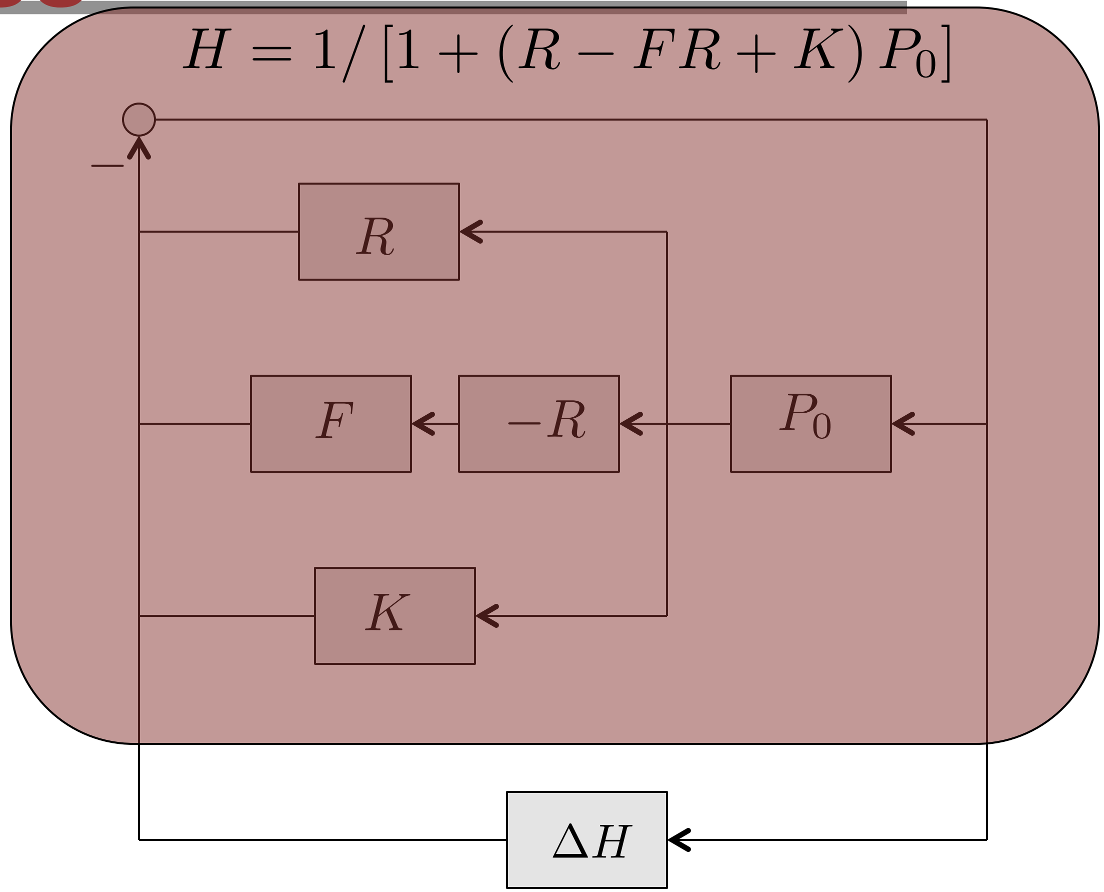



For a given $\Delta H$,

*Sufficient* condition for stability :

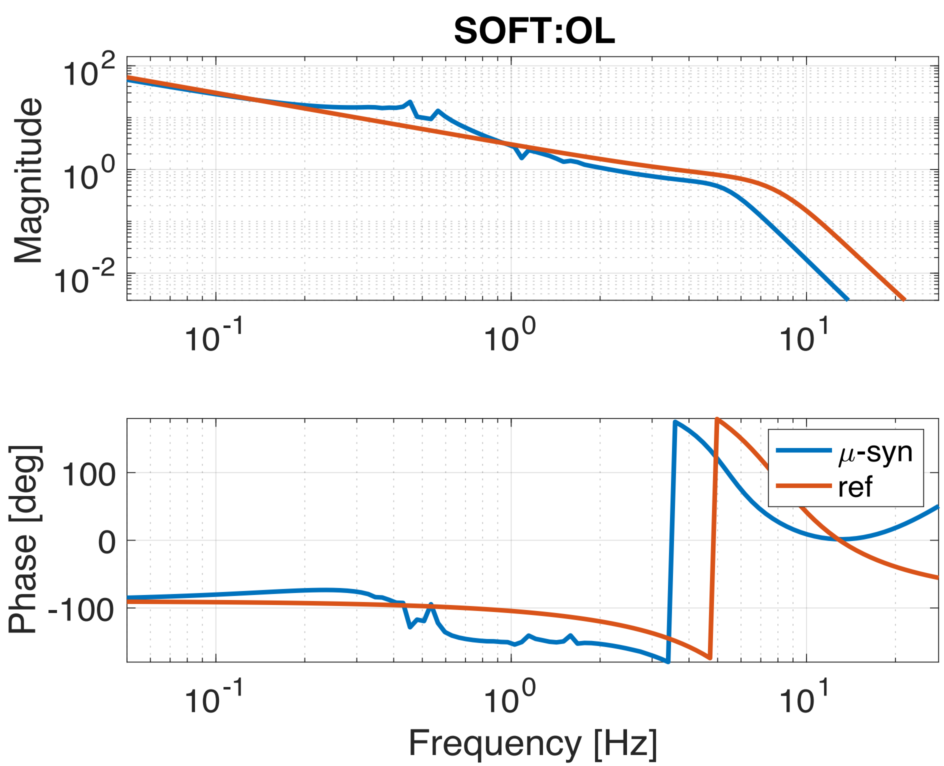
$||H \Delta H||_{\infty} < 1$,

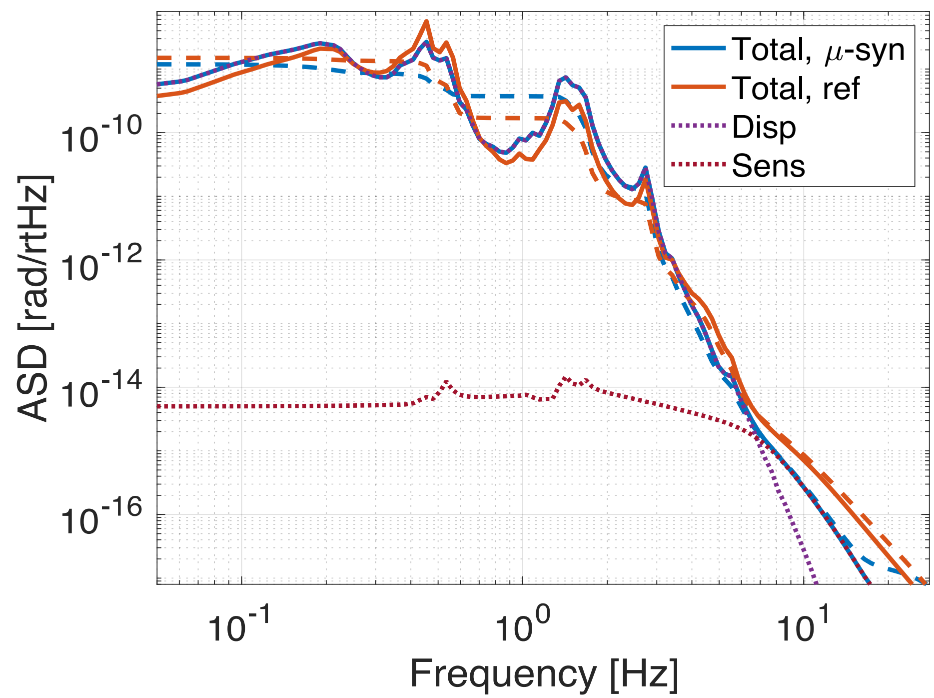
or $||\Delta H||_{\infty} < 1/||H||_{\infty}$



# SOFT MODE: OL & NB (CSOFT PIT)

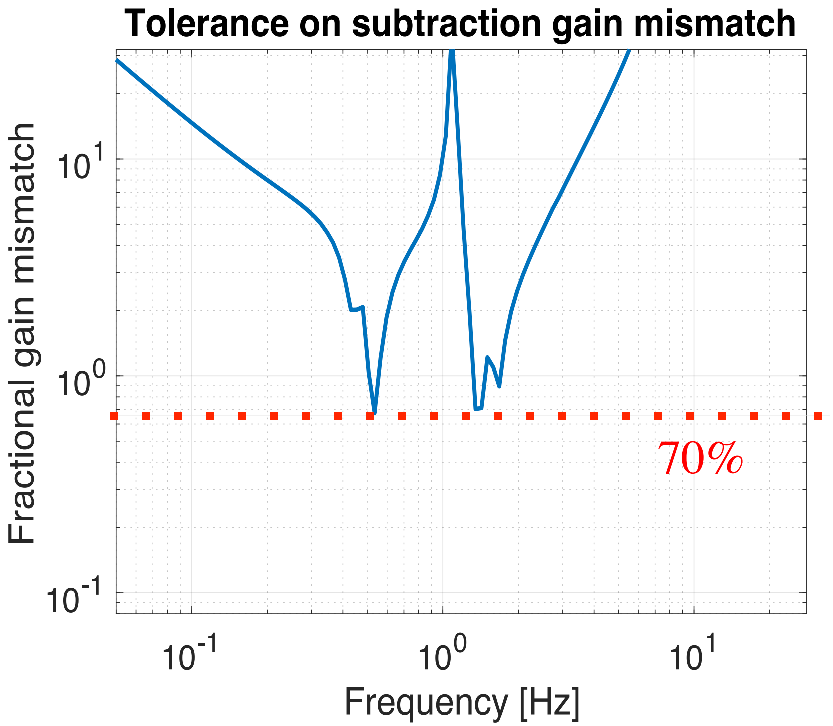
- With digital subtraction of the SS torque (*F*=1),
  => power independent plant.
- Only one set of filters needed for *P_arm* from 0 to 0.8 MW!
- Imperfect subtraction/dP-d\theta torque as uncertainties.



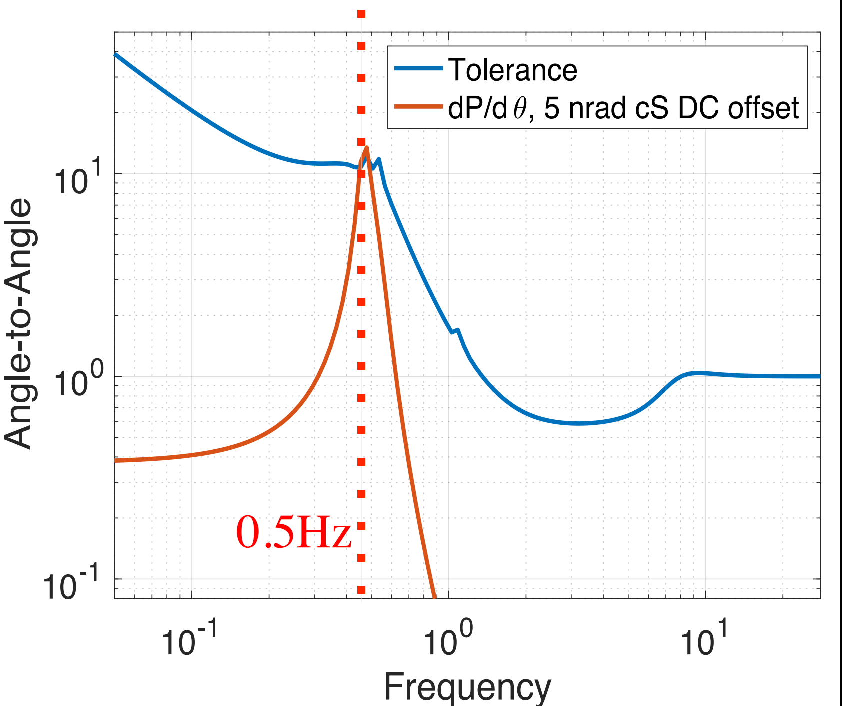



# SOFT MODE: ROBUSTNESS

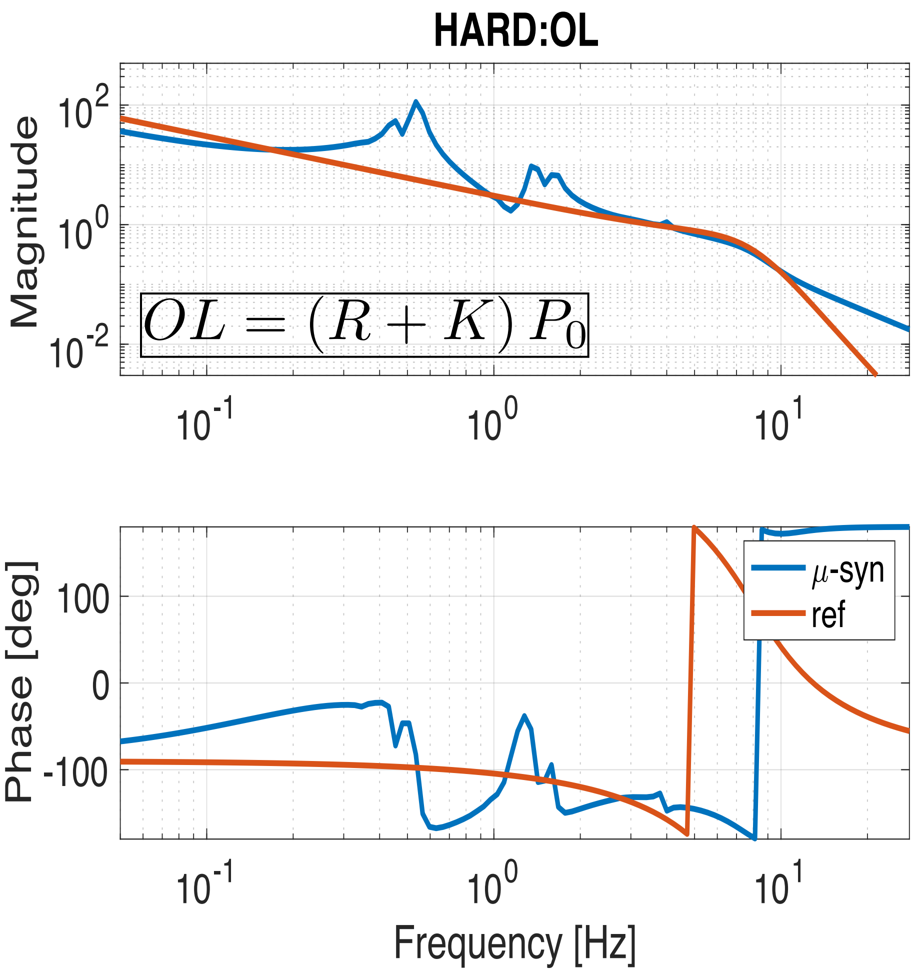
Tolerance on gain mismatch in the subtraction path

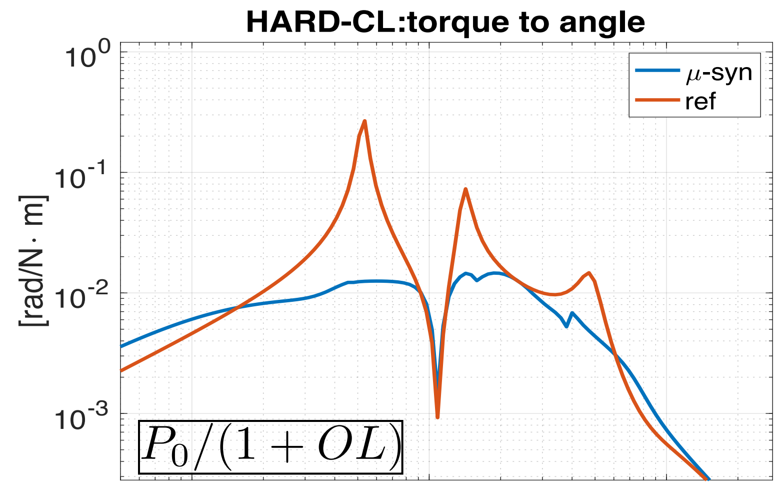


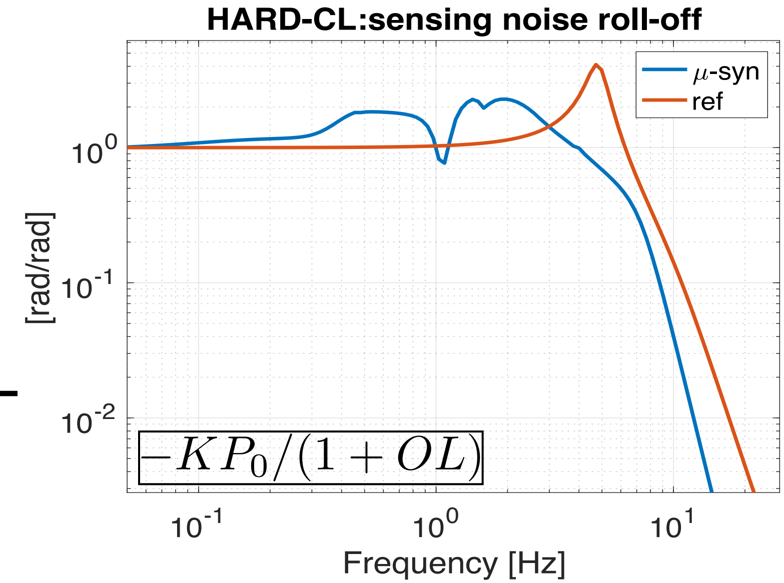

Tolerance on dP-d\theta perturbation (P_arm = 0.8 MW)



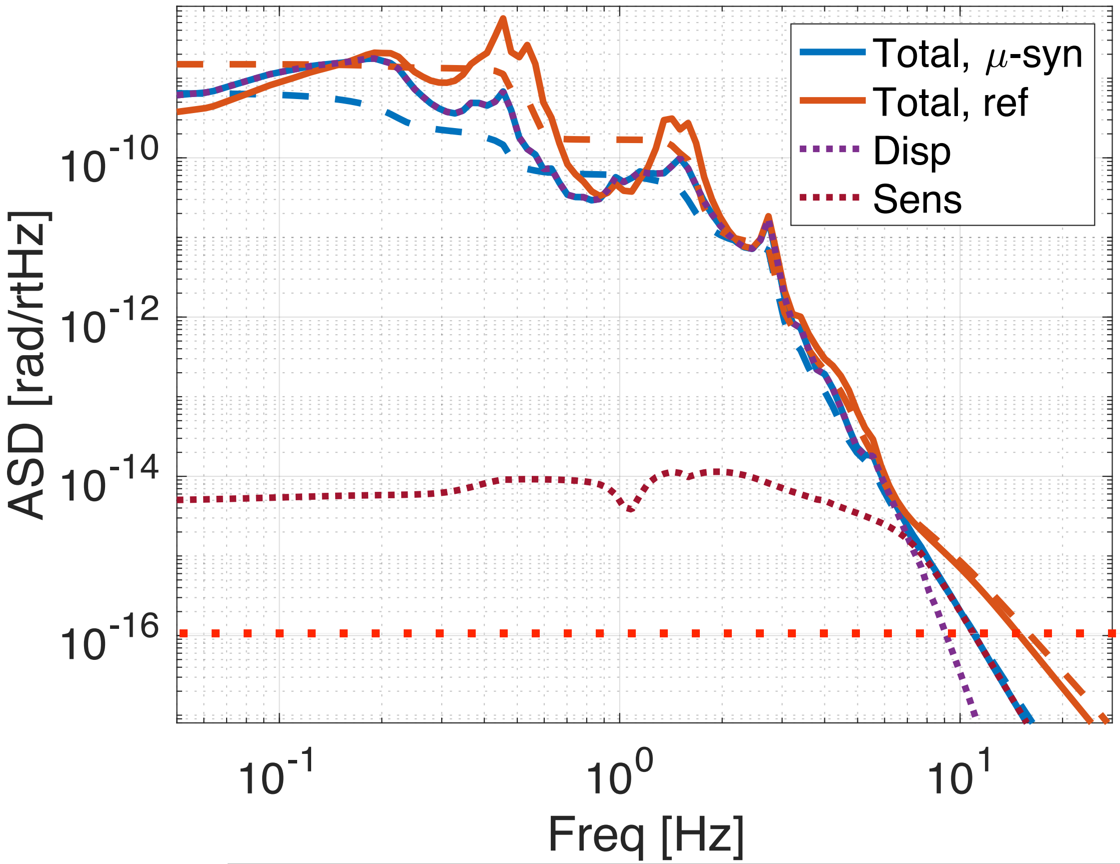
# HARD MODE: P_arm=0.75 MW



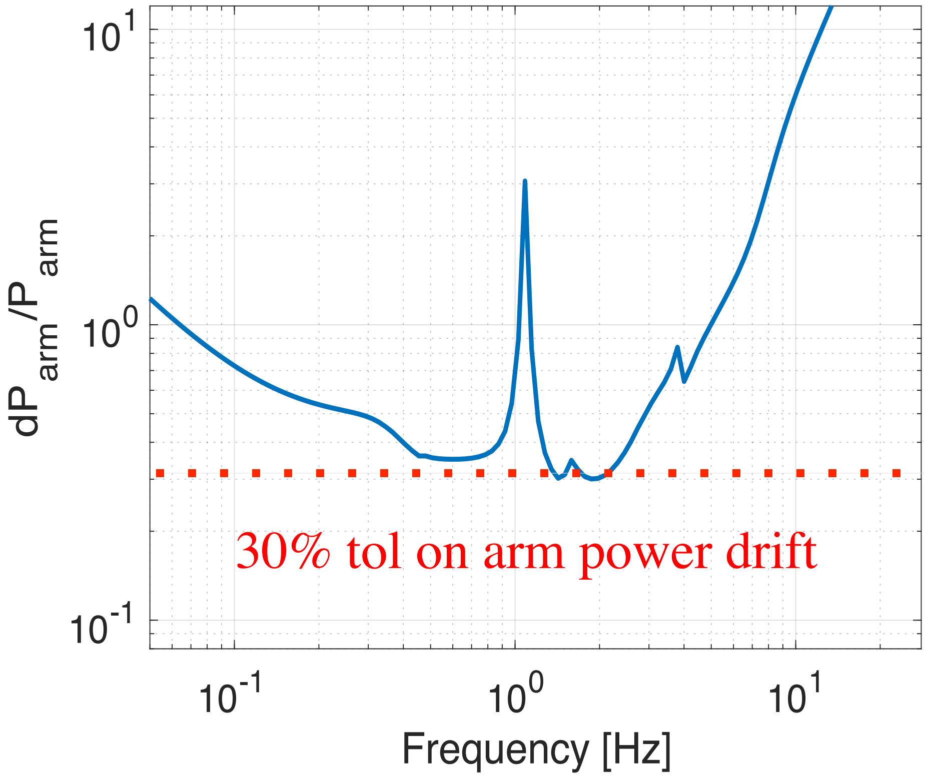



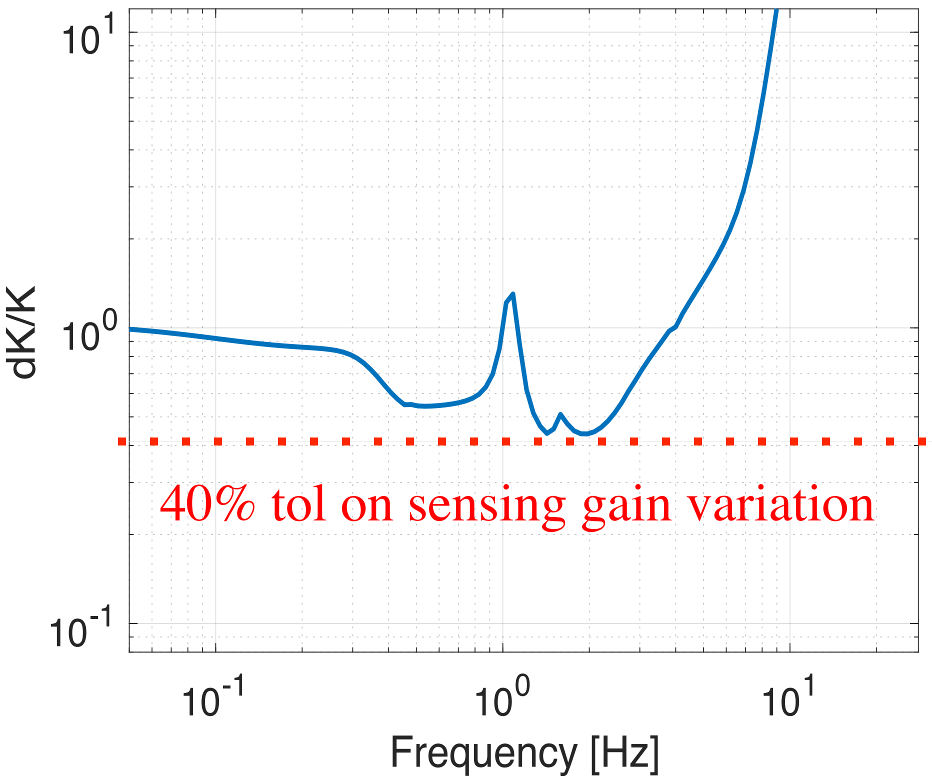
# HARD MODE: NOISE




$$\delta x_{\rm asc}(10\,{\rm Hz})$$
$$=10^{-19}\frac{\rm m}{\sqrt{\rm Hz}}\times\left(\frac{G_{a2l}}{1\,{\rm mm/rad}}\right)$$
$$<$$
$$\delta x_{\rm aLigo}(10\,{\rm Hz})$$
$$=6\times10^{-19}\,\frac{\rm m}{\sqrt{\rm Hz}}$$

# HARD MODE: ROBUSTNESS



# CONCLUSIONS

- Optimal control via H-inf / mu syntheses.
  - Only need to set the weights.
  - Stability guaranteed; performance optimal.
  - Can handle plant uncertainty.
- Arm alignment with opt. ctrl.
  - Soft mode: combined with digital sub of SS torque.
  - Hard mode: lowered both rms & sensing noise.
  - Robust against perturbations.

# EXTRA SLIDES

# MATLAB COMMANDS

- H-inf synthesis:

(https://www.mathworks.com/help/robust/ref/hinfsyn.html?searchHighlight=hinfsyn&s_tid=doc_srchtitle)

K = hinfsyn(G)

- Mu synthesis:

(https://www.mathworks.com/help/robust/ref/dksyn.html?searchHighlight=dksyn&s_tid=doc_srchtitle)

K = dksyn(G)

- Where: $K$ = optimal controller, $G$ = weight-augmented plant,

$$G = \begin{bmatrix} W_1, & W_1 P \\ 0, & W_3 P \\ -I, & -P \end{bmatrix}, \text{ with } P = \text{the original plant.}$$

# CORRECTING FOR RADIATION PRESURE EFFECTS

- SS torque modified plant:

$$P_{\rm ss}(P_{\rm arm}) = P_0/\left[1 + P_0 R\left(P_{\rm arm}\right)\right].$$

- Want a const CL torque to angle TF:

$$P_{\rm ss}(P_{\rm arm})/\left[1 + K P_{\rm ss}(P_{\rm arm})\right] = P_0/(1 + K_0 P_0),$$

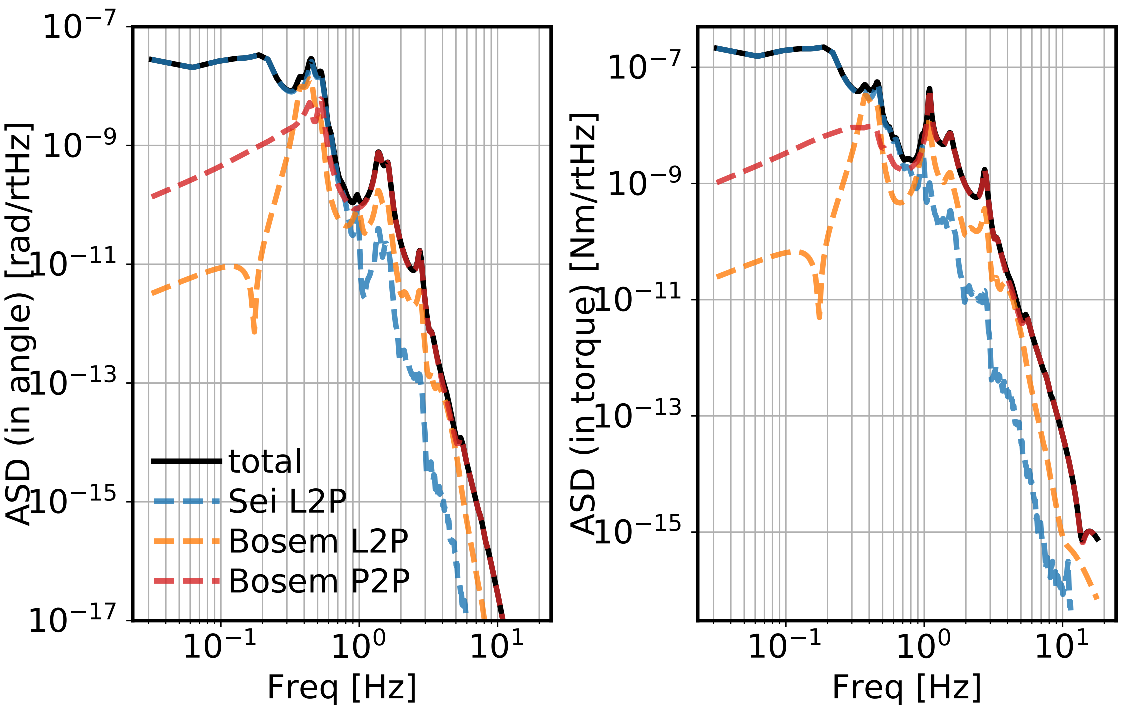
$$\text{or } S_{\rm ss}(P_{\rm arm}) = 1/\left[1 + K P_{\rm ss}(P_{\rm arm})\right]$$
$$= \left[1 + P_0 R(P_{\rm arm})\right] S_0$$

- Since $W_1(P_{\rm arm}) S_{\rm ss}(P_{\rm arm}) \simeq \gamma \simeq W_1(0) S_0$
- We have:

$$\boxed{W_1(P_{\rm arm}) = \frac{W_1(0)}{1 + P_0 R(P_{\rm arm})}}$$

# NOISE INPUT

Disp noise input:




Sensing noise:
Flat 5e-15 rad/rtHz for
Both soft and hard modes

# MODEL FOR dP-d\theta

- Radiation torque: $\tau = \frac{2P_{\rm arm}}{c} y$
- => $\frac{d\tau}{d\theta}(f) = \frac{2P_{\rm arm}}{c}\left[\underbrace{\frac{dy}{d\theta}}_{\text{Sidles-Sigg.}} + \underbrace{y({\rm dc})\frac{d{\rm RIN}}{d\theta}(f)}_{dP/d\theta}\right]$
- y(dc): DC beam offcentering; or suspension point offcentering (cS pitch only)
- With DC spot offset: $\left.\frac{d{\rm RIN}}{d(\theta/1{\rm rad})}\right|_{\rm cS}(f) \simeq \frac{2.6\times 10^5}{1+{\rm i}(f/f_+)}\left[\frac{y_{\rm cS}({\rm dc})}{0.1\,{\rm mm}}\right]$

  $\simeq \frac{5.0\times 10^4}{1+{\rm i}(f/f_+)}\left[\frac{\theta_{\rm cS}({\rm dc})}{10\,{\rm nrad}}\right]$

  (Note: dRIN can be either positive or negative depending on the dc offset)
- => $\left.\frac{d\theta}{d\theta}\right|_{dP/d\theta}(f) = -\frac{2P_{\rm arm}}{c}\frac{d{\rm RIN}}{d\theta}(f)\frac{L2P(f)}{1+G_{\rm 3rd\,ISS}(f)}$