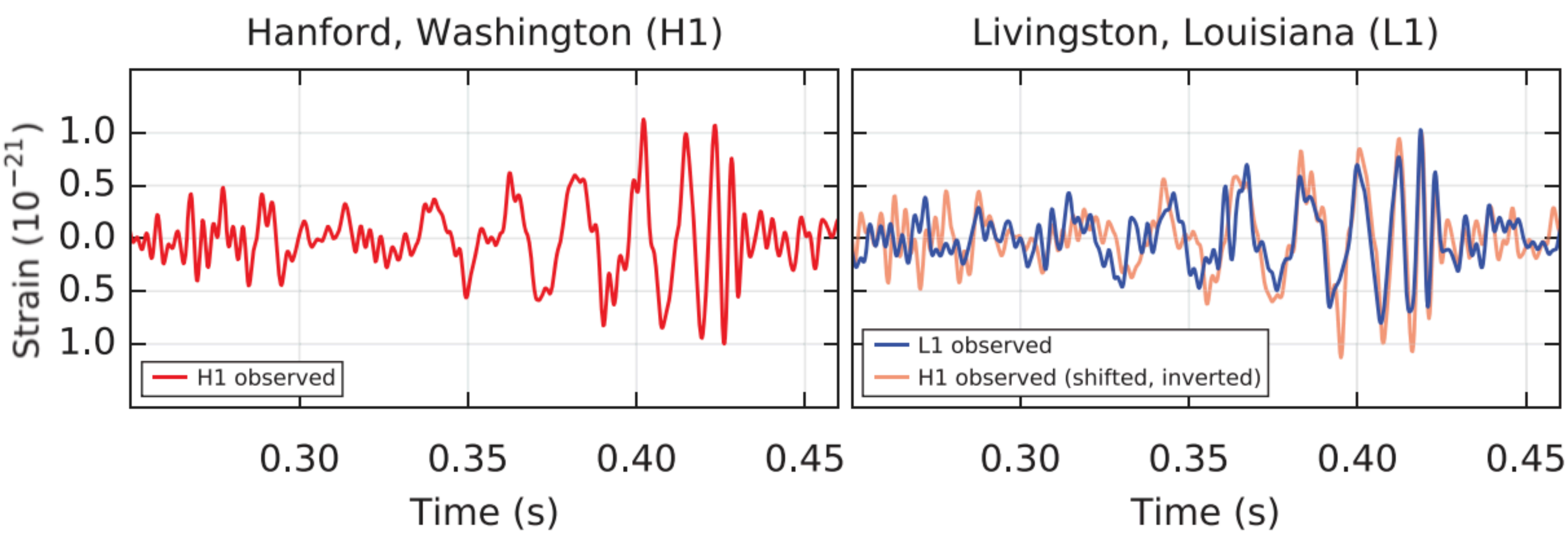
An introduction to CBC Parameter Estimation

Katerina Chatziioannou
Canadian Institute for Theoretical Astrophysics

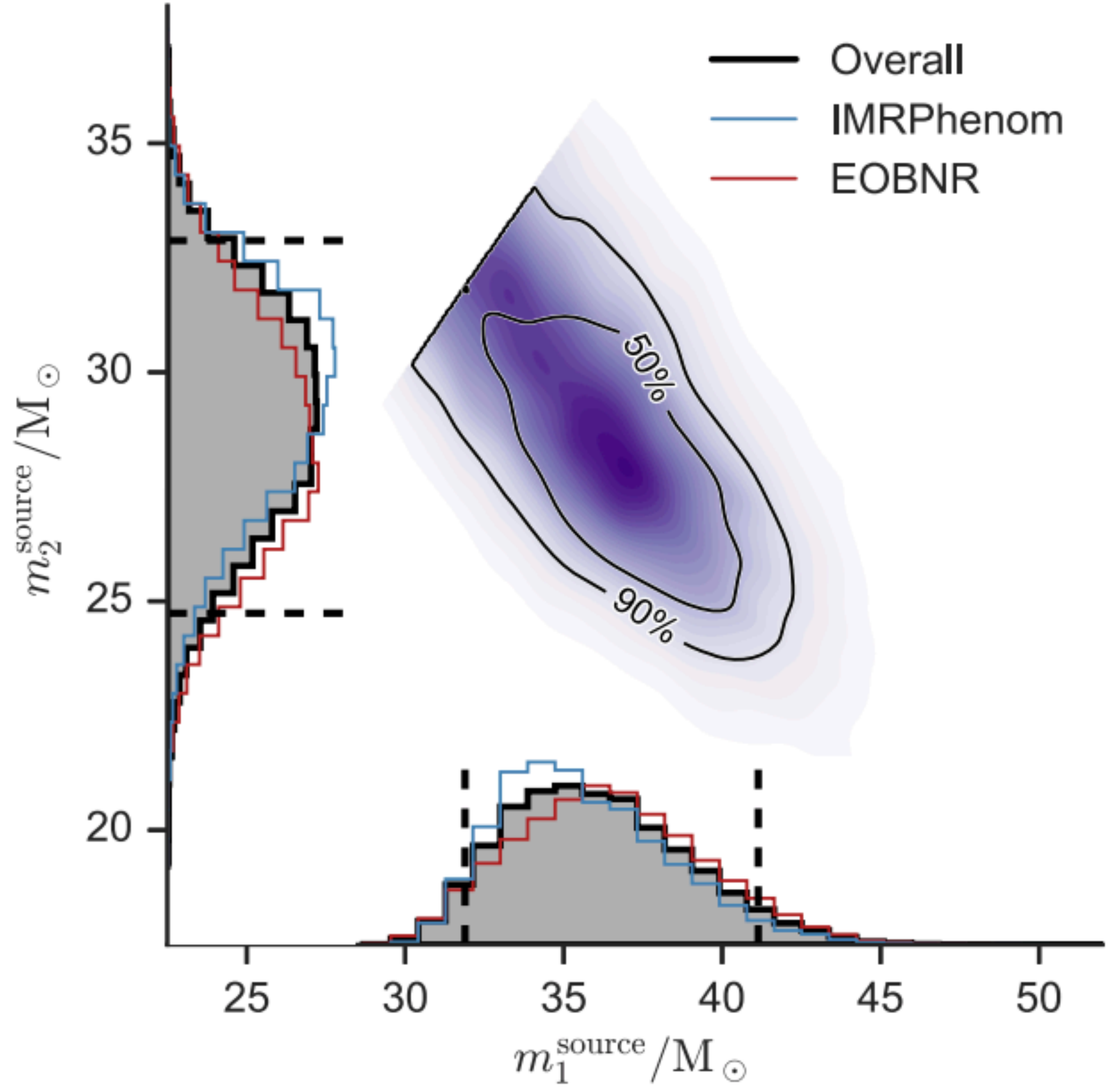
On behalf of the LIGO Scientific & VIRGO Collaborations

Open Data Workshop #1, March 26th
DCC G1800247

From raw(ish) data



LVC (PRL:116, 061102)



LVC (PRL:116, 241102)

Bayes' theorem

Initial Understanding + New Observations = Updated Understanding

Likelihood

Prior

Posterior

$$p(h'|d) = \frac{p(d|h')\,p(h')}{p(d)}$$

Evidence

Likelihood

$$p(h'|d) = \frac{p(d|h')p(h')}{p(d)}$$

Data

A gravitational wave hits the detector, and the detector records

$$d = R[h] + n$$

available data

detector response to GW

detector noise

A single data point

$$d_1 = R[h_1] + n_1$$

Noise probability $p(n_1)$

Residual probability $p(d_1 - R[h_1'])$

Are they compatible? $p(d_1 - R[h_1']) = p(n_1) = p(d_1|h_1')$

Probability of drawing $d_1 - R[h_1']$ from the noise distribution under the null hypothesis

Many data points

discrete frequency bins

$$d = \{d_1, d_2, ..., d_{N_f}\}$$
$$= \{R[h_1] + n_1, R[h_2] + n_2, ..., R[h_{N_f}] + n_{N_f}\}$$

Joint probability for the noise from all frequency bins

$$p(d|h') = p(d_1 - R[h'_1], d_2 - R[h'_2], ..., d_{N_f} - R[h'_{N_f}]) = p(n_1, n_2, ..., n_{N_f})$$

Two critical assumptions

noise correlation matrix

Gaussian noise

$$p(n_1, n_2, ..., n_{N_f}) \sim e^{-\frac{1}{2} n_i C_{ij}^{-1} n_j}$$

Stationary noise

$$C_{ij} = \frac{1}{2} S_n(f_i) \delta_{ij}$$

detector PSD (no summation)

Likelihood

Probability of obtaining data d assuming signal h and that the noise is stationary and gaussian

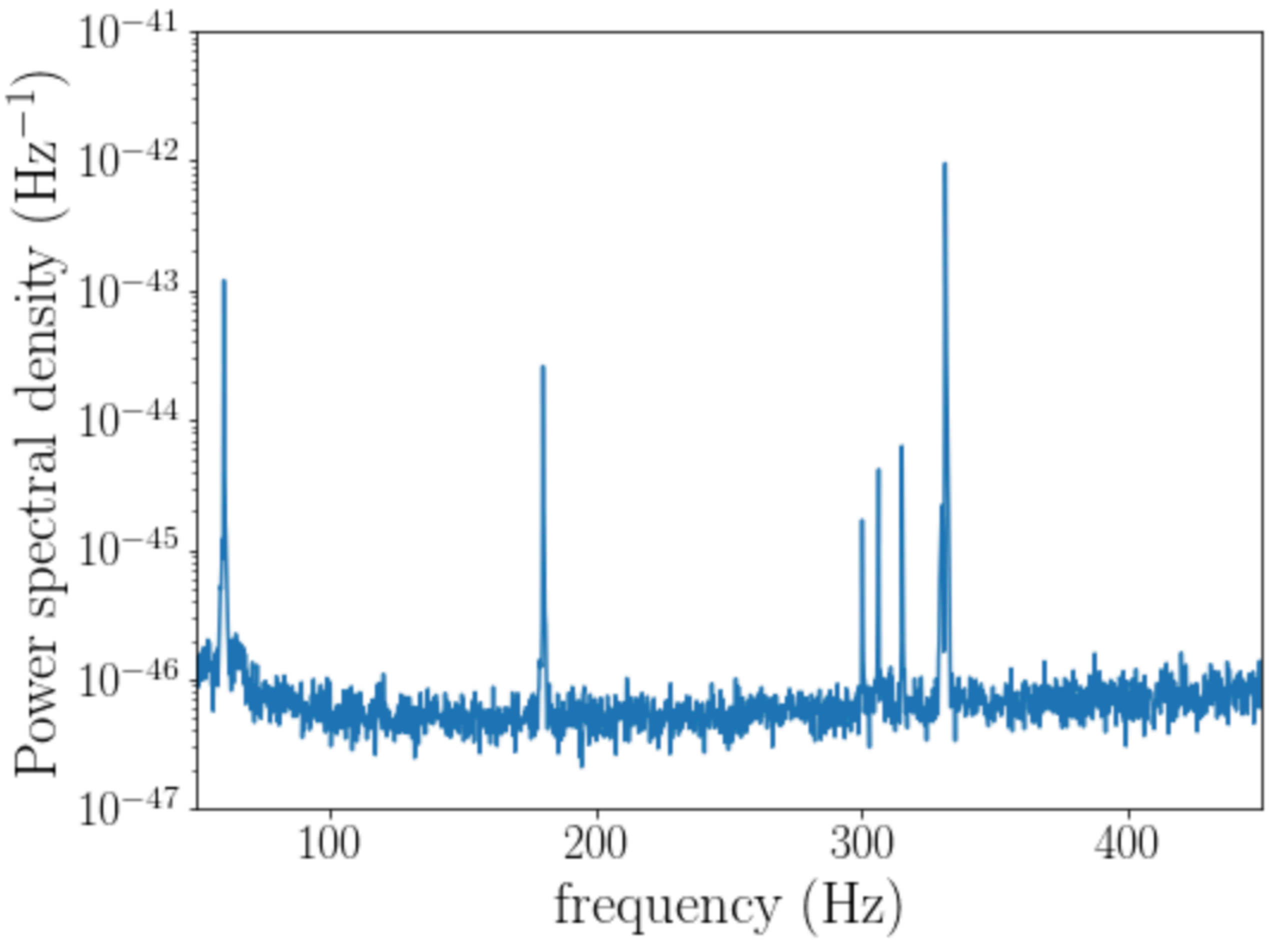
$$p(d|h) \sim e^{-\frac{1}{2}(d - R[h]|d - R[h])}$$

where the discrete frequency bins are now continuous

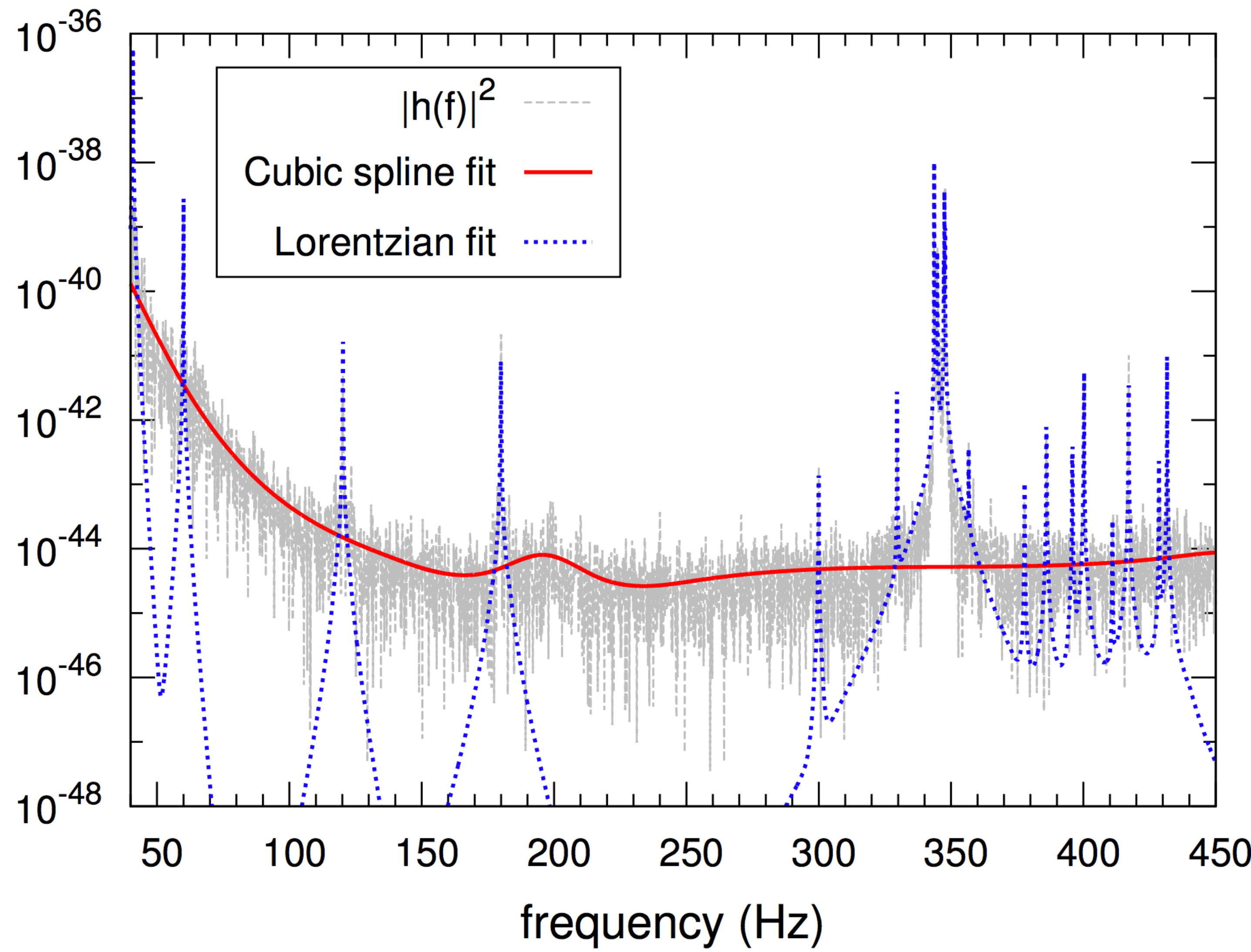
$$(a|b) = 4 \int \frac{a(f) b^*(f)}{S_n(f)} df$$

PSD Calculation

Off-source (average)



On-source

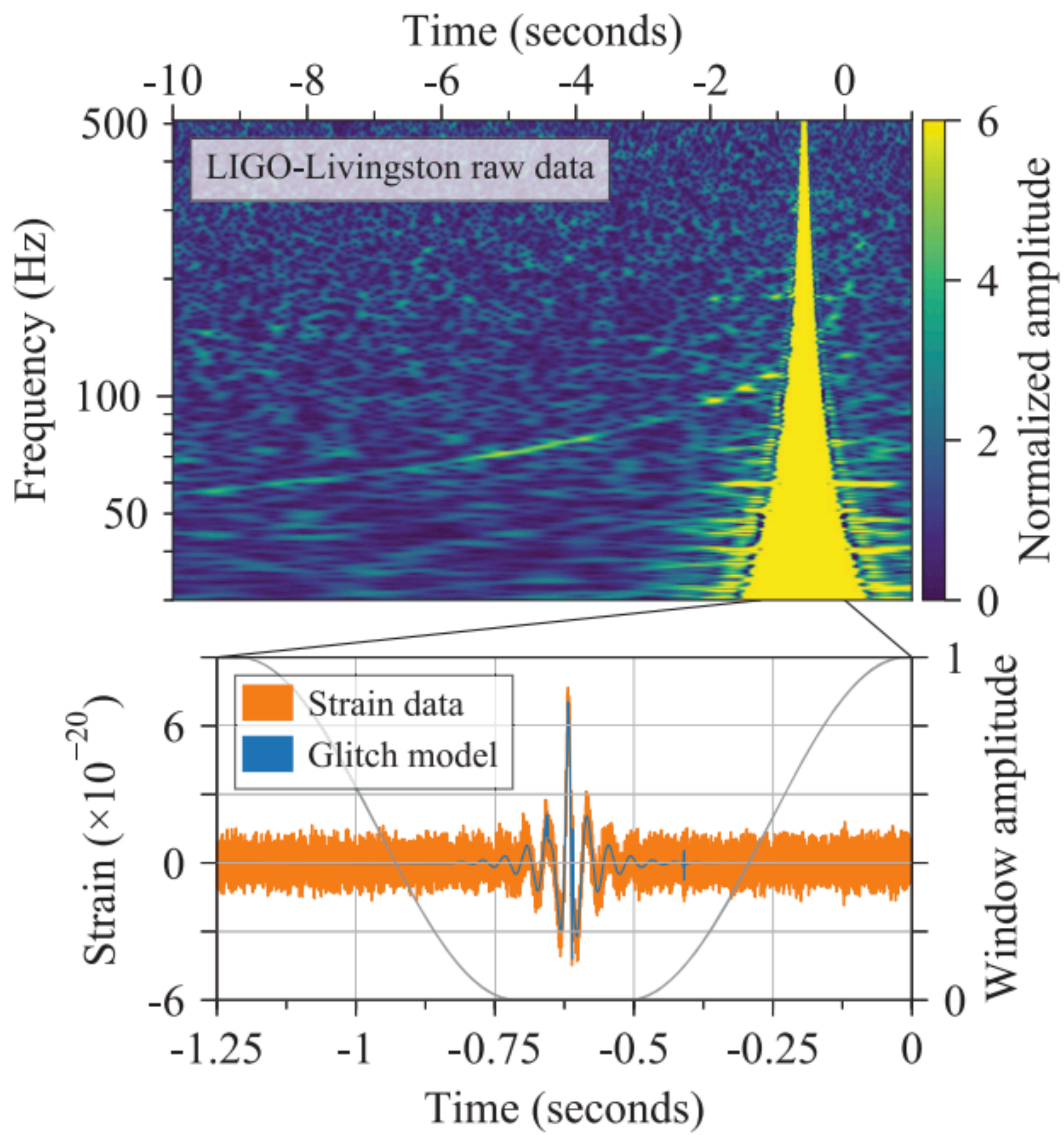


For more details see
Littenberg and Cornish (arxiv:1410.3852)

Gaussianity

$$d = R[h] + n + g$$

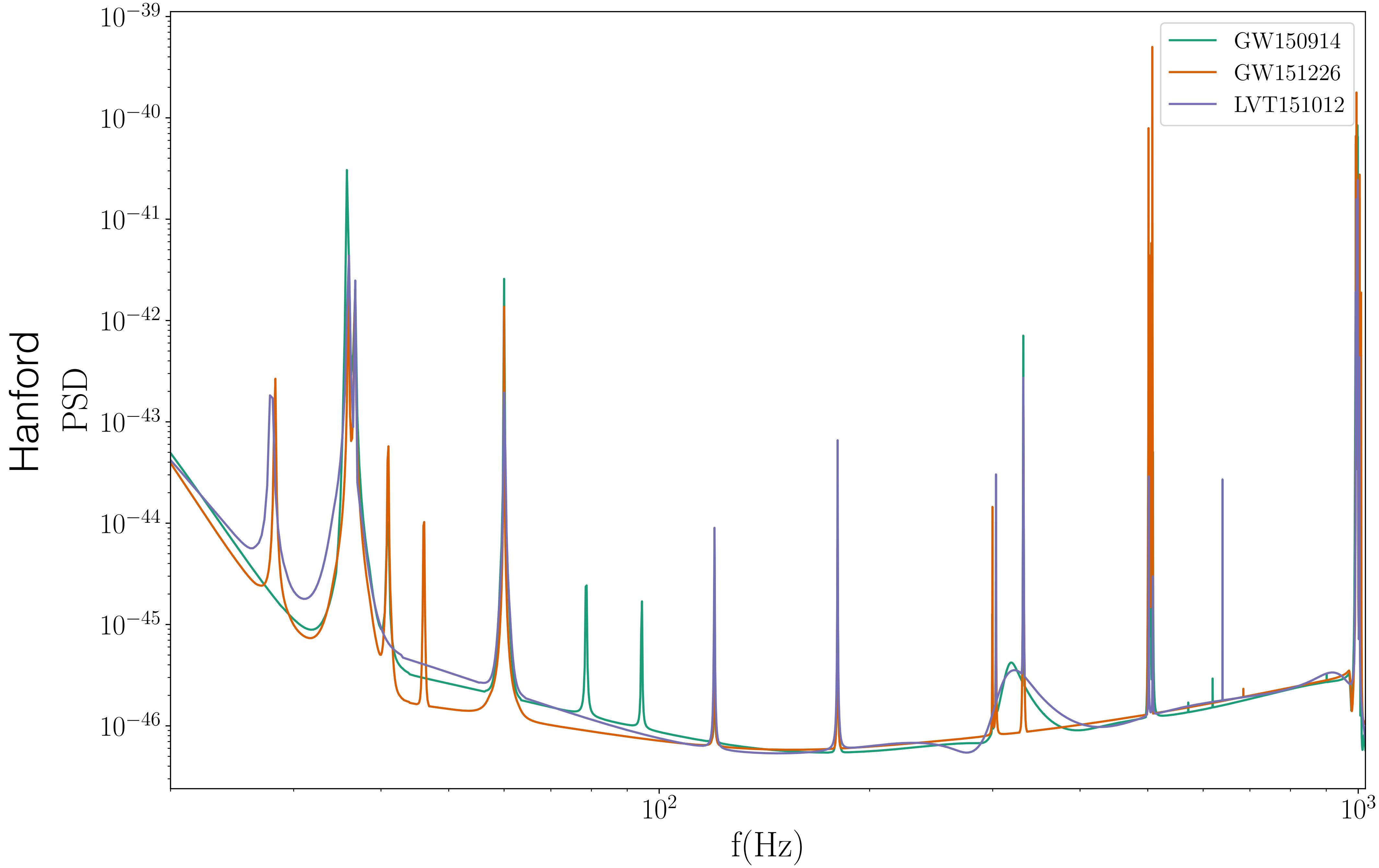
Include glitch
in the likelihood or
remove it from the data



LVC (PRL:119, 161101)

Stationarity

Recompute the PSD for every detector and for every event



For more details see
Littenberg and Cornish (arxiv:1410.3852)

Prior

Prior

$$p(h'|d) = \frac{p(d|h')p(h')}{p(d)}$$

Weak prior

Decompose the signal as a sum of bases functions

$$h' \to \sum^{N} w(\vec{y})$$

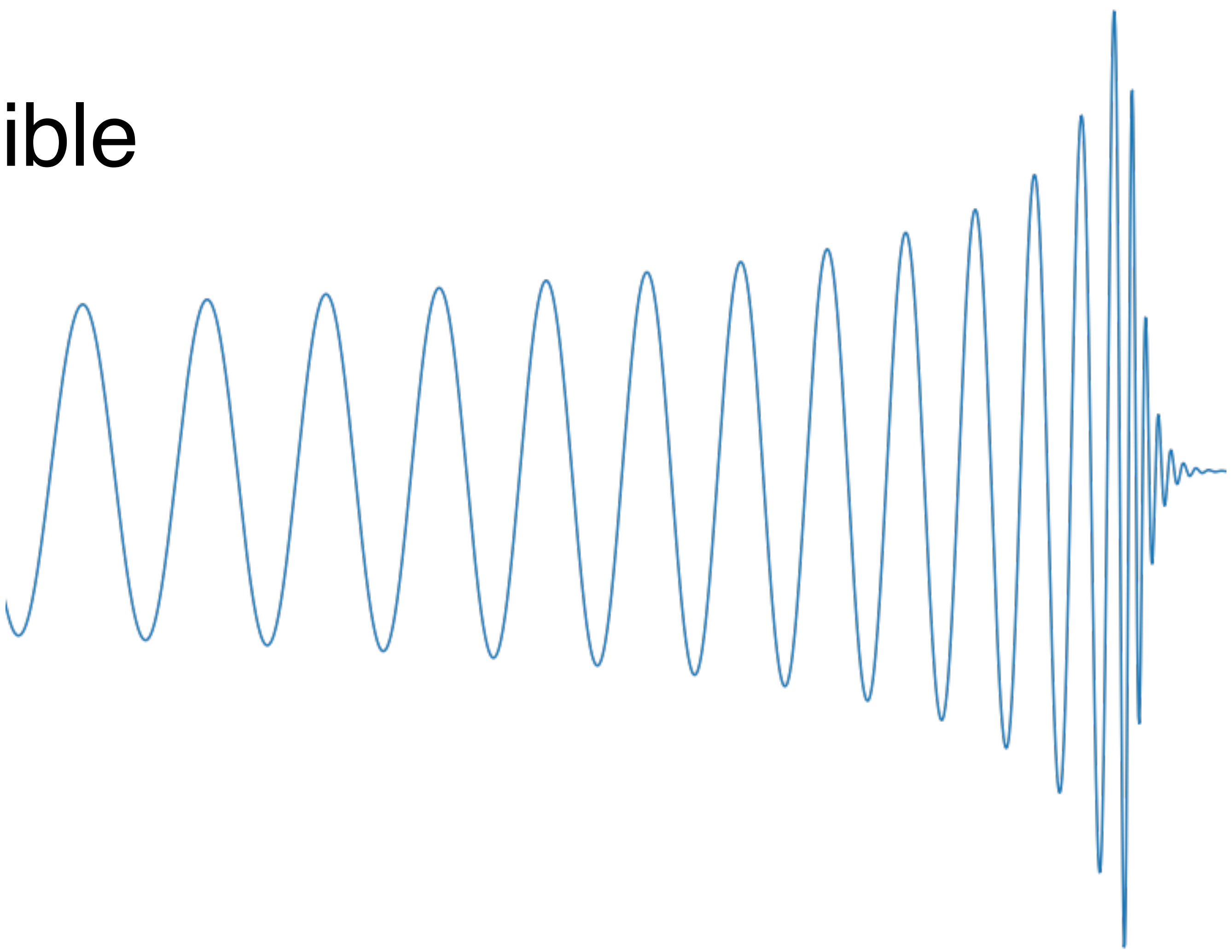
Less sensitive, more flexible

$$p(h') = \delta \left[h' - \sum^{N} w(\vec{y}) \right] p(\vec{y}, N)$$

Strong prior

More sensitive, less flexible

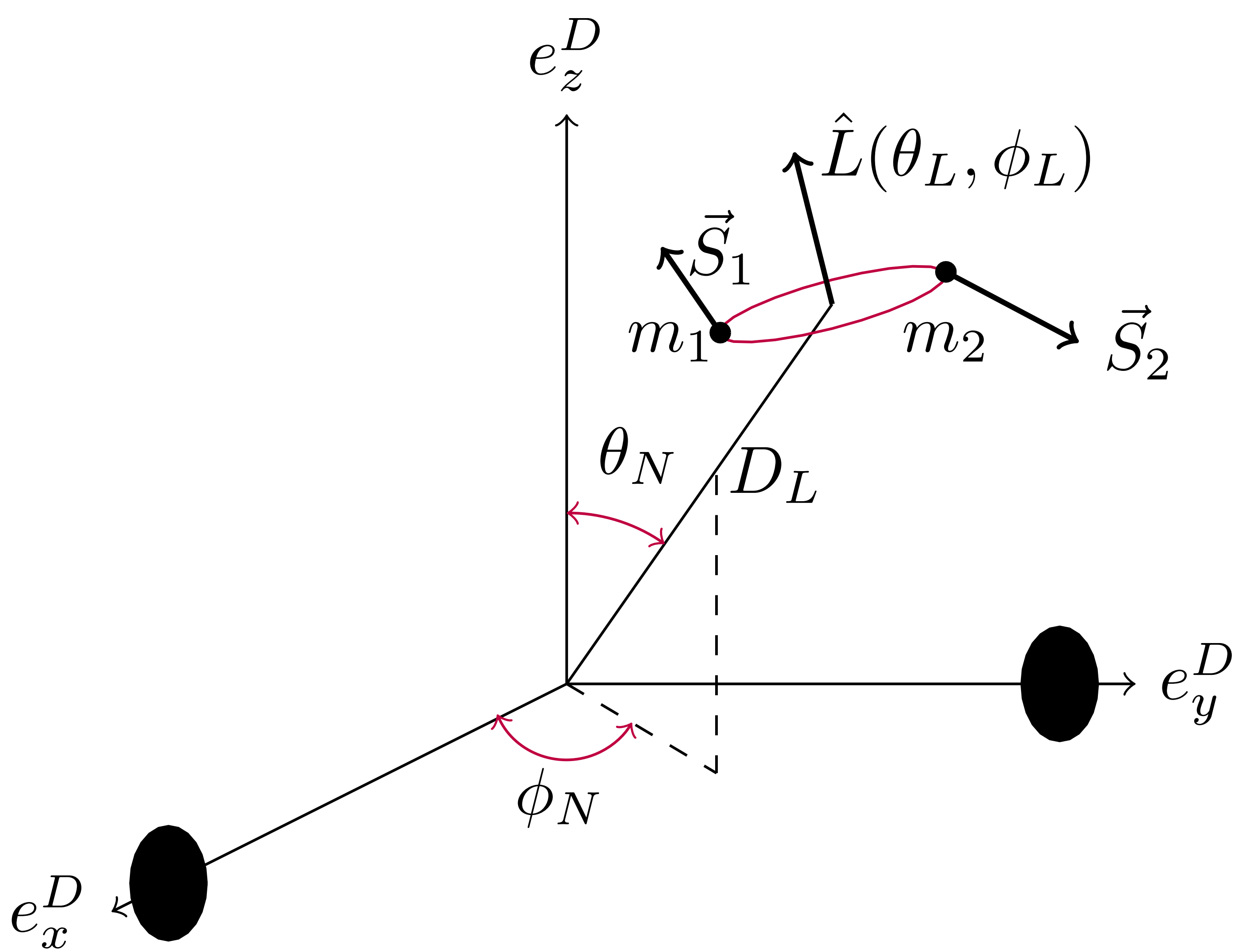
$h' \rightarrow$



Waveform models for the waves emitted from a compact binary coalescence: IMRPhenomPv2, IMRPhenomD, SEOBNRv3, SEOBNRv4…

The various models differ both in the physical effects they assume and the methods they use to describe the waveform

Compact binaries

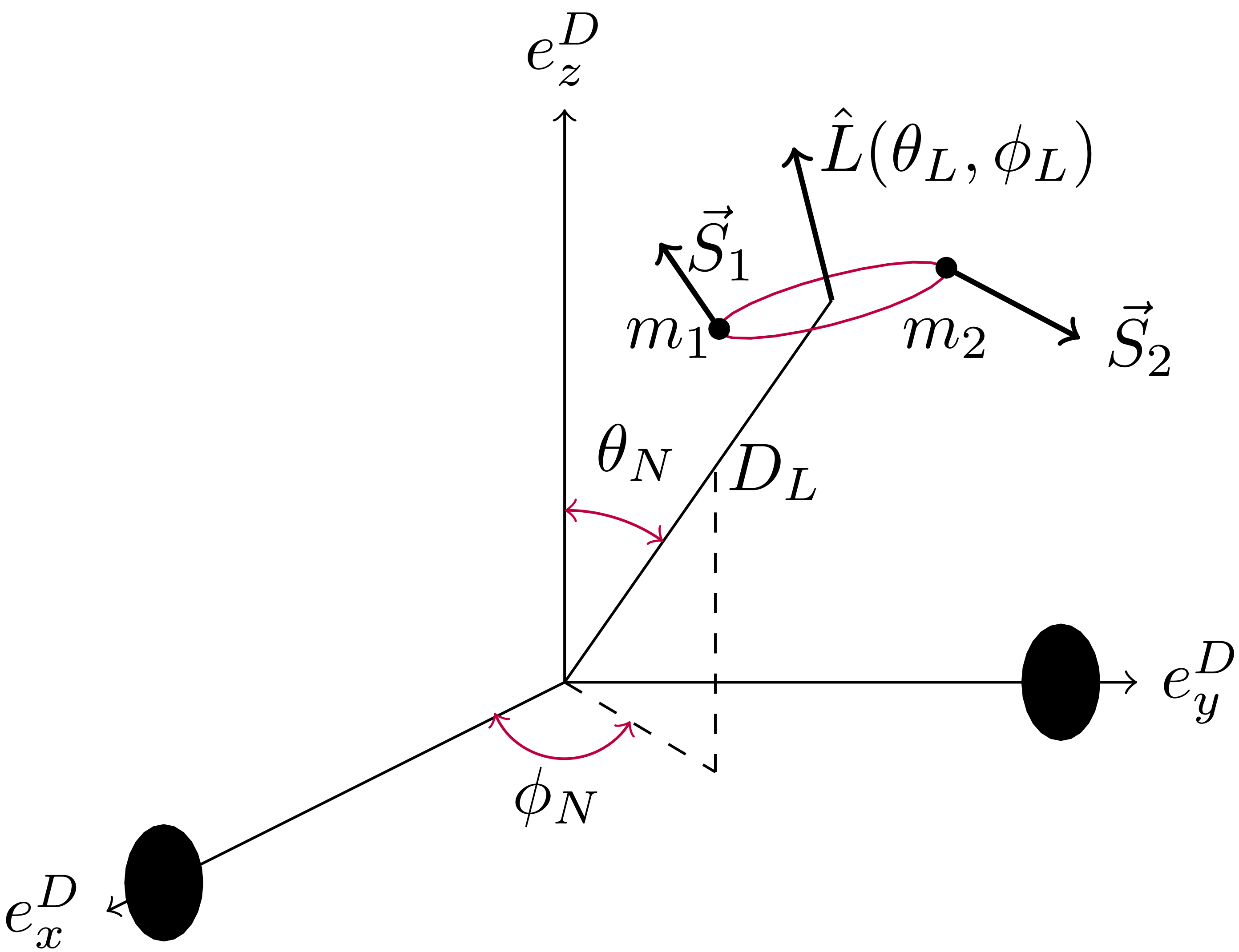


$$p(h') = \delta(h' - h'(\vec{\theta}))p(\vec{\theta})$$

priors on the system parameters

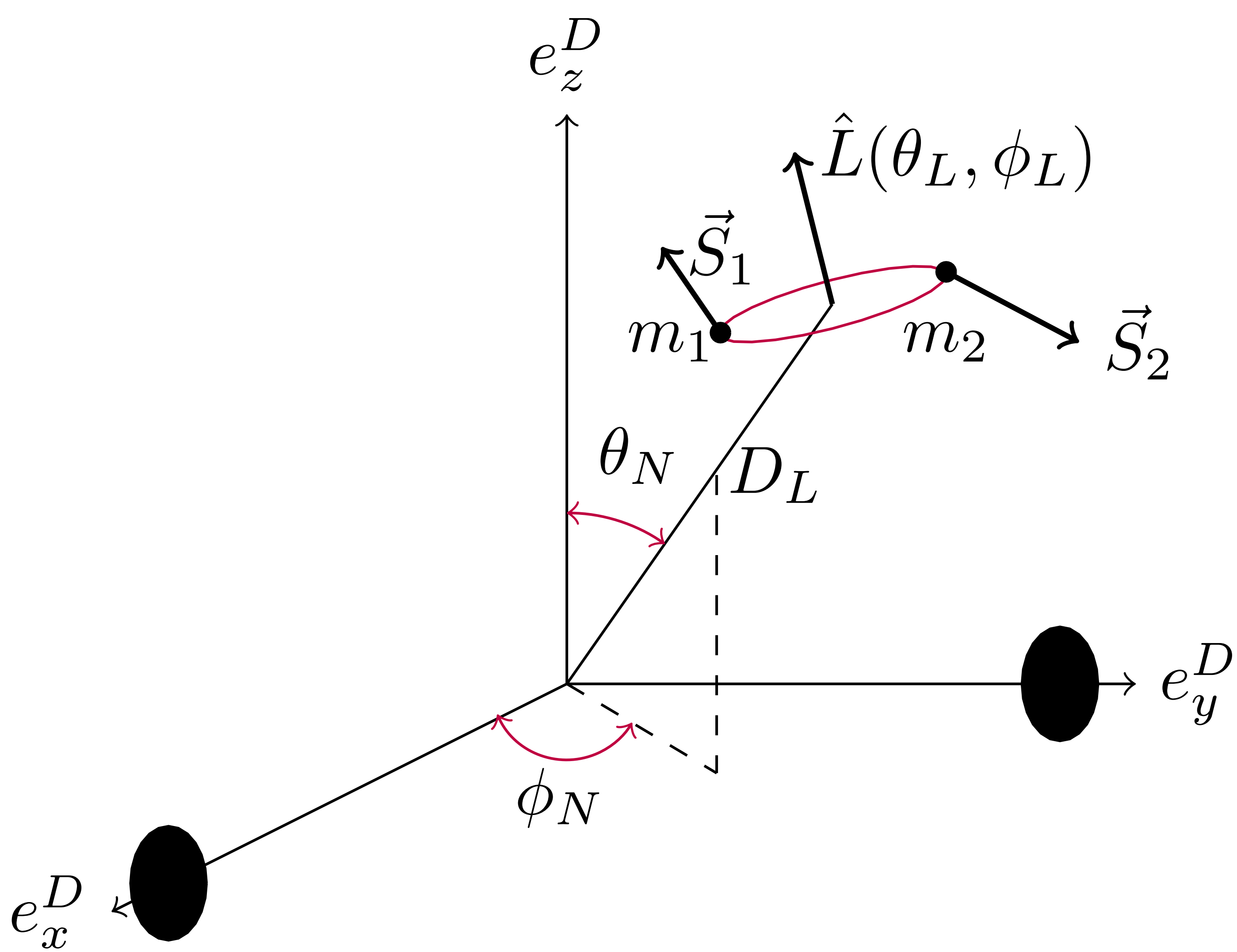
What we know about the parameters of the system *before* obtaining the data

Extrinsic parameters



D_L	Uniform in volume
θ_N ϕ_N	Uniform in the sky
θ_L ϕ_L	Uniform in direction

Intrinsic parameters

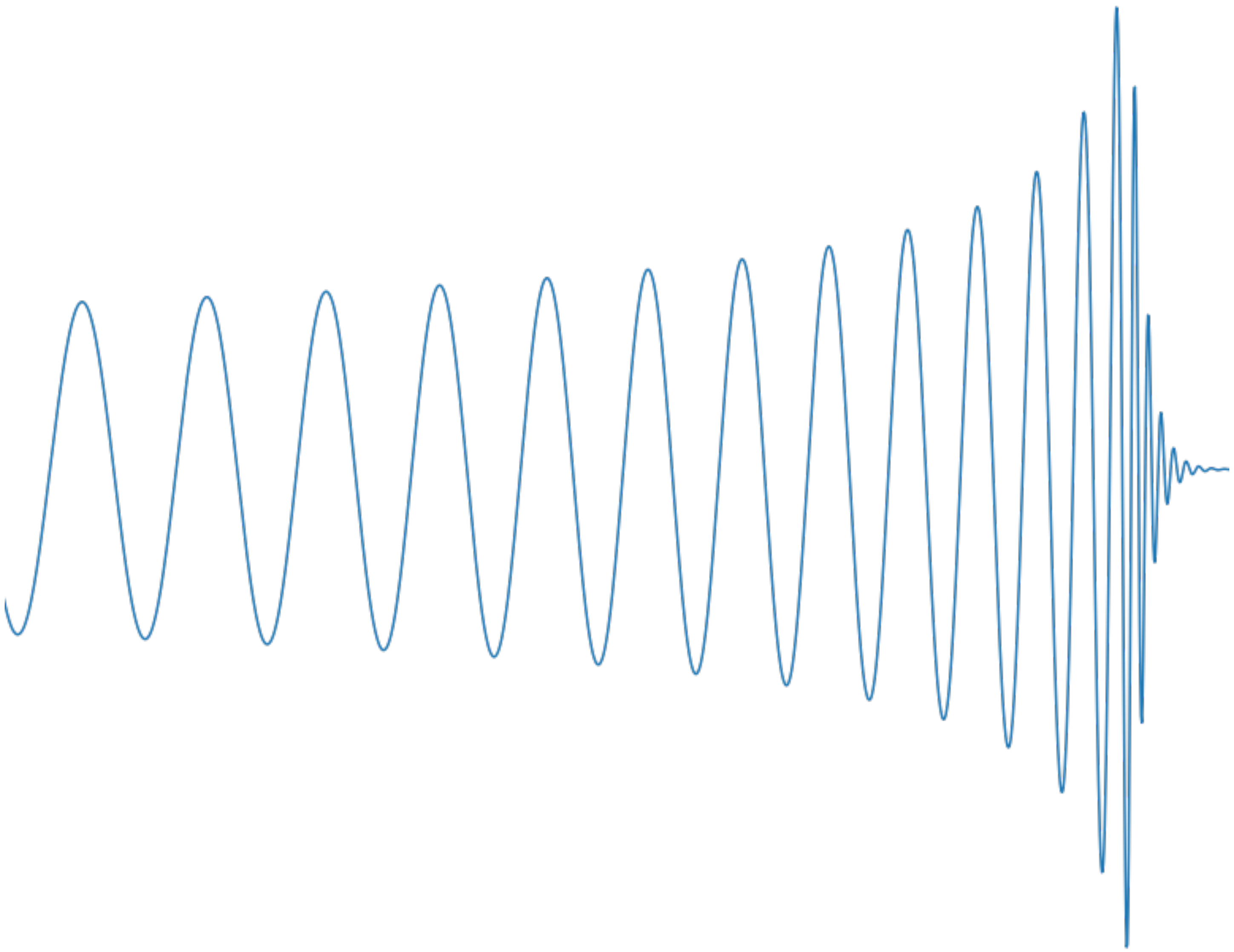


m_1 m_2	Uniform in some range
$\vec{S}_2$ $\vec{S}_1$	Uniform in direction and magnitude in $[0, m_i^2]$
λ_1 λ_2	Uniform in (0,5000)

Highly non unique and potentially influential choices

Calibration

$h' \rightarrow$



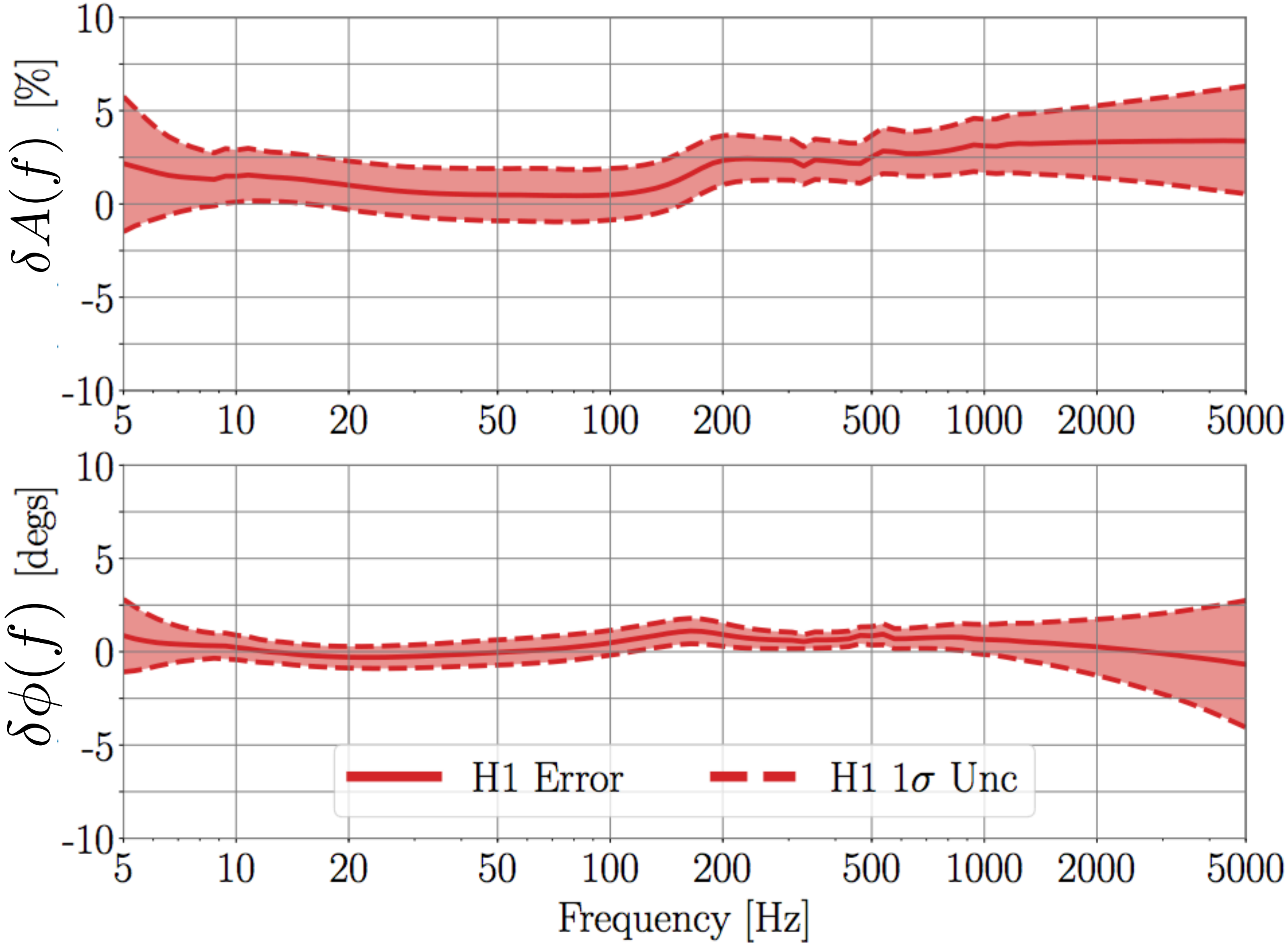
$+\delta h'$

$$h' \rightarrow h'(1 + \delta A)e^{i\delta\phi}$$

Marginalize over
an amplitude and
a phase uncertainty

Calibration

$$h' \rightarrow h'(1 + \delta A)e^{i\delta\phi}$$



$$\delta A(f) = p_s(f; \{f_i, \delta A_i\})$$

$$\delta\phi(f) = p_s(f; \{f_i, \delta\phi_i\})$$

Interpolate with cubic splines and marginalize over the calibration error

Farr+ LIGO Document T1400682-v1

Cahillane+ (PRD:96, 102001)

For more details see Alex Urban's talk

Evidence

$$p(h'|d) = \frac{p(d|h')p(h')}{p(d)}$$

Evidence

Evidence

Normalization factor for parameter estimation

Important for model selection

$$p(h'|d, M) = \frac{p(d|h', M)p(h'|M)}{p(d|M)}$$

M: any overall assumption or model (e.g. the signal is a GW, the BBH is spin-precessing, the binary components are NSs)

Odds ratio

Compare competing models, for example
'GW170817 was a BNS' vs 'GW170817 was a BBH'

$$\mathcal{O}_{ij} = \frac{p(M_i|d)}{p(M_j|d)}$$

$$= \frac{p(M_i)p(d|M_i)}{p(M_j)p(d|M_j)}$$

Bayes Factor
Likelihood of the models
Ratio of the evidences

Posterior

Posterior

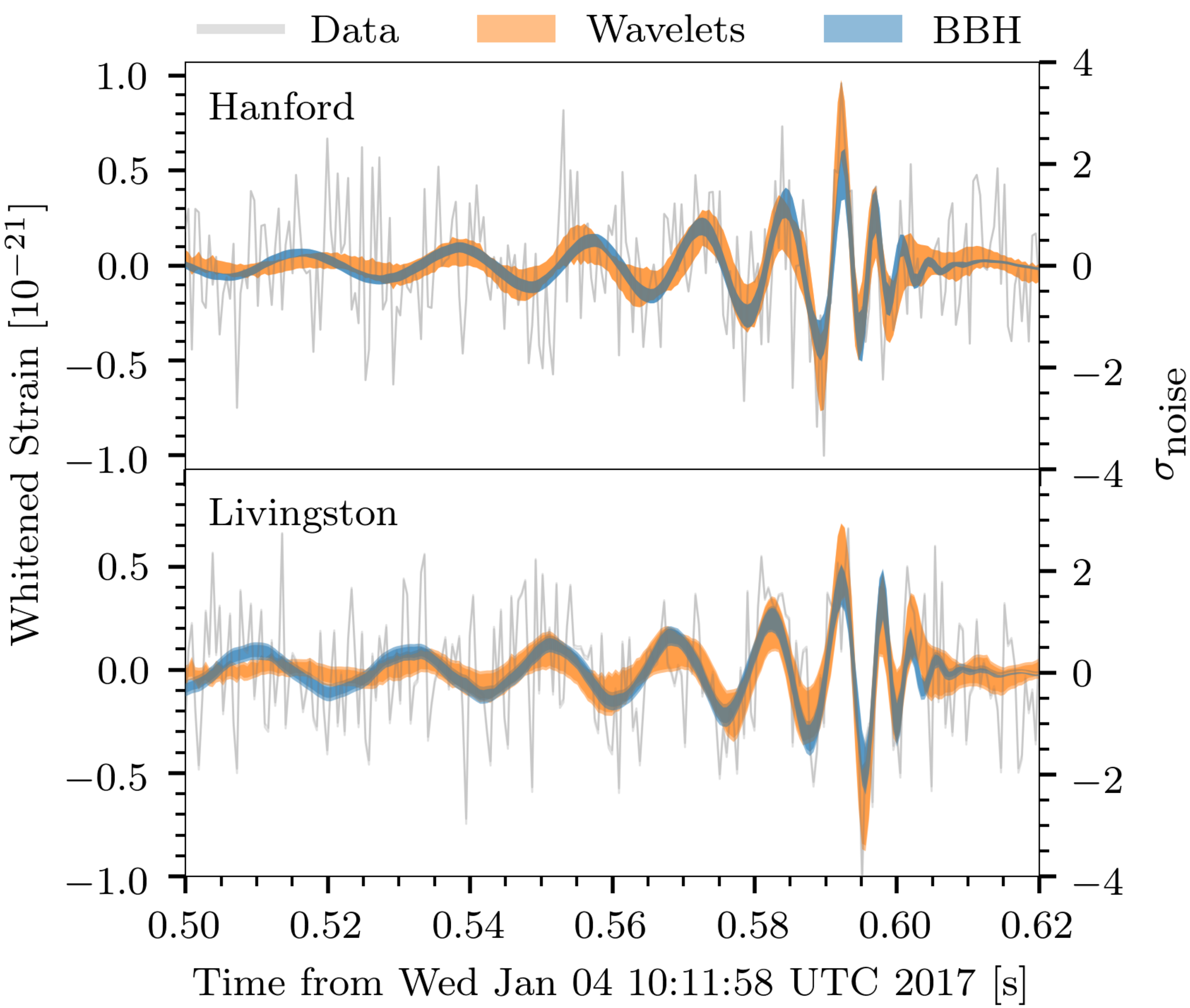
$$p(h'|d) = \frac{p(d|h')p(h')}{p(d)}$$

Bayesian Inference

The posterior $p(h'(\vec{\theta})|d)$ gives the probability density that a model $h'(\vec{\theta})$ describes the data.

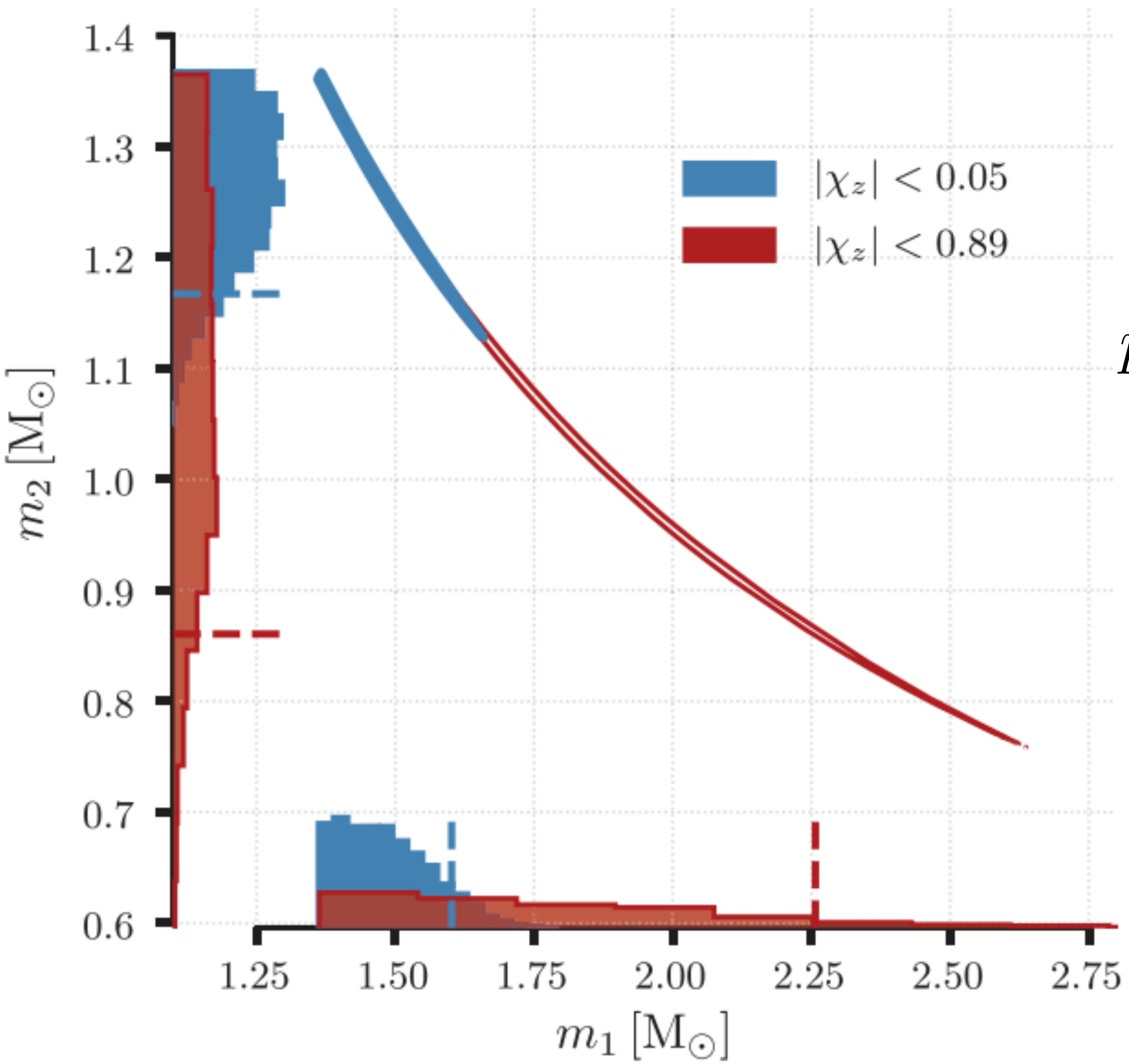
It is calculated with a likelihood and a prior and it is valid under the assumptions that were used when computing these two

Waveform posterior



LVC (PRL:118, 221101)

Marginalized posterior

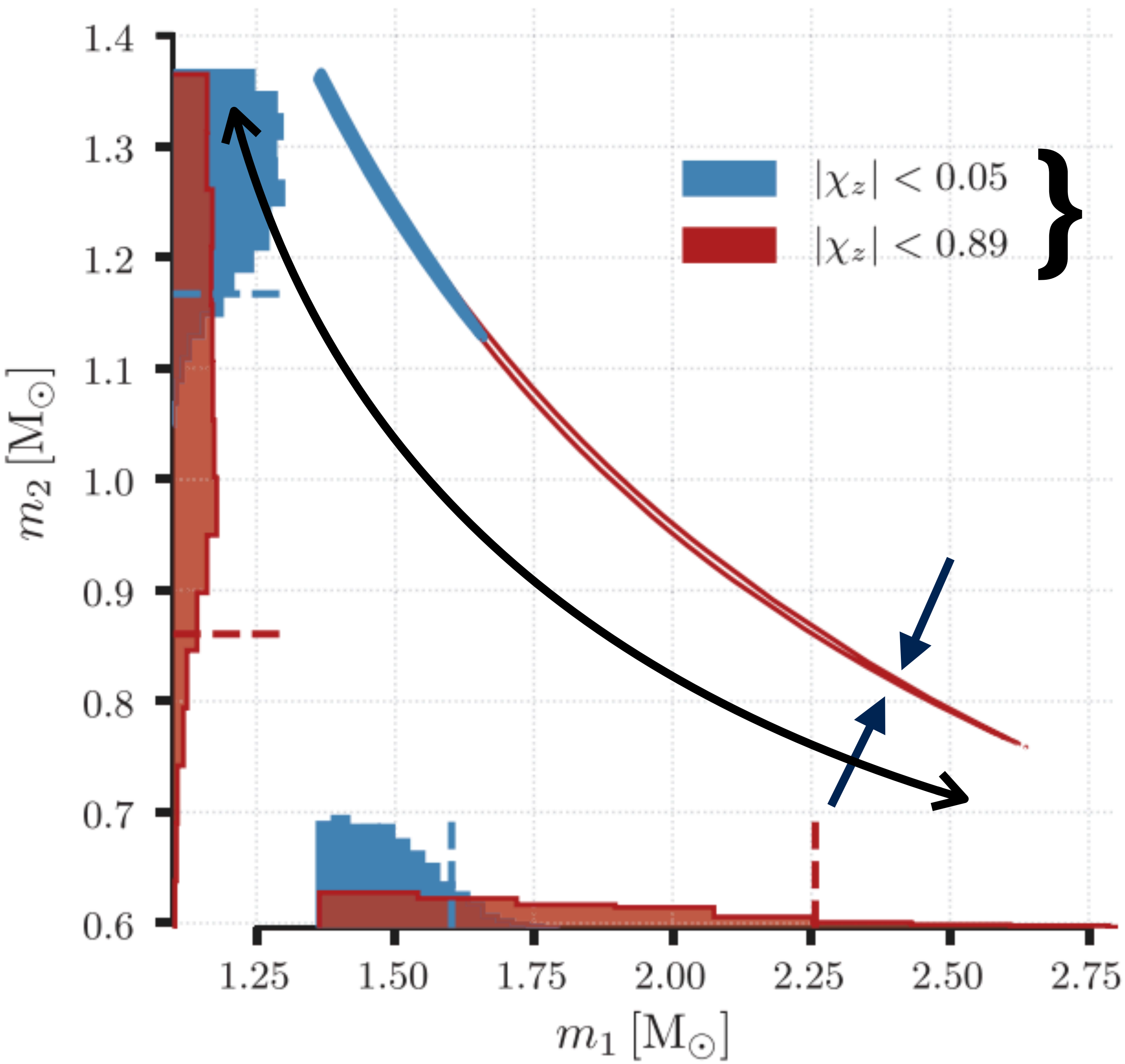


$$p(m_1, m_2|d) = \int p(\vec{\theta}, m_1, m_2|d)d\vec{\theta}$$

Integrate over all 'other' parameters

LVC (PRL:119, 161101)

Inference from the posterior



The chirp mass is measured very well

The mass ratio is measured less well

The mass ratio is correlated with the spin

LVC (PRL:119, 161101)

The likelihood
is the noise model

Prior choices can
influence results

Posterior

$$p(h'|d) = \frac{p(d|h')\,p(h')}{p(d)}$$

The evidence is unimportant
for parameter estimation
(but not model selection!)

LSC LIGO Scientific Collaboration