Non-Linear Angular Noise Coupling into Differential Arm Length

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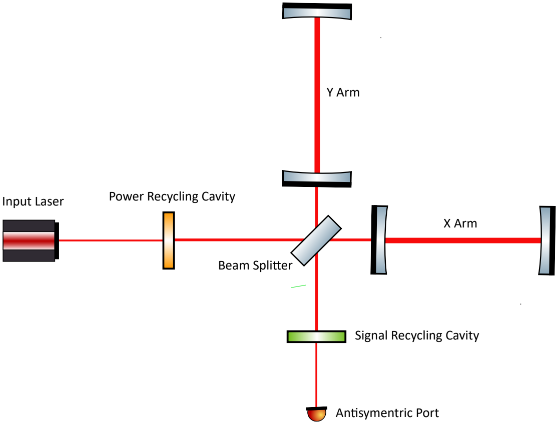
Objectives

I will talk about:

- The angular noise that couples differential arm length (DARM)
- A Fabry-Perot cavity misalignment model
- Measuring test mass angular spectra
- Results of angular misalignment coupling

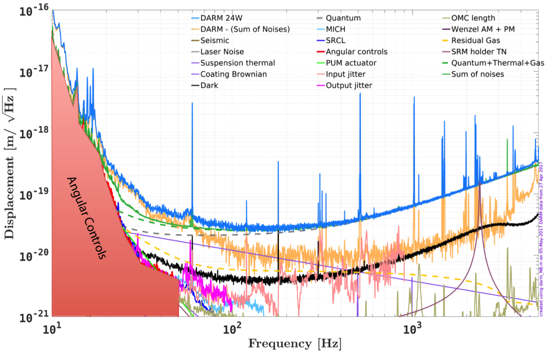
Advanced LIGO Simplified Optical Layout

- DARM is gravitational wave output signal
- Angular noises couple to DARM and reduce sensitivity



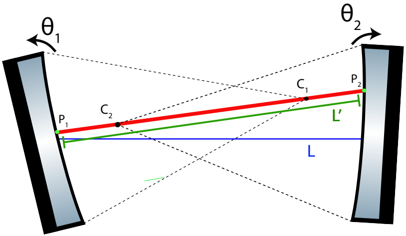
Angular Sensing and Control Noise

- Angular sensing and control noise is a major contributor to DARM below 15 Hz as shown below [1].
- Low frequency sensitivity is important for binary neutron star mergers.
- This noise budget contains linear and non-linear couplings. I want to know non-linear contribution.



What is Angle to Length Coupling?

- Angular misalignments cause a change in length of the Fabry-Perot cavity.
- The beam is constrained by the two centers of curvature.
- This change in cavity length couples to DARM.



Change in Cavity Length

The exact solution isn't simple:

$$\Delta L = \left(L\sqrt{\frac{(R_1 \sin(\theta_1) - R_2 \sin(\theta_2))^2}{(-L + R_1 \cos(\theta_1) + R_2 \cos(\theta_2))^2} + 1} - R_1 \cos(\theta_1)\sqrt{\frac{(R_1 \sin(\theta_1) - R_2 \sin(\theta_2))^2}{(-L + R_1 \cos(\theta_1) + R_2 \cos(\theta_2))^2} + 1} - R_2 \cos(\theta_2)\sqrt{\frac{(R_1 \sin(\theta_1) - R_2 \sin(\theta_2))^2}{(-L + R_1 \cos(\theta_1) + R_2 \cos(\theta_2))^2} + 1} + R_1 + R_2 \right) - L$$

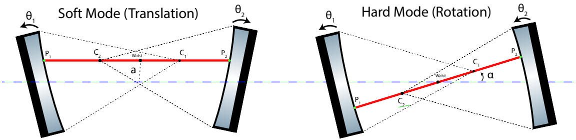
But a Taylor series gives a simpler result.

$$\Delta L = \frac{L}{2(1 - g_1 g_2)} \left(g_2 \theta_1^2 + 2\theta_1 \theta_2 + g_1 \theta_2^2 \right)$$

Where the cavity g factor is: $g_i = 1 - \frac{L}{R_i}$

Hard & Soft Basis

- LIGO uses the hard-soft basis to understand the mirror setup (pictured below).



This leads to a decoupled cavity length change equation:

$$\Delta L = C_h \theta_{hard}^2 + C_s \theta_{soft}^2$$

Two Cavity Hard & Soft Basis

We can describe the two cavity with combination of hard-soft modes.

- Differential hard: $\theta_{dh} = \frac{1}{2}\left(\theta_{xh} - \theta_{yh}\right)$
- Differential soft: $\theta_{ds} = \frac{1}{2}\left(\theta_{xs} - \theta_{ys}\right)$
- Common hard: $\theta_{ch} = \frac{1}{2}\left(\theta_{xh} + \theta_{yh}\right)$
- Common soft: $\theta_{ds} = \frac{1}{2}\left(\theta_{xs} + \theta_{ys}\right)$

Angular Coupling to DARM

The single cavity model can be extended to the dual cavity case.

$$\Delta\text{DARM} = \Delta L_x - \Delta L_y = \alpha\theta_{ch}\theta_{dh} + \beta\theta_{ch}\theta_{ds} + \beta\theta_{cs}\theta_{dh} + \gamma\theta_{cs}\theta_{ds}$$

For Livingston, the constants are equal to:

$$\alpha = -156191 \frac{\text{m}}{\text{rad}^2}$$
$$\beta = 23379.1 \frac{\text{m}}{\text{rad}^2}$$
$$\gamma = 3897.58 \frac{\text{m}}{\text{rad}^2}$$

Discussion of Mirror Static Offset

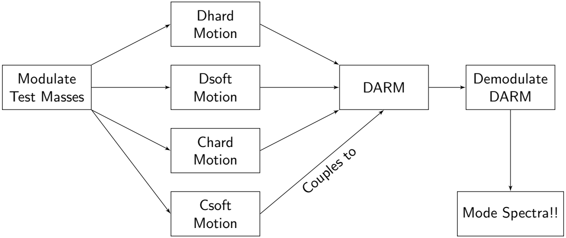
- We wish to estimate the non-linear angular coupling to DARM
- Each angle has a time series like:

$$\theta_{tot}(t) = \theta_0 + \theta(t)$$

- Therefore, we have set the static offset for each mode, θ_0, equal to zero through our experimental setup

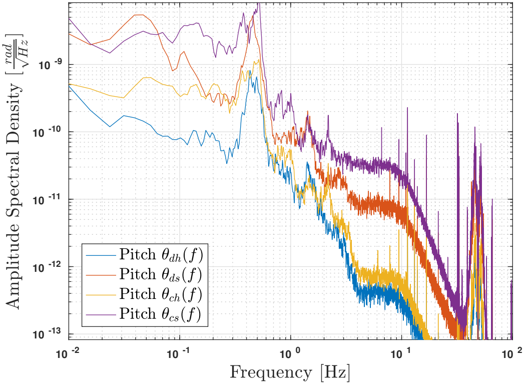
Measuring Spectra of Modes by Dithering

- Dithered (modulated) all test masses at a set frequency
- Demodulated DARM output gets answer proportional to a single modes' spectra (repeated for other modes)
- Dither amplitudes chosen a priori to get single term linear demodulated output. eg: $dmod(DARM) = C_{dh}\theta_{dh}$
- Find mode spectrum from demodulated output



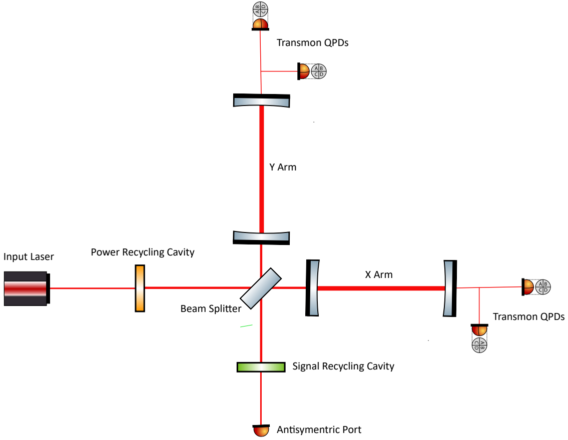
Hard-Soft Mode Spectra

- We dithered at at 48.7 Hz to measure the spectra of all the modes
- Above 3 Hz, the spectra are noise



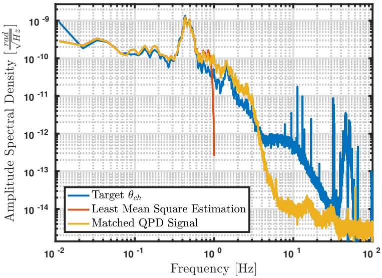
Transmon QPD Layout

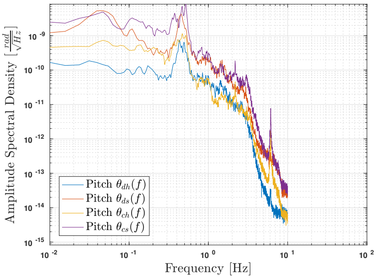
- Transmon QPD are only accurate for high frequency angle measurement



High Frequency Mode Spectra

- Used a least mean square fit to match the dither measurement and Transmon QPD signals at frequencies below 1 Hz [left]
- Transmon QPD signals provide accurate higher frequency spectra up to 10 Hz [right]

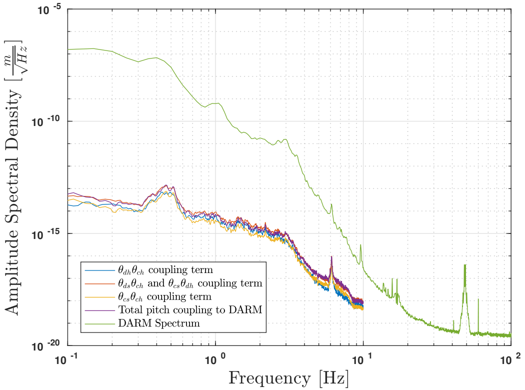




Angular Noise Estimate

$$\Delta \text{DARM} = \alpha\theta_{ch}\theta_{dh} + \beta\theta_{ch}\theta_{ds} + \beta\theta_{cs}\theta_{dh} + \gamma\theta_{cs}\theta_{ds}$$

- Found DARM spectra with the approximation: $\mathcal{F}(\theta_1(t)\cdot\theta_2(t)) \approx \theta_2^{RMS}\theta_1(f) + \theta_1^{RMS}\theta_2(f)$
- Non-linear angle to length coupling is small compared to DARM



Conclusions

Takeaways:

- Angular noise DARM coupling is given by analytic model of test mass angles
- Dithering technique can be used to find non-linear low frequency angular spectra
- Transmon QPD could be used to measure high frequency angular spectra
- From our measurements, non-linear angular noise is not a main contribution to DARM below 10 Hz

Future inquiry:

- Extend cavity length model to all mirrors in interferometer
- Validate test mass angular measurement technique using other sensors and simulations

Acknowledgements

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Full paper can be found here [2]

References

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aLIGO LLO Logbook, 2017.

Brian Seymour, Marie Kasprzack, Arnaud Pele, and Adam Mullavey.
Characterization of Angular Noise Coupling into Differential Arm Length of the LIGO Livingston Detector.
LIGO DCC, 2017.

Extra slide: Constants referenced in paper

$$C_s = \frac{L(g_1 r^2 + 2r + g_2)}{2(1 - g_1 g_2)} = 1810 \frac{m}{rad^2}$$

$$C_h = \frac{L(g_2 r^2 - 2r + g_1)}{2(1 - g_1 g_2)} = -39884 \frac{m}{rad^2}$$

$$\mu = \frac{2L}{-1 + g_1 g_2}$$

$$\alpha = \mu(-g_1 r^2 - g_2 + 2r) = -156191 \frac{m}{rad^2}$$

$$\beta = \mu(-g_1 r + g_2 r - r^2 + 1) = 23379.1 \frac{m}{rad^2}$$

$$\gamma = \mu(-r\,(g_2 r + 2) - g_1) = 3897.58 \frac{m}{rad^2}$$

$$r = .87$$