Angular Subtraction at LIGO Livingston Observatory Project Proposal

Brian C. Seymour,[1] Marie Kasprzack,[2] Arnaud Pele,[2] and Adam Mullavey[2]
[1] *LIGO SURF Student, University of Virginia*[*]
[2] *LIGO SURF Mentor, LIGO Livingston Observatory*

(Dated: May 15, 2017)

Angular noise is the limiting source of noise at low frequencies for Advanced LIGO. One source of this is when test mass rotations produce a coupling to DARM. This SURF project proposal details the plan for the summer to model and reduce this angle-to-length noise.

I. Introduction

In 1916 gravitational waves were predicted by Albert Einstein as a byproduct of his general theory of relativity which describes the relation between spacetime and matter. Just as accelerating charges emit light in classical electrodynamics, gravitational waves are emitted by accelerating masses. Gravitational waves travel at the speed of light in vacuum but, contrary to the latter, they are not obstructed by matter [1].

Gravitational waves will lead to new physics and astrophysics to examine the universe in complement to the existing information such as electromagnetic radiation and particles. Since gravity is much weaker than the other four fundamental forces, it is challenging to set up an experiment with gravity due to the large scale required to observe it. Thus, research in gravity is done by observing astrophysical signals around us, and gravitational waves will provide a new way to see more direct influences of the effects of gravity [2]. Additionally, predictions of general relativity can be compared to the gravitational wave data to test the validity limits of the theory. Lastly, gravitational wave astronomy will open a myriad of astronomical data, from examining black holes to probing past the cosmic microwave background. The cosmic microwave background may include a gravitational wave background, which would be evidence for inflation.

Laser Interferometer Gravitational-Wave Observatory (LIGO) is a collaboration dedicated to finding gravitational waves. It consists of two main facilities in Livingston, Louisiana and Hanford, Washington. These facilities detect gravitational waves with an optical system based on the interferometer developed by Michelson and Morley. Each observatory consists of a Michelson interferometer with 4 kilometer arms which measure the deformation of space produced by gravitational wave by looking for intensity variations in the interference pattern. It contains Fabry-Perot cavities which allows the light to bounce along the arms about 280 times. It uses light recycling to increase the power of a 25W laser into a 100KW laser in the arms. The laser also travels through a vacuum and dampens out environmental vibrations both actively and passively. The main optics are suspended with four stages of pendulums to passively isolate them from ground motion, and is kept at its operating point by active control through a plethora of feedback and feed-forward techniques.

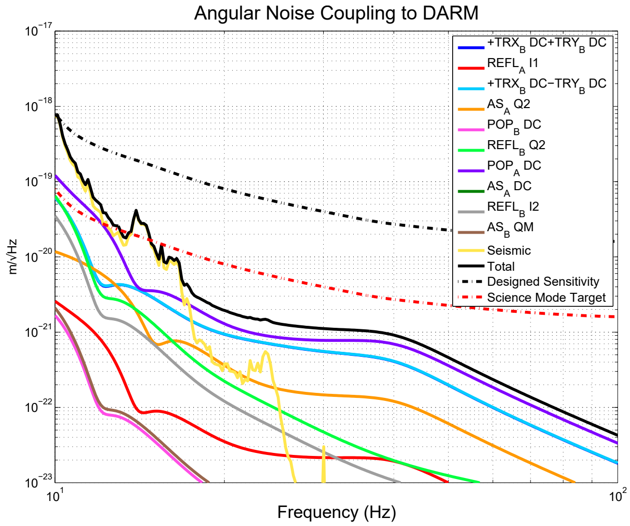

FIG. 1. The solid black line is the angular noise, which should be compared to the dotted black line of the designed sensitivity. Notice the sharp uptick in angular noise 25Hz. This noise is the limiting source of noise for the detector sensitivity at lower frequencies. This figure originated in [5].

After LIGO searched for gravitational waves in the 2000's with no clear detections. Afterwards, the detectors were upgraded via the Advanced LIGO project. With these improvements, in September 2015, LIGO detected the first gravitational wave, and they had a second detection in December 2015 [3, 4]. Both of these detections were due to binary black hole system rotating and then combining.

In order to keep the mirrors stable, LIGO uses an active alignment sensing and control system (ASC) to reduce the mirror's angular motion [5]. Currently, the low frequency sensitivity (10-15 Hz) of the LIGO Livingston instrument is limited by the angular control feedback loops. To reduce the noise in this frequency band, the angular noise to DARM coupling should be reduced below 1/10th of the design sensitivity at that frequency [5].

When aligning a laser on an optical axis with two mirrors, angular misalignments from the fundamental mode cause coupling into higher order modes [6]. The mirror

---

[*] Seymour.BrianC@gmail.com

alignment perturbations can be measured and corrected by measuring the higher order mode content of the beam [6].

In the interferometer cavity, angular rotations can create noise by coupling to cavity length. This is because the beam is not centered the mirror's center of rotation, and this distance is called static beam spot offset. When the mirror rotates slightly, this causes a change in cavity length (eq 1). The equation depends on frequency however, and for the Fourier transform of equation 1, both the beam spot offset and mirror angle depend on frequency. The convolution can be approximated as in equation 2 [5].

$$\Delta l(t) = d_{spot}(t) \times \theta_{mirror}(t) \tag{1}$$

$$\begin{aligned} \Delta L(f) &= D_{spot}(f) * \Theta_{mirror}(f) \\ &\approx d_{spot}^{RMS} \times \Theta_{mirror}(f) + \theta_{mirror}^{RMS} \times D_{spot}(f) \end{aligned} \tag{2}$$

When a beam hits a misaligned mirror, light in a higher order mode can change the amplitude of the fundamental mode. This can be related to beam spot motion in order to find the change in cavity length. The change in cavity length is in equation (5).

$$\begin{aligned} &\Delta A_{00} = \frac{-2i\alpha}{\theta_0}\left(\theta_x A_{01} + \theta_y A_{10}\right) \\ &\text{Where: } \alpha = \frac{w_{beam}}{w_0} \end{aligned} \tag{3}$$

$$\begin{aligned} A_{00}e^{i\phi} &= A_{00} + \Delta A_{00} \simeq A_{00}(1 + i\phi) \\ \phi &= \frac{-2\alpha}{\theta_0 A_{00}}\left(\theta_x A_{01} + \theta_y A_{10}\right) \end{aligned} \tag{4}$$

$$\begin{aligned} &\text{Since, } \mathrm{Re}(\phi) = -kz \\ &z = \frac{\alpha\lambda}{\pi\theta_0}\left(\mathrm{Re}\,\frac{A_{01}}{A_{00}}\theta_x + \mathrm{Re}\,\frac{A_{10}}{A_{00}}\theta_y\right) \end{aligned} \tag{5}$$

For a displacement in a single angular direction x:

$$\begin{aligned} z &= d_{spot}\theta \\ d_{spot} &= w_{beam}\,\mathrm{Re}\left(\frac{A_{01}}{A_{00}}\right) \end{aligned} \tag{6}$$

When the beam is not aligned with the cavity axis, the radiation pressure of the beam exerts a torque on the mirrors. This reduces the pendulum restoring torque by the mirror suspension which can increase misalignment. Equation (7) shows how the torque depends on the mirror angles. As laser power increases in the cavity, eigenvectors of the $\boldsymbol{K_{opt}}$ have a negative restoring spring constant. Thus, control systems designed to mitigate this are used to keep the mirror alignment at its operating point.

$$\vec{\tau}_{opt} = \frac{2P}{c}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = -\boldsymbol{K_{opt}}\begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} \tag{7}$$

II. Objective

Angular control feedback loops limit the sensitivity of the LIGO Livingston Instrument at low frequencies (10-35Hz). We would like to reduce the angular noise in both angle-to-length and angle-to-angle by examining how the cavity axis changes as the test masses rotate.

III. Approach

At first, I will cement techniques from readings, better understand LIGO instrument's systems, and data extraction from the system into Matlab. We will practice simple Fourier transform exercises and apply it to real data from the instrument. We will take and analyze the data of the angular coupling. We will test the results of these filters and iteratively work to improve them. Lastly we will compile the results of the filters, and I will write a report for the DCC and create a poster.

IV. Timeline

- **May 10th - June 20th:** Gain familiarity with techniques used by reading references provided. This includes gaining more knowledge in the following:
  - interferometric gravitational wave detector workings
  - Control systems
  - Digital signal processing and data analysis (specifically random data analysis)
  - Wiener filtering and spectral subtraction
  - Better knowledge of optics to better understand angular sensing and control
- **June 20th - June 24rd:** Learn about LIGO instrument and focus on angular control systems and incorporate knowledge gained from the readings with feedback and feedforward control systems.
- **June 25th - July 1st:** Learn about the tools to take necessary online data. Measurements templates will be created with the online LIGO tool DTT or generated with Matlab depending on the progress.
- **July 2nd - July 15:** We will on taking online measurements, importing data into Matlab, and analyzing the measurements. We are particularly interested in measuring the coupling from each angular signal into our gravitational sensing readout. This will be done by sending swept sine or white noise functions into digital excitation channels to actuate on the optics in angle. We will use the differential arm sensor (DARM) to sense the coupling.

We will also measure the coupling from the length degrees of freedom to the angular sensors to create a simple noise budget by projecting the ambient noise.

- **July 16th - July 22nd:** Work on solutions to implement on the detector, by using feedforward lessons from week 1, and more advanced Wiener filtering techniques. The measured transfer functions will be fit in the frequency domain to simulate projected noise, and subtract it from the end channel.

- **July 23rd - July 29th:** We will test those techniques after making appropriate filter design. We will implement the filters into the instrument and measure the improvement in the DARM coupling. We will send again digital excitations to measure the new coupling and construct a new noise budget

- **July 30th - August 12th:** Based on the previous results, we will repeat the procedure and design new filters if the coupling reduction is not satisfying or we will look for further improvement by trying other feedforward techniques. A clear summary of the results should be started to make sure we got all the necessary measurements to assess the residual coupling. If time allows, the student could also start a model of the noise budget to validate the results.

- **August 13th - August 19th:** Write a report to be posted in the DCC and create a poster to present their results (check what are the Caltech REU requirements). The last week(s) will be dedicated to the finalization of the documents.

[1] R. Kinney, A. Weinstein, O. Miyakawa, and R. Ward, (2005).

[2] P. Saulson, *Fundamentals of Interferometric Gravitational Wave Detectors* (World Scientific, 1994).

[3] B. P. Abbott, R. Abbott, T. D. Abbott, M. R. Abernathy, F. Acernese, K. Ackley, C. Adams, T. Adams, P. Addesso, R. X. Adhikari, and et al., Physical Review Letters **116**, 061102 (2016), arXiv:1602.03837 [gr-qc].

[4] B. P. Abbott, R. Abbott, T. D. Abbott, M. R. Abernathy, F. Acernese, K. Ackley, C. Adams, T. Adams, P. Addesso, R. X. Adhikari, and et al., Physical Review Letters **116**, 241103 (2016), arXiv:1606.04855 [gr-qc].

[5] L. Barsotti and M. Evans, LIGO Document Control Center (2009), lIGO-T0900511-v4.

[6] D. Z. Anderson, Applied Optics **23**, 2944 (1984).