Source localization for eccentric binary with ground-based detector network

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with

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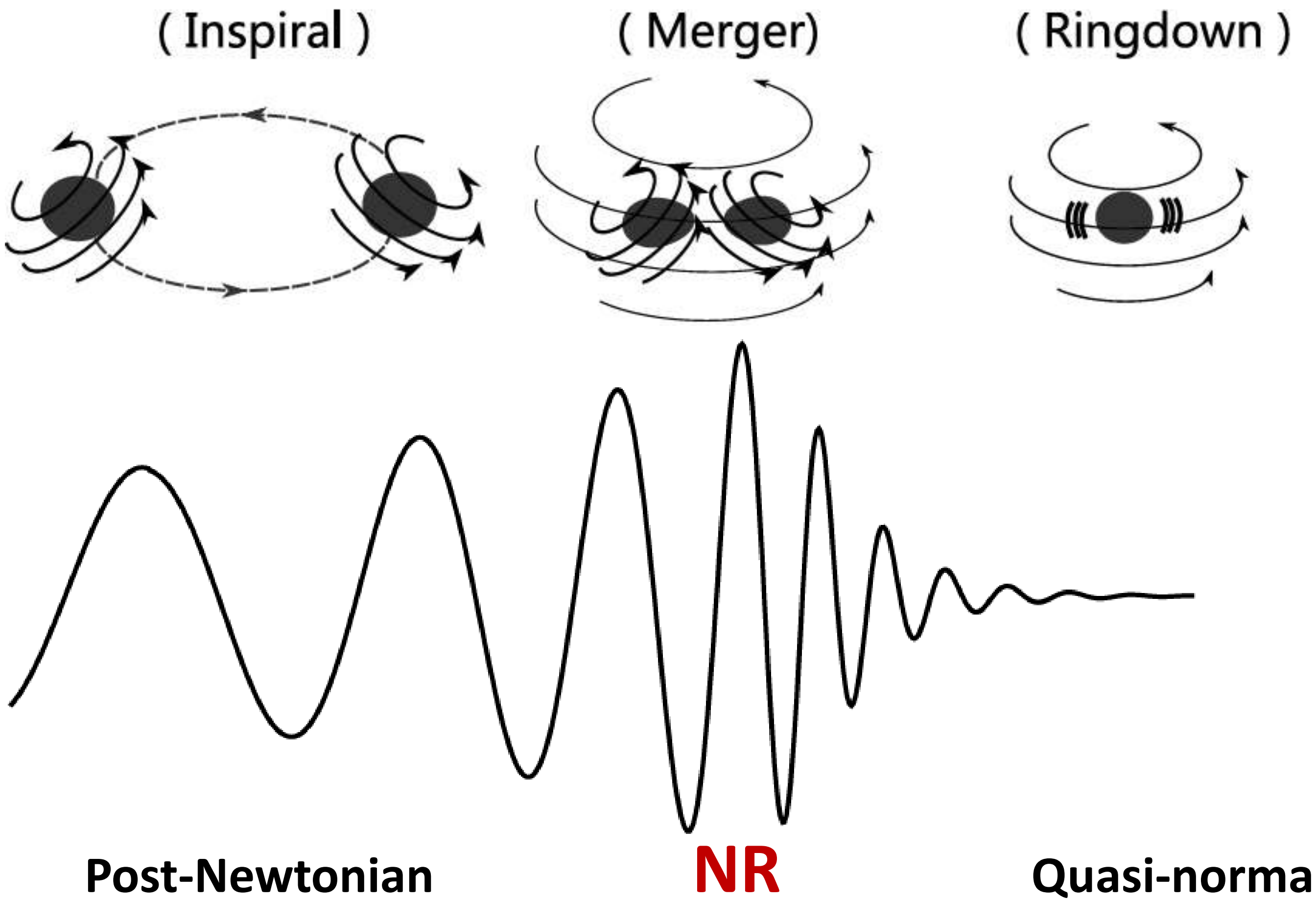
Outline

- Why eccentric binary
- Enhanced Post-Circular waveform model
- Fisher matrix on evaluating localization accuracy
- Further works

Quasi-circular binary BH coalescence



- First full BH coalescence waveform in 2005
- For equal-mass, non-spinning BBH:

  ~3% energy, ~15% angular momentum radiated, final spin~ 0.68

Quadruple waveform of binary

$$h_+(t) = \frac{4GM\eta}{c^2 r}\left(\frac{GM\pi f_{gw}}{c^3}\right)^{2/3} \frac{1+\cos^2\iota}{2} \cos\left(2\pi f_{gw} t_{ret}\right)$$

$$h_\times(t) = \frac{4GM\eta}{c^2 r}\left(\frac{GM\pi f_{gw}}{c^3}\right)^{2/3} \cos^2\iota \qquad \sin\left(2\pi f_{gw} t_{ret}\right)$$

- Phase→ Chirp mass
- Relative amplitude → inclination
- Amplitude → effective luminosity distance, source location coupled

  (that is why the importance of EM counterpart measurement)

Beyond quasi-circular

- Gravitational circularization (Peters '64)

$$\frac{e}{e_0} \propto \left(\frac{f}{f_0}\right)^{\frac{-19}{18}}$$

- It is suggested few eccentric binary in LIGO band (Kowalska+, 2010)
- Proposed formation mechanisms for eccentric binary/ merger:
  - Dynamical interaction in globular cluster/ galactic nuclei (Benacquista+ 2002, Wen 2003, O'Leary+ 2009, …)
  - Mass transfer between eccentric compact binary (Dosopoulou+ 2016)
  - …

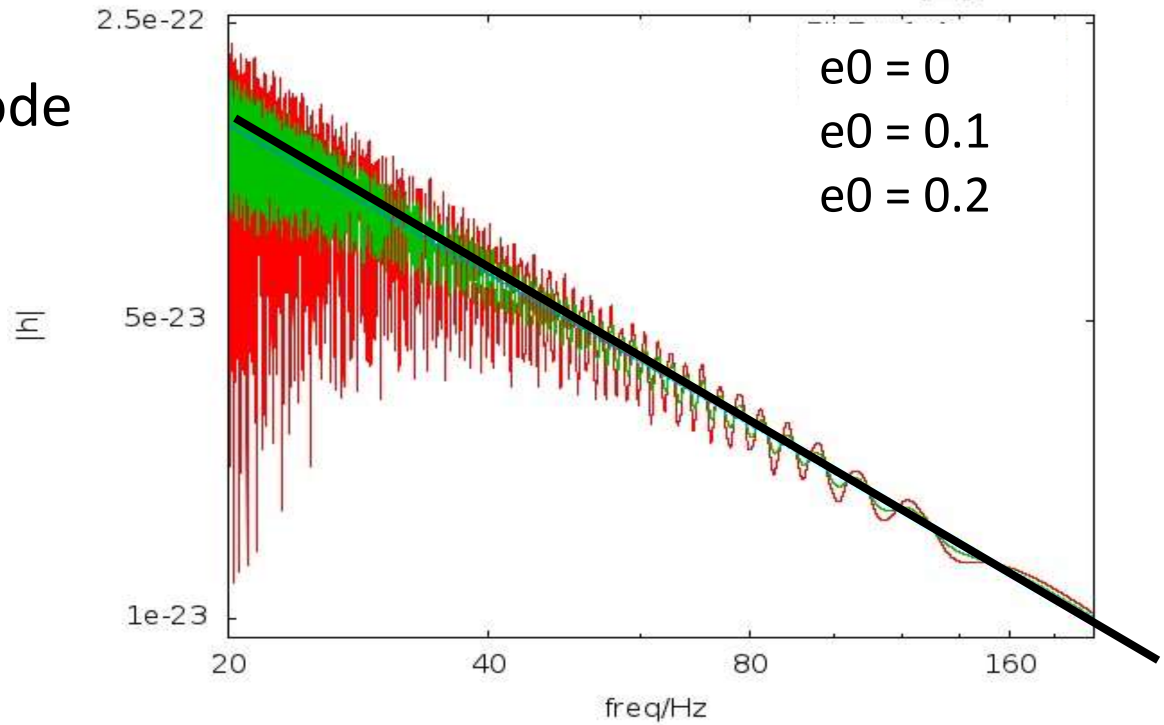
Enhanced post-circular waveform

- Fourier domain analytic waveform
  - ▶ Reduce to TaylorF2 (3.5 PN) when e→0
  - ▶ Reduce to Post-circular waveform when PN order → 0

$$h(f) = C\frac{\mathcal{M}^{5/6}}{D_L} f^{-7/6} \sum_{\ell=1}^{10} \xi_\ell \left(\frac{\ell}{2}\right)^{2/3} \exp(-i\Psi_\ell)$$

$$\Psi_\ell = 2\pi f t_c - \ell\phi_c + \left(\frac{\ell}{2}\right)^{8/3} \frac{3}{128\eta(v_{ecc})^5} \sum_{i=0}^{i=7} a_i (v_{ecc})^i$$

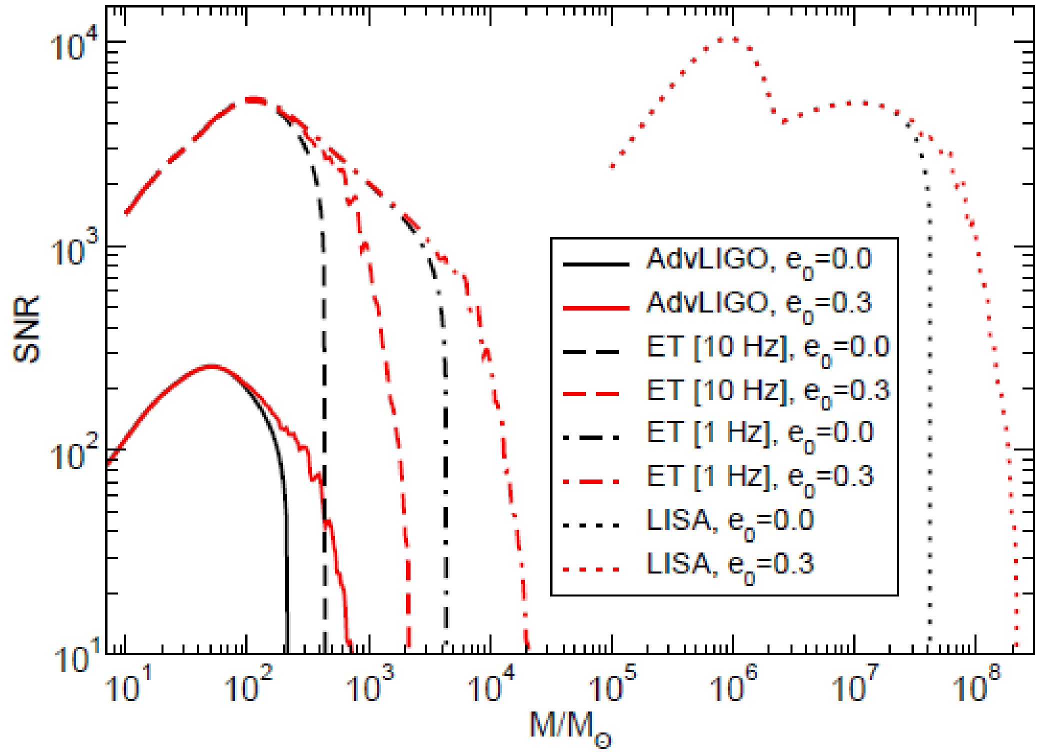
- Upto 10 harmonics mode that contribute the SNR calculation
- Kepler orbital v(e)



Yunes+. PRD80 (2009)
Huerta+ PRD90 (2014)

LSO frequency

- PN waveform, LSO frequency
  - ~ 200Hz for 20 solar mass BH-BH
  - ~ 1500Hz for NS-NS
- Higher harmonics mode allow probe for higher mass regime



Huerta+ (1408.3406)

11 parameters: $e_0(f_0), M, \eta, \iota, \beta, t_c, \phi_c, D, \theta, \phi, \psi$

$$h(f) = C\frac{\mathcal{M}^{5/6}}{D_L} f^{-7/6} \sum_{\ell=1}^{10} \xi_\ell \left(\frac{\ell}{2}\right)^{2/3} \exp(-i\Psi_\ell).$$

$\boldsymbol{\theta}, \boldsymbol{\phi}, \psi, \iota$ $\qquad$ $\boldsymbol{\theta}, \boldsymbol{\phi}, \psi, \iota, \beta, e$ $\qquad$ t_c, ϕ_c, e, η, M

$$\xi_\ell = \frac{\left(1-e^2\right)^{7/4}}{\left(1+\frac{73}{24}e^2+\frac{37}{96}e^4\right)^{1/2}} \alpha_\ell e^{-i\frac{f}{\ell}\phi_\ell}$$

$$\Psi_\ell = 2\pi f t_c - \ell\phi_c + \left(\frac{\ell}{2}\right)^{8/3} \frac{3}{128\eta(v_{ecc})^5} \sum_{i=0}^{i=7} a_i (v_{ecc})^i$$

Mismatch suggest the necessity of eccentric waveform

$$FF \equiv \max_{\mathbf{p}} \frac{(h(\mathbf{p})|s)}{\sqrt{(h(\mathbf{p})|h(\mathbf{p}))(s|s)}}$$

CBC	e_0	FF
BH-BH	0.1	0.974
	0.2	0.922
	0.3	0.858
	0.4	0.782
BH-NS	0.1	0.918
	0.2	0.803
	0.3	0.692
	0.4	0.591
NS-NS	0.1	0.774
	0.2	0.576
	0.3	0.434
	0.4	0.331

Sun+ PRD92, (2015)

Three roads to localization accuracy

- Timing triangulation $\sigma_t = \frac{1}{2\pi\rho\sigma_f}$
  - ▶ For ground-based, short duration source
  - ▶ Not fully exploit phasing information

- Fisher information matrix
- Bayesian parameter estimation

Method	Median (sq. deg.)	Mean (sq. deg.)	Standard Dev. (sq. deg.)
$A_{\rm Bayes}$	2.9	8.9	17.1
$A_{\rm TT}$	10.6	29.3	59.3
$A_{\rm F9}$	4.0	8.7	16.0
$A_{\rm F4}$	1.6	3.2	6.1

Grover+ (1301.7454)

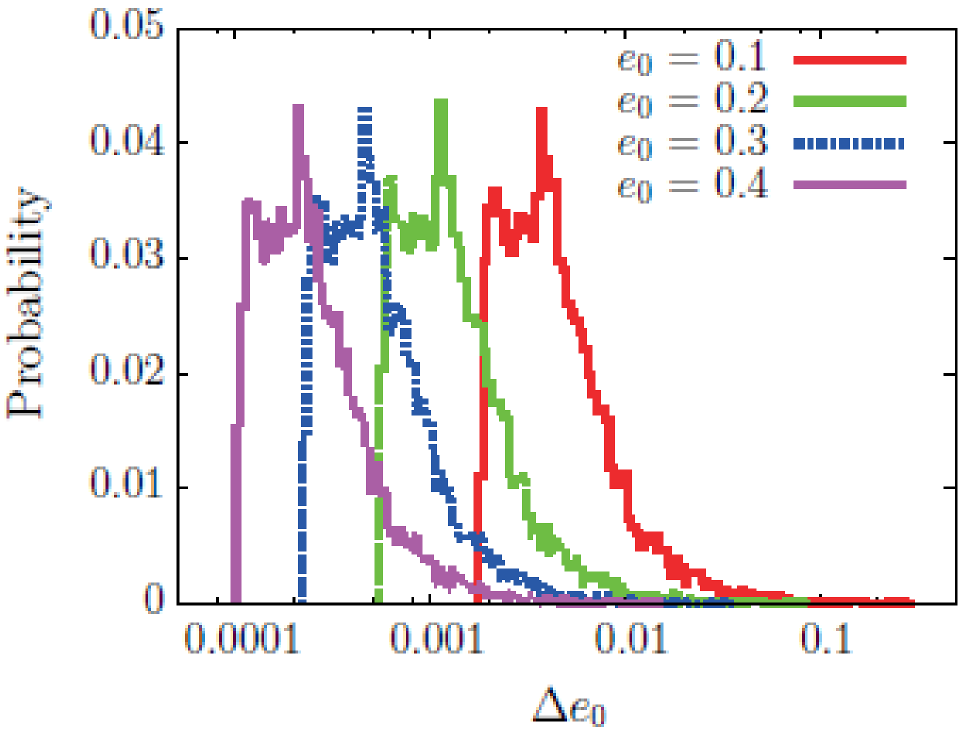
Fisher matrix analysis for single detector

- Estimate uncertainty of parameter estimation

$$\Gamma_{ab} = (\partial_a h|\partial_b h) \equiv 4Re \int \frac{\partial_b h \partial_b h^*}{S(f)} df$$

$$\Delta\theta^b = \sqrt{\Gamma_{bb}^{-1}} ;$$

- Similar pattern for other parameters



Sun+ PRD92, (2015)

Network response

- Total Fisher metric: $\Gamma_{ab} = \sum \Gamma^I_{ab} = \sum (\partial_a h^I | \partial_b h^I)$

$$h(f) = C \frac{\mathcal{M}^{5/6}}{D_L} f^{-7/6} \sum_{\ell=1}^{10} \xi_\ell \left(\frac{\ell}{2}\right)^{2/3} \exp(-i\Psi_\ell)$$

$$C = -\pi^{-2/3} \sqrt{\frac{5}{384} [\frac{1}{4} F_+^2 (1 + c_i^2)^2 + F_\times^2 c_i^2]},$$

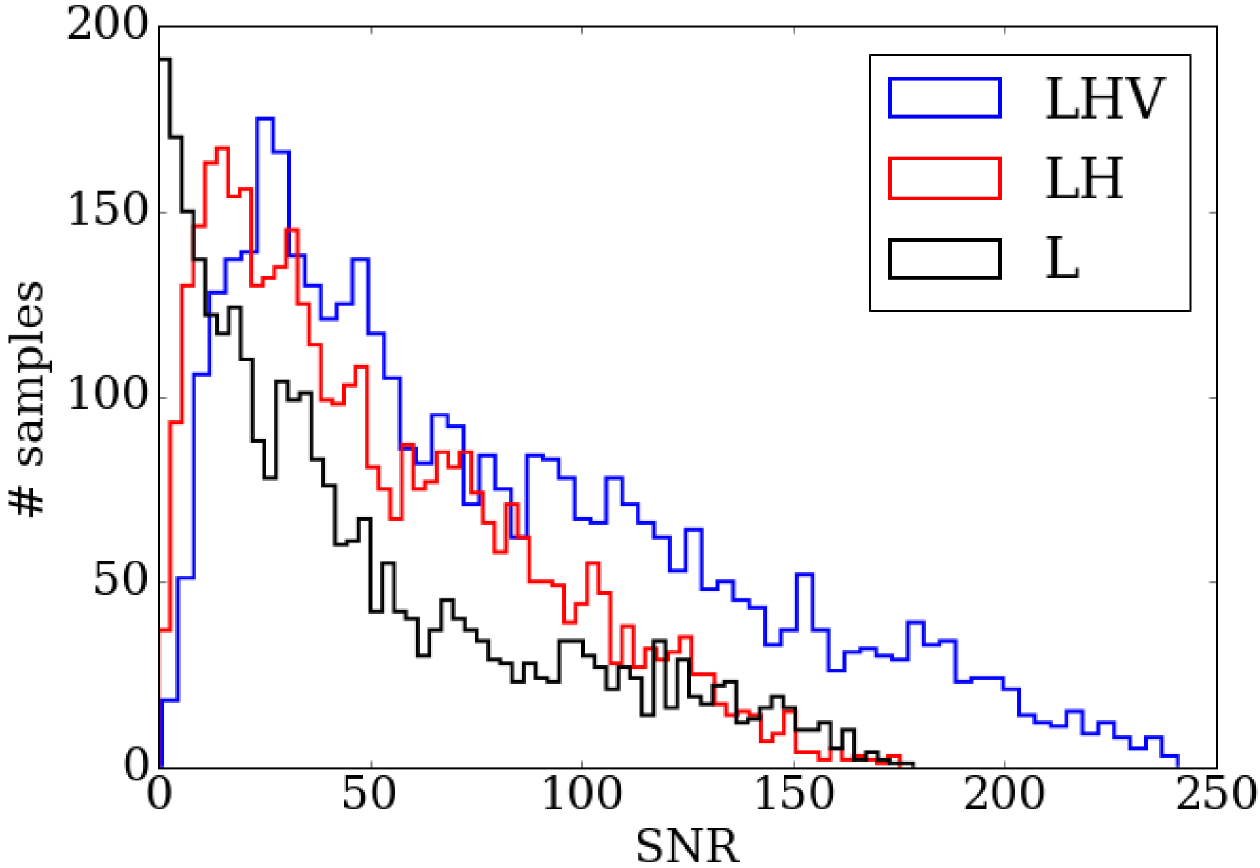
With detector's pattern function (for right-angled interferometer)

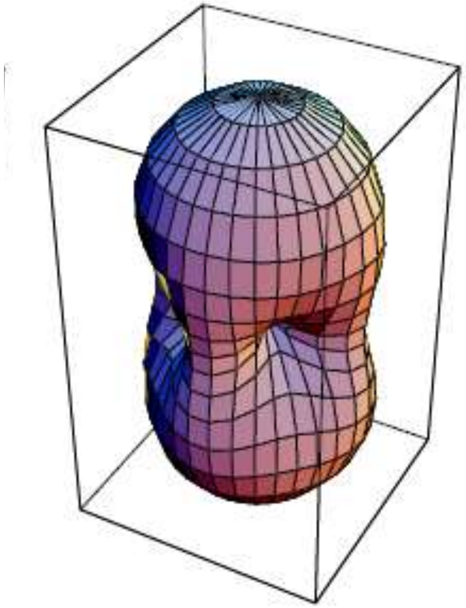
$$F_+ = \frac{1}{2} \left(e_1^j e_1^k - e_2^j e_2^k\right) \left(e_X^j e_X^k - e_Y^j e_Y^k\right)$$

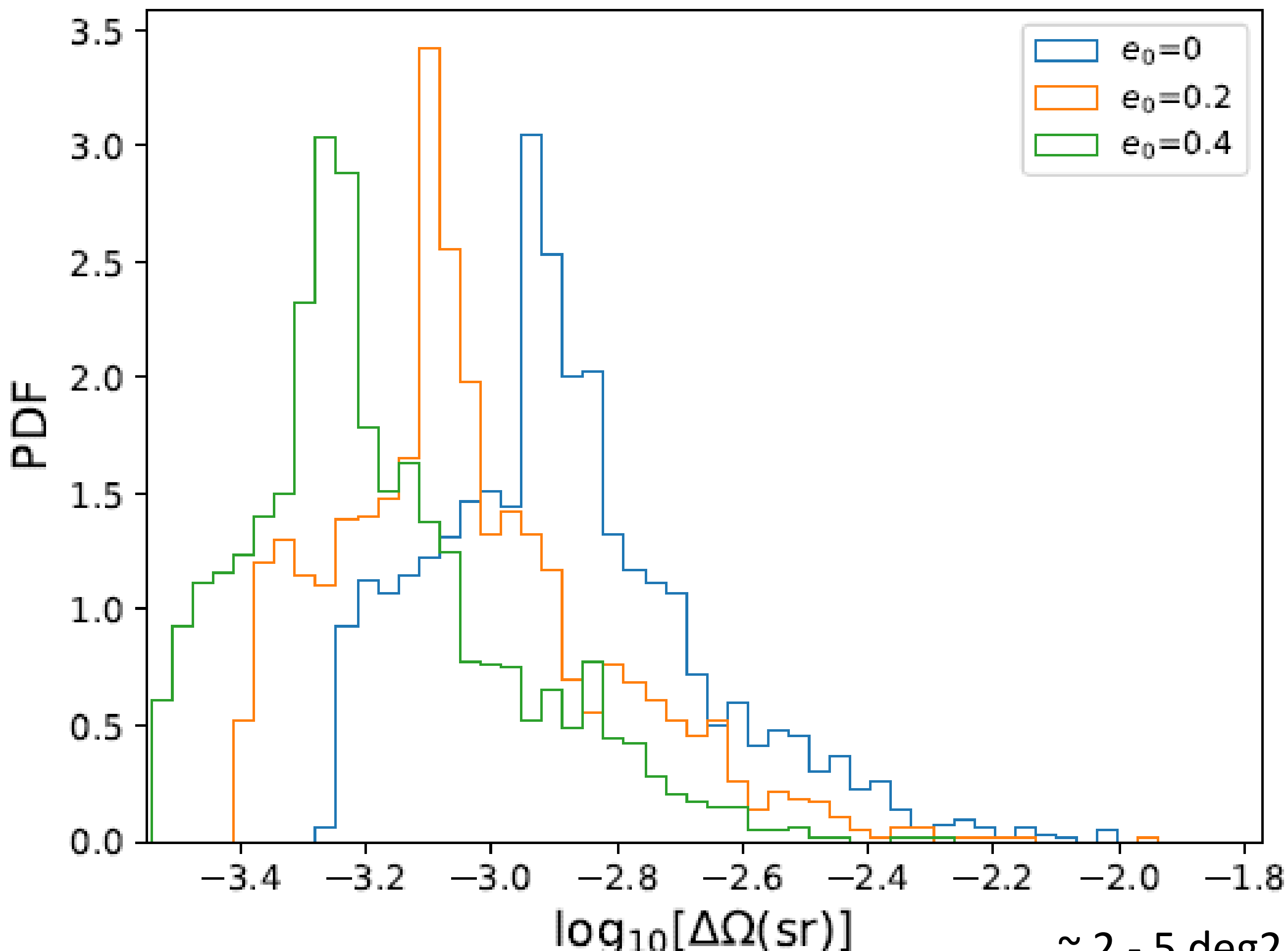
$$F_\times = \frac{1}{2} \left(e_1^j e_1^k - e_2^j e_2^k\right) \left(e_X^j e_Y^k - e_Y^j e_X^k\right)$$

Arm basis　　　　TT-frame GW basis

Avoid dead spot of single detector pattern function



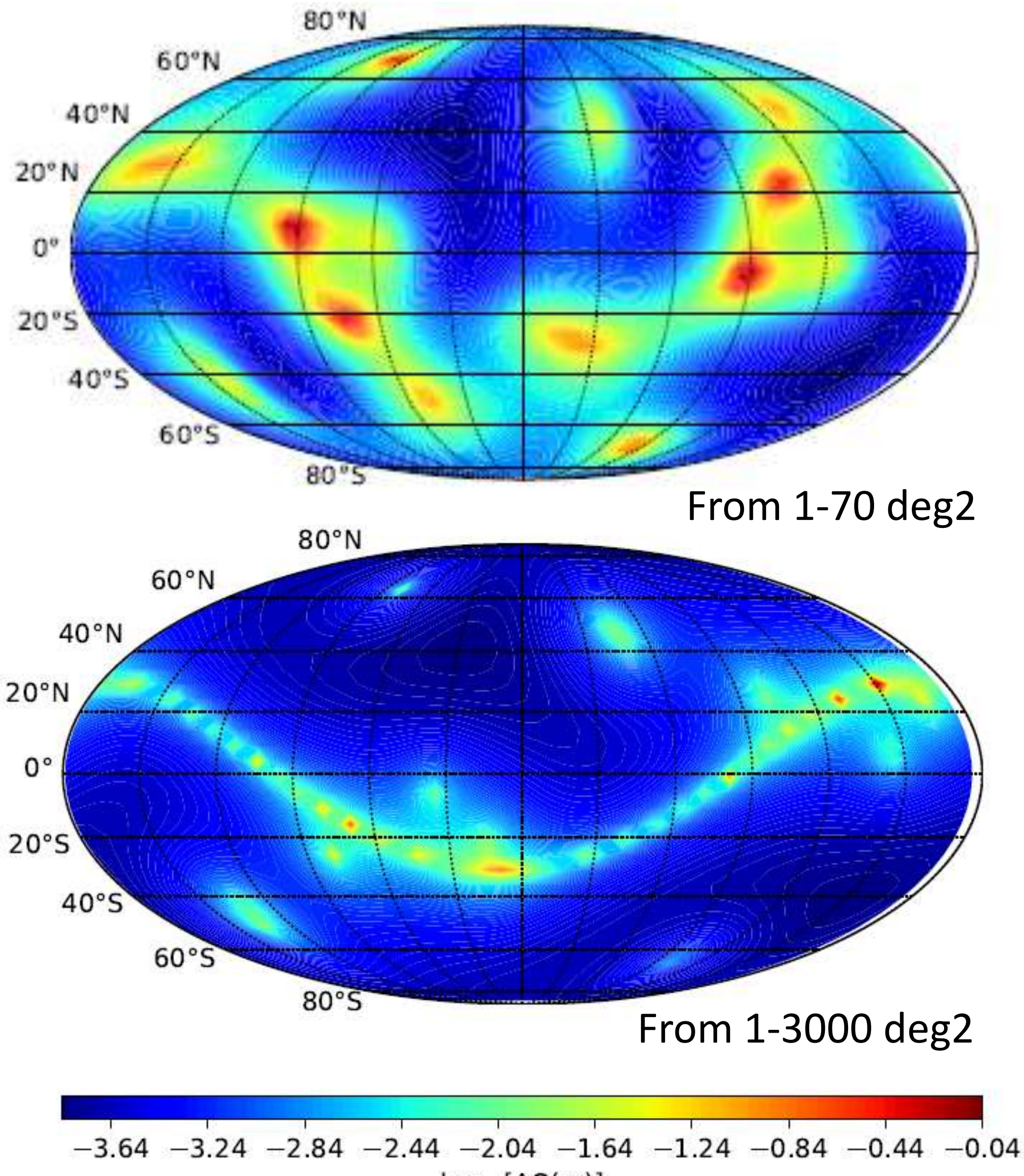




e$_0$=0
e$_0$=0.2
e$_0$=0.4
PDF
$\log_{10}[\Delta\Omega(sr)]$
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
−3.4
−3.2
−3.0
−2.8
−2.6
−2.4
−2.2
−2.0
−1.8

~ 2 - 5 deg2

Compare with timing triangulation



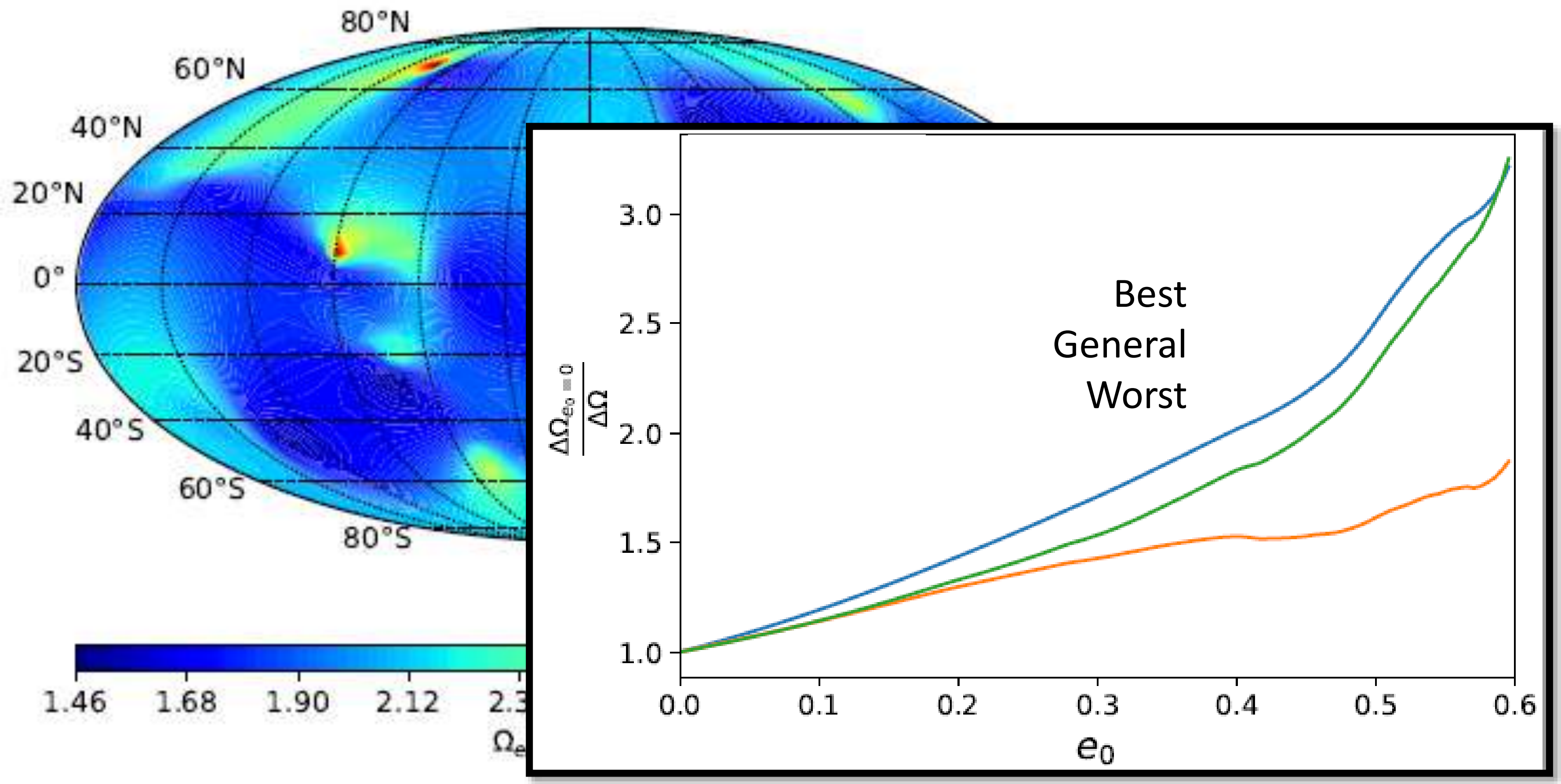
e0 = 0.4

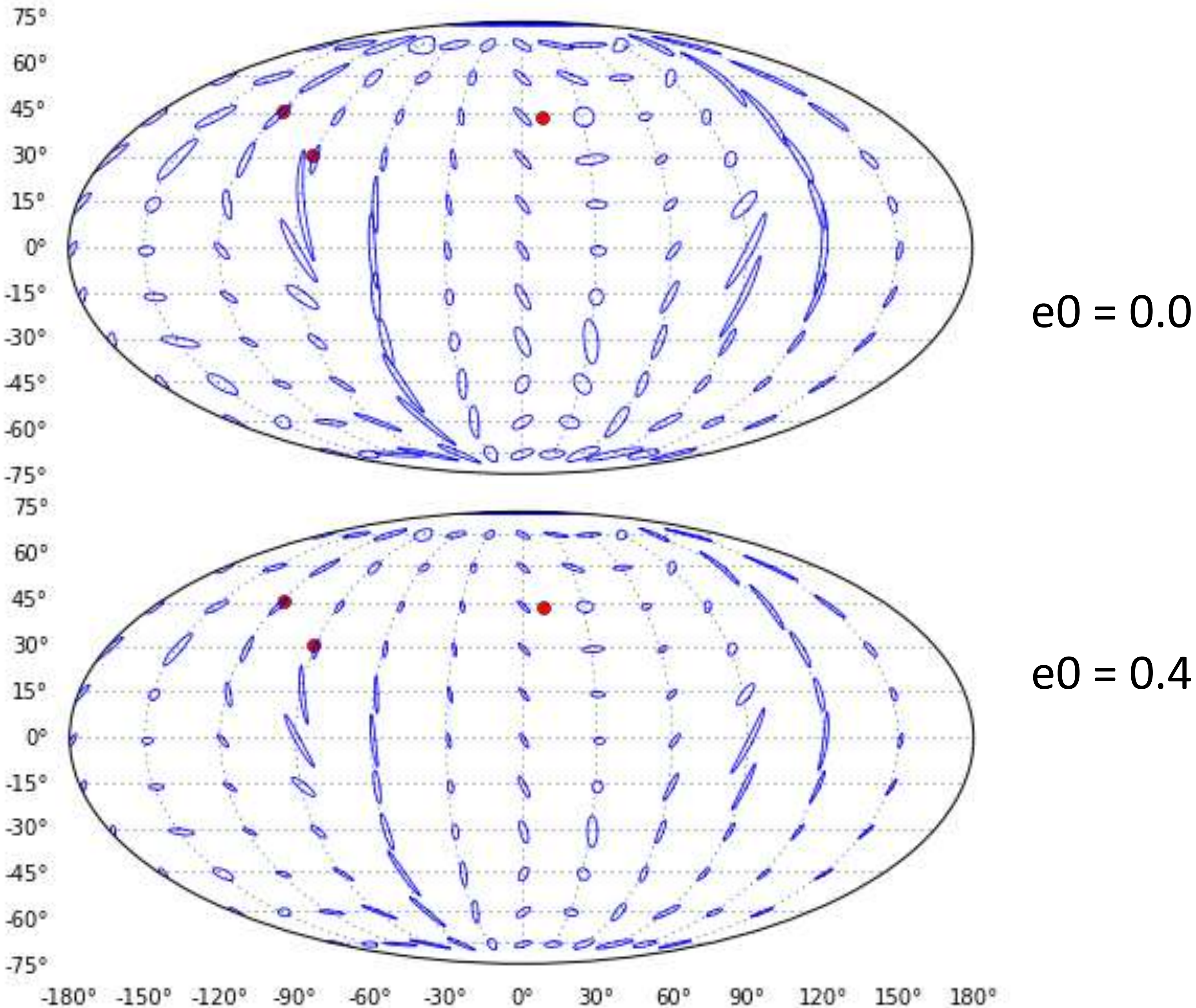
Wen & Chen (1003.2504)

$$(f,best) = \frac{1}{\rho_N^2}\frac{c^2}{f^2 A_\perp}\frac{1}{4\pi}\sqrt{\frac{1}{\rho_1^2\rho_2^2\rho_3^2/\rho_N^6}},$$

- $\mathrm{d}\Omega(e=0)$ / $\mathrm{d}\Omega(e=0.4)$ = 1.5 ~ 3

  for eccentric binary (100 solar mass)




75°
60°
45°
30°
15°
0°
-15°
-30°
-45°
-60°
-75°
e0 = 0.0
75°
60°
45°
30°
15°
0°
-15°
-30°
-45°
-60°
-75°
-180°
-150°
-120°
-90°
-60°
-30°
0°
30°
60°
90°
120°
150°
180°
e0 = 0.4

Future works

- Systematic analysis on parameter space
- Astrophysical consideration
- Fully Bayesian analysis
- Complete waveforms model :

  IMR for eccentric binary,

  Higher harmonics WF (FWF) at e=0

Thank you