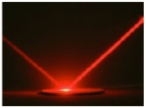

Applied Physics Laboratory

A new class of compact high sensitive tiltmeters based on the UNISA folded pendulum mechanical architecture

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12th Edoardo Amaldi Conference on Gravitational Waves
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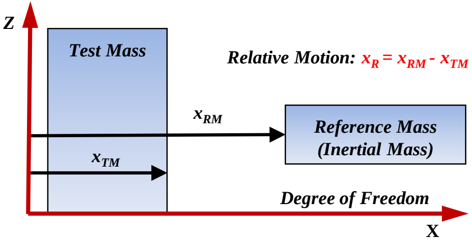
Inertial Sensors

Working Principle: measurement of the relative motion of a **test mass** with respect to an **inertial mass**.



Inertial mass: the mass of a mechanical oscillator is «**inertial**» beyond its resonance frequency

e.g. classical pendulum: $$f_o = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$$

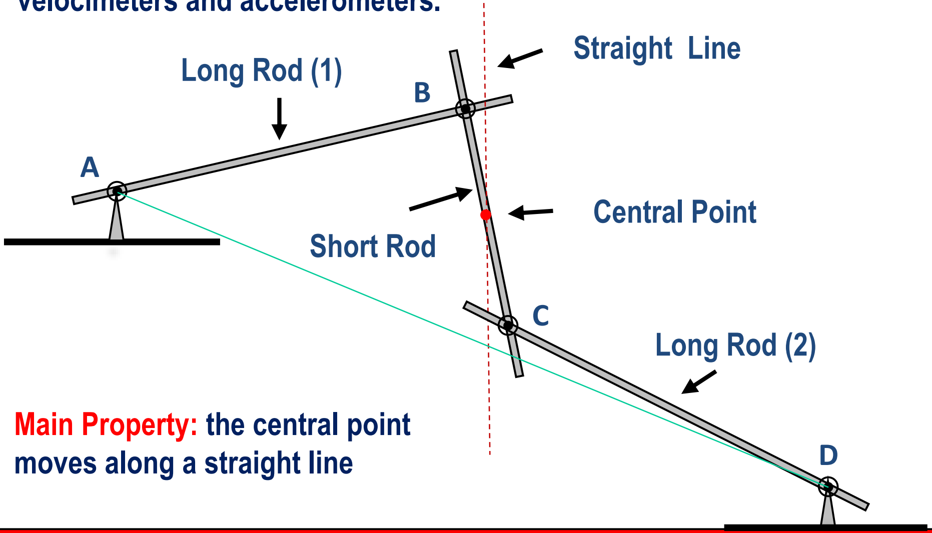
$f_o \sim 0.5\ \mathrm{Hz}$ ⇒ $l = 1\ \mathrm{m}$

$f_o \sim 0.05\ \mathrm{Hz}$ ⇒ $l = 100\ \mathrm{m}$

Main problem: implementation of low frequency oscillators

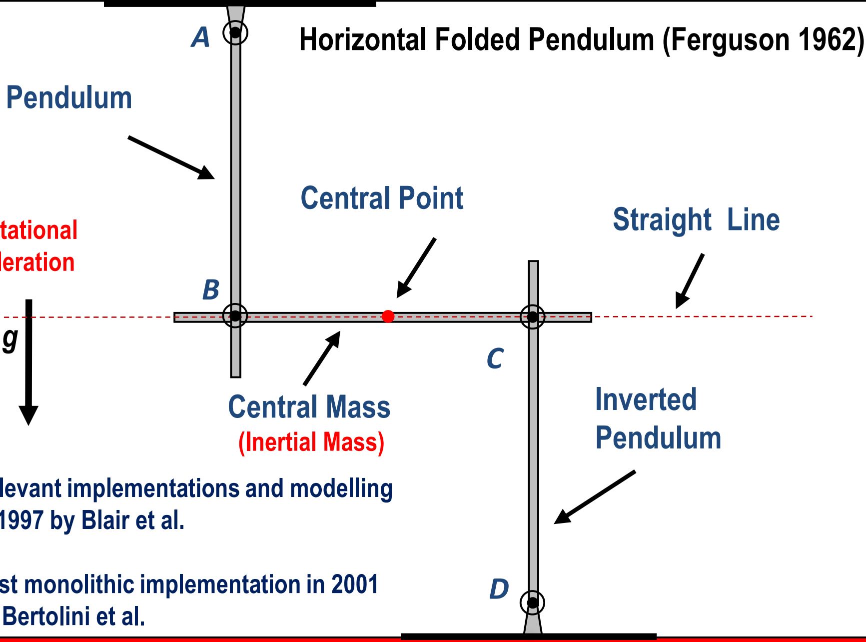
Start from the past: the Watt's Linkage

Watt's Linkage (1774): a very promising mechanical architecture for the implementation of horizontal, vertical and angular seismometers, velocimeters and accelerometers.



Main Property: the central point moves along a straight line

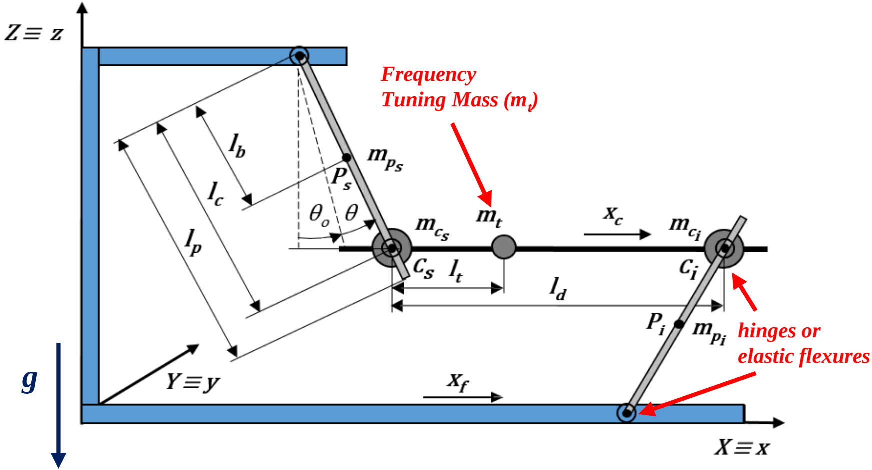
Horizontal Folded Pendulum



- Relevant implementations and modelling in 1997 by Blair et al.

- First monolithic implementation in 2001 by Bertolini et al.

Classical Horizontal Folded Pendulum Lagrangian Model (Blair et al. 1997)



XY ≡ local horizontal plane

xy ≡ folded pendulum reference system

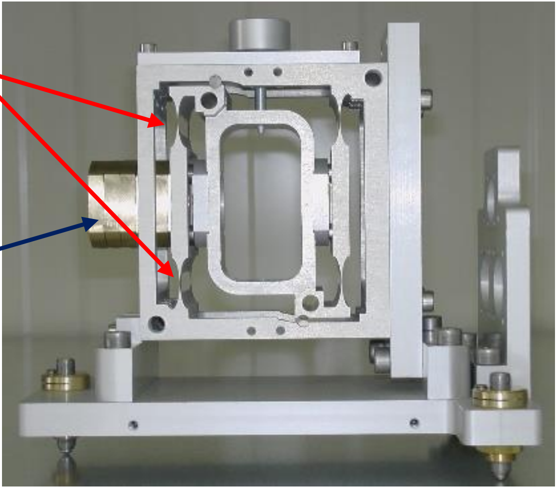
g ≡ gravitational acceleration (perpendicular to the central rod motion)

UNISA Monolithic Folded Pendulum

Mod. GE15 (2015)

Key element: elliptic flexures working in compression on the inverted pendulum

Tuning Mass



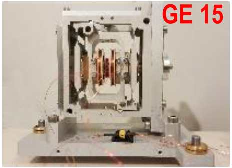
Physical Limitation:

Mechanical Thermal Noise

Monolithic design Advantages:

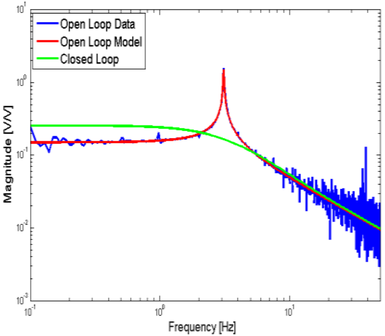
small size, directivity, compacteness, robustess, light weight, application to all environments (earth, marine, space, UHV and cryogenic).

First Property: a Folded Pendulum is dynamically fully equivalent to a second order system.

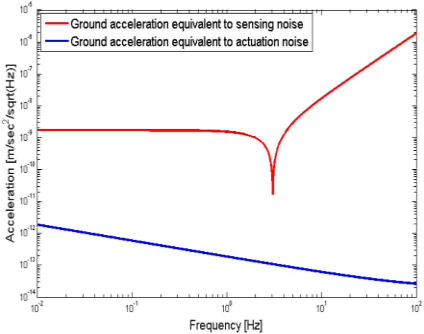


$$H(s) = \frac{X_c(s) - X_g(s)}{X_g\,(s)} = \frac{X_{output}(s)}{X_g(s)} = \frac{-(1-A_c)\,s^2}{s^2 + \frac{2\pi f_o}{Q(f_o)}s + 4\pi^2 f_o^2}$$

Open (seimometer) and Closed Loop (accelerometer) Transfer Functions



Sensitivity and Noise Budget



(Barone, Giordano, Acernese, Romano, Gennai, Passuello, Boschi, Cerretani, Passaquieti, GWADW 2016 , Isola d'Elba)

Folded Pendulum Relevant Properties - II

Second Property: The folded pendulum resonance frequency is equivalent to that of a spring-mass oscillator.

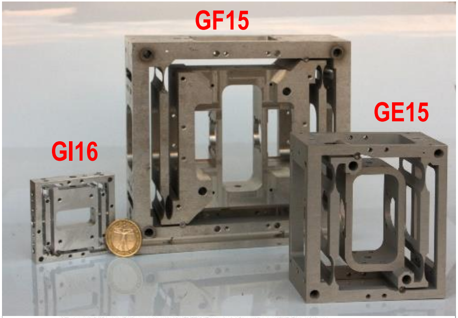
$$f_o = \frac{\omega_o}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{\left[\left(m_{p_s}-m_{p_i}\right)\frac{l_p}{l_c}+\left(m_{c_s}-m_{c_i}\right)\right]\frac{g_{eq}}{l_c}+\frac{k_\theta}{l_c^2}}{\left(m_{p_s}+m_{p_i}\right)\frac{l_p^2}{3l_c^2}+\left(m_{c_s}+m_{c_i}\right)}} = \frac{1}{2\pi}\sqrt{\frac{K_{g_{eq}}+K_{e_{eq}}}{M_{eq}}} = \frac{1}{2\pi}\sqrt{\frac{K_{eq}}{M_{eq}}}$$

Equivalent Gravitational Elastic Constant ≡ $K_{g_{eq}}$ < = > 0

Equivalent Elastic Constant ≡ $K_{e_{eq}} > 0$

Note: suitable combinations of physical and geometrical parameters allow in theory to set the resonance frequency to 0 Hz (ideal inertial mass), in practice to frequencies as low as 60 mHz (up to now).

The UNISA Folded Pendulum Class of Sensors



UNISA Monolithic Modular Folded Pendulums: GE15, GF15 and GI16

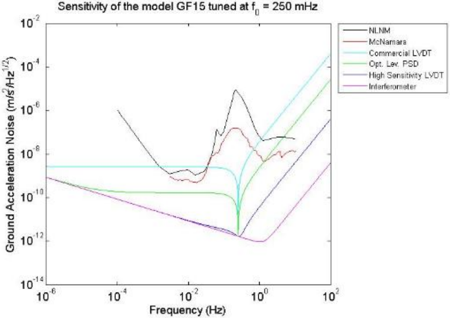
Band 1µHz ÷ 1 kHz

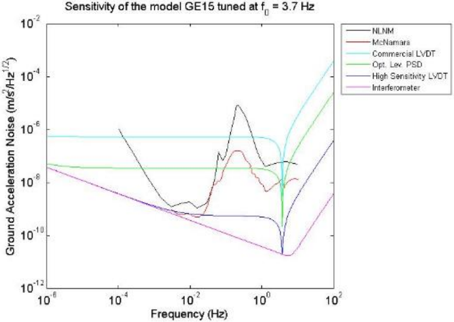
Resonance Frequency 60 mHz ÷ 10 Hz

Readout LVDT, capacitive sensor, optical lever, optical fibre bundle, interferometer, etc.

Readout Noise $10^{-14} \div 10^{-6}$ m/Hz$^{1/2}$

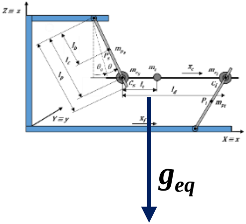
Quality Factor Q > 16000 (UHV), Q > 2000 (air) (resonance frequency dependent)





Generalized Folded Pendulum: Effects of Folded Pendulum Tilts

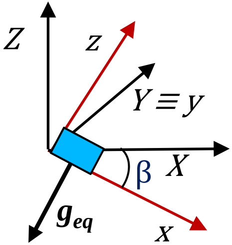
Generalized Folded Pendulum Lagrangian Model (Barone et al. 2011)



XYZ ≡ local reference system

(XY ≡ local horizontal plane)

xyz ≡ reference system integral to the folded pendulum



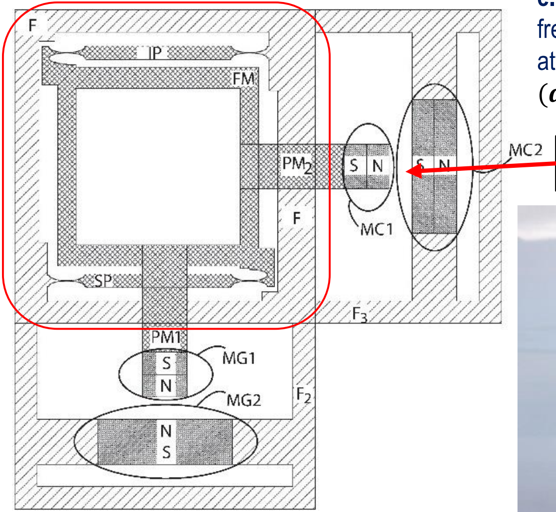
$$g_{eq} = g + a_{ext}$$

$$g_{eq} = g \cdot \cos\beta + a_{ext}$$

g_{eq} ≡ equivalent (effective) gravitational acceleration (sum of the components of all the accelerations perpendicular to the motion of the central rod)

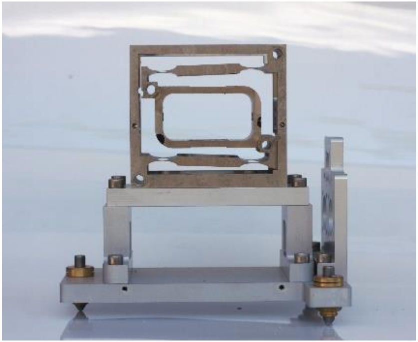
β ≡ tilt relative to the local horizontal (β = 90° for a vertical folded pendulum)

Basic Principle of UNISA Vertical Folded Pendulum with frequency tuning and offset regulation (β = 90°).



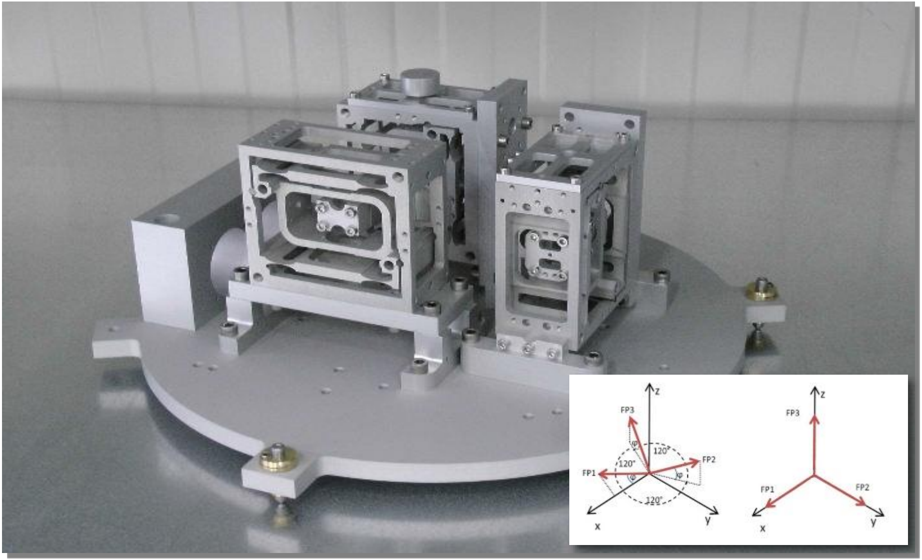
e.g. Magnetic Force: resonance frequency reduction (or increase) with attractive (or repulsive) magnetic force (a_{ext})

$$g_{eq} = g \cdot \cos\beta + a_{ext} = a_{ext}$$



UNISA Triaxial Folded Pendulum (3 x GE15)

Triaxial mechanical inertial sensor - “xyz“ configuration.



Folded Pendulum as Tiltmeter

$$f_o = \frac{\omega_o}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{\left[\left(m_{p_s} - m_{p_i}\right)\frac{l_p}{l_c} + \left(m_{c_s} - m_{c_i}\right)\right]\frac{g_{eq}}{l_c} + \frac{k_\theta}{l_c^2}}{\left(m_{p_s} + m_{p_i}\right)\frac{l_p^2}{3l_c^2} + \left(m_{c_s} + m_{c_i}\right)}} = \frac{1}{2\pi}\sqrt{\frac{K_{g_{eq}} + K_{e_{eq}}}{M_{eq}}} = \frac{1}{2\pi}\sqrt{\frac{K_{eq}}{M_{eq}}}$$

Equivalent Gravitational Elastic Constant $\equiv K_{g_{eq}} < = > 0$

Equivalent Elastic Constant $\equiv K_{e_{eq}} > 0$

Interpreting the Generalized Folded Pendulum Model:
the resonance frequency of a Folded Pendulum is function of its orientation with respect to the local horizontal.

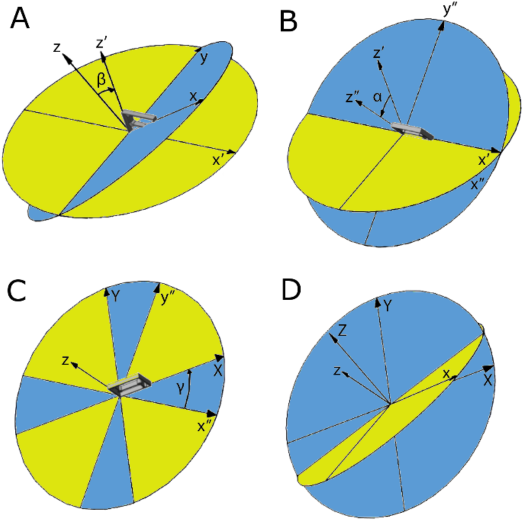
A new approach for tilt measurement: the Folded Pendulum orientation can be obtained by inversion of the resonance frequency function!!

We used the Tait-Bryan angle rotation system (sequence (β α γ)

Roll angle = α

Pitch angle = β

Yaw angle = γ



Tait-Bryan Rotation matrix

$$R = R_\gamma R_\alpha R_\beta = \begin{pmatrix} \cos\gamma & \sin\gamma & 0 \\ -\sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ \sin\beta & 0 & \cos\beta \end{pmatrix} =$$

$$= \begin{pmatrix} \cos\gamma\ \cos\beta + \sin\gamma\sin\alpha\sin\beta & \sin\gamma\ \cos\alpha & -\cos\gamma\sin\beta + \sin\gamma\ \sin\alpha\ \cos\beta \\ -\sin\gamma\ \cos\beta + \cos\gamma\sin\alpha\sin\beta & \cos\gamma\cos\alpha & \sin\gamma\ \sin\beta + \cos\gamma\ \sin\alpha\cos\beta \\ \cos\alpha\sin\beta & -\sin\alpha & \cos\alpha\cos\beta \end{pmatrix}$$

Result: components of the gravitational acceleration along the direction of the folded pendulum integral reference system xyz

$$\vec{g}_{xyz} = \begin{pmatrix} \sin\gamma\ \sin\alpha\ \cos\beta \\ \sin\alpha\cos\beta \\ \cos\alpha\cos\beta \end{pmatrix} g \qquad \gamma = 0$$

Folded Pendulum Resonance Freqeuncy for a generic tilt

$$f_o(\alpha,\beta) = \sqrt{\frac{K_{eq}(\alpha,\beta)}{M_{eq}}} = \sqrt{\frac{K_{g_{eq}}(\alpha,\beta) + K_{e_{eq}}}{M_{eq}}} =$$

$$\sqrt{\frac{\left[\frac{1}{2}(m_{p_s} - m_{p_i})\frac{l_p}{l_c} + (m_{g_s} - m_{g_i}) - 2m_g \frac{M_{eq}}{K_{e_{eq}}}\frac{g}{l_d}\sin\beta\right]\frac{g}{l_c}\cos\alpha\cos\beta + K_{e_{eq}}}{M_{eq}}}$$

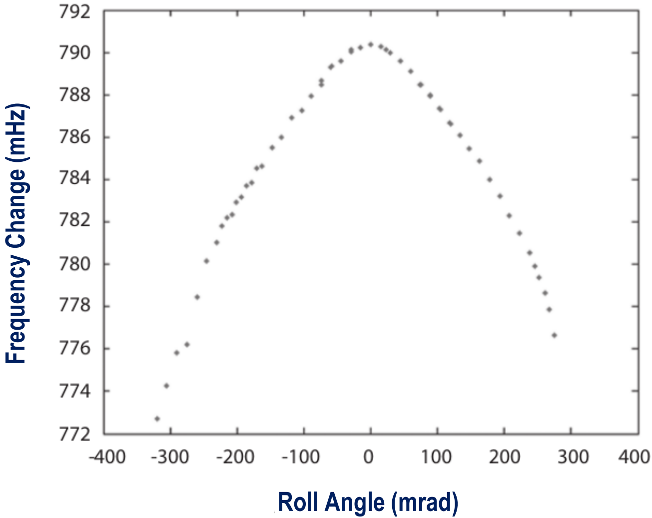
Amplification term. Close to zero for all the present implementations of the UNISA Folded Pendulum.

Modulation terms in presence of roll and pitch angles.

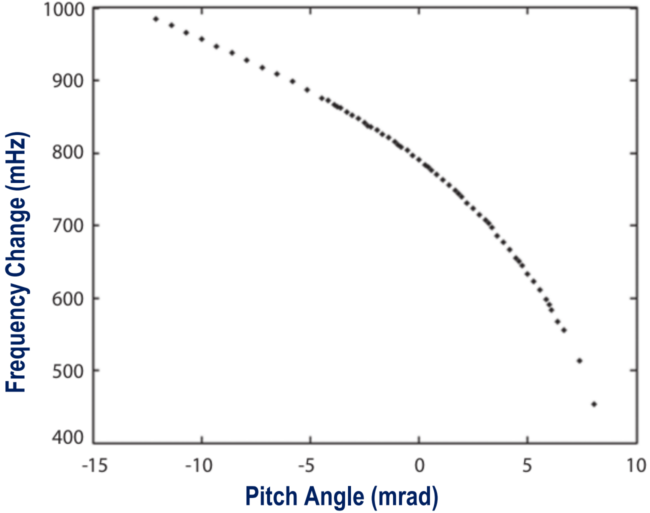
New term due to distribution mass change in presence of pitch angles.

Note: the resonance frequency is always an even function for the roll).

UNISA GC12 Model: Resonance Frequency vs. Roll Angle

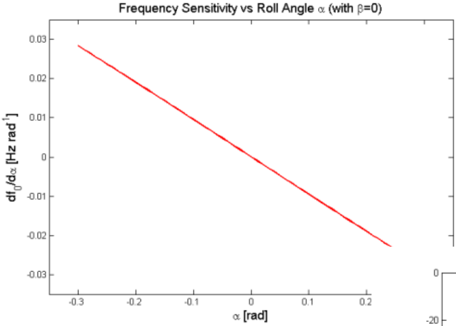


Note: optimized **asymmetric** folded pendulum implementations may produce larger changes



Note: optimized folded pendulum implementations may produce larger frequency changes

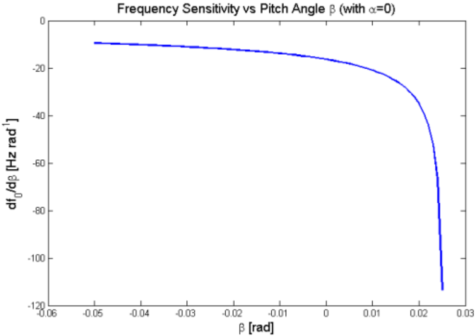
Sensitivity of the UNISA Folded Pendulum GC12 to roll and pitch angles



Preliminary tests, with a not optimized folded pendulum, show a pitch sensitivity of the order of 10 nrad

0.010 mrad/Hz) (theoretical)

30 mrad/Hz (measured) ,



Horizontal Positioning of a generic plane xy

I solution:

1) Position on the xy plane two folded pendulums orthogonally oriented;

2) Tilt the plane rolling the first folded pendulum until its resonance frequency becomes maximum (or minimum);

3) Tilt the plane rolling the second folded pendulum until its resonance frequency is again maximum (or minimum);

4) Eventually repeat points 2) and 3);

II solution:

1) Position on the xy plane two folded pendulum orthogonally oriented;

2) Measure their resonance frequency;

3) Analytically solve the system of two analytical frequency functions (one for each sensor) obtaining the roll and pitch angles of the plane;

4) Apply the tilt corrections to the plane;

5) Eventually repeat 2) 3) 4);

Note: the same operation can be done numerically using the folded pendulum calibration curves.

Conclusions and Future Perspectives

An Extended Folded Pendulum Model has been developed to predict the behavior of a folded pendulum generically oriented in the space.

According to the model, the resonance frequency allows to determine the tilt of the sensors (and of its integral plane) with respect to the local horizontal.

Preliminary experimental tests with a not optimized UNISA Folded Pendulum (Model GC 12) confirm that the resonance frequency is an even function of the roll and an odd function of the pitch.

Conclusions and Future Perspectives

New versions of the UNISA folded pendulum are being developed and tested as tiltmeters to optimize their sensitivity and dynamical behavior.

In parallel, tests of plane alignment procedures are being performed in order to check their robustness and the quality of the horizontal alignment.

Results of these activities are expected in the next months.

Thank you for you attention