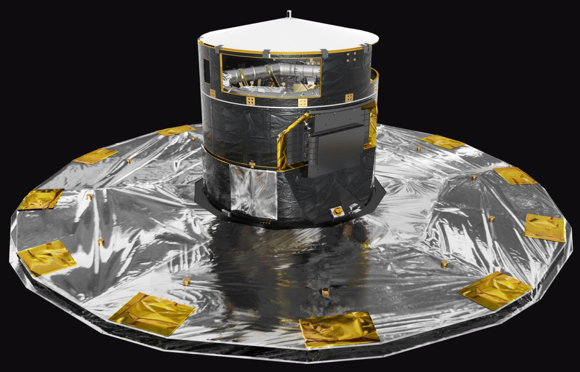
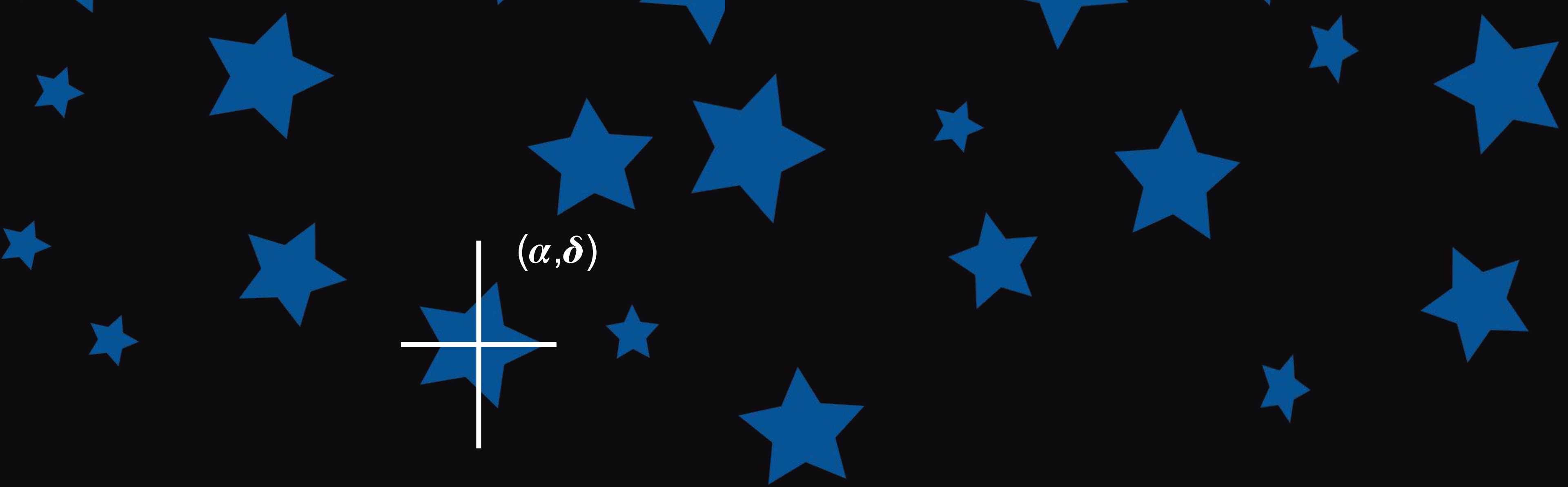
Astrometric detection of gravitational waves with Gaia

Christopher Moore
In collaboration with Deyan Mihaylov, Anthony Lasenby, Gerry Gilmore, and Jonathan Gair



Amaldi 12
12th July 2017



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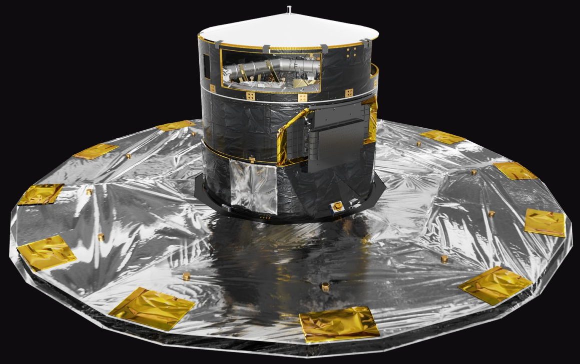

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Outline

Deflecting starlight with gravitational waves

Introducing Gaia

Comparing Gaia and PTAs for GW detection

Detecting individual sources with Gaia

Detecting stochastic GW backgrounds with Gaia

Conclusions

The Astrometric Effect of GWs

The Earth and a distant star are at rest in flat space, they are joined by the null geodesics

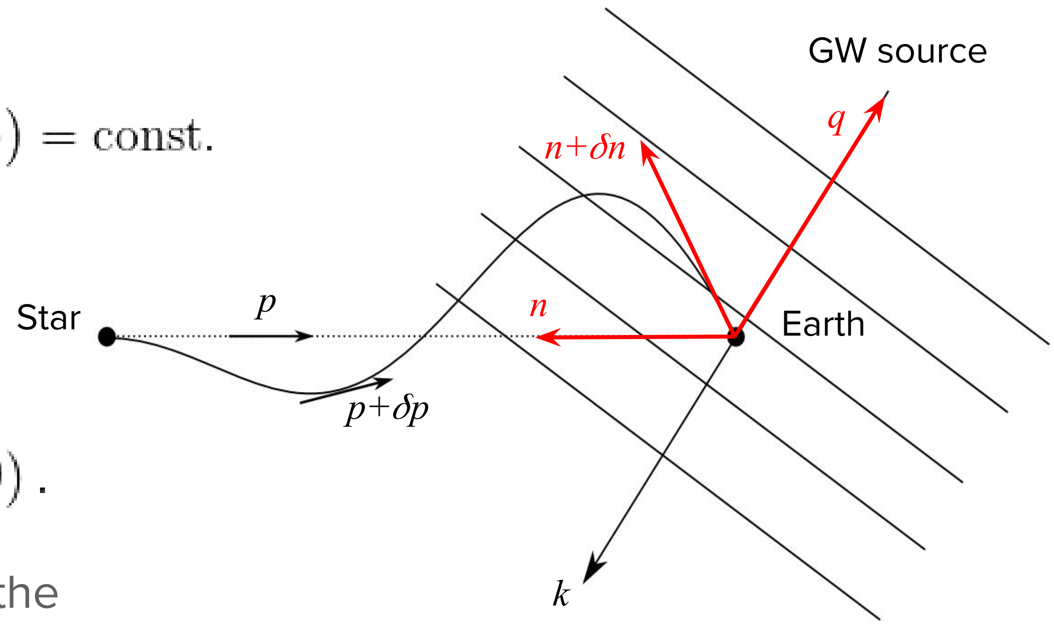
$$\frac{\mathrm{d}^2}{\mathrm{d}\lambda^2}\left(x^\mu(\lambda)\right) = 0\,, \quad p^\mu \equiv \frac{\mathrm{d}}{\mathrm{d}\lambda}\left(x^\mu(\lambda)\right) = \mathrm{const.}$$

The Earth has an orthonormal tetrad adapted to its 4-velocity

$$\epsilon^\mu_{\hat{a}}\epsilon^\nu_{\hat{b}}\eta_{\mu\nu} = \eta_{\hat{a}\hat{b}}\,, \qquad \epsilon^\mu_{\hat{0}} = (1,0,0,0)\,.$$

An observer on the Earth measures the **frequency** and **astrometric position**

$$\Omega = p_\mu \epsilon^\mu_{\hat{0}}\,, \qquad n_{\hat{i}} = p_\mu \epsilon^\mu_{\hat{i}}/\Omega\,.$$

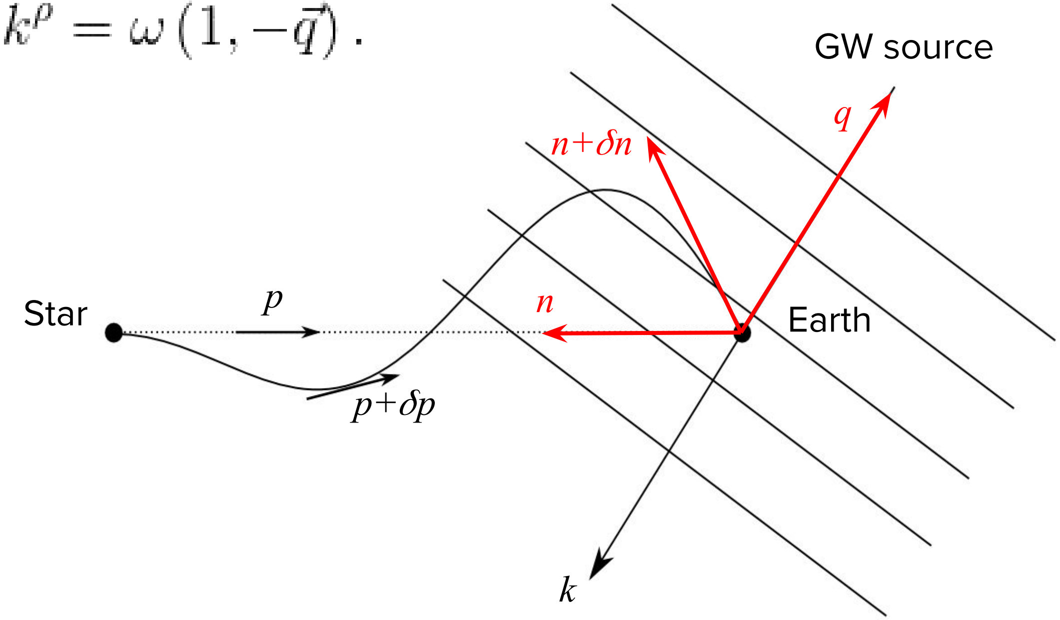


Book & Flanagan (2011)

The Astrometric Effect of GWs

Now consider perturbing this setup with a GW

$$h_{\mu\nu} = \Re\left\{H_{\mu\nu}\exp\left(\mathrm{i}k_\rho x^\rho\right)\right\} , \qquad k^\rho = \omega\left(1, -\vec{q}\right) .$$



Book & Flanagan (2011)

The Star and Earth follow **timelike geodesics** in the perturbed metric

The photons follow **null geodesics** in the perturbed metric

The observer's tetrad is **parallel transported** (using the perturbed metric) along Earth’s worldline

The Astrometric Effect of GWs

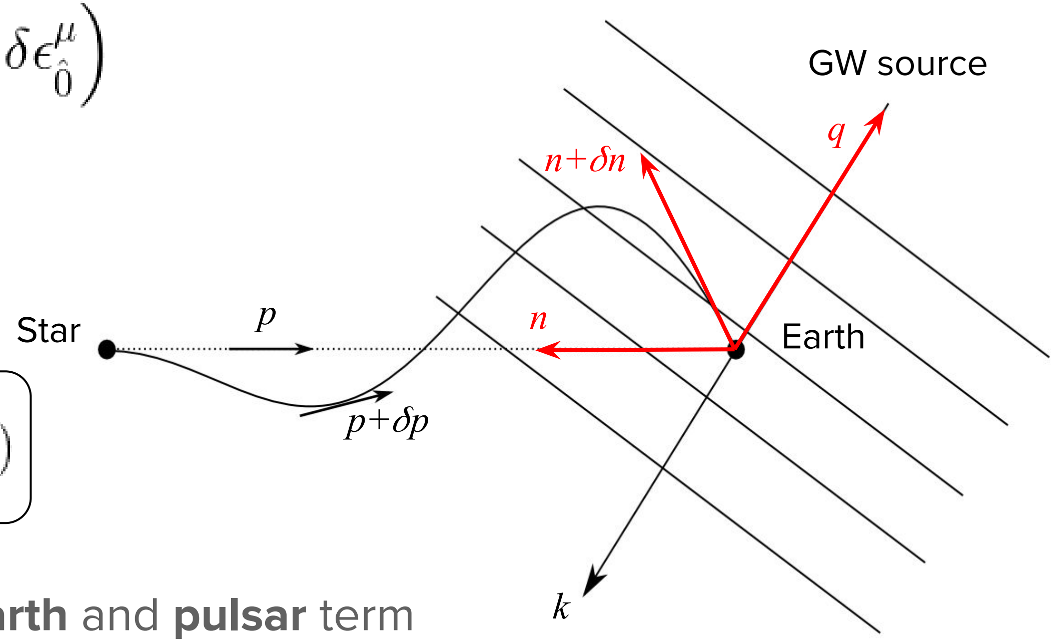
The effect on the measured **frequency**

$$\Omega \equiv p_\mu \epsilon^\mu_{\hat{0}} \to (p_\mu + \delta p_\mu)\left(\epsilon^\mu_{\hat{0}} + \delta\epsilon^\mu_{\hat{0}}\right)$$

Usually quantified via the **redshift**

$$1 + z \equiv \Omega_{\mathrm{emit}}/\Omega_{\mathrm{obs}}$$

$$\boxed{z = \frac{n^i n^j}{2\left(1 - n_k q^k\right)}\left(h_{ij}(\mathrm{E}) - h_{ij}(\mathrm{S})\right)}$$



The redshift depends on both the **Earth** and **pulsar** term

The pulse time-of-arrivals are changed, the induced **timing residual** is given by

Book & Flanagan (2011)

$$R(t) = \int^t z(t')\,\mathrm{d}t'$$ - This will be important later

The Astrometric Effect of GWs

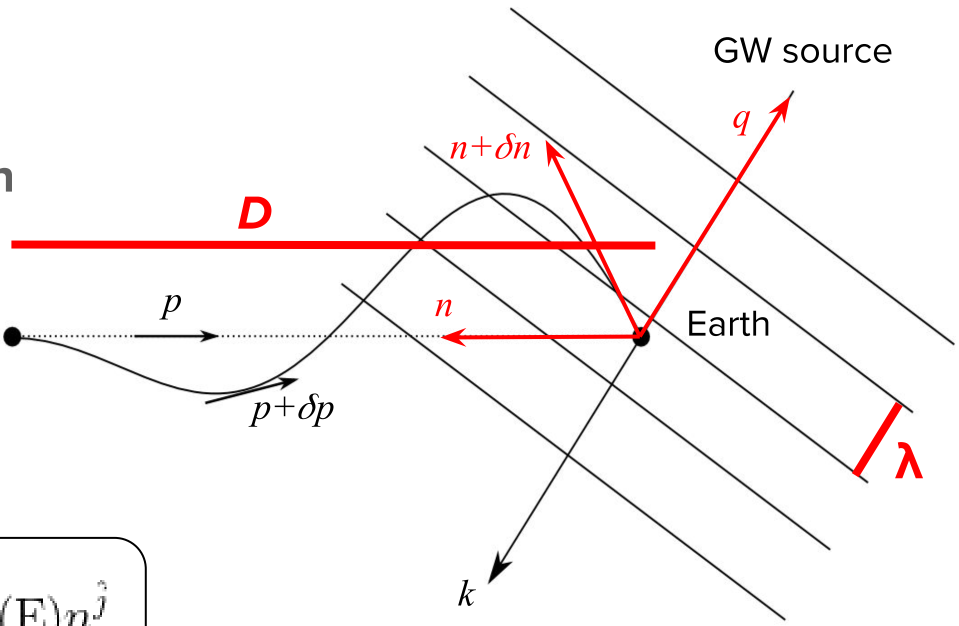
The effect on the **astrometric position**

$$n_{\hat{i}} = \frac{p_\mu \epsilon^\mu_{\hat{i}}}{\Omega} \rightarrow \frac{(p_\mu + \delta p_\mu)(\epsilon^\mu_{\hat{i}} + \delta\epsilon^\mu_{\hat{i}})}{\Omega/(1+z)}$$

The deflection also depends on the **Earth** and **star** term, but not symmetrically

At leading order in (λ/D) the deflection simplifies, and depends only on the **Earth** term

$$\delta n_{\hat{i}} = \frac{n_{\hat{i}} - q_{\hat{i}}}{2(1 - n_{\hat{k}} q^{\hat{k}})} h_{\hat{j}\hat{k}}(\mathrm{E}) n^{\hat{j}} n^{\hat{k}} - \frac{1}{2} h_{\hat{i}\hat{j}}(\mathrm{E}) n^{\hat{j}}$$



Result can be straightforwardly generalised to cosmological FRW spacetimes provided GW wavelength is much less than horizon scale

The Astrometric Effect of GWs

Randomly distributed on the sky are 2000 stars

Orthographic projections of the Northern (**top**) and Southern (**bottom**) hemispheres

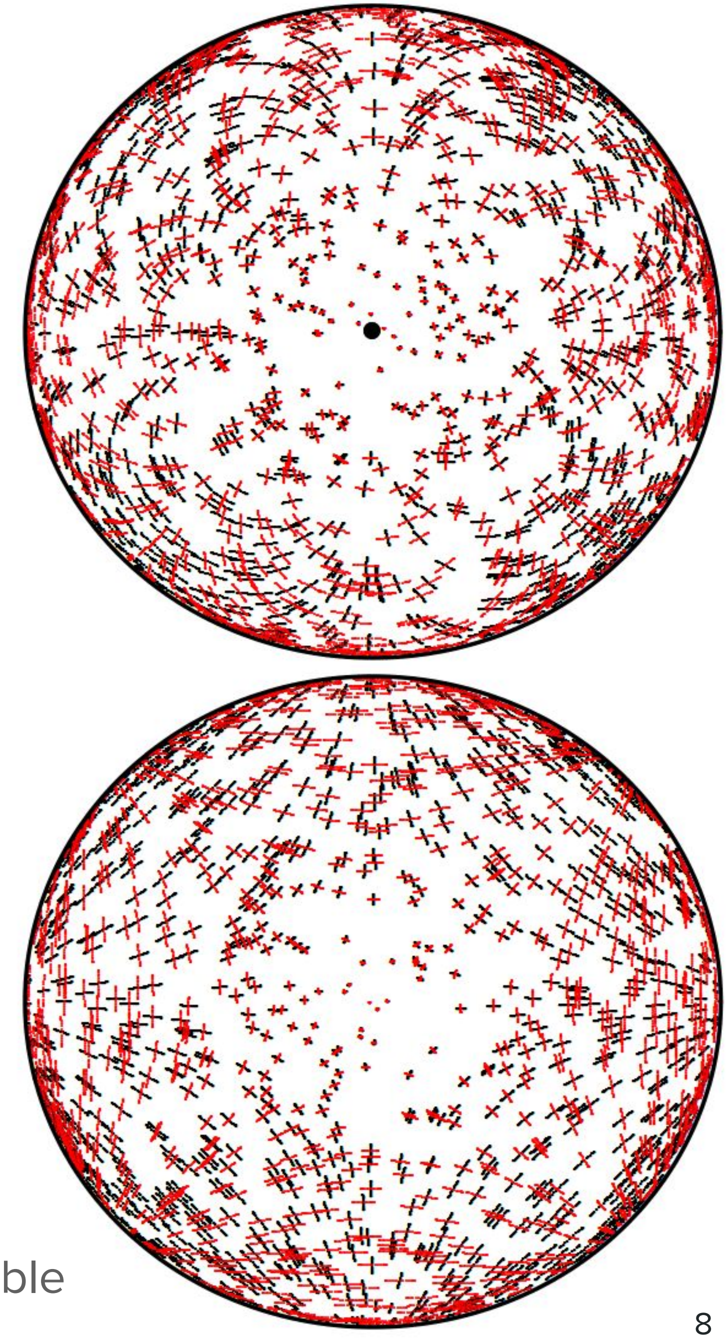
A GW is incident on the Earth from the North pole (Indicated by the black dot •)

The GW is linearly polarised (**plus** and **cross**)

Stars move periodically back and forth at the GW wave frequency

The typical size of the angular deflections (in radians) is the GW strain amplitude.
For clarity, the incident GW has amplitude $A=0.1$

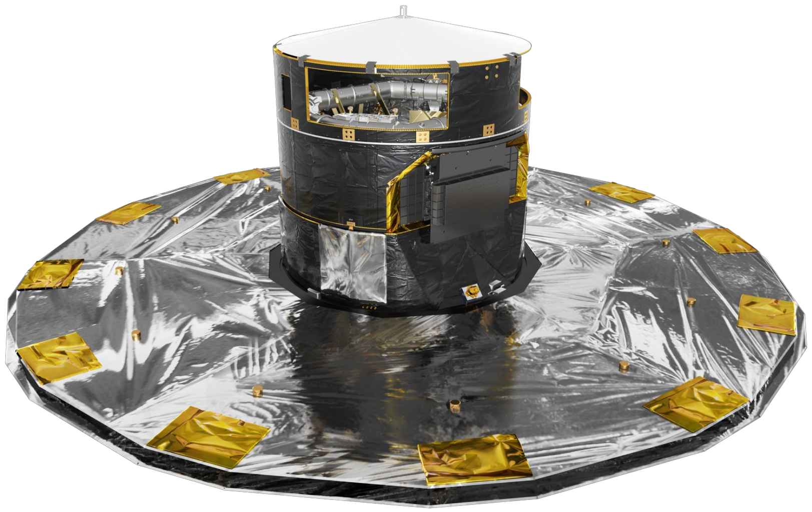
The 4-fold, spin-2 symmetry of the GW is clearly visible



Introducing Gaia

ESA space-astrometry telescope (natural successor to Hipparcos)

Launched in 2014, currently orbiting about the L2 point



A 5-10 year mission, aiming to build both astrometric and photometric maps of over one billion stars

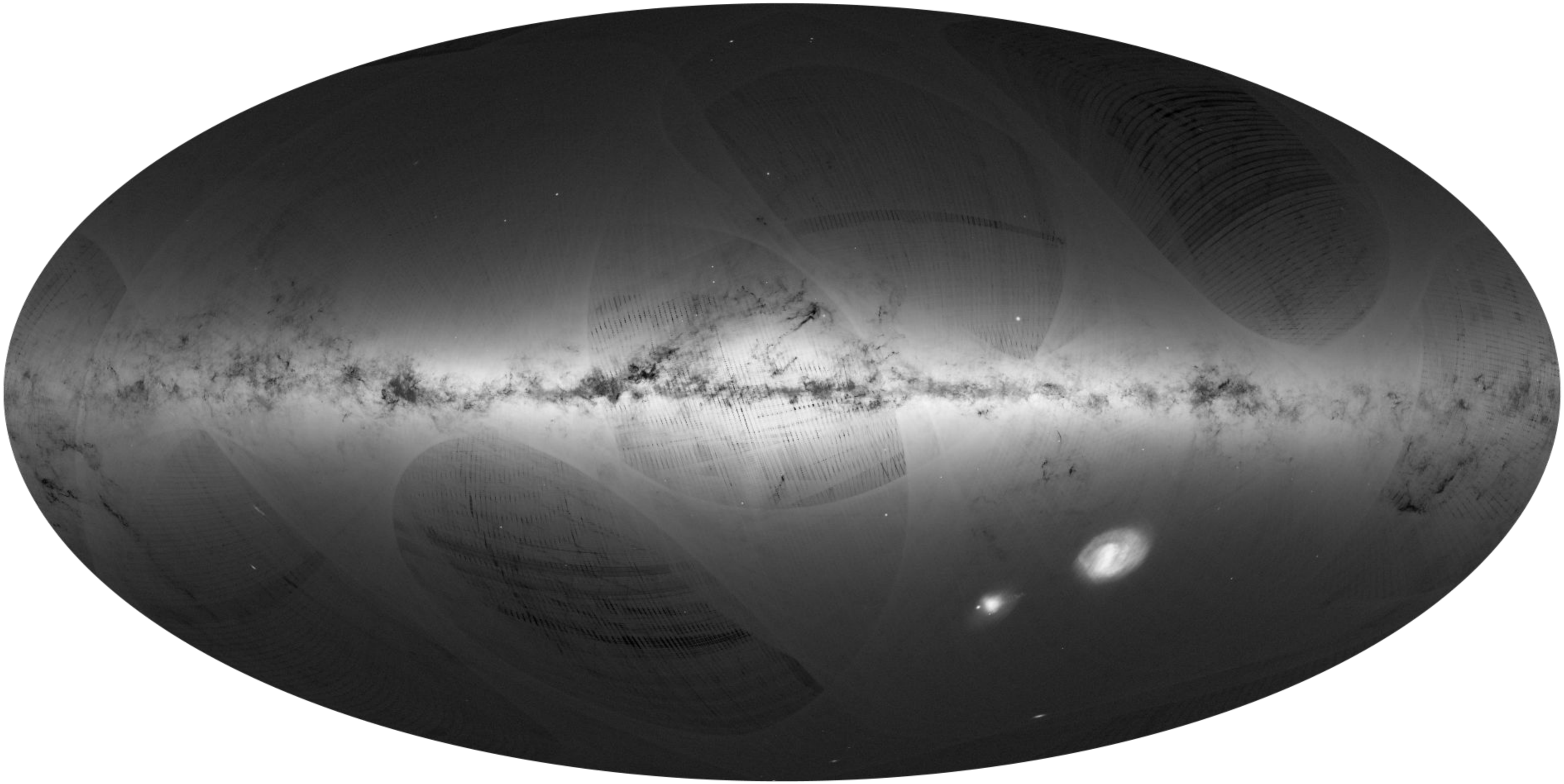
Takes around 80 observations of each star over a 5 year period

Astrometric accuracies down to a few tens of μas★

Wikipedia Fun Fact:

★ A microarcsecond (μas) is about the size of a period at the end of a sentence in the Apollo mission manuals left on the Moon as seen from Earth

Introducing Gaia: 1st data Release, September 2016



GW sources for Gaia

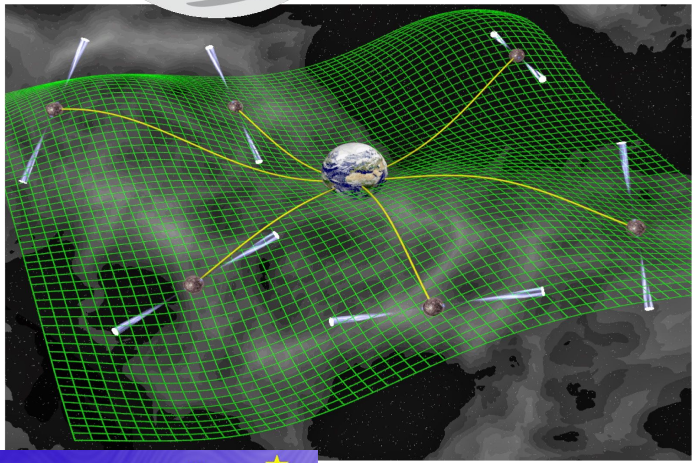
The basic 5 year Gaia mission makes around 80 measurements of each star

The timing, i.e. cadence and total observation time, set the sensitive bandwidth to GWs $(10^{-8} - 3\times10^{-7})\,\mathrm{Hz}$

Obvious similarities in bandwidth and target sources between **Gaia** and **PTAs**

Individually resolvable BBHs with total mass $(10^{7}\text{-}10^{10})M_{\odot}$ in early inspiral stage

Stochastic background formed from the superposition of many such binaries



GW data analysis for Gaia

A simplified astrometric data set consists of *N* position measurements of *M* stars

$$\mathcal{S} = \{\vec{s}_{I,J} | I = 1, 2 \ldots M; J = 1, 2 \ldots N\}$$

Each measurement contains the background star position, noise, and (possibly) a GW

$$\vec{s}_{I,J} = \vec{n}_I(t_J) + \vec{r}_{I,J} + \vec{h}_{I,J}(\Psi)$$

Stars move! For each star a quadratic is fit to the data and subtracted (c.f. pulsar timing model)

$$\vec{s}_{I,J} \rightarrow \vec{s}_{I,J} - \underline{n}_I(t_J)$$

Noise is assumed to be identical, independent and Gaussian in each measurement

$$\mathrm{Ex}\left[\vec{r}_{I,J} \cdot \vec{r}_{I',J'}\right] = \sigma^2 \delta_{II'} \delta_{JJ'}$$

The **likelihood** may be written as... (where |●| denotes norm of a vector on the sphere)

$$P(\mathcal{S}|\vec{\Psi}) \propto \exp\left(-\sum_{I,J} \frac{|\vec{s}_{I,J} - h_{I,J}(\Psi)|^2}{2\sigma^2}\right)$$

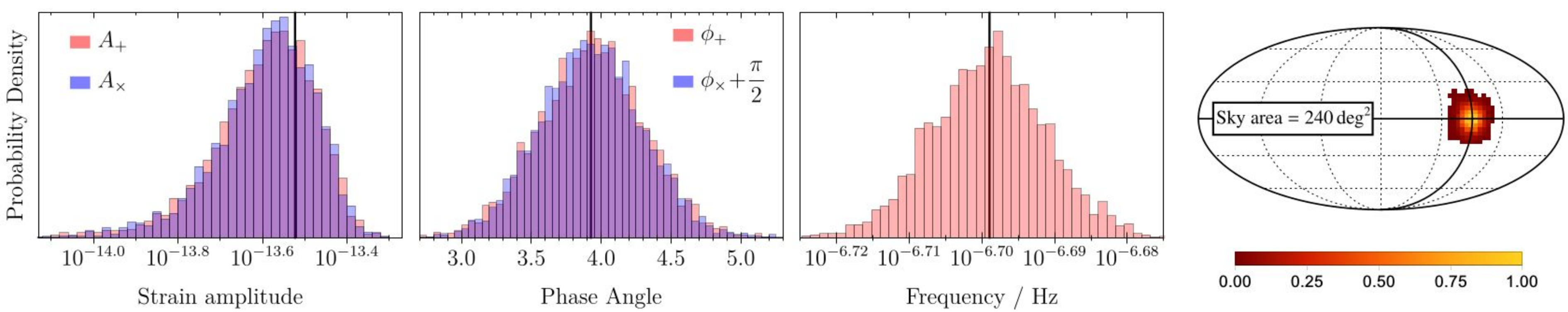
GW Parameter Estimation with Gaia

Mock data representative of the final Gaia data release was generated

Injected GW from a supermassive BH binary

System was assumed to be an equal mass binary with total mass $10^9 M_\odot$ and orbital frequency 1.8×10^{-8} Hz at 20 Mpc, viewed *face on*. This gives a circularly polarised GW with $\omega_{GW} = 2\times10^{-7}\ \mathrm{s}^{-1}$, and strain amplitudes $A_+ = A_\times = 3\times10^{-14}$

1-D marginalised posteriors on gravitational wave parameters are shown (black lines indicate injected values)



Bayes' factor $\mathcal{B} = 10^{4.20}$

GW data analysis for Gaia: Compression

The data set is huge! For Gaia $M \simeq 10^9$, $N \simeq 10^2$

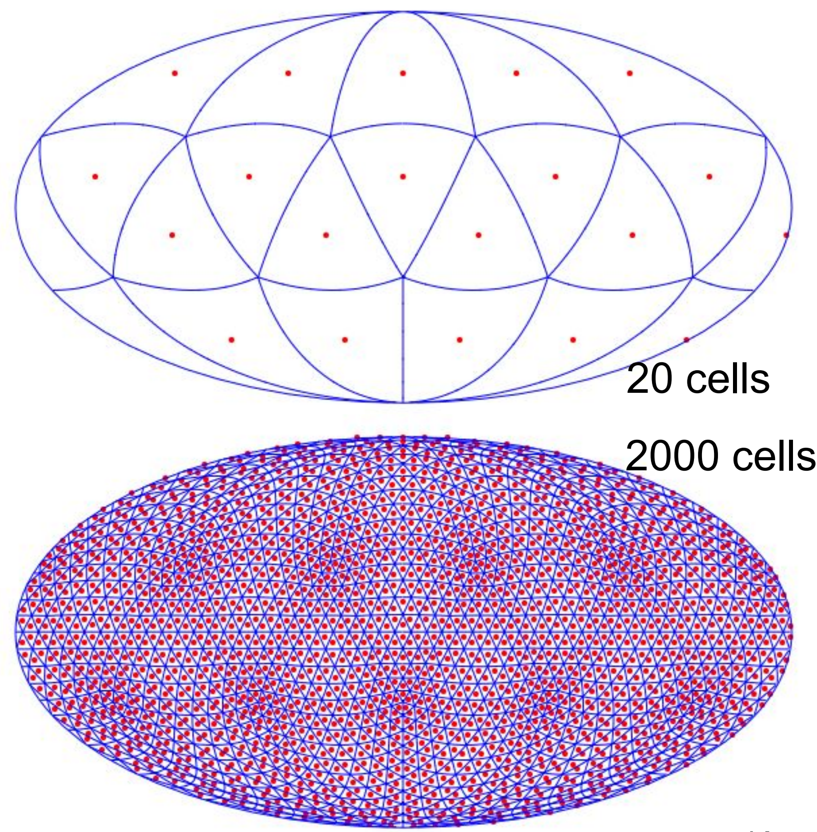
$$P(\mathcal{S}|\vec{\Psi}) \propto \exp\left(-\sum_{I,J} \frac{\left|\vec{s}_{I,J} - h_{I,J}(\Psi)\right|^2}{2\sigma^2} \right)$$

The likelihood is a sum over both M and N, and this is computationally expensive

Nearby stars have parallel deflections

Compress the data into a **virtual** data set with $\tilde{M} \ll M$ stars

$$\tilde{\mathbf{s}}_{\tilde{I},J} = \frac{1}{|\mathcal{V}_{\tilde{I}}|} \sum_{I \in \mathcal{V}_{\tilde{I}}} \mathbf{s}_{I,J} \qquad \frac{1}{\tilde{\sigma}^2_{\tilde{I},J}} = \sum_{I \in \mathcal{V}_{\tilde{I}}} \frac{1}{\sigma^2_{I,J}}$$



The compression would be lossless if

(i) the noise was independent and Gaussian

(ii) if all stars in a cell have parallel deflections

GW data analysis for Gaia: Compression

A mock data set with $M = 10^5$ stars takes 2-3 days to search with the techniques described

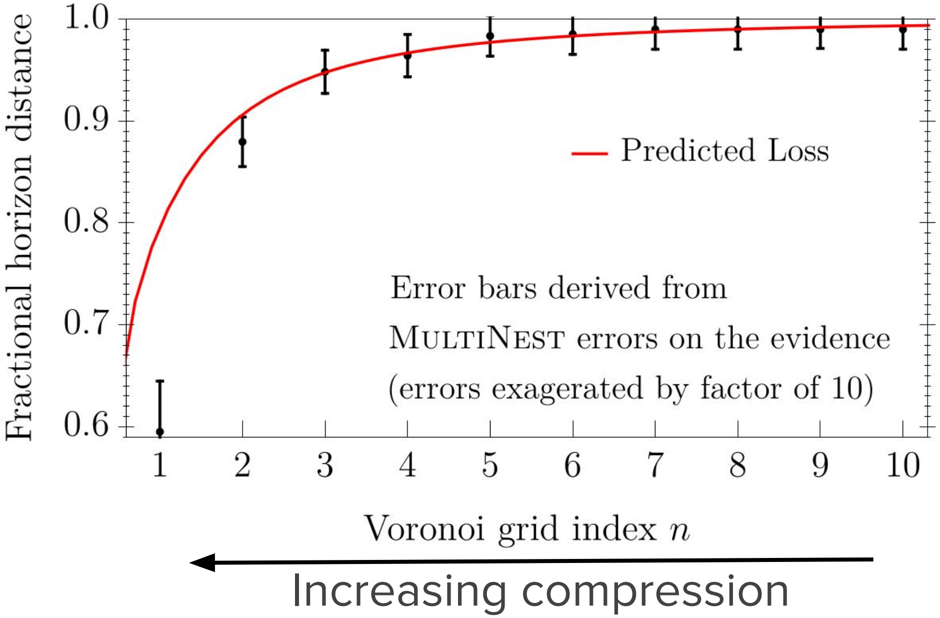
This data was compressed onto a series of grids of varying resolution

The recovered Bayes' factors are lower, i.e. sources must be closer for detection

Horizon distance estimated as distance which gives a Bayes' factor of $10^{1.5}$

We can compress Gaia's $>10^9$ real stars into $\simeq 10^3$ virtual stars with a loss of horizon range of $<1\%$

The $M = 10^5$ data set gave a large Bayes' factor of $10^{4.2}$



The n^{th} grid contains $\tilde{M} = 20n^2$ cells

Gaia's frequency sensitivity

Number of stars $= 10^9$
Number of measurements $= 75$
Mission lifetime $= 5$ years
Noise in each measurement $= 100\,\mu$as

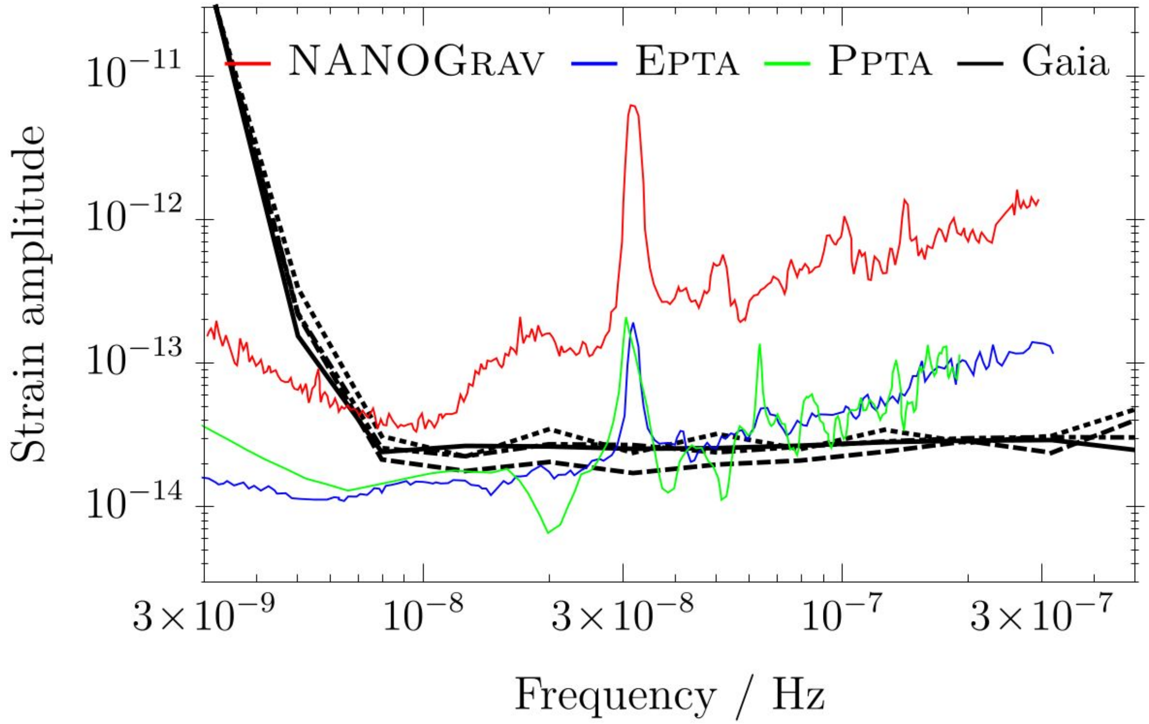
Mock Gaia data created with injections at different amplitudes and frequencies

Critical amplitude for detection estimated as the amplitude that gave $\mathcal{B}=10^{1.5}$

Different black curves use different Gaia time samplings

Gaia has a **flat sensitivity curve** for $f \gtrsim 10^{-8}$ Hz

Gaia complementary to pulsar timing efforts, especially at higher frequencies



Arzoumanian *et al*. (NANOGrav) ApJ 794, 141 (2014)
Babak *et al*., (EPTA) MNRAS 455, 1665 (2016)
Zhu *et al*., (PPTA) MNRAS 444, 3709 (2014)

Gaia’s directional sensitivity

Formula for the astrometric deflection ➔

$$\delta n_{\hat{i}} = \frac{n_{\hat{i}} - q_{\hat{i}}}{2(1 - n_{\hat{k}} q^{\hat{k}})} h_{\hat{j}\hat{k}}(\mathrm{E}) n^{\hat{j}} n^{\hat{k}} - \frac{1}{2} h_{\hat{i}\hat{j}}(\mathrm{E}) n^{\hat{j}}$$

Stars collocated with (or antipodal to) the GW source have no astrometric deflection

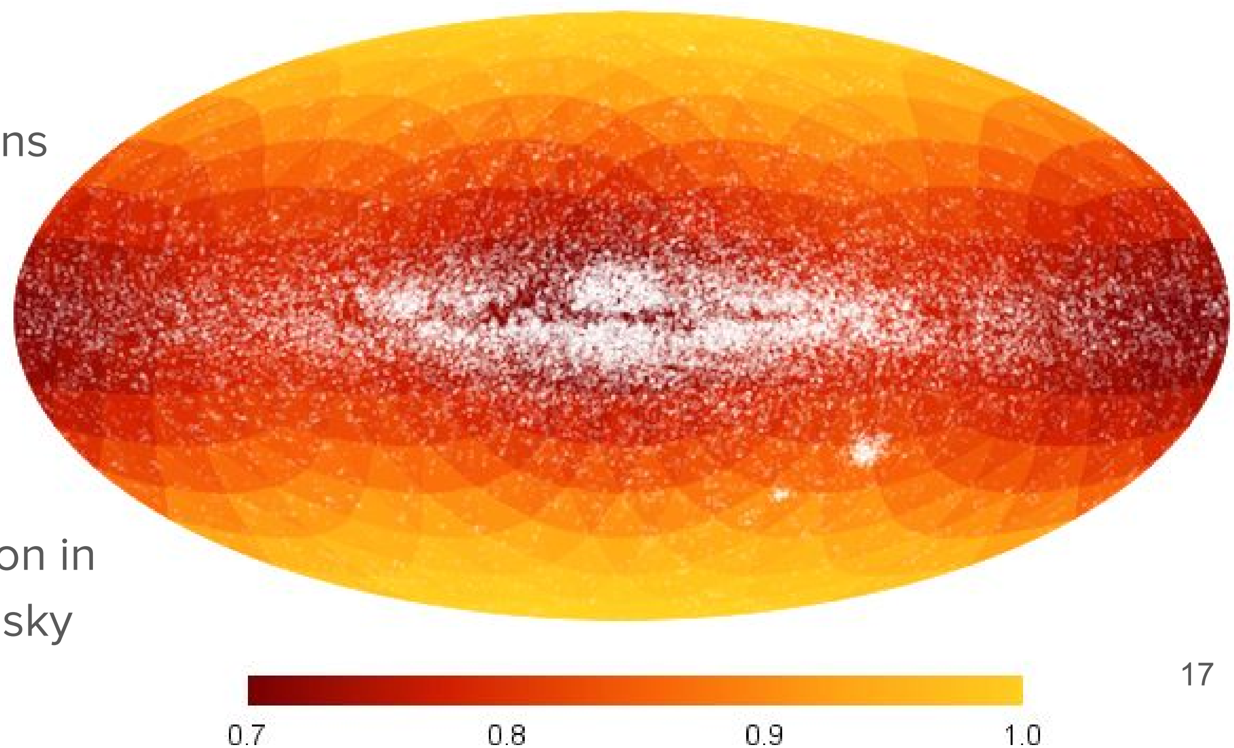
$$\text{Let } \vec{q} \cdot \vec{n} = \cos\gamma$$
$$|\delta\vec{n}| \propto \sin\gamma$$

Conversely, stars at $90°$ to the GW source have maximum deflections

Mock data using the locations of the $1.14{\times}10^{9}$ stars in **Gaia DR1** catalogue

Inject GW sources at 500 sky locations

Plot shows fractional variation in the **horizon range** over the sky

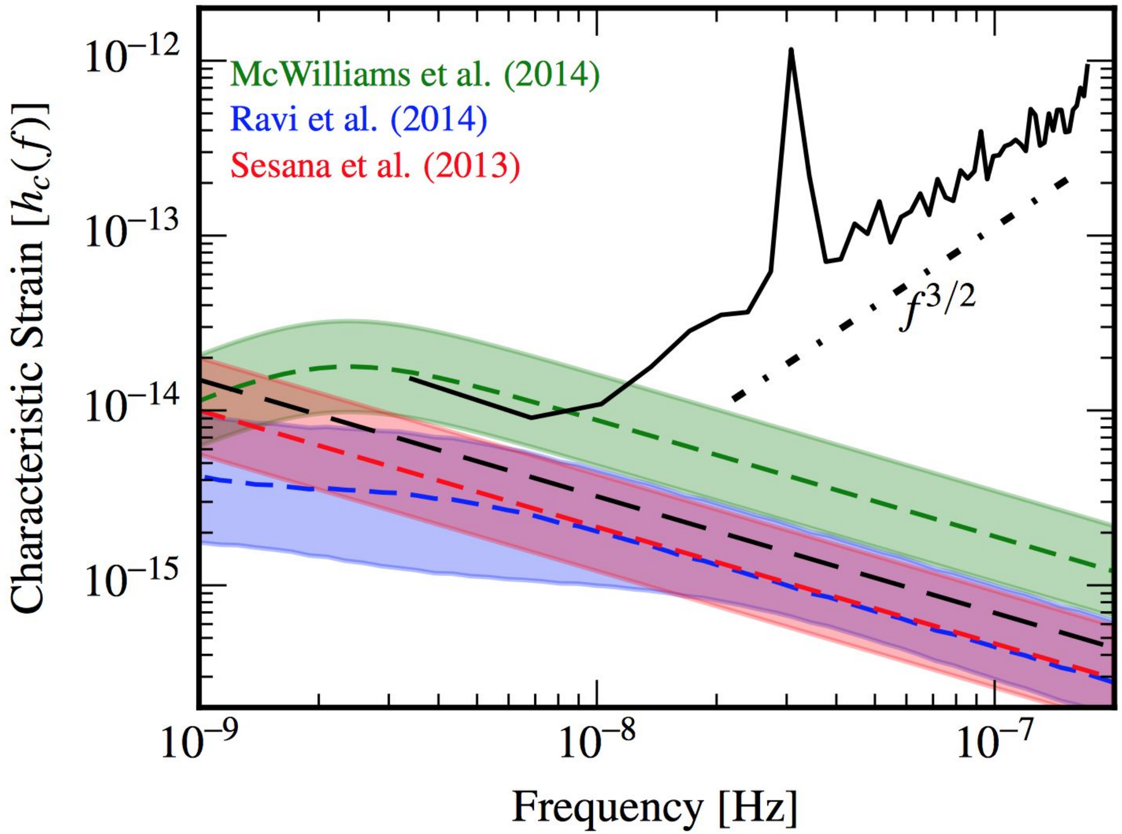


Stochastic Background of GWs

For PTAs there is a good chance that the first detection will not be a single source, but a red **stochastic background**

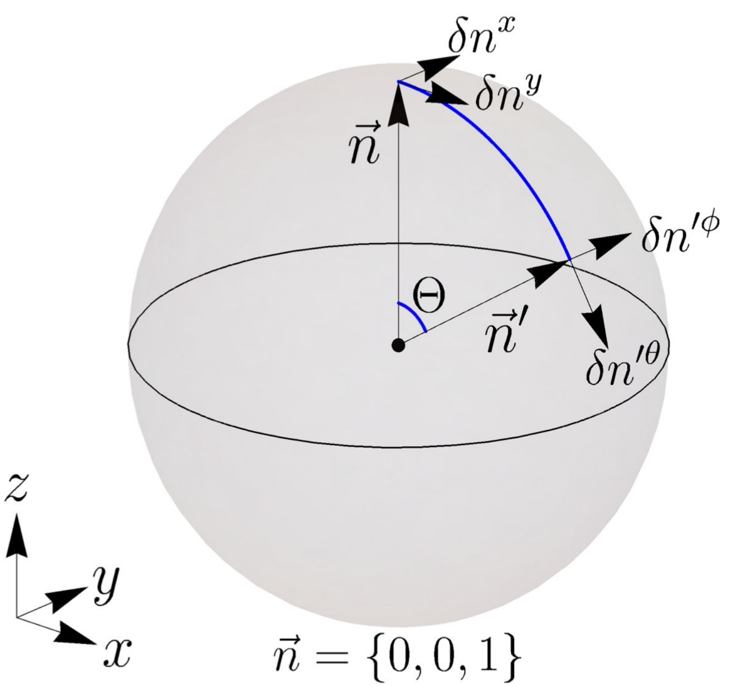
Pulsar timing residuals now form a correlated Gaussian process on the sky, with a characteristic correlation function

Upper limits placed by all three PTA collaborations



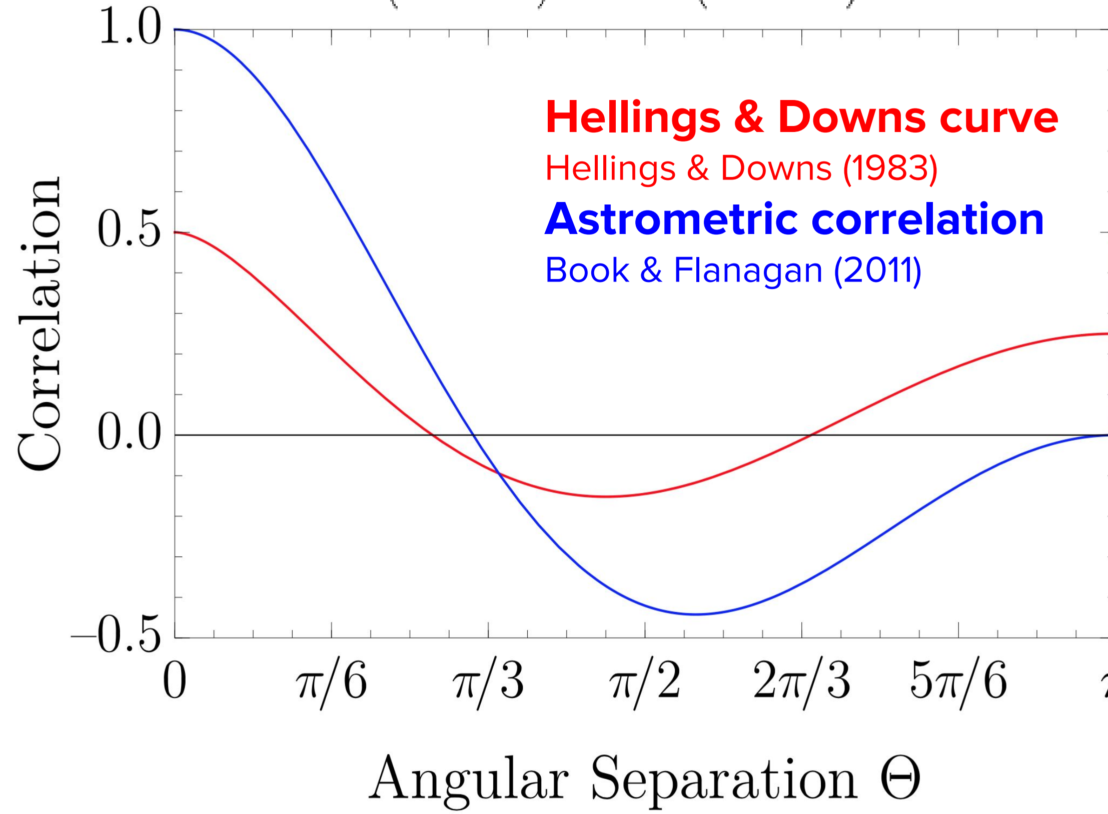
Arzoumanian et al. 2015 (NANOGrav)

Stochastic Background of GWs



$$\vec{n} = \{0, 0, 1\}$$
$$\vec{n}' = \{\sin(\Theta), 0, \cos(\Theta)\}$$

$$\mathrm{corr}\left(\delta n^x \delta n'^\theta\right) = \mathrm{corr}\left(\delta n^y \delta n'^\phi\right) = \mathrm{curve}$$
$$\mathrm{corr}\left(\delta n^x \delta n'^\phi\right) = \mathrm{corr}\left(\delta n^x \delta n'^\phi\right) = 0$$



Stochastic Background of GWs

Draw 2 realisations of stochastic background on the sky

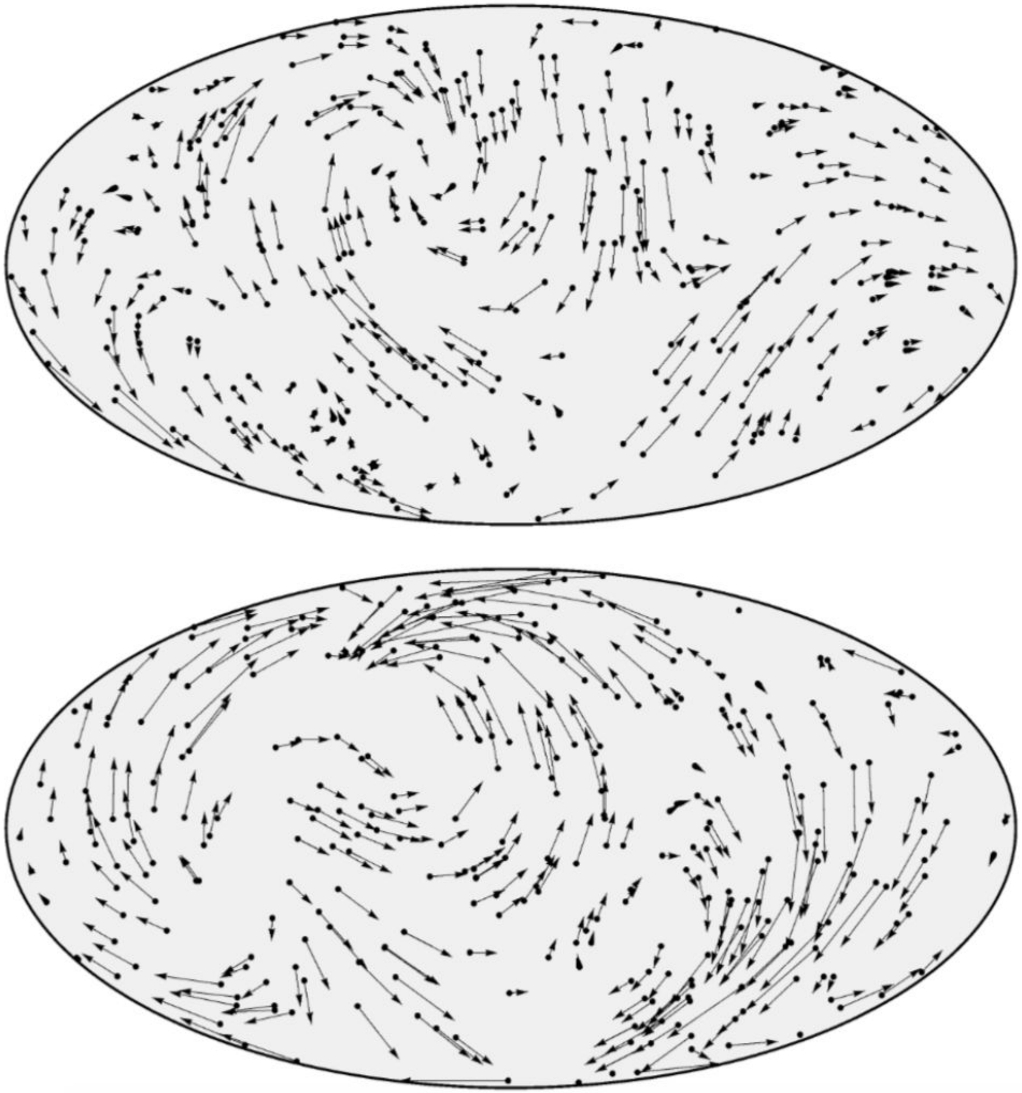
Deflection patterns shown in **Mollweide projection**

Astrometric deflection pattern varies smoothly over the sky

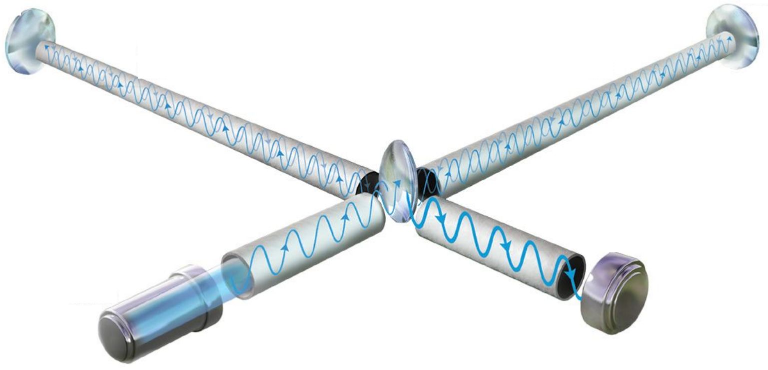
Characteristic **curl** pattern

Reminiscent of claimed BICEP2 pattern, but on large angular scales

(Quadrupolar, $l=2$, $\theta \simeq 90\,\mathrm{deg}$)

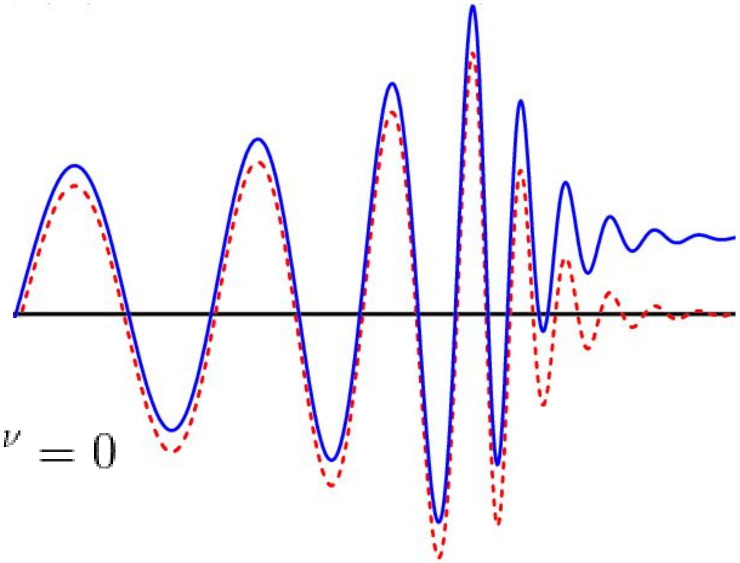


The astrometric signature of GW memory



Normally described in terms of the effect on two freely falling test masses

A gravitational wave burst causes the separation of two masses to oscillate (distance measured via interferometry)



The initial and final separations are not the same

One way to describe this is as a permanent change to the metric

$$g_{\mu\nu}(t \to -\infty) = \eta_{\mu\nu}$$
$$g_{\mu\nu}(t \to +\infty) = \eta_{\mu\nu} + h_{\mu\nu}$$

where

$$h_{0\mu} = h_{\mu\nu} k^{\nu} = 0$$
$$h^{\mu}_{\mu} = 0$$

The astrometric signature of GW memory

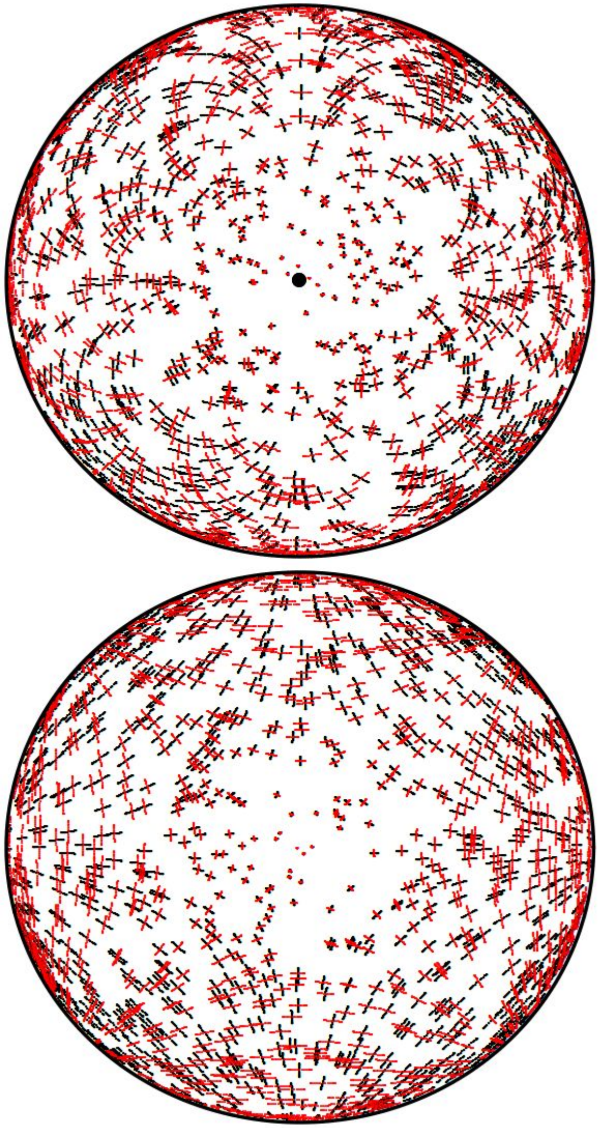
As the GW burst passes the astrometric positions oscillate at the GW frequency

If the GW burst has memory the astrometric positions change permanently

As a consequence, an initially uniform distribution of distant objects (e.g. quasars) Becomes non-uniform

$$\rho \to \rho' = \rho - \nabla_{S_2} \cdot \vec{\delta n}$$

If we assume initial isotropy then astrometric measurements provide a way, at least in principle, to measure the memory effect from a single, late time measurement



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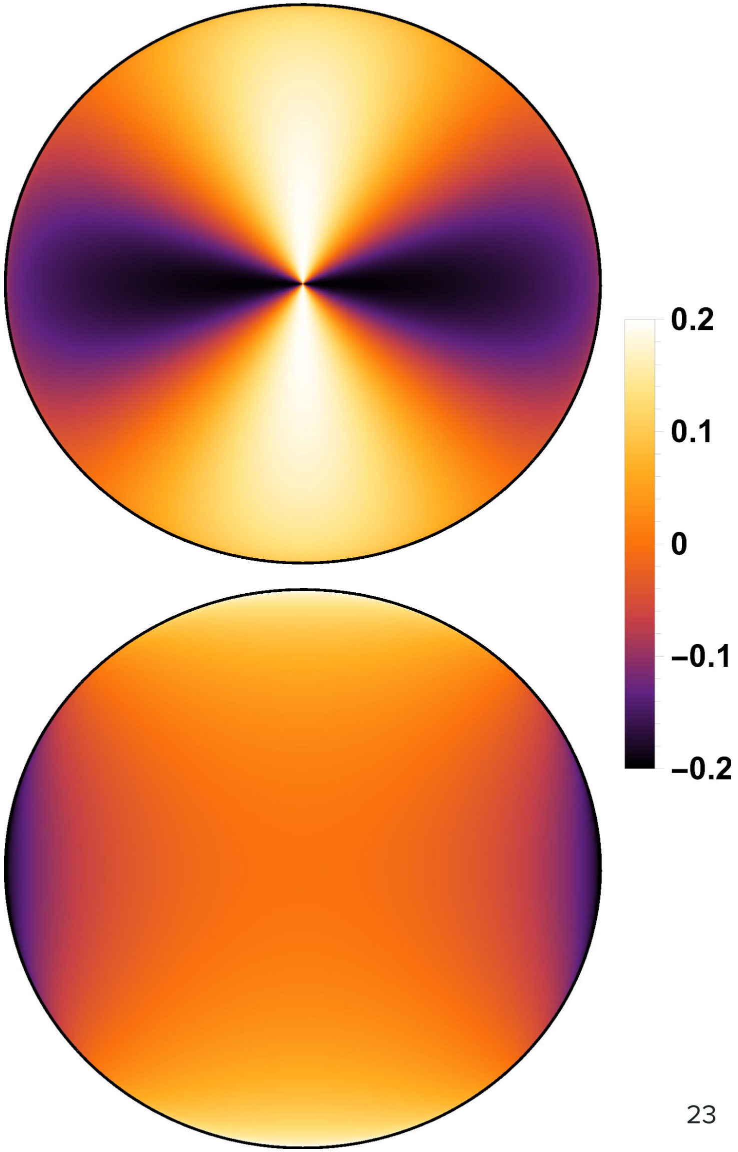


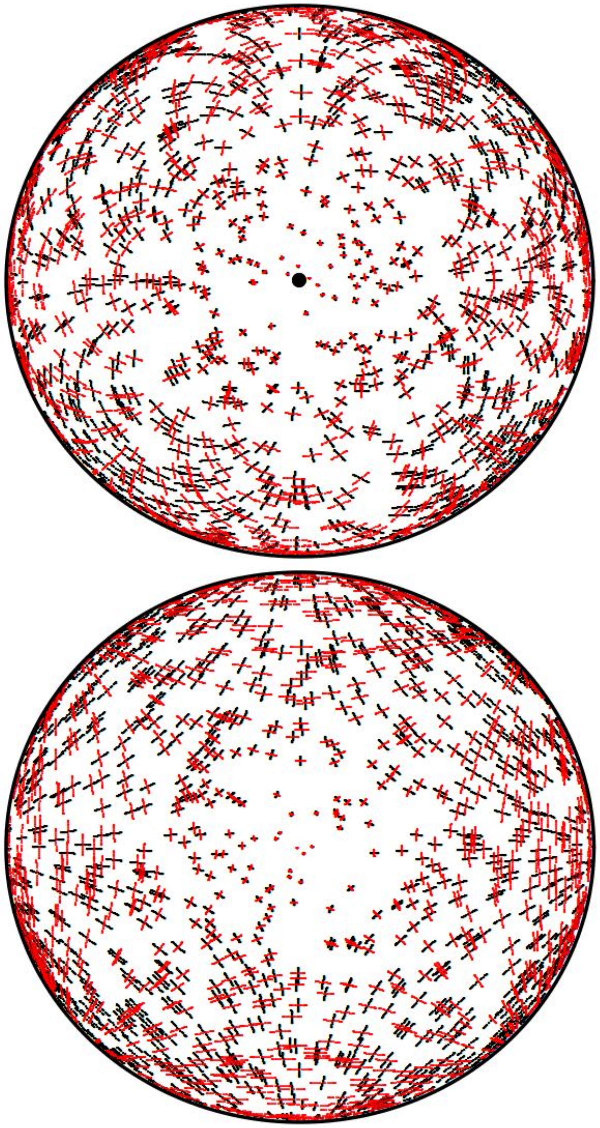
Conclusions

Gravitational waves change the apparent position of stars

Astrometry with Gaia provides a new way to search for GWs with frequencies down to $10\,\mathrm{nHz}$

Work on Gaia data analysis pipeline is progressing, we have demonstrated how to compress the data and search efficiently for individual sources

Gaia is complementary to pulsar timing across a range of frequencies, but best contributions will be at “high” frequencies



A comment on correlated noise

Two correlations to worry about: **temporal** and **spatial** $\mathrm{Ex}\left[\mathbf{r}_{I,J} \cdot \mathbf{r}_{I',J'}\right] = \sigma^2 \delta_{II'} \delta_{JJ'}$

Temporal correlations:

These are a major factor in PTA analysis ("**red noise**")

There is no intrinsic process to the star which can shift the astrometric position

Not a major concern for Gaia. In between successive measurements (typically several weeks) the spacecraft rotates, and each measurement uses different part of CCD array

Spatial correlations:

These do exists in the Gaia data, and they reduce the effective number of independent stars

However, only slightly. Correlations are $\sim 3\%$ for colocated stars, decreasing to zero correlation for stars separated by $0.7°$