Developing a Phenomenological Frequency-Domain Waveform Model for Black Hole Kicks

Interim Report Two
(Dated: August 8, 2017)

Objective: We aim to develop a kicked gravitational waveform model in the frequency-domain that can be used to place projected constraints on measurements of kick velocities with future ground- and space-based gravitational wave detectors.

I. BACKGROUND

Black hole binaries are generically expected to contain black holes that are unequal in both mass and spin that result in a high amount of asymmetry. The gravitational waves (GWs) that are emitted during the evolution of these asymmetric systems are then radiated anisotropically and are beamed in some preferential direction. Thus, there is a net linear momentum flux at infinity that, by conservation of momentum, causes the binary to recoil in the direction opposite to that of the beamed GWs. This effect is present throughout the entirety of the binary evolution but becomes significant during *very* late inspiral and merger-ringdown, causing the remnant black hole to recoil significantly at some velocity. This velocity is called the recoil (or kick) velocity [1] and results in a process known as a "black hole kick."

It was recently determined that black hole kicks could be directly detected using space-based and future ground-based GW detectors [2]. In the case of space-based instruments, these measurements could be of particular importance to event rates for supermassive black hole binaries mergers if the remnant black holes of these mergers receive kicks that exceed the escape velocity of the host galaxy [3]. In this project, we hope to place constraints on how well we could measure black hole kicks using GW observations.

II. APPROACH

In order to determine how well we will be able to measure black hole kicks, we need to begin with a GW waveform that encapsulates the redshifting (blueshifting) of the GWs emitted from binaries during inspiral, merger, and ringdown. Ref. [2] provides a model for the kick velocity as a function of *time*. However, we need the velocity as a function of frequency such that it can be incorporated into frequency-domain waveforms, as these approximates are fast and allow us to study parameter space. In particular, we have an analytic phenomenological frequency-domain waveform, IMRPhenomD [4–6], that we need to modify to include black hole kicks. This can be done by increasing (decreasing) the mass of the binary and increasing (decreasing) the luminosity distance as a function of velocity in the frequency-domain; this is equivalent to red (blue) shifting the GWs emitted from the binary as a function of the kick velocity.

Once we have a kicked frequency-domain waveform, we can perform a Fisher Analysis to determine constraints on these kicks. However, this study requires derivatives of the frequency-domain waveform with respect to binary intrinsic and extrinsic parameters, as well as the kick velocity. In order to complete this analysis with the highest accuracy, we require these derivatives to be done analytic. Thus it is important to obtain the kick velocity analytically as a function of frequency and the binary parameters. In order to incorporate the frequency dependance, we need time as a function of frequency so that we can take $v(t) \to v(t(f)) \to v(f)$.

We will perform this Fisher Analysis for various future ground- and space-based instruments and for a variety of binaries such that we are covering large regions of the parameter space. This will allow us to understand how well each detector can measure black hole kicks and can help determine sensitivity requirements for third generation ground-based detectors.

III. PROGRESS

At the beginning of the project, we thought that obtaining the time evolution of the frequency of the GW in PhenomD would be relatively straightforward. It is easy to obtain $f(t)$ given a time-domain waveform by differentiating the argument with respect to time, but we do not have a waveform analytically in the time-domain. Thus we determined that it would be reasonable to complete an approximate analytic inverse Fourier transform (IFT) of our frequency-domain waveform. This can be done simply in some cases using the Stationary Phase Approximation (SPA) [7]. This approach appeared promising, but provided unphysical results in which time did not increase monotonically. We determined that this was due to a number of the assumptions that were made in order to use the SPA. In particular, we determined that after the inspiral regime of PhenomD, the amplitude of the waveform was oscillating too rapidly with respect to the phase for the SPA to be valid. We then explored whether or not our IFT could be completed using an asymptotic technique called the method of steepest descents [8]. However, our integral was too complicated to be expanded in a convergent sum, so the method of steepest descent would not provide an easier way to obtain our time-domain waveform.

We determined that the best way to proceed was to try and calculate the inverse Fourier transform of the

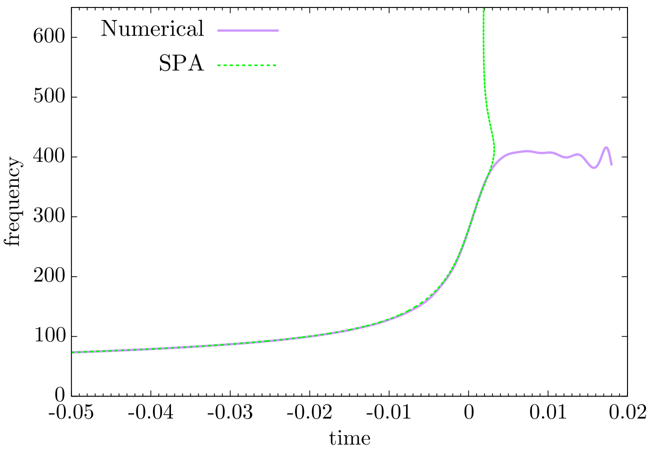

FIG. 1. Plot of $f(t)$ demonstrating the divergent behaviour of the SPA compared to numerical data.

frequency-domain waveform analytically in an attempt to recover the phase in the time-domain. From the phase, we could then take derivate with respect to time to find frequency as a function of time. Initially, we believed that we would only require the Fourier transform in the merger-ringdown section of the waveform. We were able to find an analytic approximant to the Fourier transform in this section, but it did not display the kind of behaviour that we expected the time-domain waveform to exhibit. This forced us to re-evaluate our approach and recognize that we would have to complete the entire Fourier transform analytically in order to recover the expected behaviour.

In order to ensure that the frequency-domain waveform was the same as that provided by the LIGO Algorithm Library (LAL), we determined that we should compare directly the waveforms that are output from LAL and from my own Mathematica script. We considered the PhenomD time-domain output from LAL and performed a numerical inverse Fourier transform of PhenomD to obtain the waveform in the time-domain. We then numerically calculated the phase and it's time derivative through both processes which matched exactly after frequency-domain windowing was implemented in the Mathematica script to reflect that done in LAL.

In an effort to determine why our results using the SPA did not match results from LAL, especially at early times when the SPA is valid, we re-derived the expressions that we were using to calculate $t(f)$ from the frequency-domain using the SPA, and discovered that we had dropped factors of 2π. Upon implementation of the missing factors, the SPA and LAL computations matched exactly for all frequencies below that of ringdown, at which point the SPA breaks down and diverges, as can be seen in Figure 1. Thus, the frequency as a function of time can be represented using the SPA for $f < f_{\rm RD}$, where $f_{\rm RD}$ is the ringdown frequency, and as some calculated final frequency for $f > f_{\rm RD}$. This prescription for $t(f)$ is thus analytic and can be implemented into the frequency-domain directly.

IV. FUTURE DIRECTIONS

In the future, we plan to:

- test that kicks in the frequency and time-domain yield the same expected behaviour
- modify PhenomD waveform to include precession
- confirm that kicks in the frequency-domain behave appropriately for this new waveform
- perform Fisher Analysis to determine projected constraints that can be placed on the velocity of a black hole kick for thousands of sources.

This analysis should lead to publishable results.

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