Towards a Phenomenological Frequency-Domain Waveform Model for Black Hole Kicks

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Abstract: Generic black hole binaries emit gravitational waves anisotropically due to mass and spin asymmetries. Gravitational waves carry linear momentum away from the binary in a preferential direction which causes the binary to recoil during the late inspiral and merger phases of evolution. Black hole recoils (or kicks) result in emitted gravitational waves that are Doppler shifted during merger and ringdown. Gravitational wave observations of such a Doppler shift will allow for the first direct detections of black hole kicks. We extend existing phenomenological analytic frequency-domain waveform approximants to model gravitational waves from a kicked back hole binary. Our kicked frequency-domain model is quick to calculate and can be used to (i) explore kick detectability over large regions of parameter space, (ii) address possible degeneracies between kicks and other binary parameters, and (iii) place projected constraints on black hole kick velocities with current and future detectors.

I. INTRODUCTION

Generic black hole binary systems are expected to consist of two black holes with unequal masses and nonzero spins. As a result, generic binaries will contain some amount of asymmetry that leads to an anisotropic emission of gravitational waves throughout the binary evolution. Gravitational wave beaming leads to a net linear momentum flux that forces the binary to recoil by conservation of momentum. This recoil, or “kick,” is present throughout the duration of the binary’s evolution and becomes significant during the late inspiral and merger phases.

As gravitational waves are emitted from a kicked binary during the final stages of coalescence, they are Doppler shifted along the line of sight. Black hole kicks are only significant at the end of the evolution of the binary, so gravitational waves that are emitted during the early inspiral will not be significantly Doppler shifted. Thus, there will be a relative dephasing of the resulting kicked waveform from the “non-kicked” waveform one would expect to measure from a binary system at rest with the same intrinsic properties.

If the kick velocity is large enough that the kicked waveform deviates significantly from the non-kicked waveform, and if gravitational wave detectors have a high enough signal-to-noise ratio (SNR) during the merger-ringdown phases, then black hole kicks should be resolvable. Indeed, Gerosa and Moore [1] determined that black hole kicks, in principle, could be directly detected using space-based and future ground-based gravitational wave detectors.

The prospect of a black hole kick detection is enticing for a number of reasons. Electromagnetic candidates have been identified that display certain signatures of black hole kicks [2, 3], but there are still no unambiguous detections. For instance, general relativity (GR) predicts that gravitational waves carry linear momentum, but this prediction has yet to be tested. Gravitational waves can provide the first direct detections of black hole kicks, further testing and confirming aspects of GR in the strong-field regime.

Though we expect that gravitational wave detectors will be able to detect black hole kicks, there have been no studies to date about potential degeneracies between kicks and other binary parameters. In order to understand how well we will be able to distinguish black hole kicks, it is important to address these degeneracies. However, in order to address degeneracies and determine how well we could constrain black hole kicks using gravitational wave detections, we need a kicked frequency-domain waveform that is quick to calculate and that will allow us to explore a large region of parameter space. Currently, frequency-domain waveform approximants contain no information amount black hole kicks. With an analytic frequency-domain waveform approximant, we will be able to explore degeneracies and constraints through a Fisher analysis.

This project resulted in the development of such an analytic kicked frequency-domain waveform approximant that will be used to place constraints on black hole kick velocities and explore possible degeneracies present in the waveform. The remainder of this paper will discuss the methods by which this waveform was developed, present current results, and provide some future directions for this research.

II. APPROACH AND METHODS

In order to create a kicked frequency-domain waveform, we first have to understand how a black hole kick will effect the emitted gravitational waves. The black hole binary will be moving with some non-constant kick velocity throughout the coalescence of the binary, therefore the kick velocity is a function of time. Fortunately, numerical relativity simulations suggest that kick velocities can be represented in the time domain generically as Gaussian (in the acceleration) with a width that is proportional to the total mass of the system [4, 5]. This observation considerably simplifies gravitational wave modeling and will be used as a working assumption.

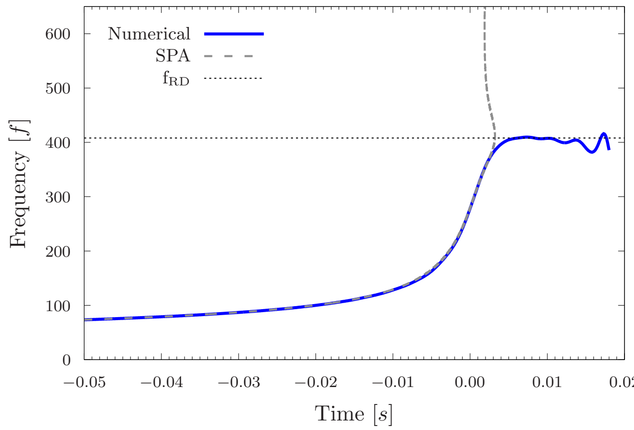

FIG. 1. Plot of frequency as a function of time. This demonstrates the divergent behavior of the analytic SPA (dashed grey line) for frequencies greater than the ringdown frequency (horizontal dotted line). The numerical results (solid blue line) for frequency as a function of time are plotted for comparison.

In order to develop a kicked waveform model, we introduce the kick velocity into the waveform through a Doppler shift of the mass. The mass is the only binary parameter that is degenerate with redshift and is thus the parameter that will contain this kick information. However, in order to be implemented into a frequency-domain waveform approximant, we need the kick velocity as a function of frequency instead of time. To do this, it is sufficient to find a relationship between time and frequency to obtain the velocity as a function of time. The waveform that we are interested in modifying to include black hole kicks is the nonprecessing spin-dependent inspiral-merger-ringdown frequency-domain waveform approximant PhenomD [6, 7].

A. Stationary Phase Approximation

It is instructive to begin with an overview of the relationship between time and frequency for time-domain waveforms[1]. Let $h(t) = Ae^{i\phi(t)}$ be a gravitational waveform in the time-domain. The frequency as a function of time is defined by

$$f(t) \equiv \frac{1}{2\pi}\frac{d}{dt}\phi(t). \tag{1}$$

If we had an analytic time-domain waveform, it would be simple to calculate frequency as a function of time. However, we are using frequency-domain waveforms that are fast to calculate and allow us to explore kick detectability for a variety of different noise curves.

Let us now define our frequency-domain waveform as $\tilde{h} = \mathcal{A}e^{i\Psi(f)}$. In the frequency-domain, the phase is given as [8, 9]

$$\Psi(f) = 2\pi f t_*(f) - \phi(t_*(f)) + \frac{\pi}{4} \tag{2}$$

where t_* is some constant time about which the integrand of the Fourier transform was expanded.

Differentiating this with respect to frequency, we obtain

$$\frac{d\Psi(f)}{df} = 2\pi t_*(f) + 2\pi f\frac{dt_*}{df} - \frac{d\phi(t_*)}{dt_*}\frac{dt_*}{df} \tag{3}$$

which can be reduced using Eq. 1 to

$$\frac{d\Psi(f)}{df} = 2\pi t_*(f). \tag{4}$$

We can then solve Eq. 4 for $t_*(f)$ which gives

$$t_*(f) = \frac{1}{2\pi}\frac{d\Psi(f)}{df}.$$

A mathematical asymptotic analysis technique called the stationary phase approximation (SPA) allows for the parametrization of time as a function of frequency[2] [9, 10]. Thus, in the SPA, $t_*(f) = t(f)$ and we obtain an analytic function for time as a function of frequency as given by

$$t(f) = \frac{1}{2\pi}\frac{d\Psi(f)}{df}. \tag{5}$$

B. Implementation

To check that the SPA reproduces numerical results, we performed a numerical inverse Fourier transform of the frequency-domain PhenomD waveform. We then calculated the phase from the h_+ and $h_\times$ components of the time-domain waveform to obtain $f(t)$ from Eq. 1. The comparison between the SPA and the numerical calculation is shown in Fig. 1. Where the amplitude begins changing more rapidly than the phase, the SPA diverges from the numerical approximation. However, as shown numerically, the frequency can be approximated as a constant in time for all times after ringdown. As shown in Fig. 2, the kick velocity in the time- and frequency-domains have very different profiles. In particular, the velocity as a function of time is not smooth as we have

[1] The derivation that is shown here is following that given in Ref. [8].

[2] The SPA only holds in regimes in which the amplitude is changing more slowly than the phase

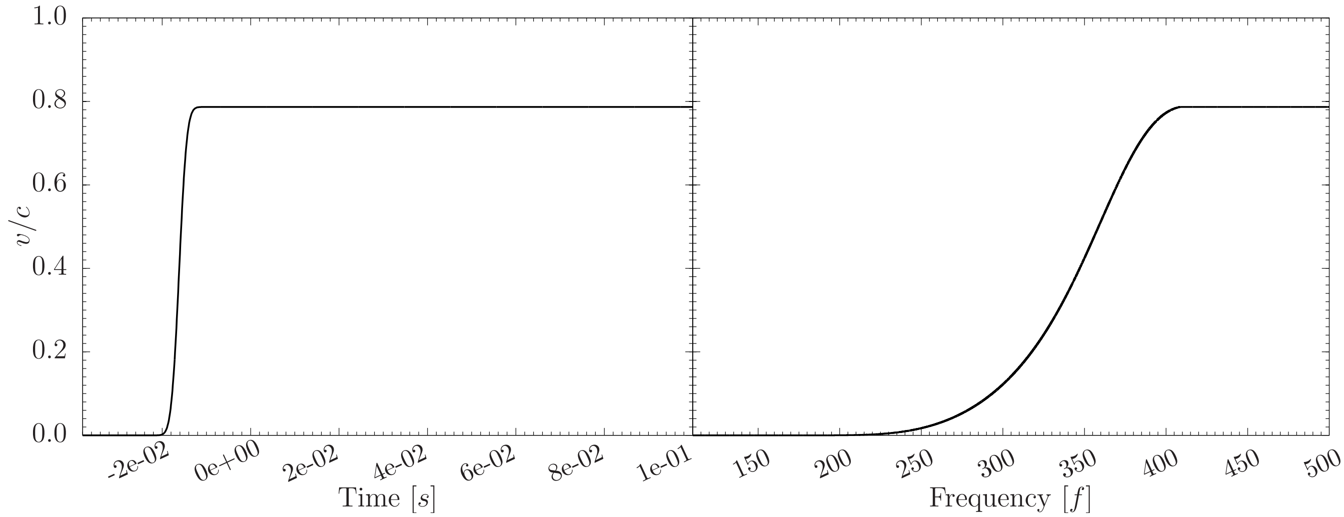

FIG. 2. Left: Kick velocity as a function of time. Right: Kick velocity as a function of frequency. Note that the scaling on the time and frequency axes is demonstrative of the time-scale over which the kick is imparted. The kicks represented here are nonphysical kicks of ~ 0.8c imparted to a binary with total mass $40M_\odot$.

demanded that frequency is constant after ringdown. We expect that this feature will not impact our future analyses.

We implement black hole kicks into PhenomD using the relationship between time and frequency given by the SPA in Eq. 5 and with $f = f_{\rm RD}$ the ringdown frequency for all times after ringdown. We redshift the mass as a function of the kick velocity in the following way:

$$m \to m' \equiv m(1 + v_k(f)). \tag{6}$$

However, since

$$\mathcal{A} \propto \frac{D_{\rm L}^{5/6}}{\mathcal{M}}, \tag{7}$$

we must also redshift the luminosity distance as

$$D_{\rm L} \to D'_{\rm L} \equiv D_{\rm L}(1 + v_k(f))^{6/5} \tag{8}$$

because we wish for the overall amplitude of the waveform to remain the same at early times where the kick is insignificant.

III. RESULTS

Upon implementing these two redshifts in the frequency-domain waveform, we obtain the waveform shown in Fig. 3. Note that the kicked waveform matches the non-kicked waveform in the inspiral where the kick velocity is approximately zero. During the intermediate regime, the kicked waveform begins to deviate from the non-kicked waveform, and diverges drastically near the merger-ringdown frequency. The kicked waveform overlaps the redshifted-mass waveform for all frequencies larger than the ringdown frequency, where the binary begins moving with a constant velocity. This will allow us to perform a Fisher matrix analysis with high accuracy, as our waveform and it's derivatives are analytic.

IV. FUTURE DIRECTIONS

Armed with a kicked frequency-domain approximant, we are now in a position to complete a Fisher analysis that will allow us to address waveform degeneracies and to explore kick detectability for various gravitational wave detectors. However, before performing this analysis, we will modify PhenomD to include precession (PhenomP), as done in Ref. [11]. This will provide us with a more realistic waveform to determine how well we will be able to constrain kick velocities.

In addition to the improvement of our current waveform model, we will also test our kicked frequency-domain waveform against the kicked time-domain waveform model of Ref. [1] to determine how well any deviations could be resolved by future gravitational wave detectors. If we find that there are large deviations between the time- and frequency-domain kicked waveforms, we will have to determine how to more accurately approximate time as a function of frequency for times after merger.

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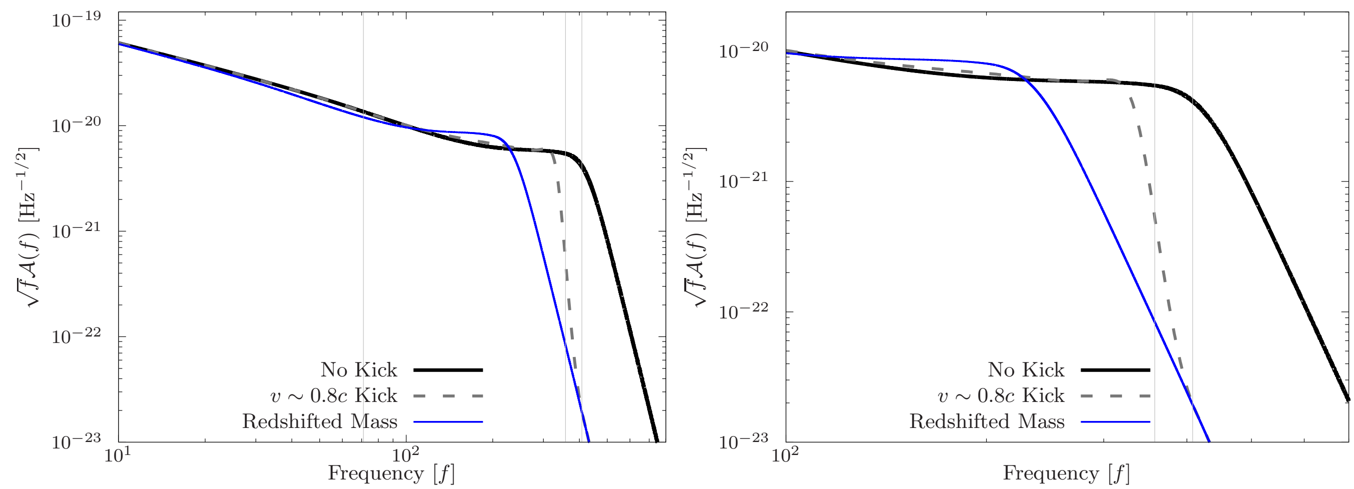

FIG. 3. Gravitational wave amplitude as a function of frequency for a binary with total mass $40M_{\odot}$ and anti-aligned spins ~ 0.1 at a distance of 1Mpc. The black line is the non kicked waveform as provided by PhenomD. The blue line is the same waveform using the final redshifted mass value. The dashed grey line is our kicked waveform model. The vertical grey lines are, respectively, the frequency of transition from the inspiral to the intermediate regime, the frequency of transition from the intermediate to the merger-ringdown regime, and the ringdown frequency [6, 7]. To better appreciate the details, the plot on the right shows a restricted region of the plot on the left in which is it easy to see the agreement between the redshifted waveform and the kicked waveform for large frequencies.

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[1] D. Gerosa and C. J. Moore, Phys. Rev. Lett. **117**, 011101 (2016), arXiv:1606.04226 [gr-qc].
[2] S. Komossa, Adv. Astron. **2012**, 364973 (2012), arXiv:1202.1977 [astro-ph.CO].
[3] D. Gerosa and A. Sesana, Mon. Not. Roy. Astron. Soc. **446**, 38 (2015), arXiv:1405.2072 [astro-ph.GA].
[4] C. O. Lousto and Y. Zlochower, Phys. Rev. **D77**, 044028 (2008), arXiv:0708.4048 [gr-qc].
[5] B. Bruegmann, J. A. Gonzalez, M. Hannam, S. Husa, and U. Sperhake, Phys. Rev. **D77**, 124047 (2008), arXiv:0707.0135 [gr-qc].
[6] S. Husa, S. Khan, M. Hannam, M. Pürrer, F. Ohme, X. Jiménez Forteza, and A. Bohé, Phys. Rev. **D93**, 044006 (2016), arXiv:1508.07250 [gr-qc].
[7] S. Khan, S. Husa, M. Hannam, F. Ohme, M. Pürrer, X. Jiménez Forteza, and A. Bohé, Phys. Rev. **D93**, 044007 (2016), arXiv:1508.07253 [gr-qc].
[8] K. Yagi and L. C. Stein, Class. Quant. Grav. **33**, 054001 (2016), arXiv:1602.02413 [gr-qc].
[9] N. Yunes, K. G. Arun, E. Berti, and C. M. Will, Phys. Rev. **D80**, 084001 (2009), [Erratum: Phys. Rev.D89,no.10,109901(2014)], arXiv:0906.0313 [gr-qc].
[10] Bender and Orszag, *Advanced Mathematical Methods for Scientists and Engineers I* (1999).
[11] M. Hannam, P. Schmidt, A. Bohé, L. Haegel, S. Husa, F. Ohme, G. Pratten, and M. Pürrer, Phys. Rev. Lett. **113**, 151101 (2014), arXiv:1308.3271 [gr-qc].