Advanced LIGO Calibration Uncertainty for Precision Astrophysics

Craig Cahillane

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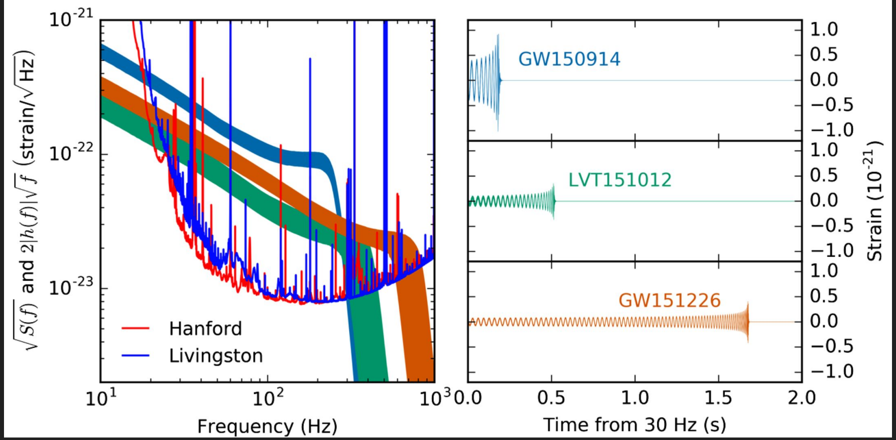
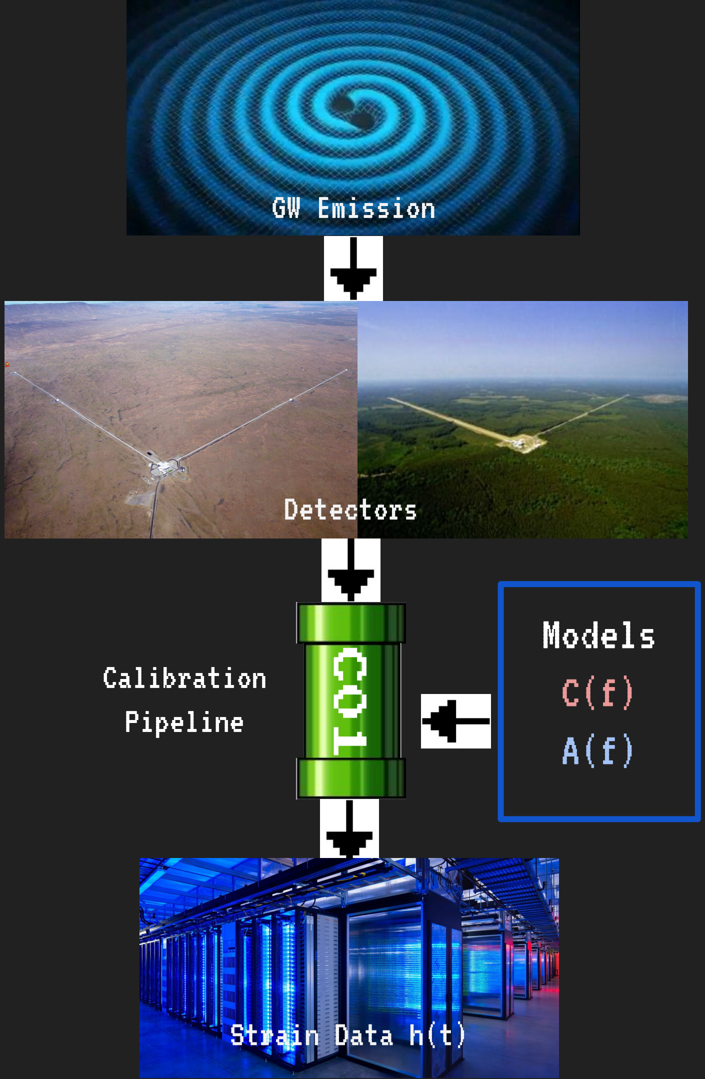
O1 + O2 Calibration Uncertainty Budgets

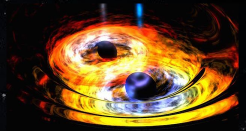
- What is calibration?
  - Production of GW strain data from our detector data
- Why calibration uncertainty?
  - It's the project I was handed when I was a first year
  - No one cared until we made a detection
  - Now everyone cares
  - Imperative for precision astrophysics



Taken from [1]



Motivation for Low Calibration Uncertainty

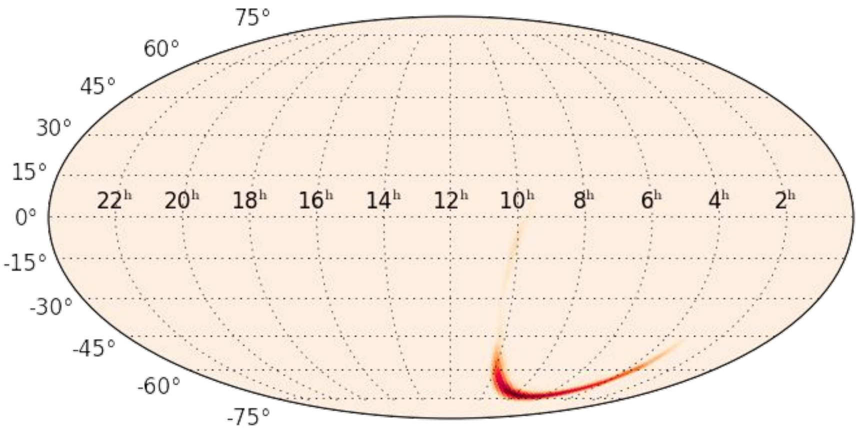


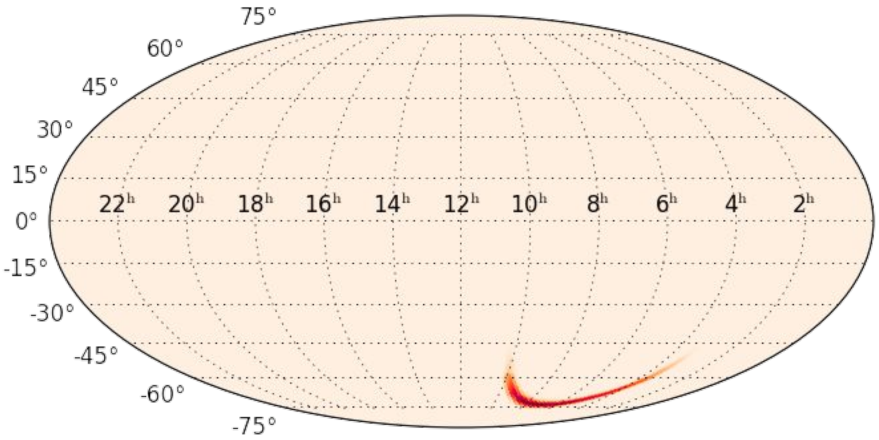
- We don’t want to just make detections, we want to do astrophysics with these detections
  - Source parameters
    - Black hole masses, spins, luminosity distance, inclination, sky location, etc
  - Merger rates
    - Event rate, universal mass distribution, binary star formation
  - Tests of general relativity
    - Strong-field non-linear regime
  - Cosmology
    - Hubble constant measurements
- The accuracy and precision to which we know GW strain data affects all of the above
- Right now we aren’t calibration uncertainty limited, we are SNR limited
  - This won’t always be the case, when we start getting SNR ~ 700 detections in Super Advanced LIGO

Impact of Cal Uncertainty on GW150914 Sky Location

GW150914 90% sky area with 10%, 10 degrees cal uncertainty = 231 square degrees

GW150914 90% sky area with NO cal uncertainty = 153 square degrees

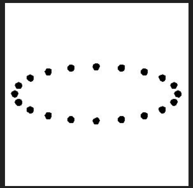




Plots from Chris Berry

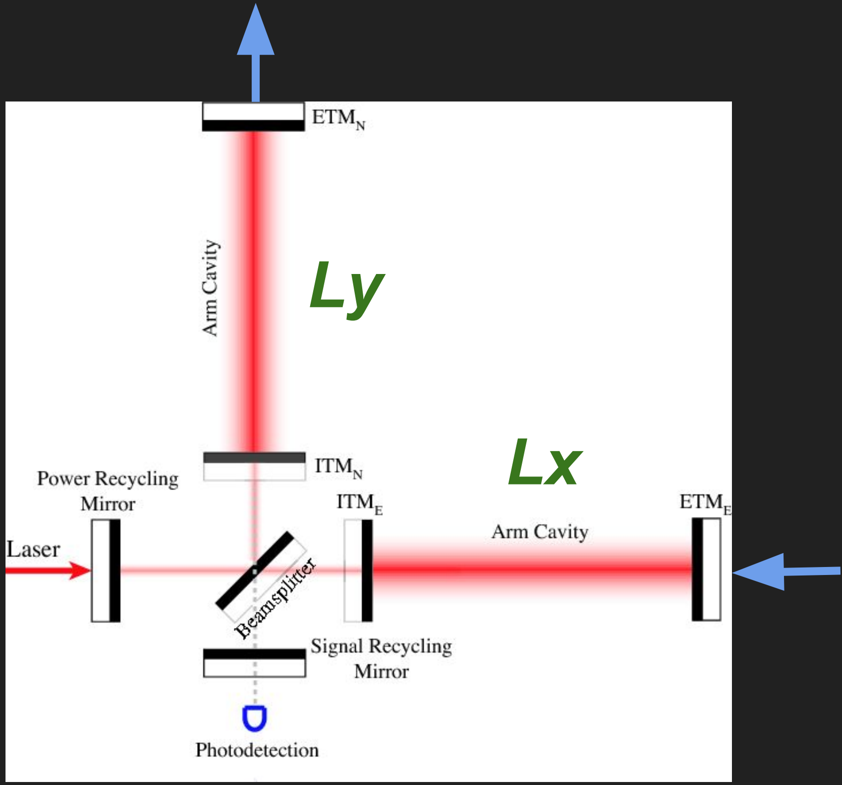
Gravitational Waves and Interferometers

- When a GW hits test particles, it stretches and squeezes them in a quadrupolar way



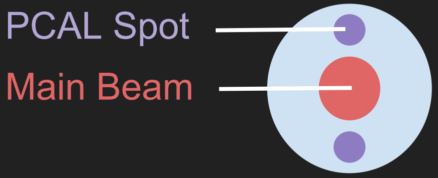
Ring of test particles hit by plus polarized GW

- An interferometer's End Test Masses (ETMs) are like the above test particles
  - A GW changes the distances L_x and L_y
- When on resonance, or "locked", an interferometer is hyper-sensitive to differential arm motion
  - DARM = Differential Arm Motion
  - $L_{DARM} = L_x - L_y$
- We control this motion with the DARM control loop

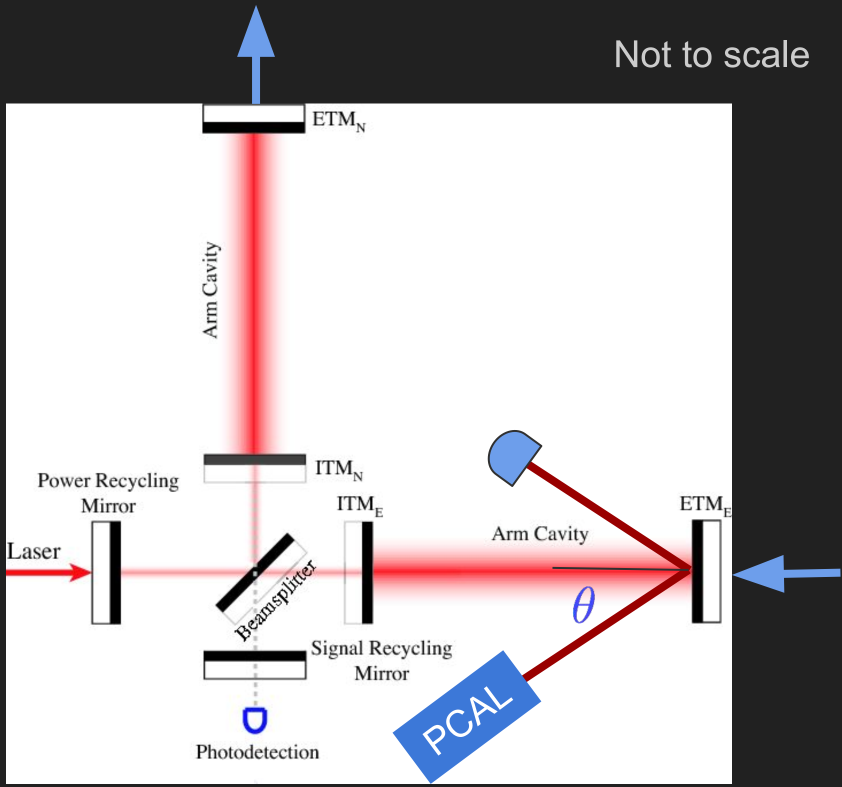


What is Calibration?

- Push on end mirrors by known amount with the photon calibrator laser (PCAL) [8]
  - This laser's power is extremely well known (~2 Watts)
  - Imposes a fundamental limit on our test mass motion uncertainty of ~0.8%
- When we push on one end test mass, it simulates a gravitational wave incident on our detector
  - Light in the cavity is phase shifted into the antisymmetric port onto our photodetector
- This photodetector readout gives us our calibration from meters of test mass motion to arbitrary counts

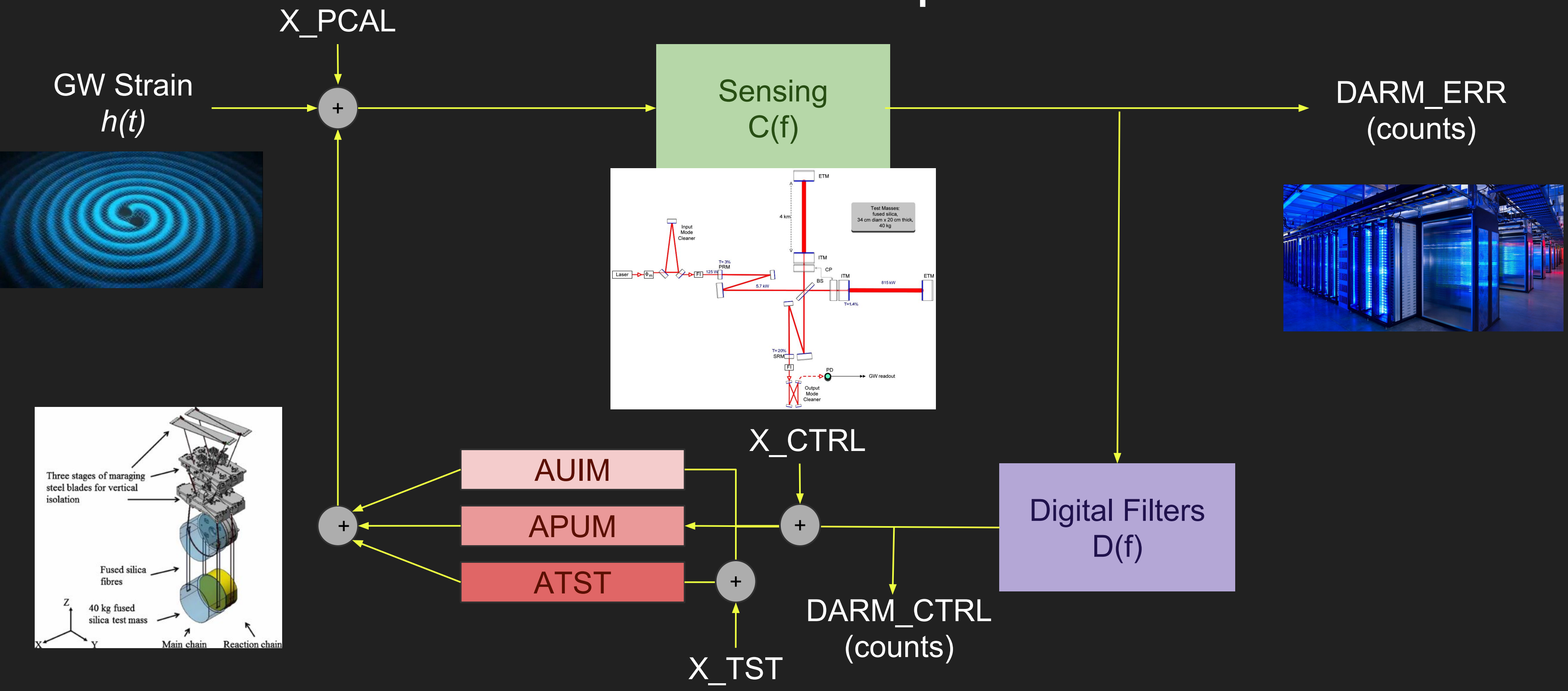


Not to scale



$$\theta = 8.75^\circ$$
$$\lambda = 1047 \text{ nm}$$

DARM Loop



$$G(f) = C(f)\,D(f)\,A(f)$$

$$R^{-1}(f) = \frac{1 + G(f)}{C(f)}$$

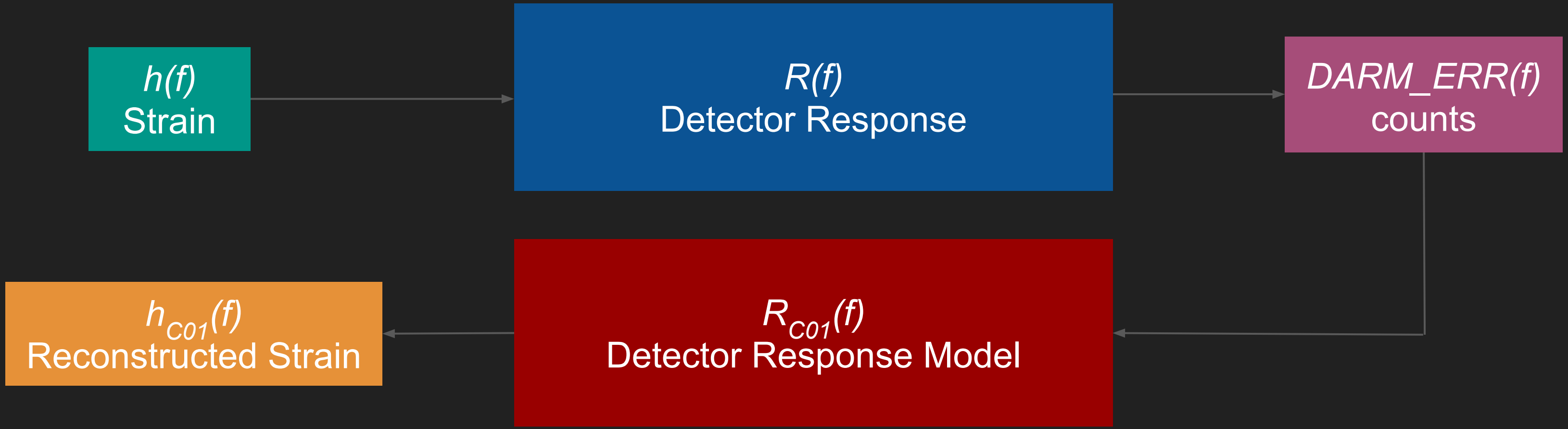
DARM Response

- The detector response function *R(f)* is the transfer function from DARM_ERR counts to GW strain:

$$h(f) = R(f)\, d_{err}(f)$$

- This means that uncertainty in strain is equivalent to uncertainty in the response:

$$\sigma_h(f) = \sigma_R(f)$$



Sensing Model *C(f)*

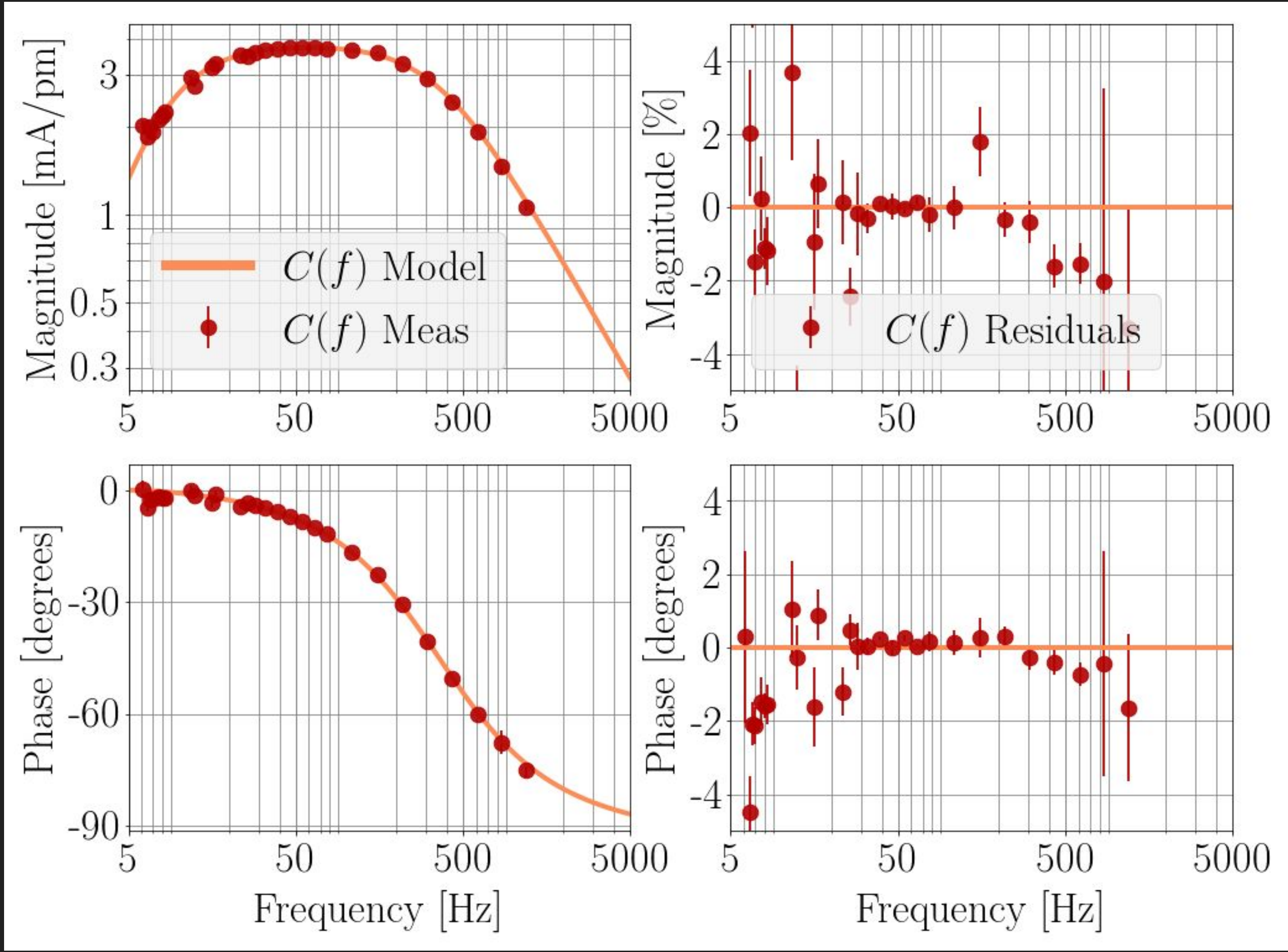
- Through the work of Buonanno and Chen, Robert Ward, Evan Hall, and Kiwamu and myself, the calibration group has a physical model approximating our interferometer response
  - Buonanno and Chen modeled a signal-recycled interferometer using quantum optics [2].
  - Robert Ward converted the above into dual-recycled Fabry-Perot interferometer model [3].
  - Evan Hall showed the above model described detuning of the interferometer [4].
  - Kiwamu and I simplified the model down to the simple pole and optical spring we have today.

Calibration Sensing Model C(f)

$$\frac{f^2}{f^2 + f_S^2 - i f f_S Q^{-1}} \frac{\kappa_C(t) H_C}{1 + i f / f_{CC}(t)}$$

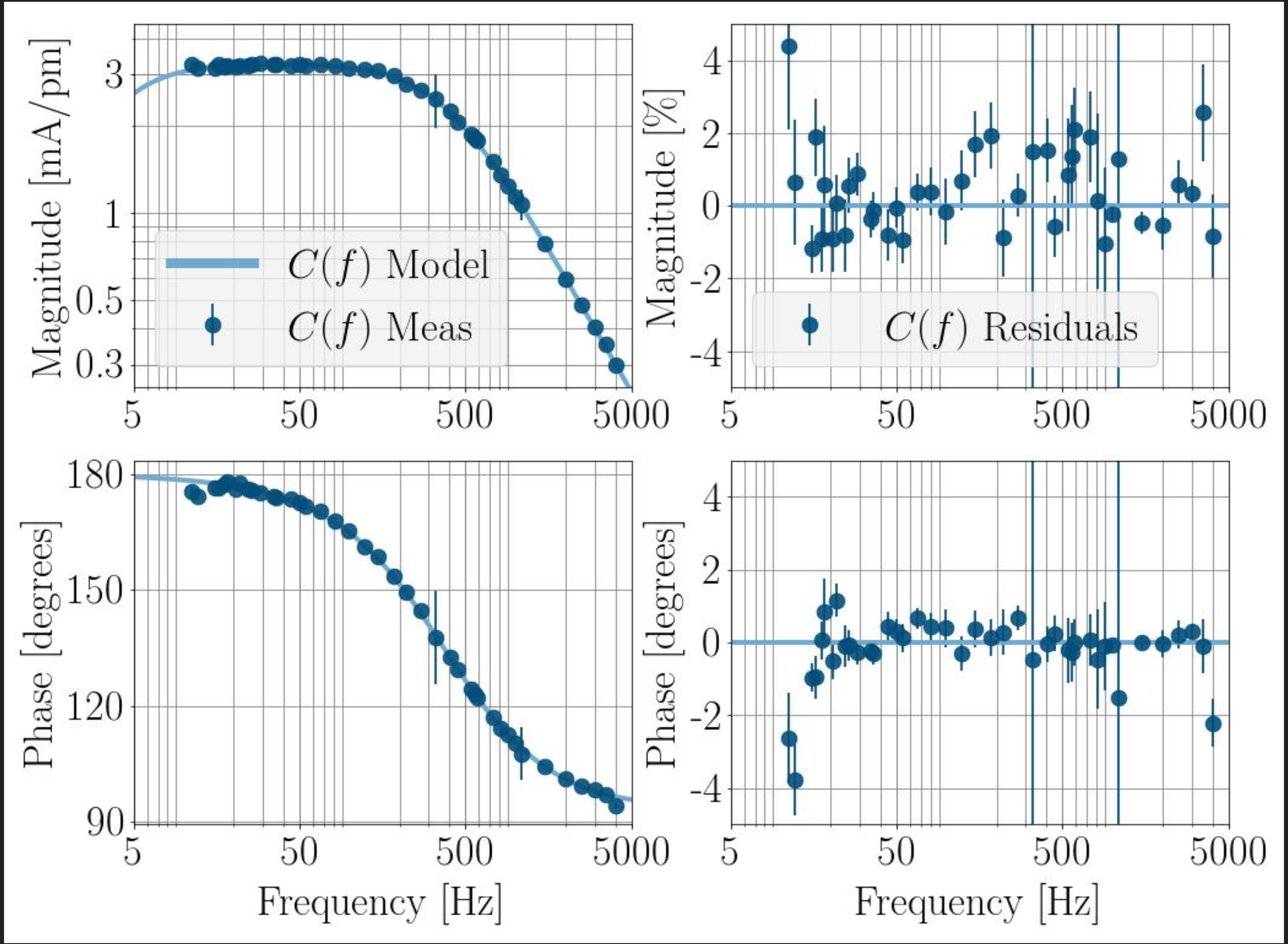
$$H_c = \text{Optical Gain}$$
$$\kappa_C(t) = \text{Gain Time-Dependent Scalar}$$
$$f_{cc}(t) = \text{Coupled Cavity Pole}$$
$$f_S = \text{Optical Spring Frequency}$$
$$Q = \text{Optical Spring Q}$$

LHO



Meas Date:
Jan 4, 2017

LLO



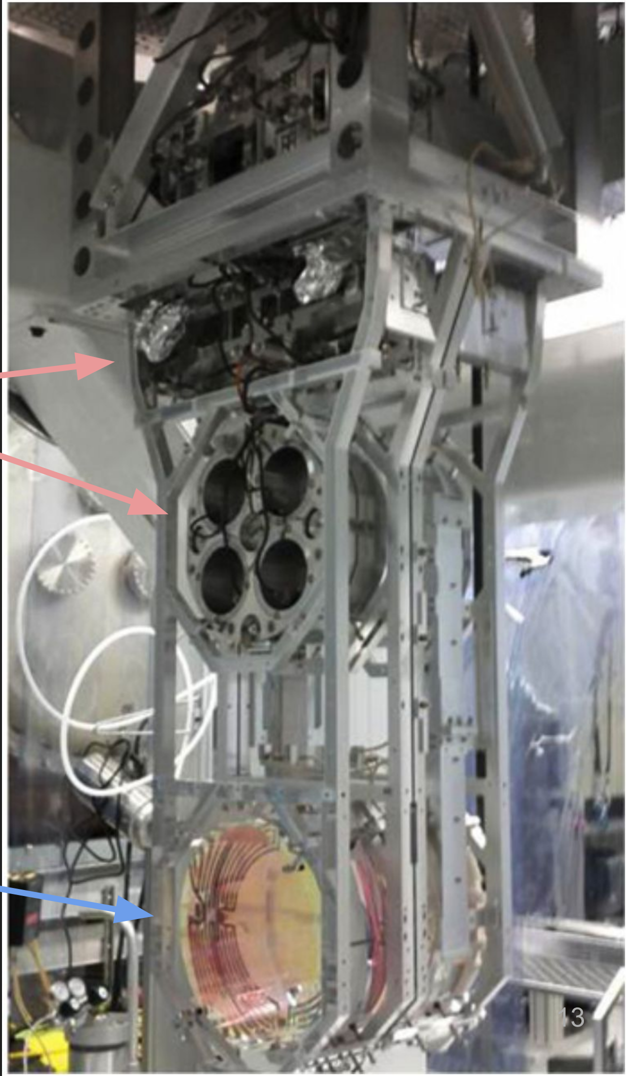
Meas Date:
Nov 26, 2016

Actuation Model *A(f)*

- We also have a complete model of our suspensions.
  - This is important because we actuate on our suspensions to keep the interferometer locked
  - The photon calibrator (PCAL), actuates on end optics using radiation pressure
    - This laser is our fundamental limit on calibration uncertainty
- With the model of the suspensions and the model of the interferometer, we have a complete physical model of our detector DARM control loop.

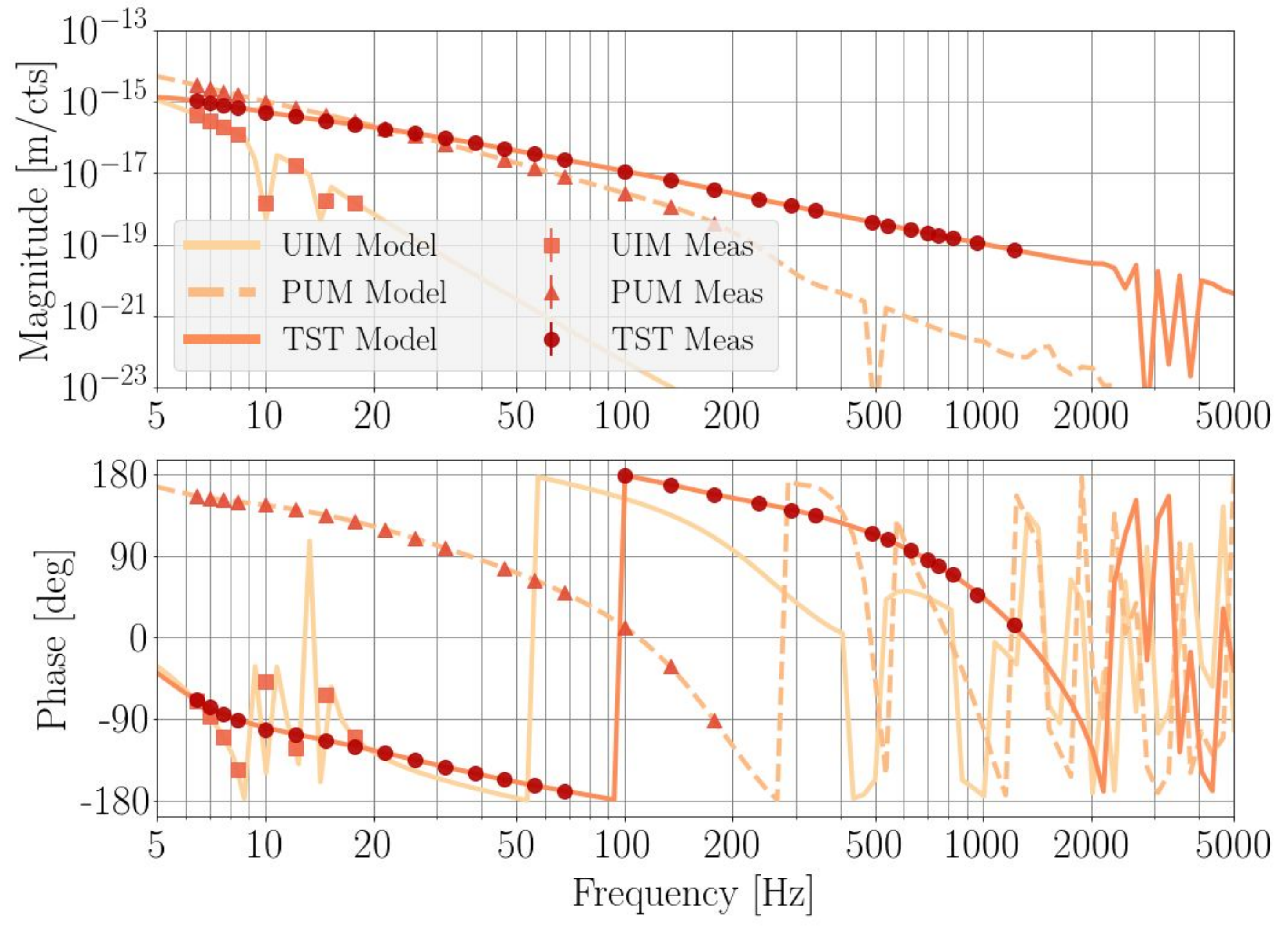
Optical Sensor, Electromagnetic Coil Actuators

Electrostatic Drive



From [5]

LHO



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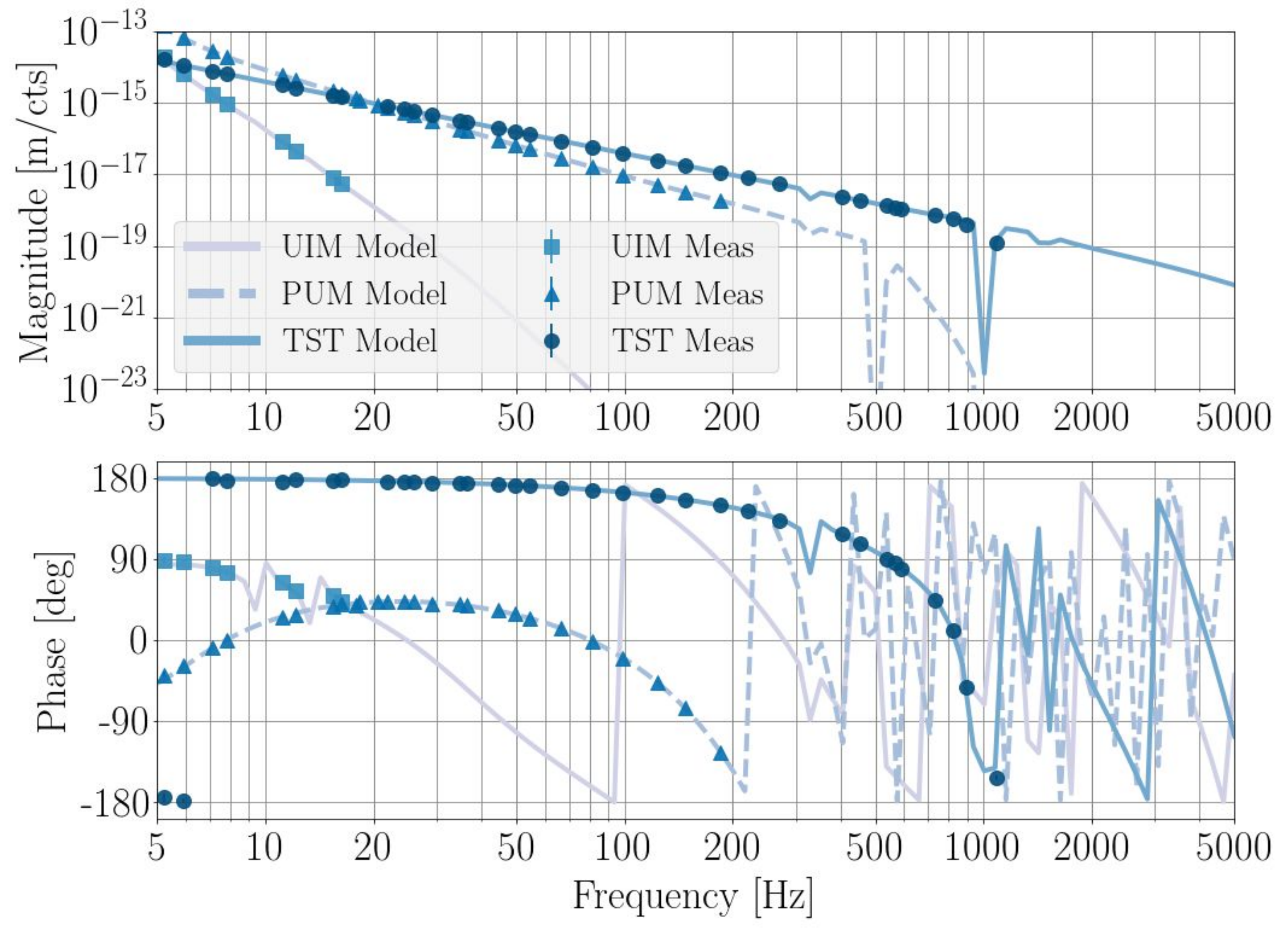


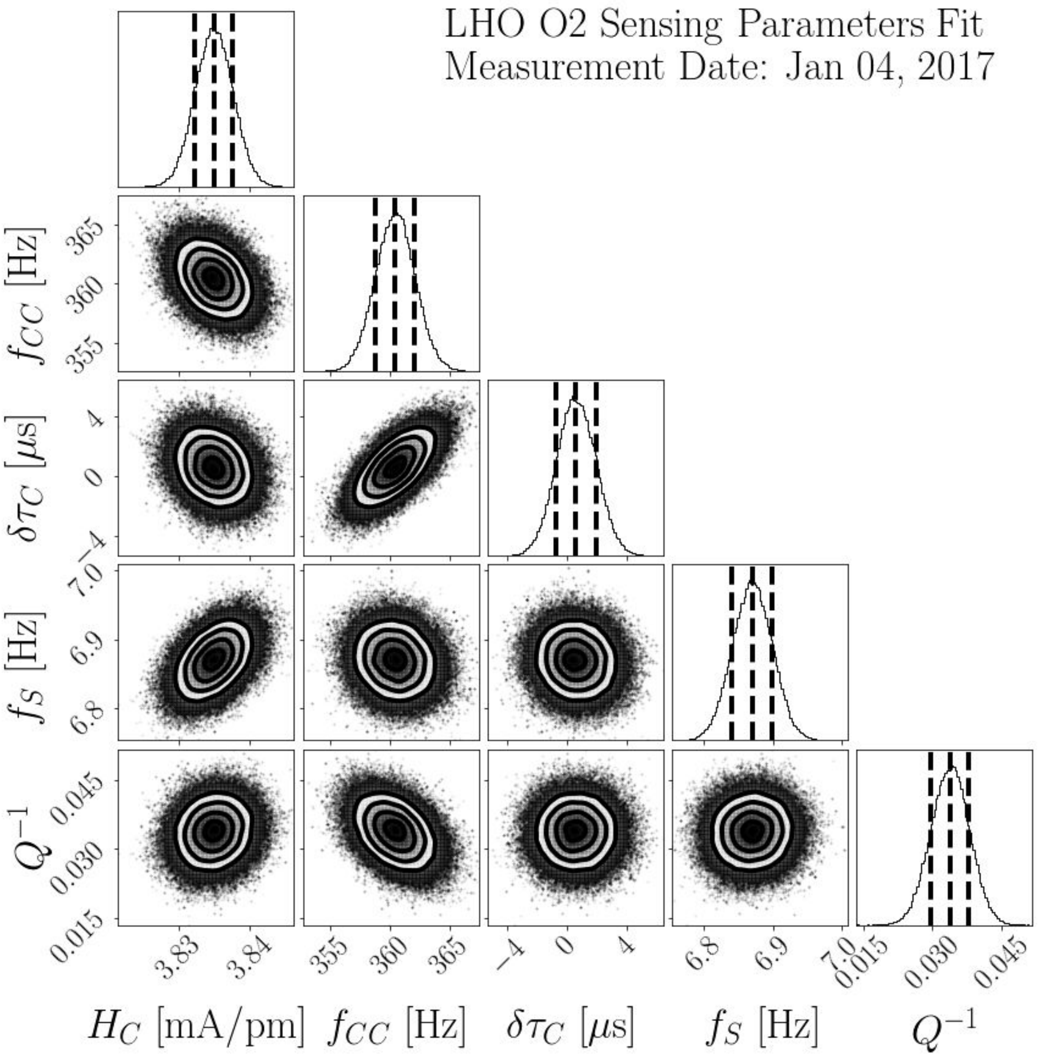
Meas Date:
Nov 26, 2016

Sensing Model Parameter Estimation

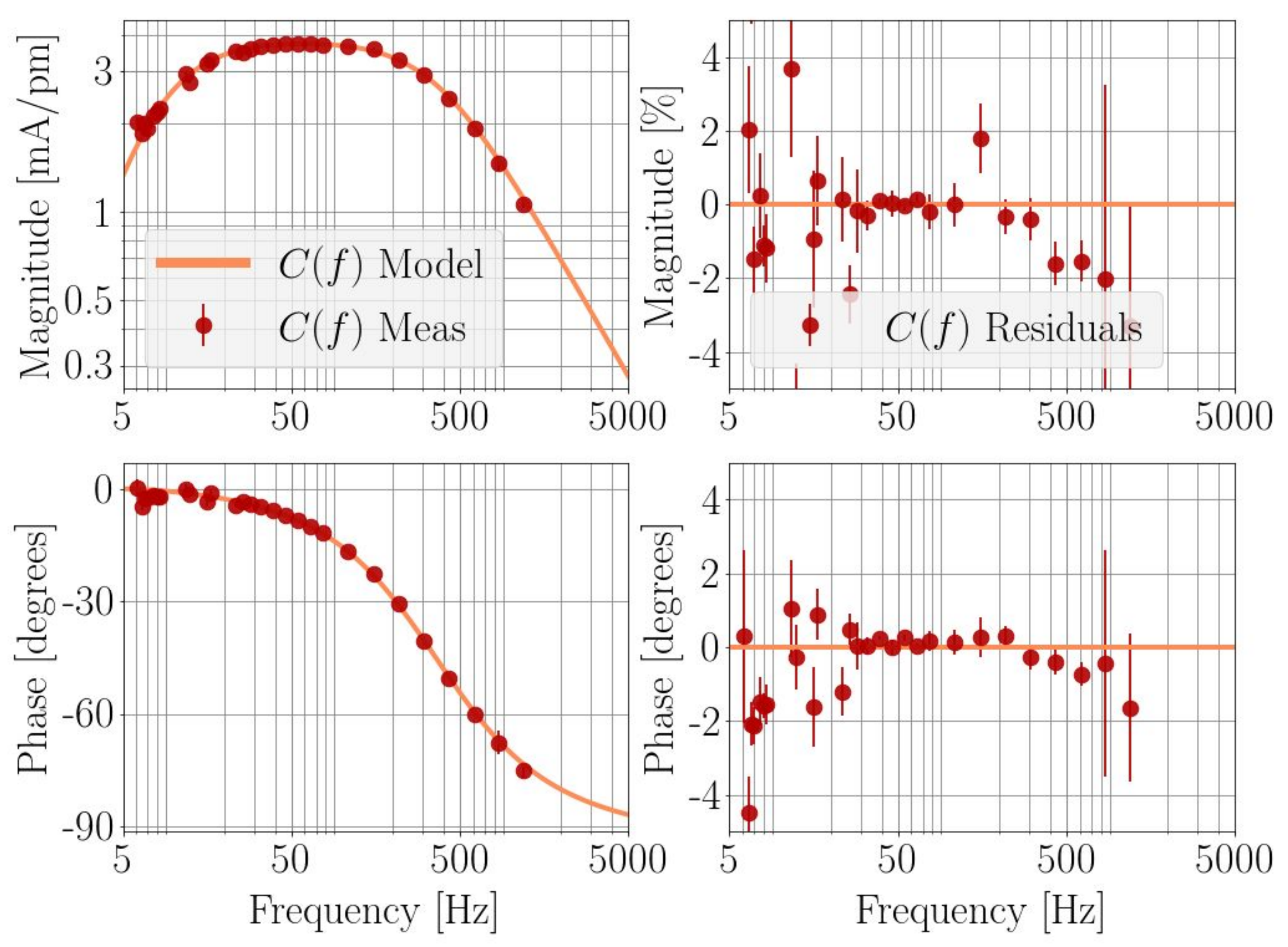
- We have calibration parameters $\vec{\lambda}$ which describe the state of our detector.
  - Optical gain
  - Coupled cavity pole
  - Time delay
  - Optical spring frequency
  - Optical spring inverse Q

- We have a calibration model $M(\vec{\lambda})$ and measurements $\vec{d}$.
- We use a Markov Chain Monte Carlo (MCMC) method to find the most likely parameter values $\vec{\lambda}$ given our data $\vec{d}$ and model $M(\vec{\lambda})$:

$$\log \mathcal{L}(\vec{d}|M, \vec{\lambda})$$



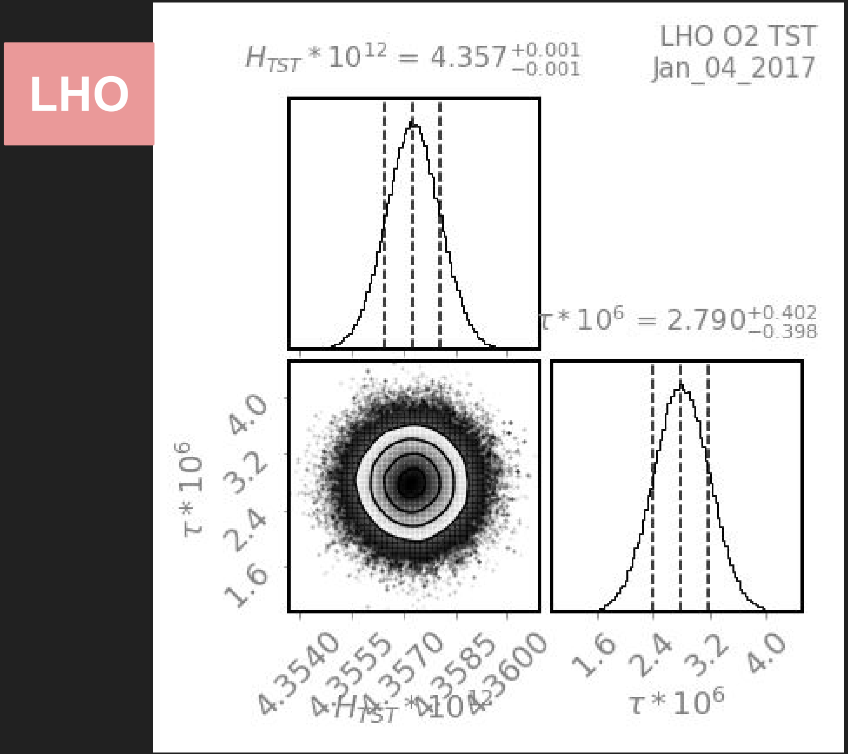
LHO

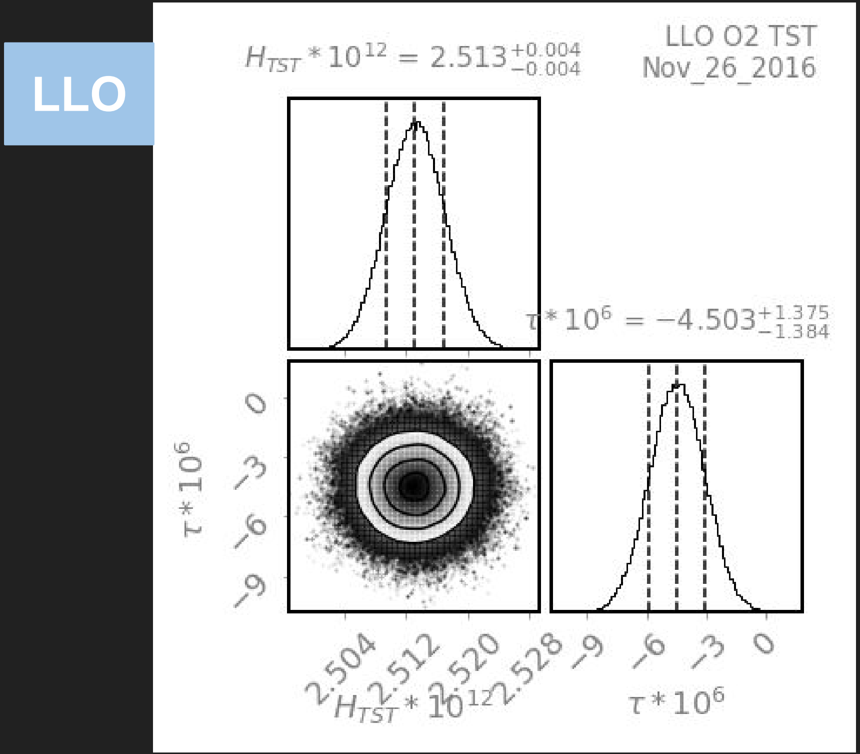


Magnitude [mA/pm]
3
1
0.5
0.3
$C(f)$ Model
$C(f)$ Meas
5
50
500
5000
Magnitude [%]
4
2
0
-2
-4
$C(f)$ Residuals
Phase [degrees]
0
-30
-60
-90
Frequency [Hz]
Phase [degrees]

Actuation Model Parameter Estimation

- Just two parameters here: Gain and Delay
- Do this for all three stages of actuation: A_{UIM}, A_{PUM}, and A_{TST}.





Estimating Unmodeled Deviations from Measurement

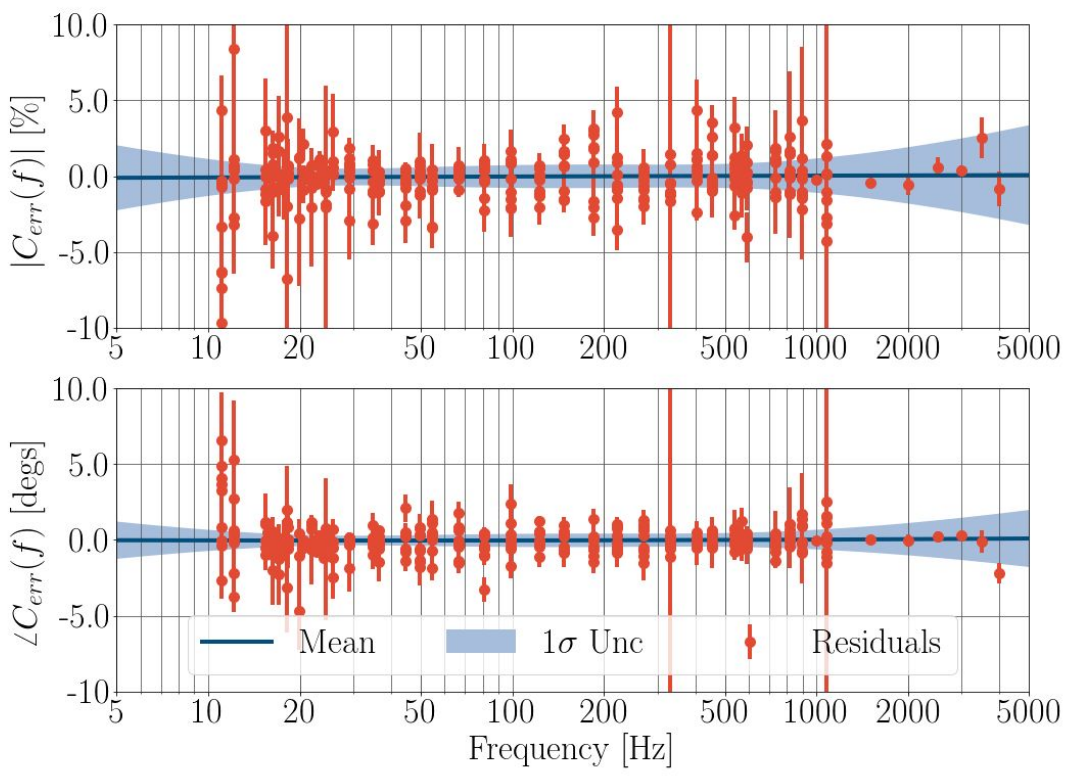
- Want to find deviations from the calibration model for the sensing and actuation functions.
  - Known as systematic biases, or systematic errors
- Also need rigorous uncertainty estimation in this systematic bias

$$f(\vec{x}) = \mathcal{GP}(m(\vec{x}), k(\vec{x}, \vec{x}'))$$

- Gaussian Process Regression
  - Mean Function: $m(\vec{x})$
  - Covariance Kernel: $k(\vec{x}, \vec{x}')$
  - “A Gaussian Process is a collection of random variables, any finite number of which have a joint Gaussian distribution.” [6]
  - Uses training data $\vec{x}$ and covariance kernel $k(\vec{x}, \vec{x}')$ to create a distribution over functions $f(\vec{x})$
  - With this function distribution, we may rigorously sample to get potential fits to our training data

Gaussian Process Regression

- Fit to residuals (meas/model) for our four functions C(f), A_U(f), A_P(f), A_T(f)
- Shown: LLO Sensing Gaussian Process Regression Results



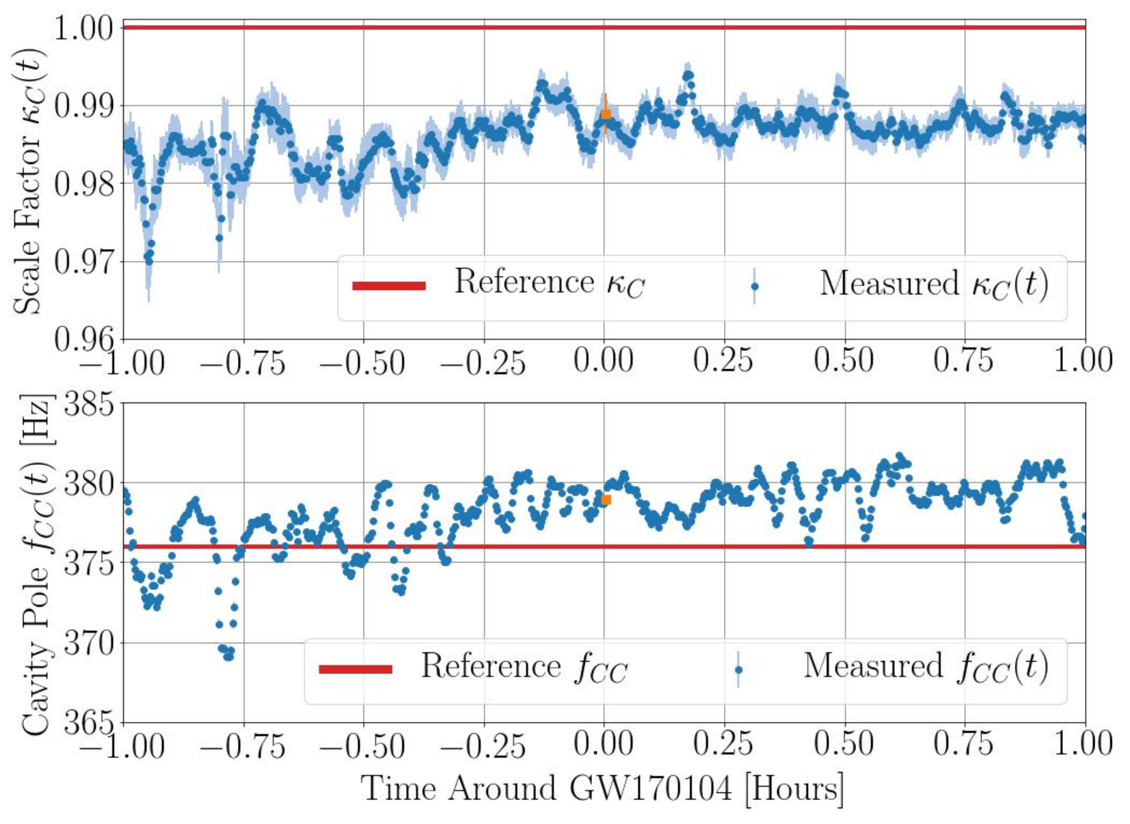
- Assumptions
  - Functions are smooth
    - Can be described by simple lines
  - Uncertainty is gaussian
  - Time dependence of measurements is removed
    - Can stack measurements

Time Dependent Parameter Uncertainty

We track changes in the interferometer in real time using calibration lines.

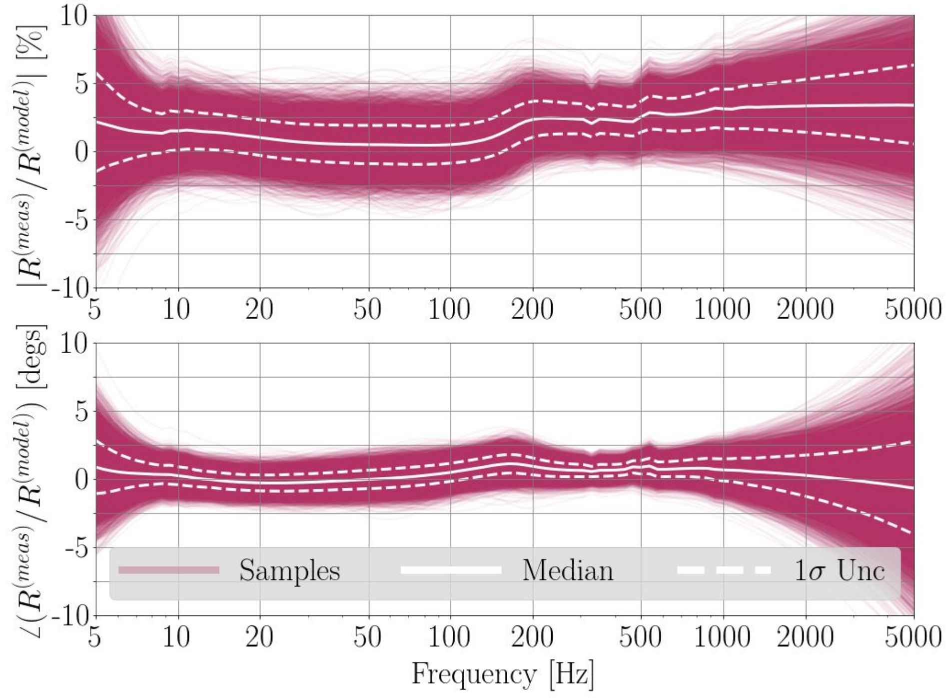
- Optical Gain
- Coupled Cavity Pole
- Electrostatic Drive Strength
- Electromagnetic Coil Drive Strength

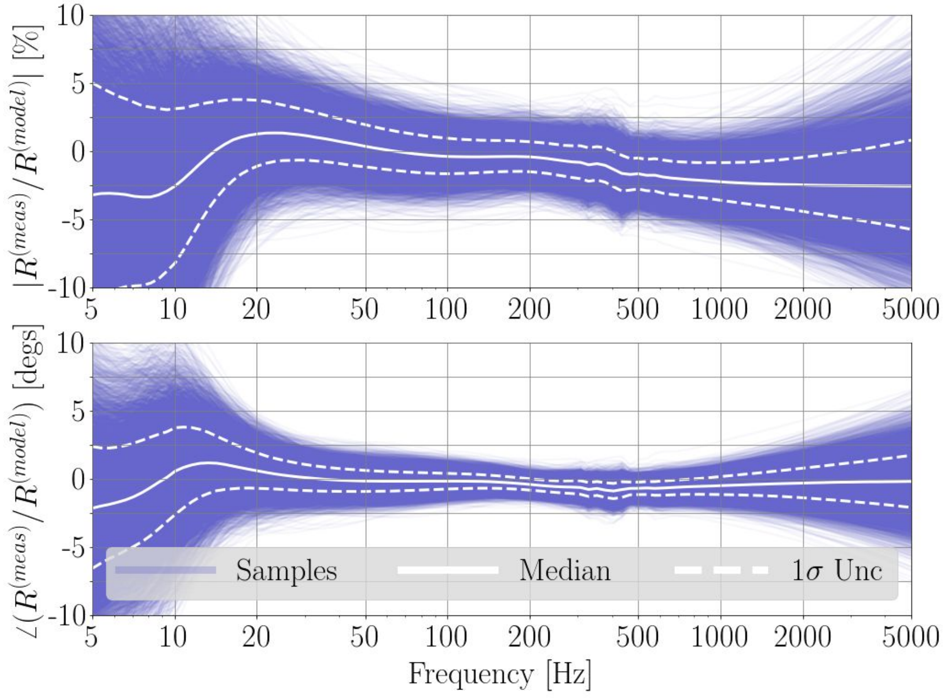
Using the coherence of our calibration lines, we can calculate uncertainty in the lines themselves, and propagate forward to the time-dependent detector parameters.



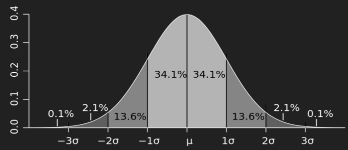
GW170104 Uncertainty Budgets

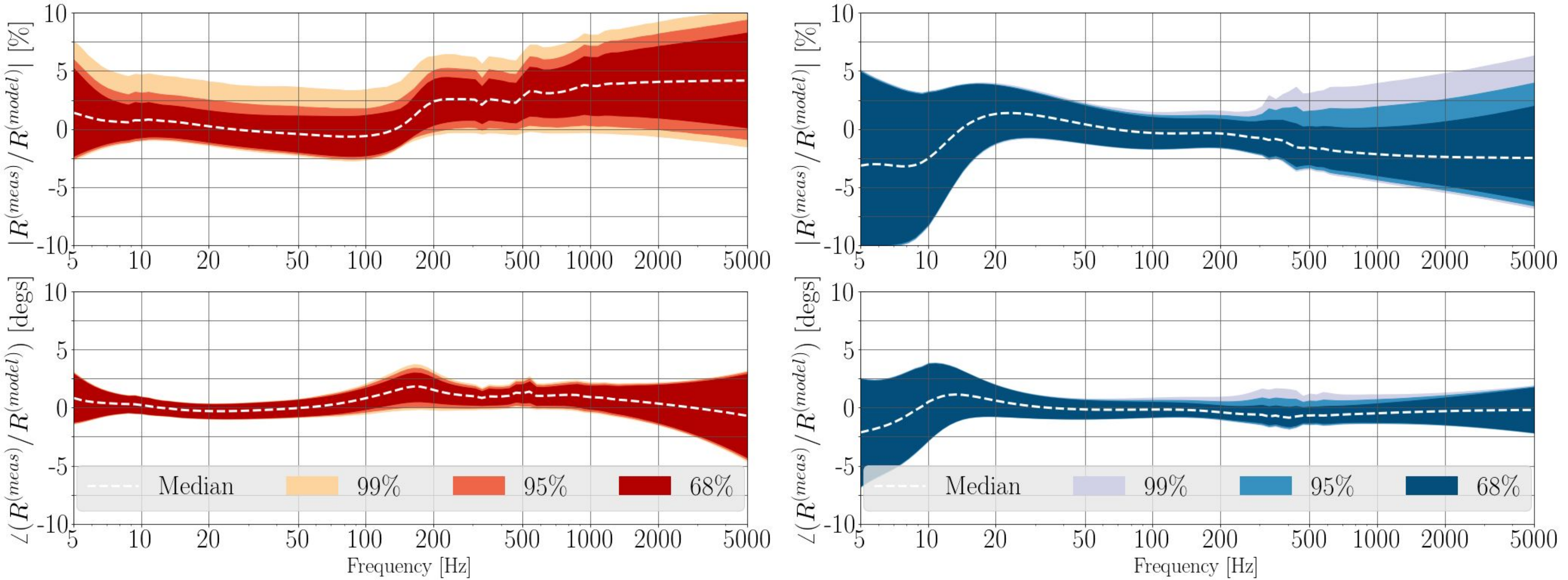
Extreme Uncertainties - 20-1024 Hz	Hanford	Livingston
1σ Magnitude [%]	-1.0 to +4.6	-3.7 to +3.7
1σ Phase [degrees]	-0.9 to +1.8	-1.5 to +1.9



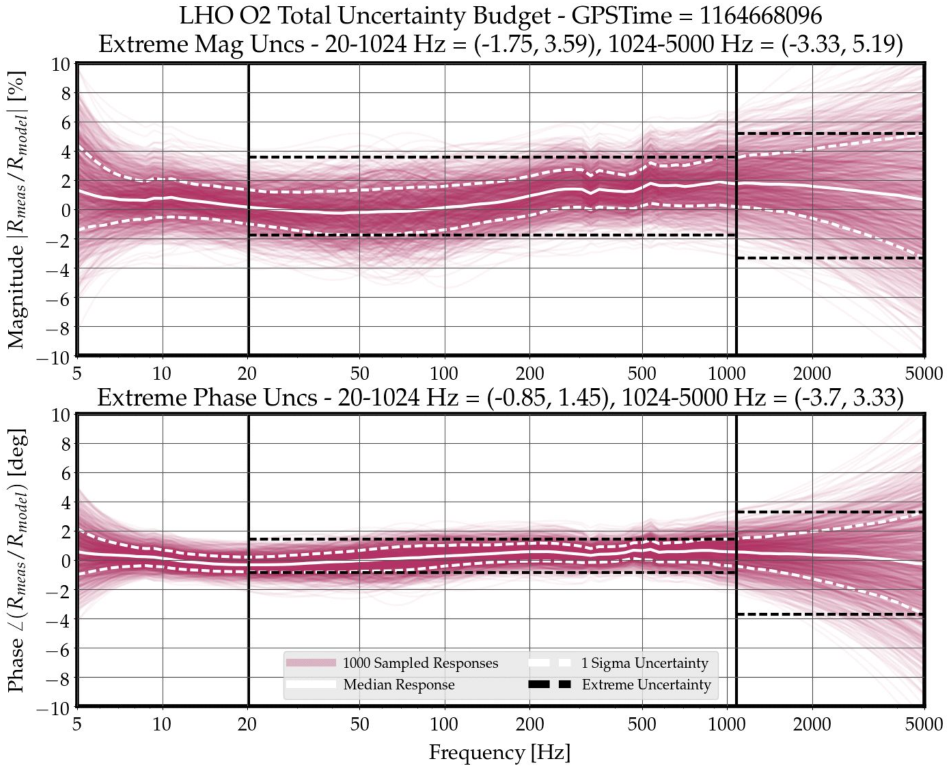


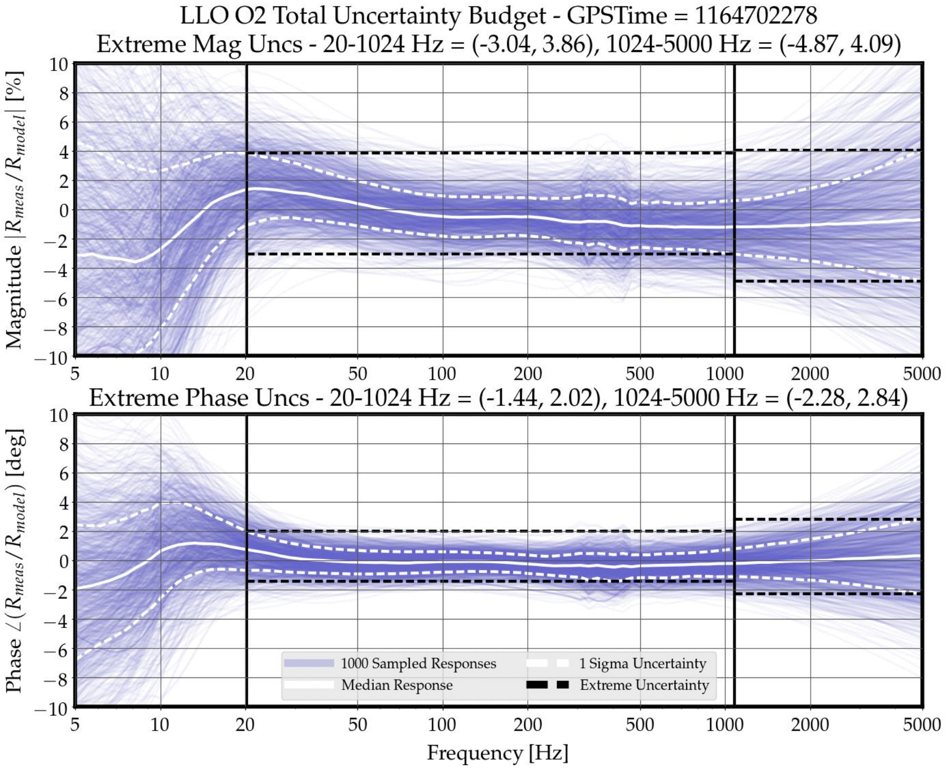
Nov - Jun O2 Uncertainty Budget Percentiles





Nov - Jun O2 Uncertainty Budget Movie





Conclusion

- The uncertainty in gravitational wave strain data is improved from 10% and 10 degrees to at least 7.4% and 3.4 degrees from 20 to 1024 Hz for both detectors.
- The uncertainty budget is frequency dependent and quantifies known systematic biases
- This information from the uncertainty pipeline is getting incorporated into astrophysical parameter estimation pipelines
- Future work to further push down calibration uncertainty and eliminate systematic errors is underway

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