EXTREME GRAVITY 2:
A PERSPECTIVE ABOUT THE NUMERICAL RELATIVITY AND OUTLOOK

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NUMERICAL RELATIVITY AND GRAVITATIONAL WAVES

"I have bet **these numerical relativists** that gravitational waves will be detected from black-hole collisions before their computations are sophisticated enough to simulate them. I expect to win…"

Reference: K.S. Thorne, Spacetime Warps and the Quantum World: Speculations About the Future," in R.H. Price, ed., *The Future of Spacetime* (W.W. Norton, New York, 2002).

BBH Mergers, before *September 14, 2005*

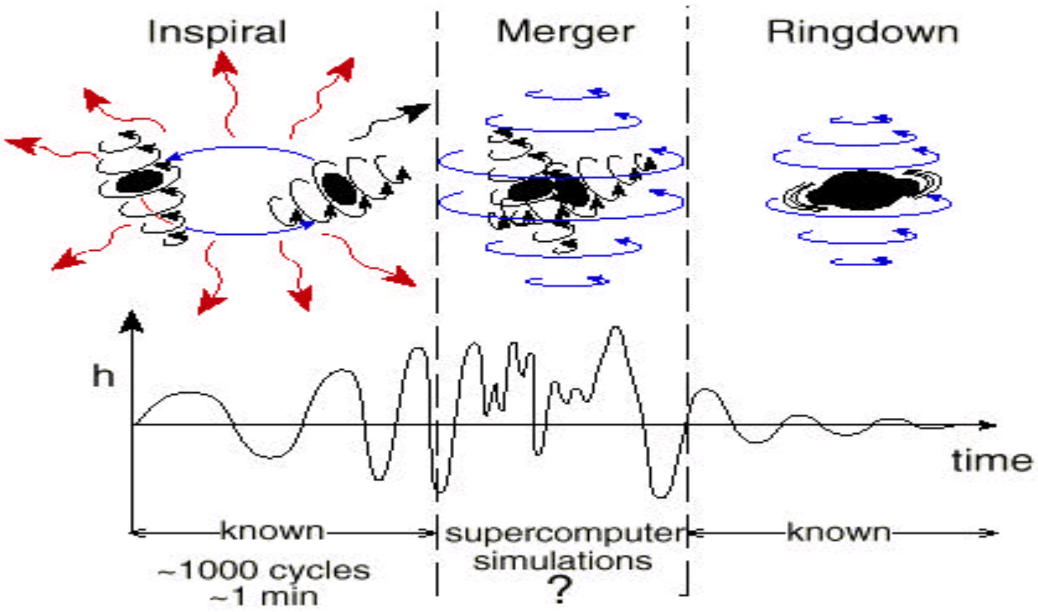

Image credits: Kip Thorne

NUMERICAL RELATIVITY AND GRAVITATIONAL WAVES

"I have bet **these numerical relativists** that gravitational waves will be detected from black-hole collisions before their computations are sophisticated enough to simulate them. I expect to win…"

"…. but hope to lose, because the simulation results are crucial to interpreting the observed waves."

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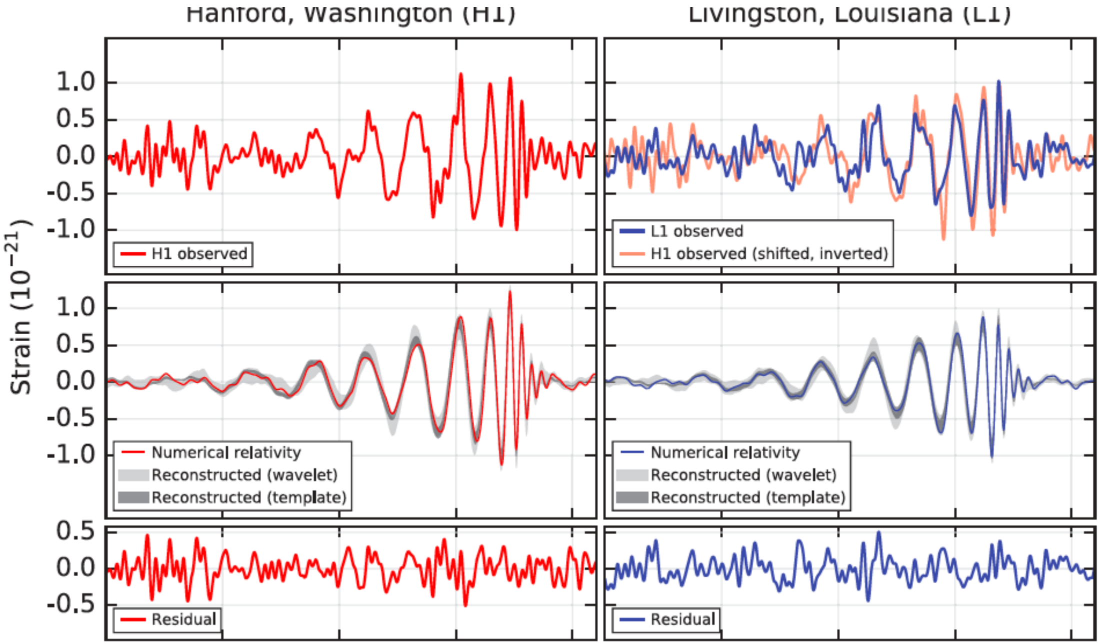
GW150914, September 14, 2015



Abbott *et al.* (LVC) PRL. 116, – February 11, 2016

IT REQUIRED 50+ YEARS OF EFFORTS …

NR is about solving the Einstein's Field Equations numerically **without any approximation.**

When they are written down to be explicitly coded up, GR equations have **hundreds of terms** depending on formulations, so it took 50+ years to the solution …

2005

First successful inspiral and merger [Pretorius 2005] **- It can be done!**

Moving punctures [Campanelli +2006, Baker+2006] **- Enable many NR groups**

Spectral Einstein Code (SXS) (inspiral) [Boyle+2006] (merger) [Scheel+2008]**- Focus on NR accuracy**

Two very different NR approaches: Spectral Methods and Moving Punctures

+12 years

Many groups and codes: SXS (SpEC), RIT (LazEv/ETK), GSFC(Hahndol/ETK), GT (Maya/ETK), AEI (CCATIE/ETK),Jena/Cardiff/Palma/Vienna (BAM), AEI/Palma (Llama/ETK), UIUC (Lean/ETK), etc

Building the NR-GW community:

- **Numerical INJection Analysis (NINJA)** NR-- DA project [Aylott+2009]
- **NINJA-2** >NR-PN hybrids [Ajith+2012], blind-injections [Aasi+2014]
- **NR-AR** comparisons by different groups, NR-EOB [Hinder+2013]

12+ YEARS OF SOLID, HARD, WORK

NR is needed to compute accurate gravitational waveforms in the "late" Inspiral and Merger dynamics of BBHs.

- **NR waveforms used to calibrate AR models:**
  - Phenom models: B[Ajith+2009], C[Santamaria+2010], P [Hannam+2013], D[Khan+2015, Husa+2015]
  - EOB models (SEOBNR): v1[Taracchini+,2012], v2 [Taracchini+,2013], v3 [Pan+,2013], v4 [Bohe+2016]

- **Catalogs of NR waveforms** (1000+):
  - SXS[Mroué +2013, Chu+2015], + surrogate models [Blackman+2017].
  - Gatech [Jani+2016],
  - RIT [Healy+2017]
  - NR Injection Infrastructure in LAL [Schmidt+2017]

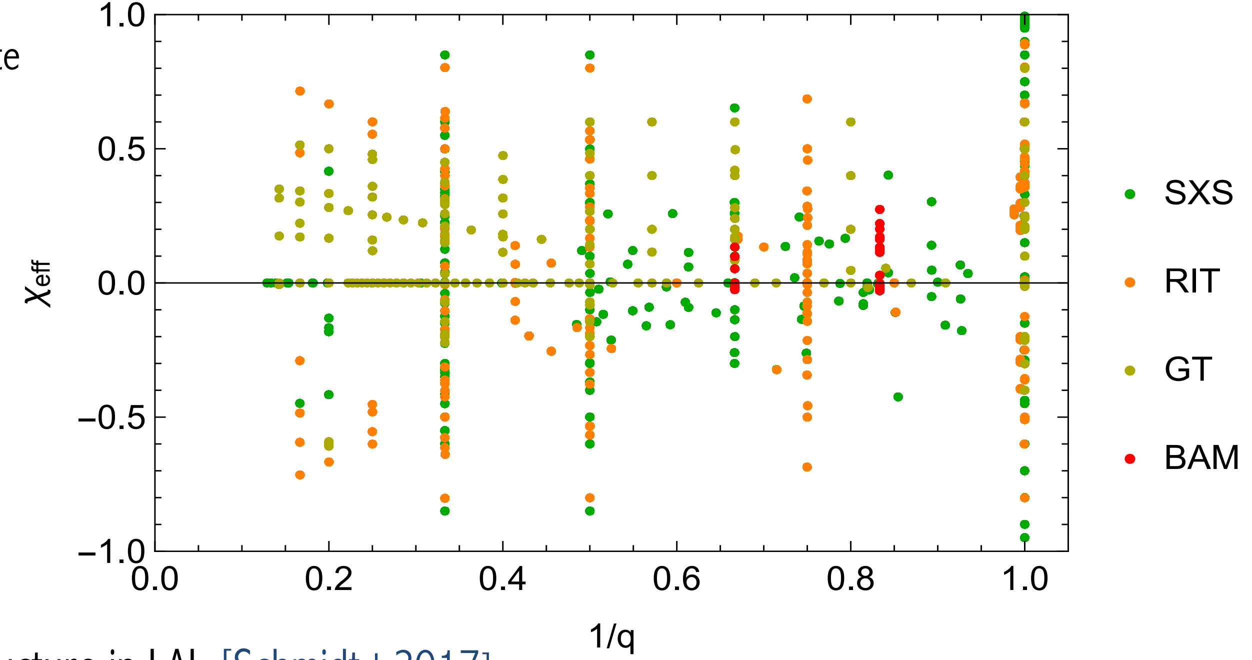
- **Direct comparison to observations** (see many LVC papers)
  - Code comparison by SXS/RIT show overlap 99.9% [Lovelace+2016]
  - Parameter Estimation using NR [Lange+2017]
  - Final BH Remnant Properties [Healy +2017]

CATALOGS OF NUMERICAL RELATIVITY WAVEFORMS

NR/LSC teams assembled ~2000+ NR waveforms (including precessing ones):

- SXS [Mroué +2013, Chu+2015]: 316 waveforms (soon 1000+) + surrogate (700+) [Blackman+2017] www.black-holes.org/waveforms
- RIT [Healy+2017]: 200 (soon 500+); http://ccrg.rit.edu/~RITCatalog
- Gatech [Jani+2016]: 452 waveforms; www.einstein.gatech.edu/catalog

8-dimensional parameter space: mass-ratio, spins, eccentricity

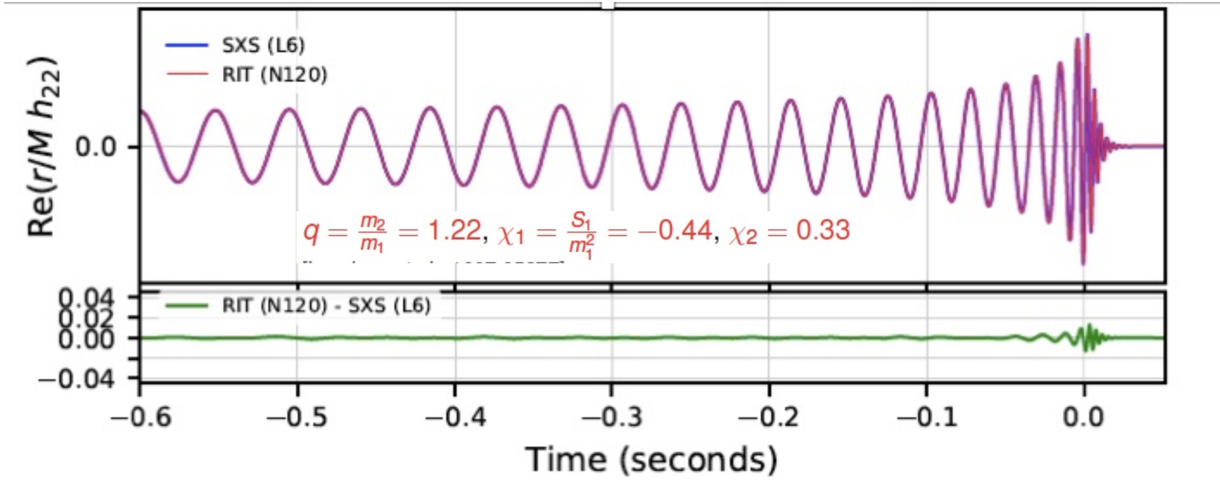


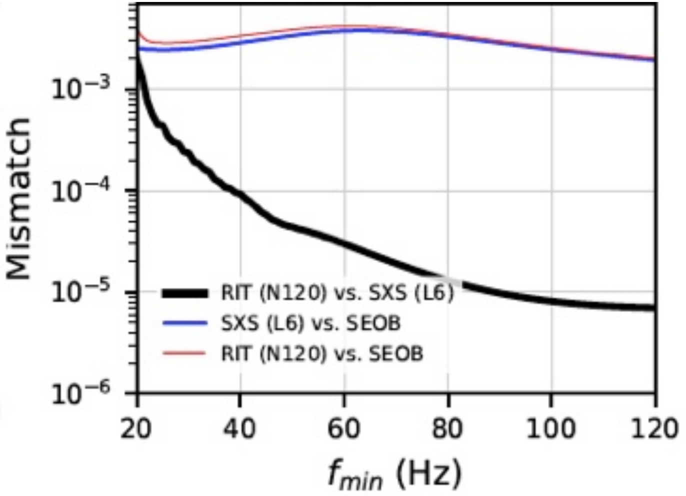
Integrated in the NR Injection Infrastructure in LAL [Schmidt+2017]

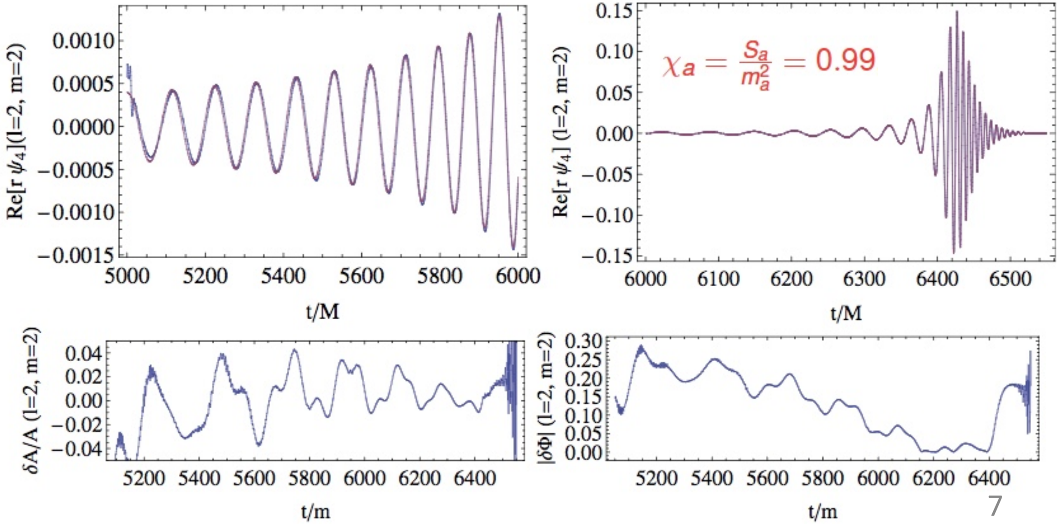
Image credits: Jacob Lange, RIT.

HOW NR DO AT PRESENT?

- NR models for GW150914 show overlap 99.9% [Lovelace+2016]
- Even for very large spins of 0.99 [Zlochower+2016]
- Mismatch [Lange+2017]: HR (0)
  MR (3.90E-05)
  LR (5.27E-05)

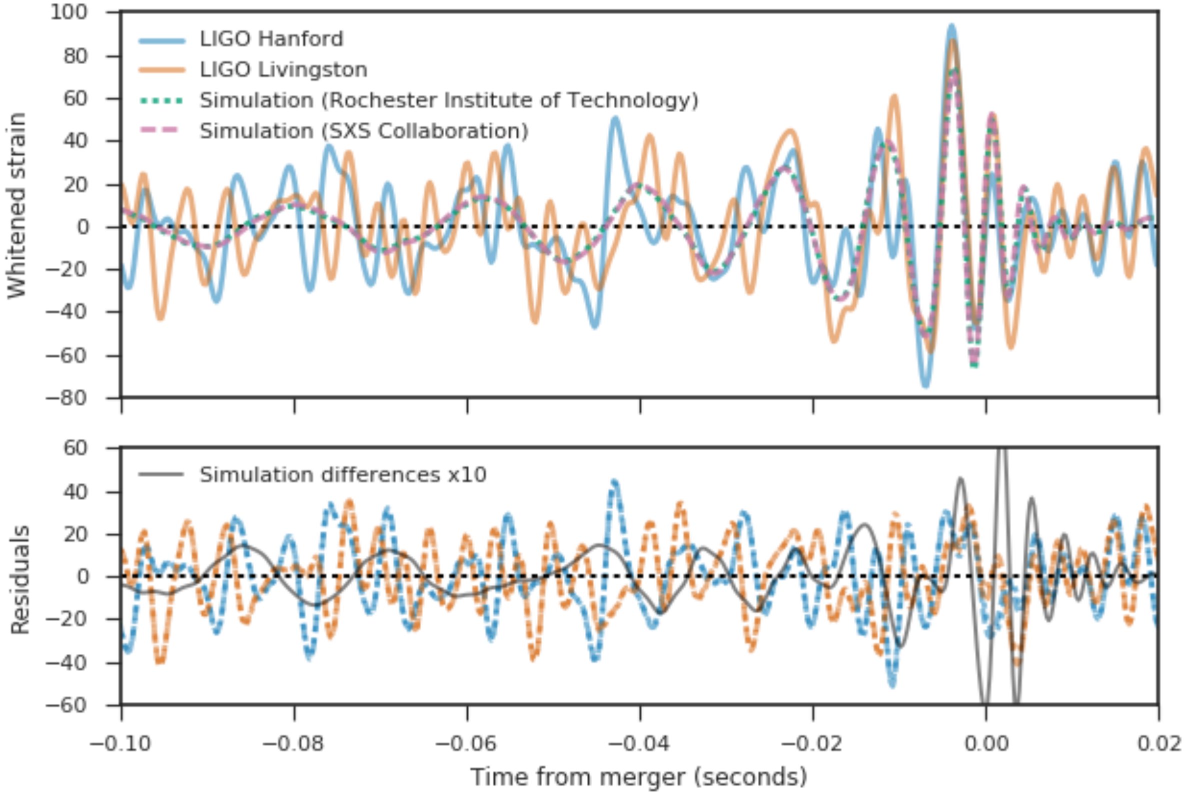






NR FOLLOW-UPS OF GW170104:

Images credits: Andrew Williamson (RIT)



$q=0.5246$, $\chi_1=(\ 0.1607, -0.1023, -0.0529\ )$, $\chi_2=(\ -0.3623, 0.5679, -0.3474\ )$

Even for the low resolution, precessing, run the NR accuracy is 10 times better than residuals!

With medium, high resolutions runs, and improved extraction we can easily get an **extra factor x5**

Top panel shows the whitened data (with Livingston data shifted by -2.93ms and sign flipped), and the whitened strain from the two simulations overlaid. Bottom panel shows the **residuals, and the difference between the two simulations multiplied by 10 in grey.**

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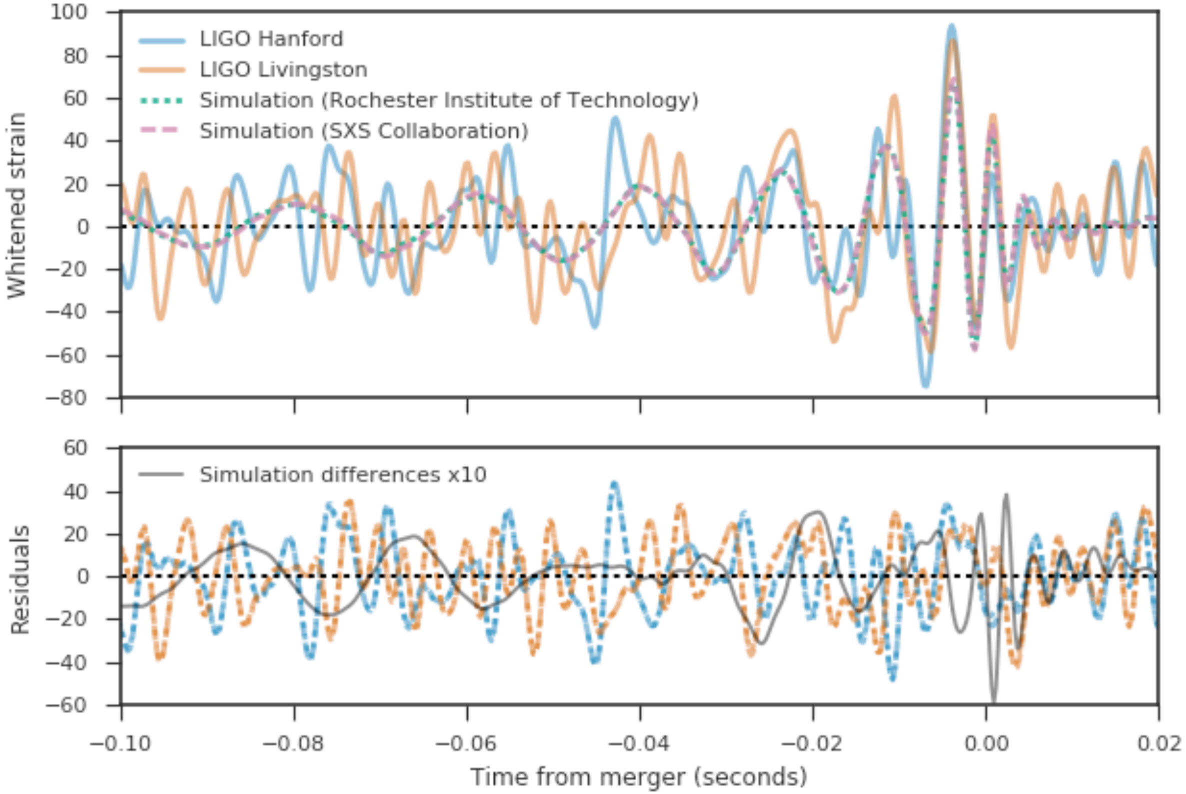

q=0.7147, χ_1=(0,0, 0.2205), χ_2=(0,0, -0.7110).

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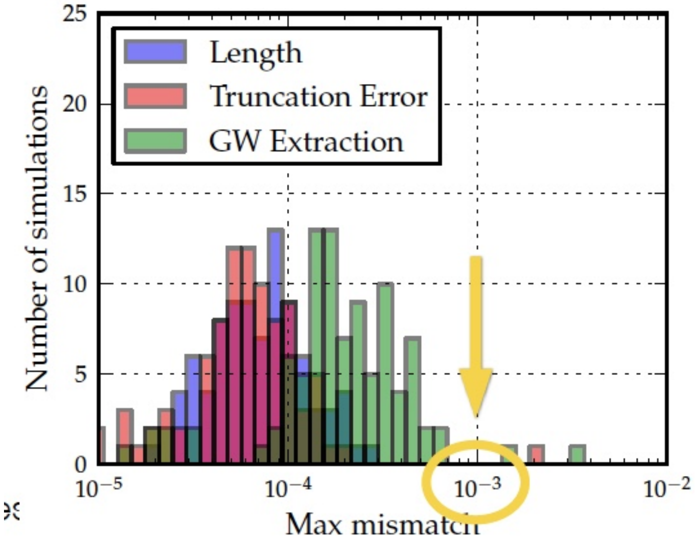
TOTALLY INDEPENDENT APPROACHES AND CODES!!!

	LazEv	SpEC
Initial data		
Formulation of Einstein constraint equations	conformal method using Bowen-York solutions [37–39]	conformal thin sandwich [38, 40]
Singularity treatment	puncture data [41]	quasi-equilibrium black-hole excision [42–44]
Numerical method	pseudo-spectral [45]	pseudo-spectral [46]
Achieving low orbital eccentricity	post-Newtonian inspiral [47]	iterative eccentricity removal [48, 49]
Evolution		
Formulation of Einstein evolution equations	BSSNOK [50–52]	first-order generalized harmonic with constraint damping [11, 53–55]
Gauge conditions	evolved lapse and shift [56–58]	damped harmonic [59]
Singularity treatment	moving punctures [12, 13]	excision [60]
Outer boundary treatment	Sommerfeld	minimally-reflective, constraint-preserving [53, 61]
Discretization	high-order finite-differences [62, 63]	pseudo-spectral methods
Mesh refinement	adaptive mesh refinement [64]	domain decomposition with spectral adaptive mesh refinement [46, 59]

Image from [Lovelace+2016];
See also Larry Kidder's talk at IAP 2017 to appreciate the meaning of this!

NR ERRORS – WAVEFORM EXTRACTION

Image credits: Chu+2015

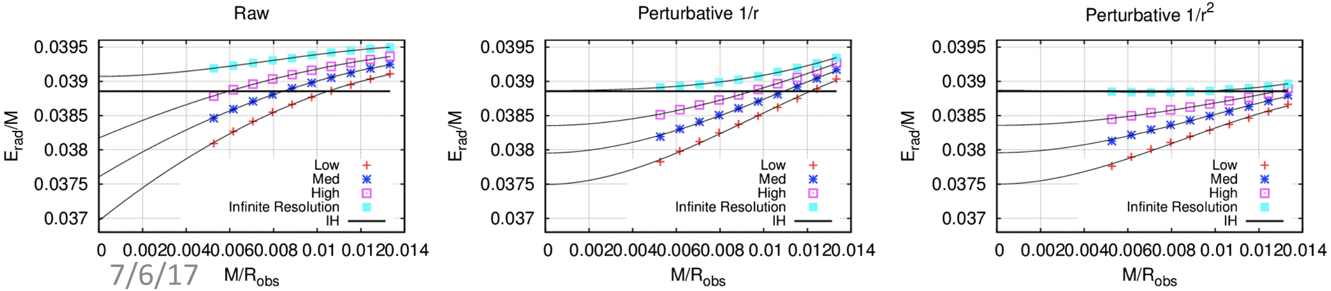


- NR source of errors, mostly due to finite extraction radius, resolution and sum over modes [Chu+2015]

- Extract information at finite radii:
  - Newman-Penrose scalar: $\lim_{r\to\infty} r\psi_4 = \lim_{r\to\infty} r(\ddot{h}_+ - i\ddot{h}_\times).$
  - Extrapolate to infinity via perturbative expansion

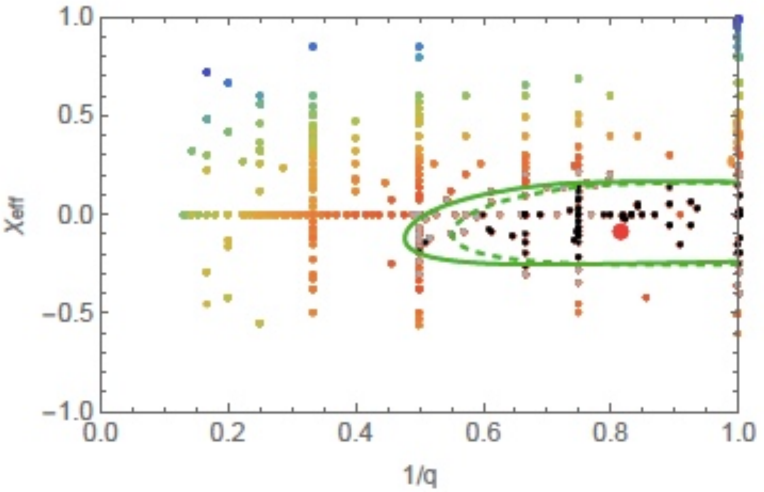
- Now improved to error ≲1E-4 [Nakano+2015] with new more accurate extraction to order $1/r^2$ (including spins)

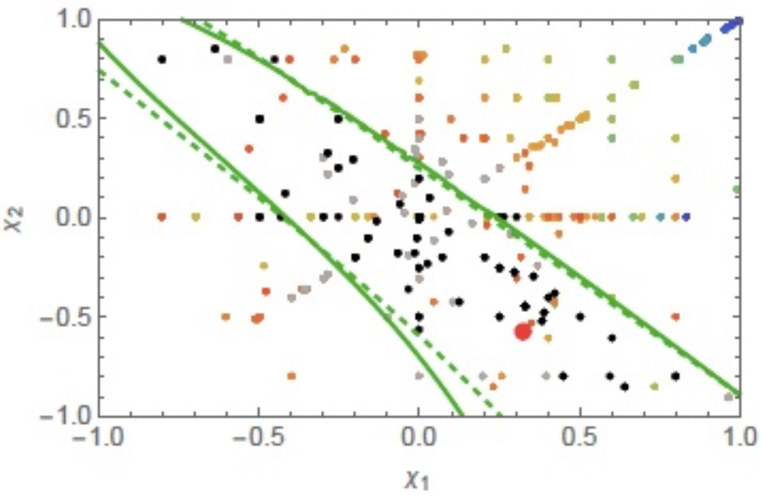
$$r\psi_4^{\ell m}|_{r=\infty} = r\psi_4^{\ell m}(t,r) - \frac{(\ell-1)(\ell+2)}{2r}\int dt[r\psi_4^{\ell m}(t,r)] + \frac{(\ell-1)(\ell+2)(\ell^2+\ell-4)}{8r^2}\int\int dtdt[r\psi_4^{\ell m}(t,r)] - \frac{3M}{2r^2}\int dt[r\psi_4^{\ell m}(t,r)] + \mathcal{O}(1/r^3).$$



HIGHER WAVEFORM MODES

- Important for both PE [Lange+2017]
  - Bayesian method that directly compares GW data to NR simulations
  - Using l= 3 modes gain more information from the signal and can better constrain the parameters





- Also important to test GR: mode mixing unique to GR vs non-GR

Images credits: J. Lange, RIT.

HOW WELL NR DO AT EXTRACTING HIGHER WAVEFORM MODES

Matching for various waveform modes for GW150914

ℓ	m	$N100$	$N110$	$N120$	$\langle h^{L6}_{\ell m} \vert h^{L6}_{\ell m} \rangle$
2	0	0.8854	0.8863	0.8870	9.82
2	1	0.9905	0.9914	0.9908	16.78
2	2	0.9980	0.9980	0.9980	927.74
3	0	0.7822	0.8146	0.8356	1.02
3	1	0.9517	0.9569	0.9582	1.52
3	2	0.9978	0.9980	0.9981	28.59
3	3	0.9927	0.9933	0.9933	42.17
4	0	0.3603	0.3581	0.3554	0.05
4	1	0.7910	0.8348	0.8616	0.17
4	2	0.9074	0.9425	0.9562	1.79
4	3	0.9844	0.9909	0.9938	2.50
4	4	0.9863	0.9886	0.9901	40.95
5	0	0.3638	0.4050	0.4458	0.01
5	1	0.2994	0.3652	0.4227	0.01
5	2	0.6108	0.6176	0.6392	0.14
5	3	0.7813	0.8709	0.9197	0.32
5	4	0.9705	0.9815	0.9879	2.49
5	5	0.9315	0.9552	0.9696	4.94

Table from Lovelace+2016];

For nearly equal mass, NR do quite well already

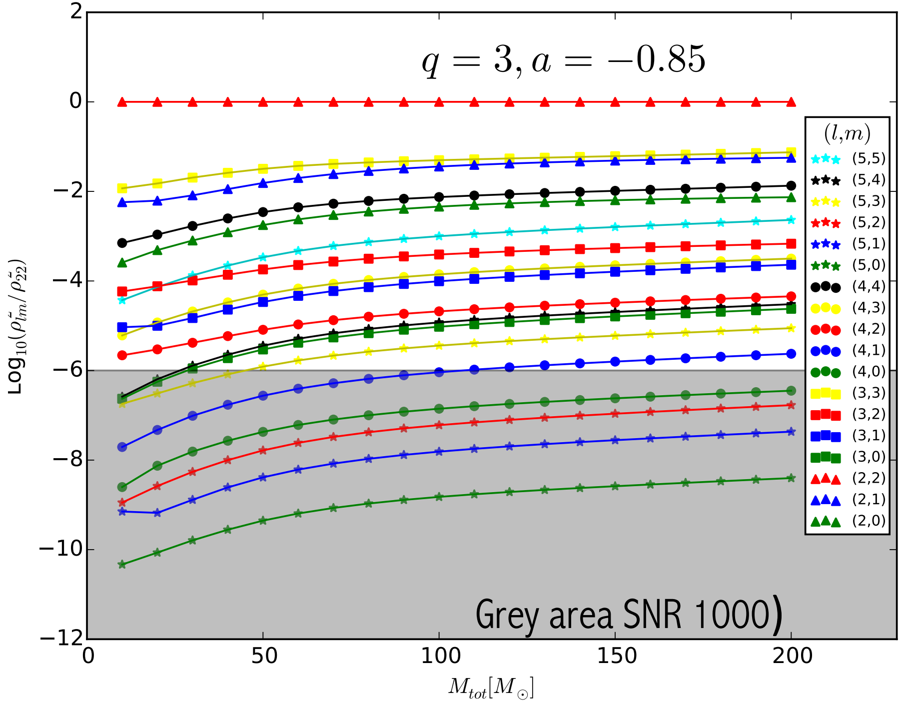
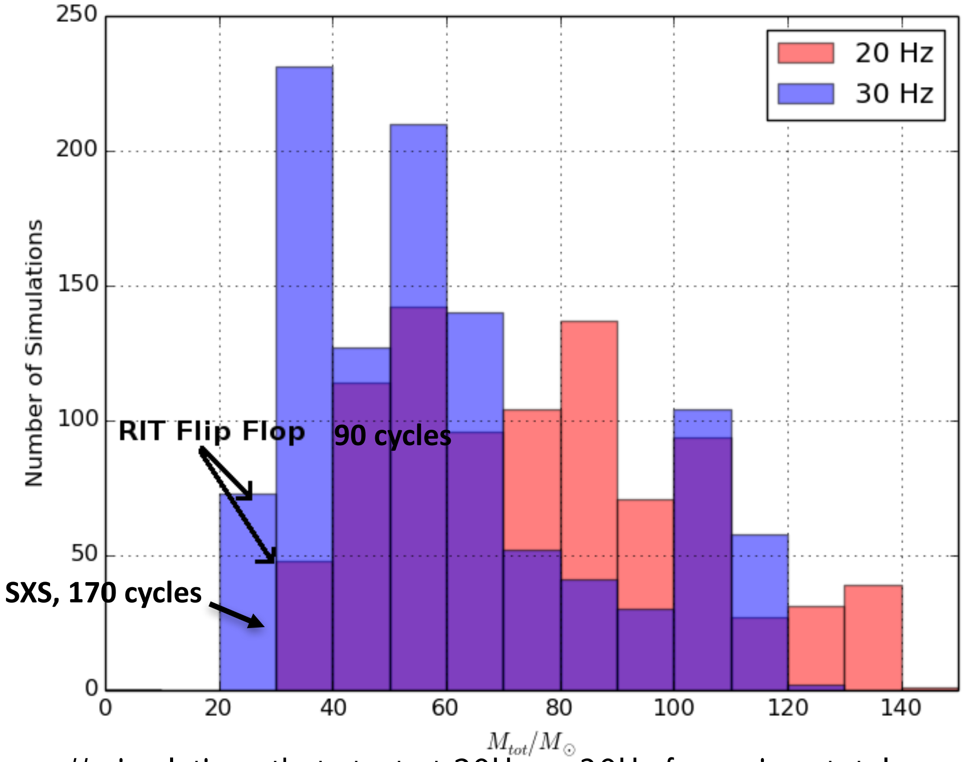


Image credits: Lange & O'Shaughenessy, RIT.

From SXS Catalog: https://arxiv.org/pdf/1304.6077.pdf

HOW WELL NR WAVEFORMS DO IN LENGTH

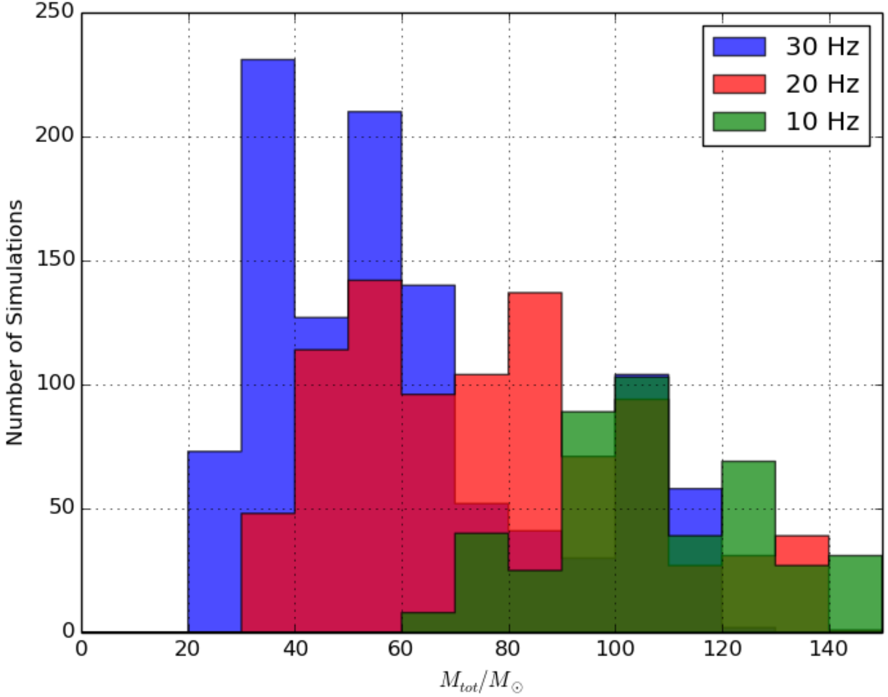


simulations that start at 20Hz or 30Hz for a given total mass

- For comparable mass, $M_{tot}/M_{\odot} \gtrsim 50$ is covered by today's simulations, with ~20 orbits.
- As $M_{tot}/M_{\odot}$ becomes smaller, the duration of the signal increase very quickly, and for Mtot ~ 30 and below, one needs hybrids.
- Some high-mass ratio waveforms and long waveforms are now available, but they are still quite computationally challenging, so one needs hybrids.
- Little on eccentricity (without or with spin), but expect a lot of work in progress in 5 years!

Images credits: James Healy (RIT)

IF WE HAD 10HZ NOW



now include 10Hz

Images credits: James Healy (RIT)

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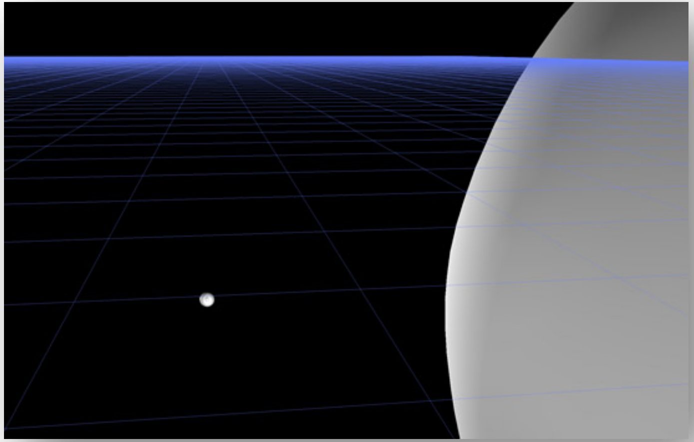
SOME CORNERSTONE GR SIMULATIONS

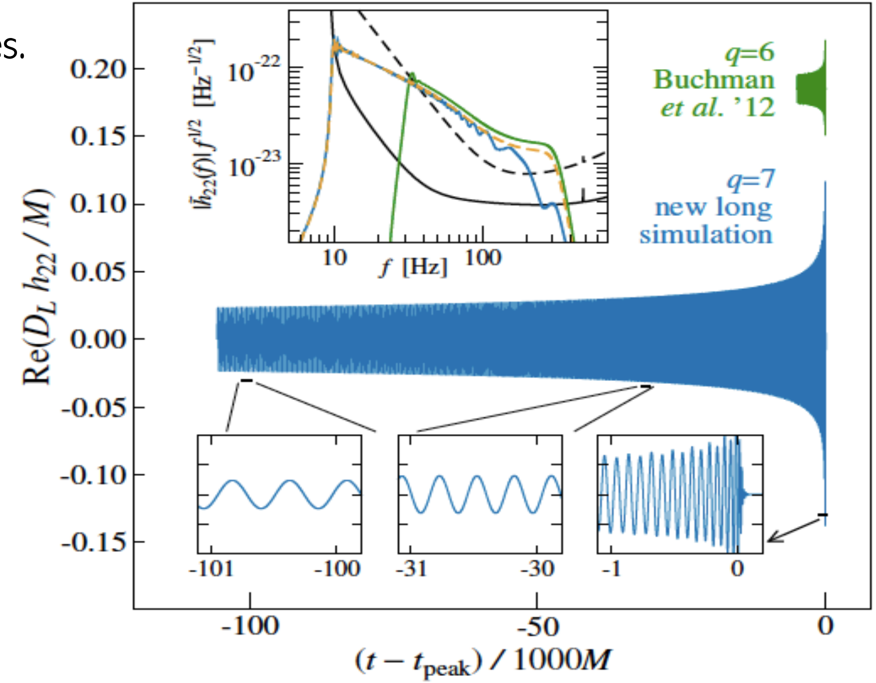
Computational speed depends strongly on BHs parameters and length

- High q, high spin yet very expensive (~several months of running time)
- Long sims need higher resolution, too!
- Use of hybrid models and/or perturbative approaches.



Extreme mass ratios 1:100 with LazEv

[Lousto et al, PRL 2010]



Very long NR simulations (350 orbits) with SpEC

[Szilagyi et al, PRD, 2015]

FUTURE WORK IN NR SIMULATIONS OF BBH SYSTEMS

- Expand parameter space of catalogs
  - 8-dimensional parameter space: mass-ratio, spins, eccentricity
  - Need more waveforms for high q, high spin, eccentricity and many orbits

- Improve the accuracy and efficiency of simulations
  - High q, high spin, and many orbits are still too expensive
  - With better measurements, we need better accuracy
  - Exploit parallelism to improve scaling and run time, e.g. MPI vs task driven parallization
  - Develop new techniques: Discontinuous Galarkin methods in SpEC (SXS), Curvilinear coordinates (RIT/WVU), Multipatch methods (RIT).

- Getting some remaining details correct
  - definitions of masses and spins NR vs PN/EOB
  - waveform extraction

- Explore non GR theories

PROSPECTS OF NON-GR NR SIMULATIONS

- Precision tests of GR requires NR waveforms for BBH systems in non-GR theories
- Essentially, no inspiral-merger-ringdown NR waveforms available of same quality as for GR
- Too many theories, many are ill-posed, and each one requires significant work to explore!
  - A lot of old literature, some can be valuable …
- Use some criteria to discard ill-posed theories, so we can discard them[Berti+ 2015]
  - Cosmology motivated nonGR theories (e.g. by metric theories) are the same as GR for BBH mergers
  - Scalar tensor gravity: BBH waveforms essentially indistinguishable [Healy+2011]
- Some theories are derived as low-energy limits of some (unknown) fundamental theory of quantum gravity, and as such carry some weight – can we fix them instead? [Cayuso+ 2017] (see also Lehner talk at IAP2017)
  - Linearize dynamical Chern-Simons gravity (dGS) [Stein+,2017];
  - Einstein-Maxwell-Dilaton (EMD) [Hirschmann+2017]
  - f(R) theories casted as Klein-Gordon [Cao+2017]

DISCUSSION/CONCLUSION

- BBH simulations stunningly successful in past years
  - More waveforms that can be carefully analyzed: some higher modes and precession
  - A few "hard" simulations: high q, high spin, and many orbits
- Really complex codes, many error sources, but very successful code comparison!
- NR is essentially OK even when LIGO SNR will improve to ~100, but need to improve efficiency and accuracy to deal with high q, high spins and long waveforms.
- For SNR ~1000 (Voyager, 3G detectors, LISA, etc) we need more accuracy:
  - Waveform errors must be ~ 1E-6 ? What is good for PE, what is good for testing GR?
  - More accuracy, higher modes, needed to test GR vs non-GR
  - IMR BBH? High mass-ratio still largely unexplored!
  - Eccentricity? Totally unexplored at the accuracy needed, longer simulations to match with AR models!
- NR predictions from modified theories are challenging:
  - NR waveforms currently lacking, but some work started.
  - Need to encourage work and community building, to move faster!
  - A lot of old literature!!!