Temperature Control and Coupled Oscillator Modeling for LIGO Voyager R&D

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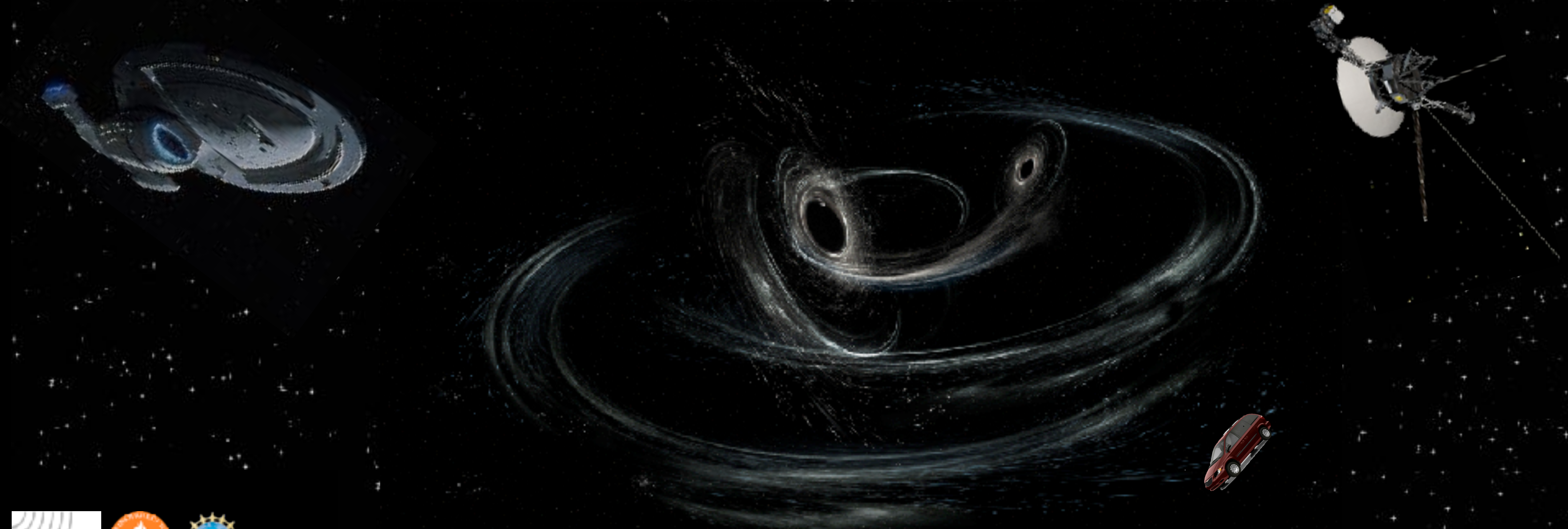
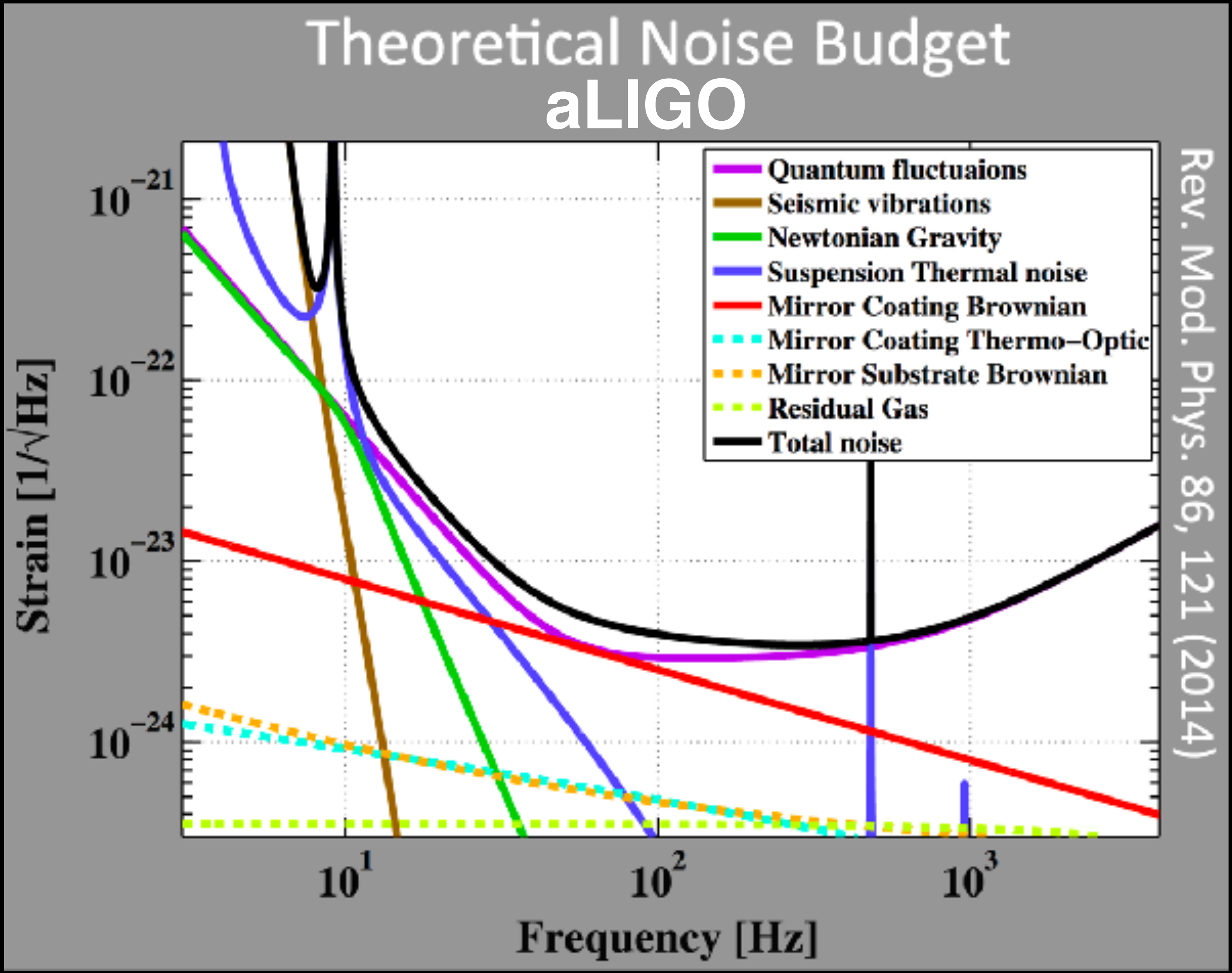


Why LIGO Voyager?

- aLIGO limited by mirror coating brownian noise in 100Hz range
- Cryogenic operation reduces thermal noise



Rev. Mod. Phys. 86, 121 (2014)

Brownian Thermal Noise

$$x^2(\omega) = \frac{4 k_b T k \phi}{\omega[(k - m\omega^2)^2 + k^2 \phi^2]}$$

Remembering that…

$$Q = \frac{1}{\phi}$$

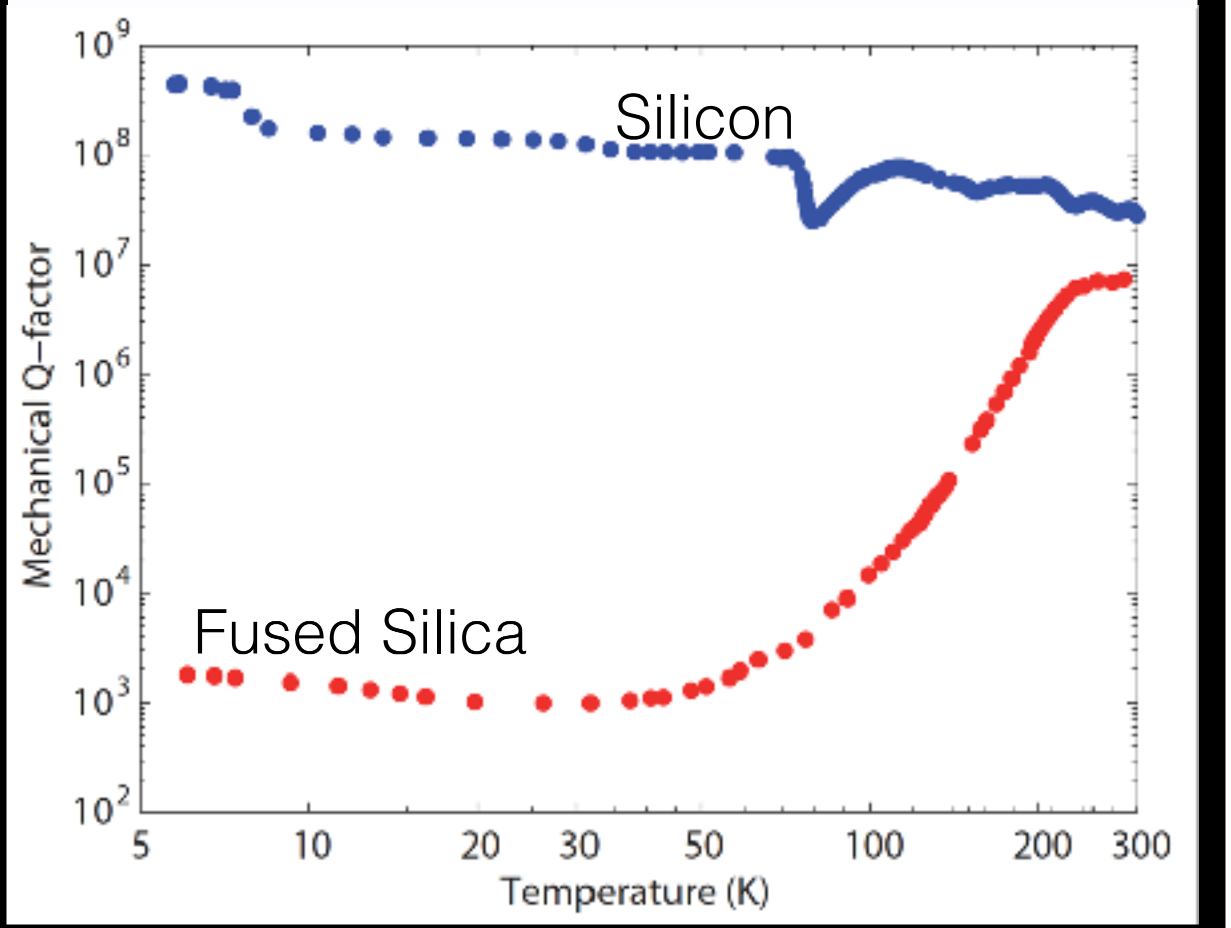
Yields…

$$\langle x_{th}^2 \rangle \propto \frac{T}{Q}$$

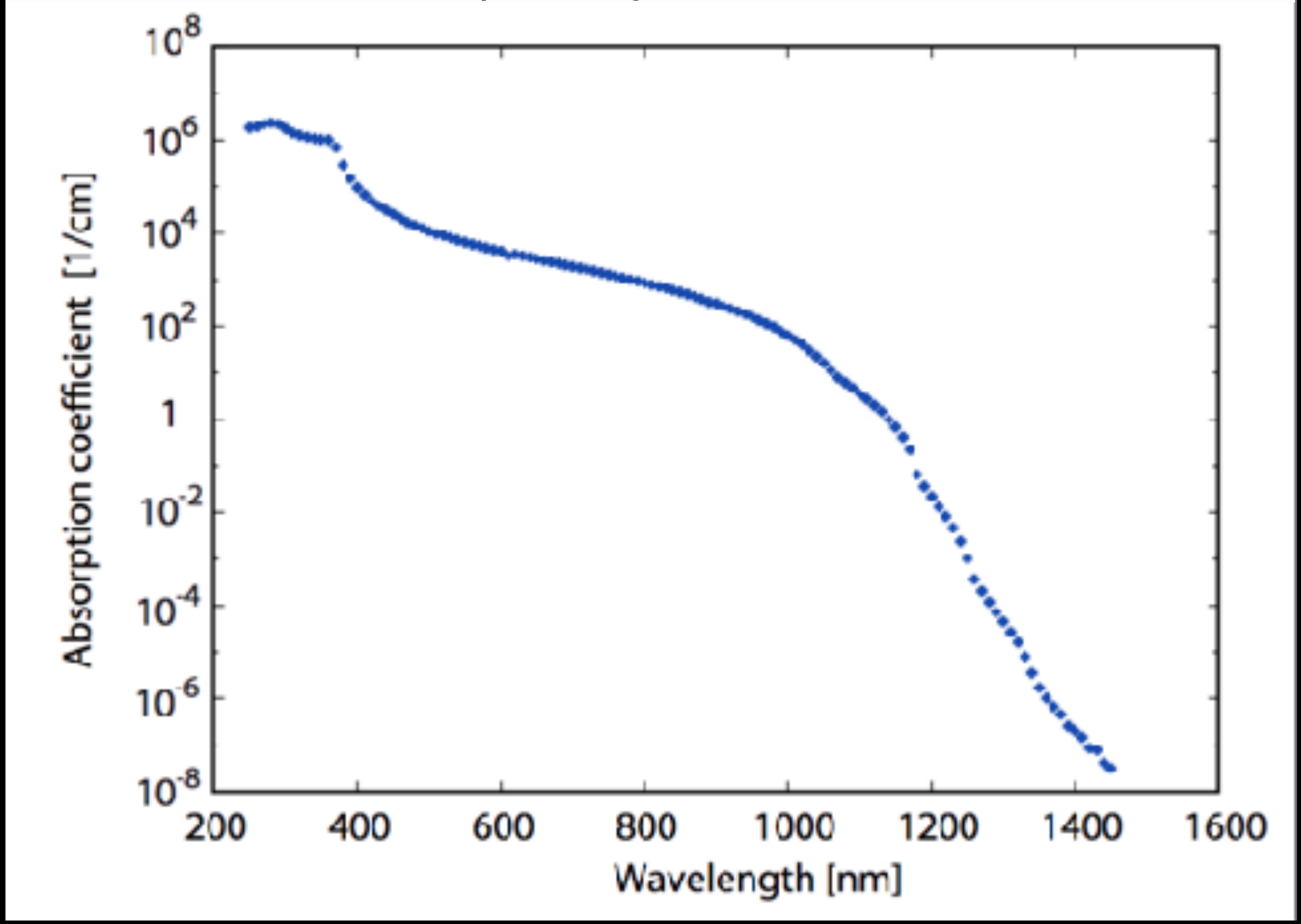
What is LIGO Voyager?

- Silicon has higher Q factor than fused silica at cryogenic temperatures
- Absorbs 1550nm light at 10^{-8}cm^{-1}- favorable for reducing Brownian thermal noise

Q Factor of Silicon and Fused Silica



Absorptivity of Silicon.

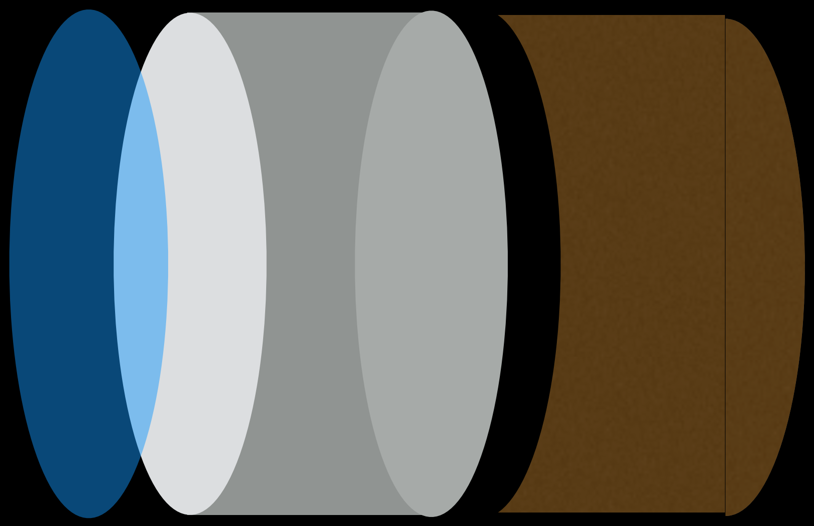


Schnabel. et. al.. Unknown

What is LIGO Voyager?

A few more relevant differences…

- Voyager will operate at 123K
- Voyager may use amorphous silicon as reflective coatings (not layered tantalum)
- Voyager may use emissive coatings for radiative cooling



Emissive coating covers barrels of optic

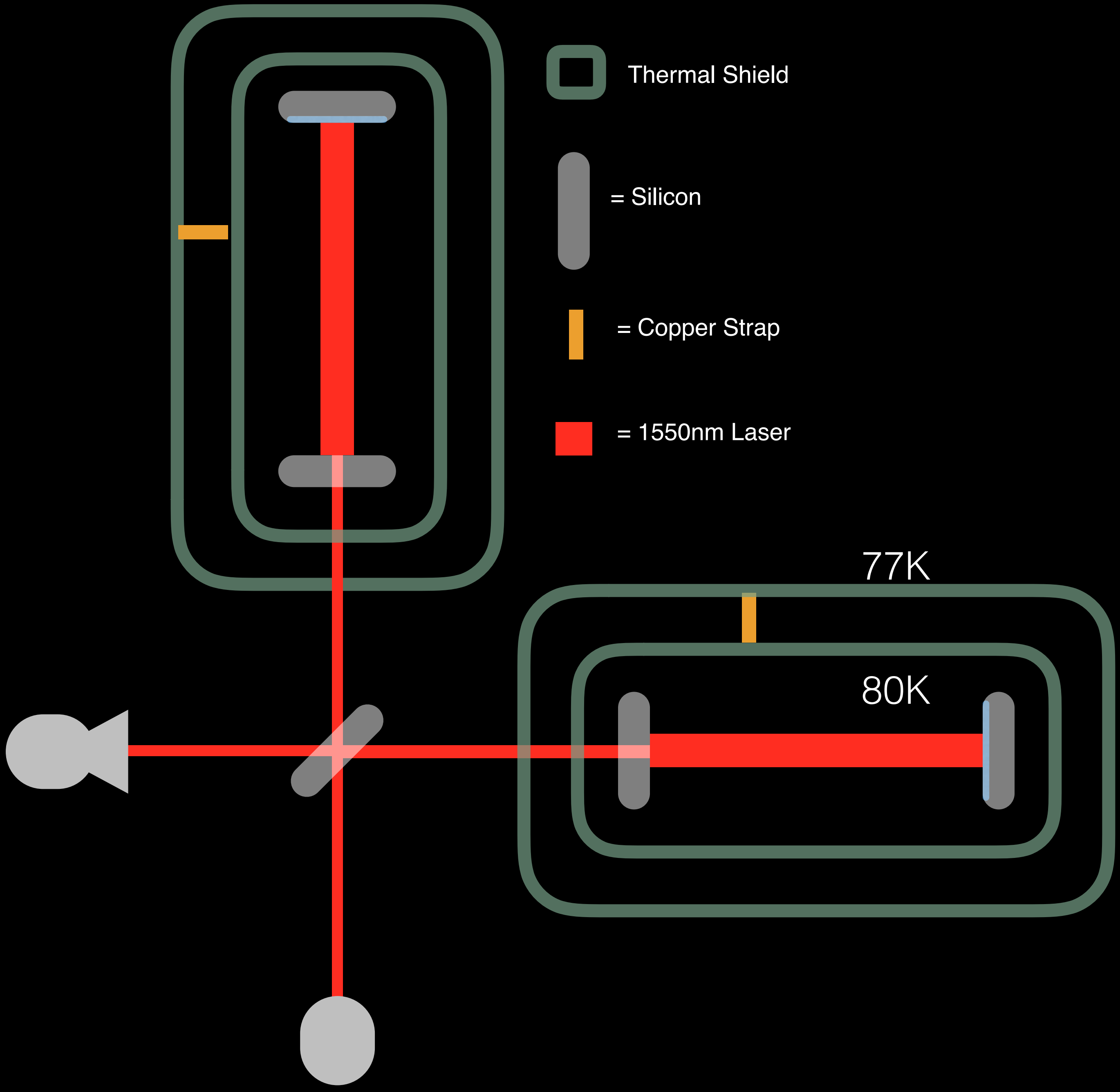
Reflective coating covers front of optic

What is LIGO Voyager?

Voyager intends on implementing these upgrades into the current aLIGO infrastructure.

A few questions

- How do we track the temperature of silicon for cryogenic Q measurements?
- How can we determine the loss angle of the coatings?
- How do we keep the test masses cold without mechanically coupling them to the environment?



Simplified Voyager Upgrade Schematic

What is LIGO Voyager?

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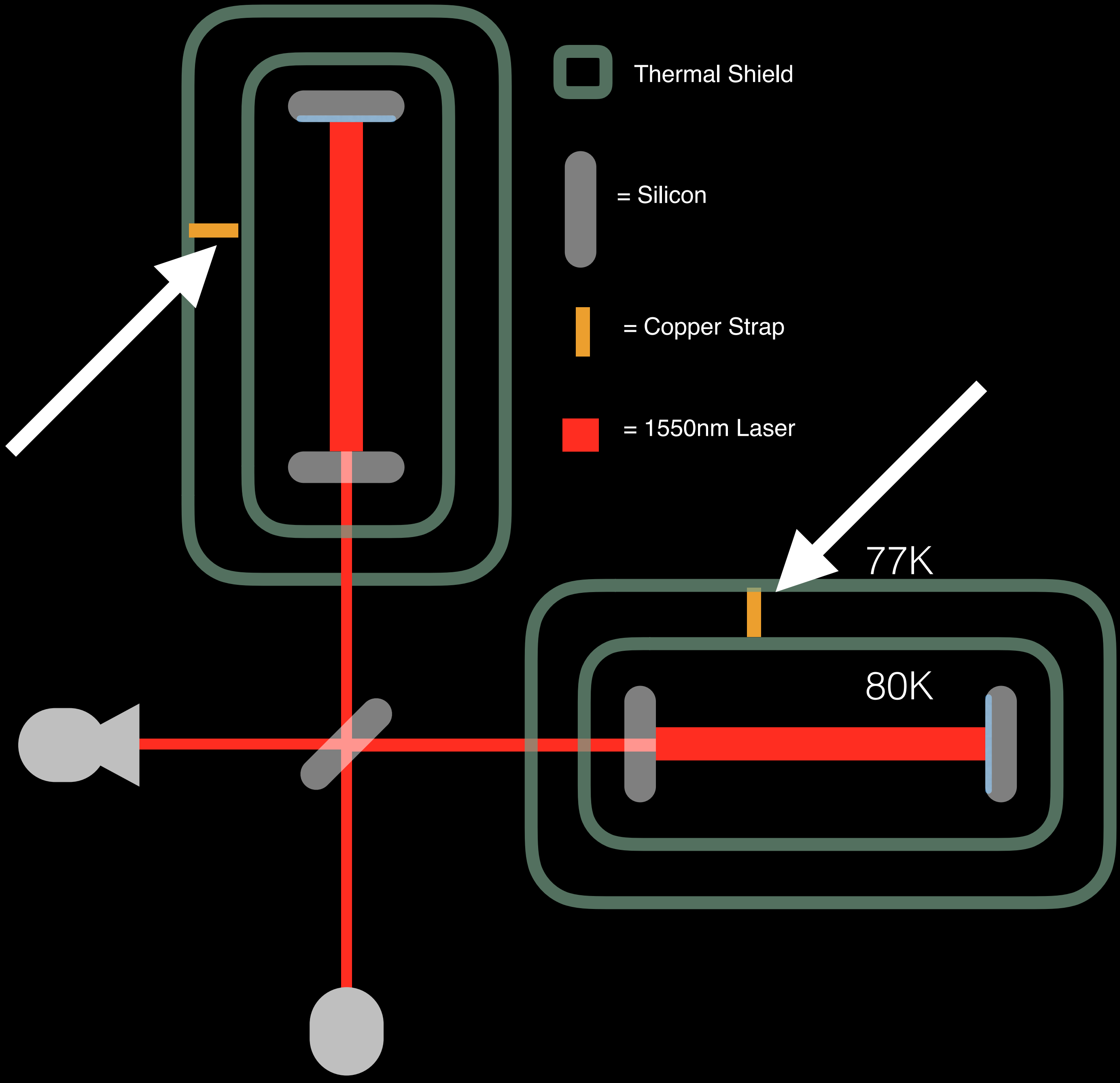

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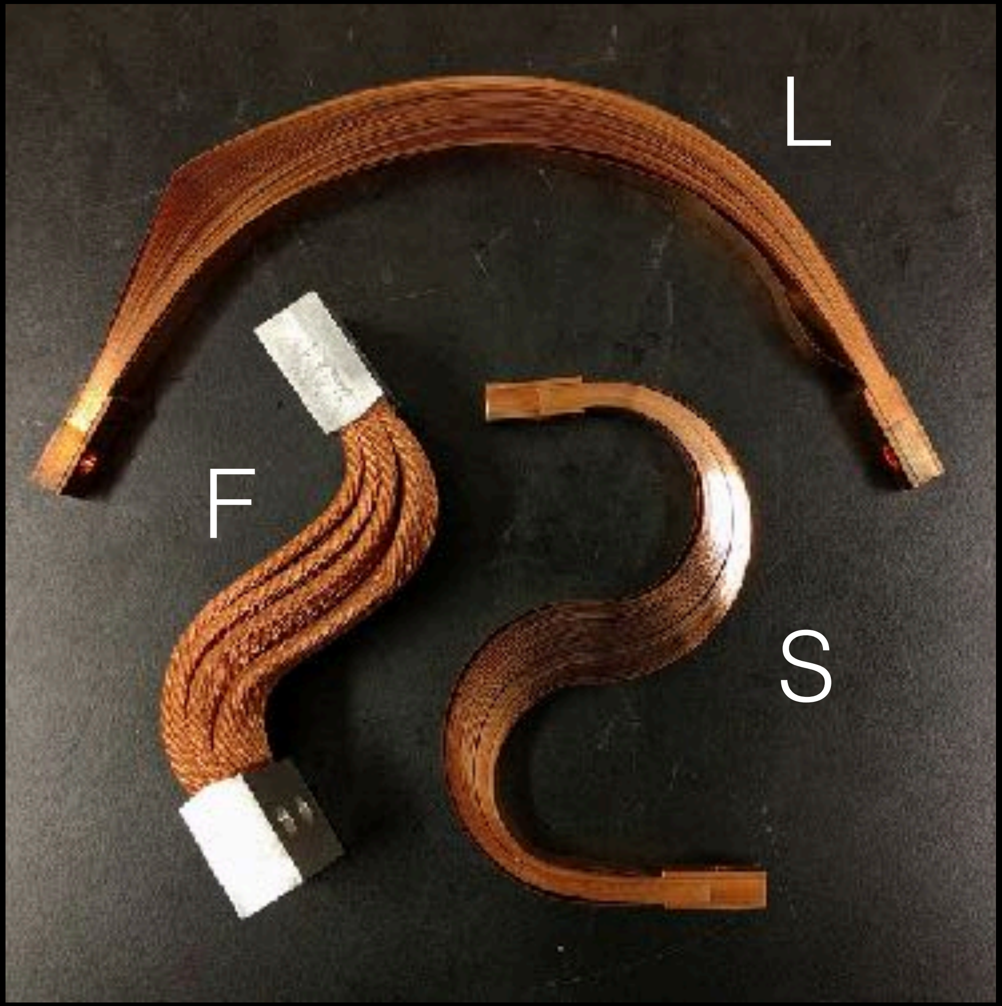
Thermal Shielding and Copper Straps

- Thermally couples the shields the liquid nitrogen

		CHARACERTISTICS
STRAPS	S	70 straps 13x.075mm 200mm long
	L	70 straps 13x.075mm Between 214 and 240mm long
	Fancy	252 coils 1.27mm in diameter 100mm long



Thermal Conductivity = ???

Thermal Conductivity

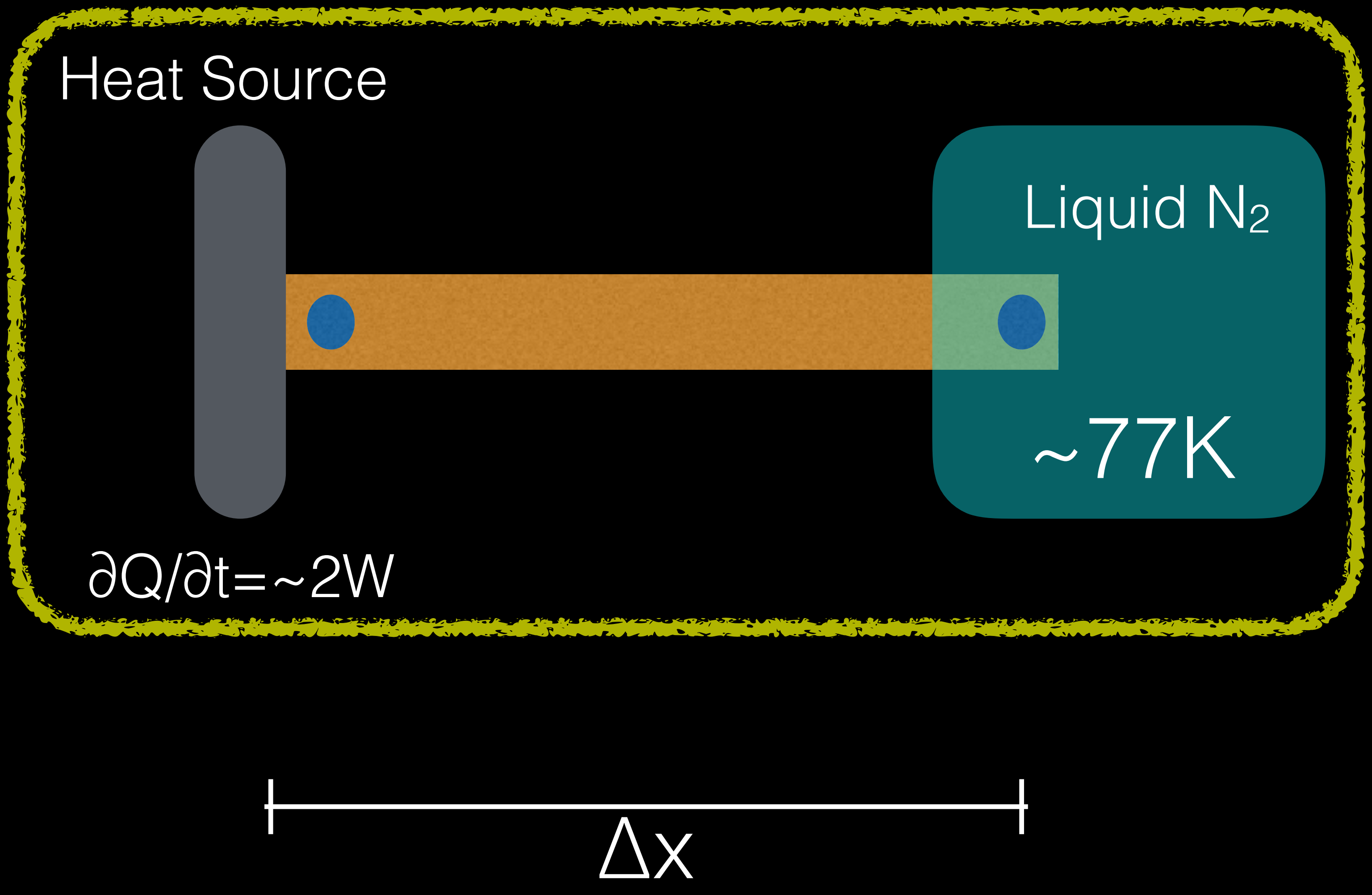
How much heat does this strap transfer?

$$\frac{\partial Q}{\partial t} = -kA\,\frac{\Delta T}{\Delta x}$$

- Dependent on material and temperature
- Dependent on the purity of the copper
- Must know temperature gradient, heat applied, and geometry of material
- Conductivity must be experimentally determined

$$k = -\dot{Q}_R \left(1 + \frac{\Delta T_R}{\Delta T_{amb}}\right) \frac{\Delta x}{A\,(\Delta T_R + \Delta T_{amb})}$$

Thermal Conductivity



● = Thermometer

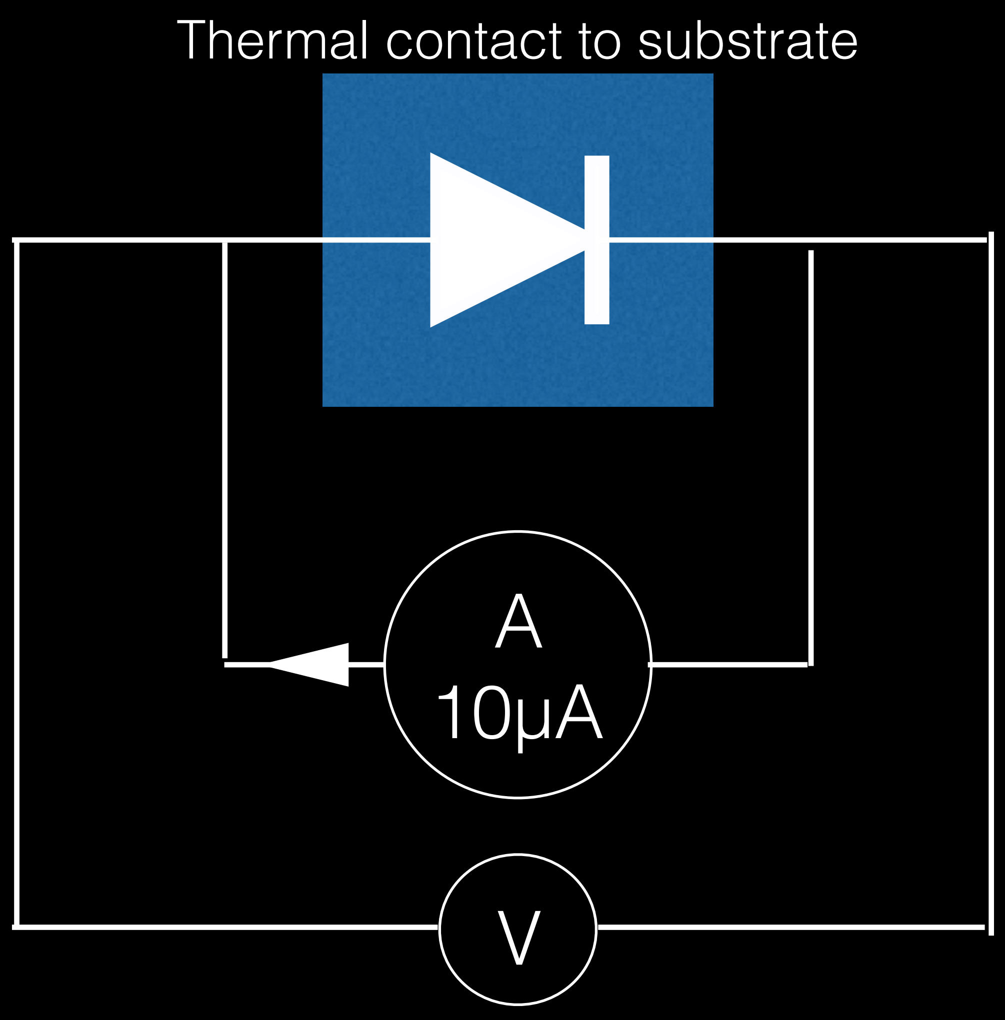
▨ = Strap

☐ = Thermal Insulation

Thermal Conductivity

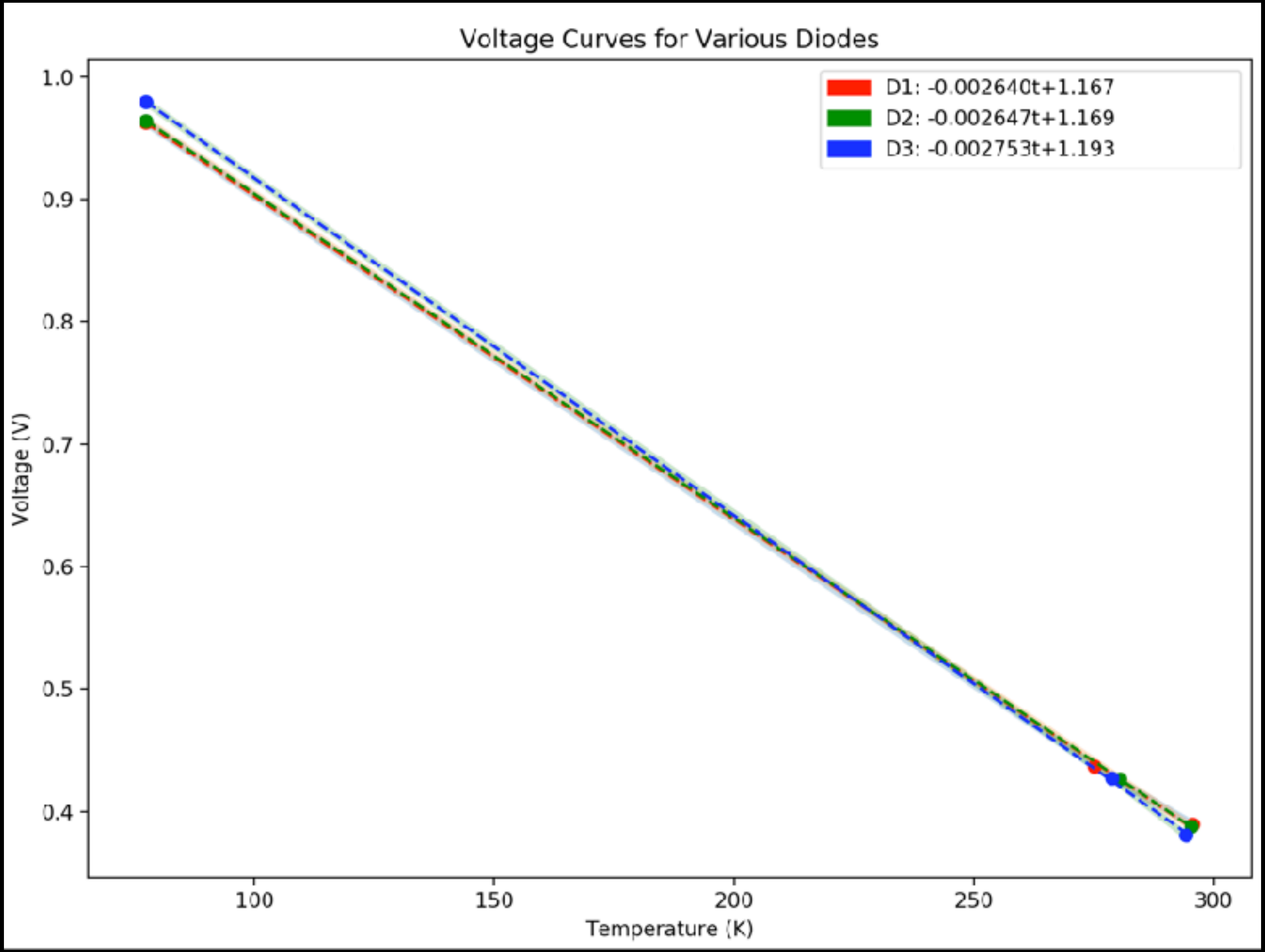
Temperature Sensing Diode Calibration

- Have linear voltage response to changes in temperature
- Use four-lead method to eliminate lead-resistance
- Calibrate at room, ice bath, liquid N_2 temperature, extrapolate

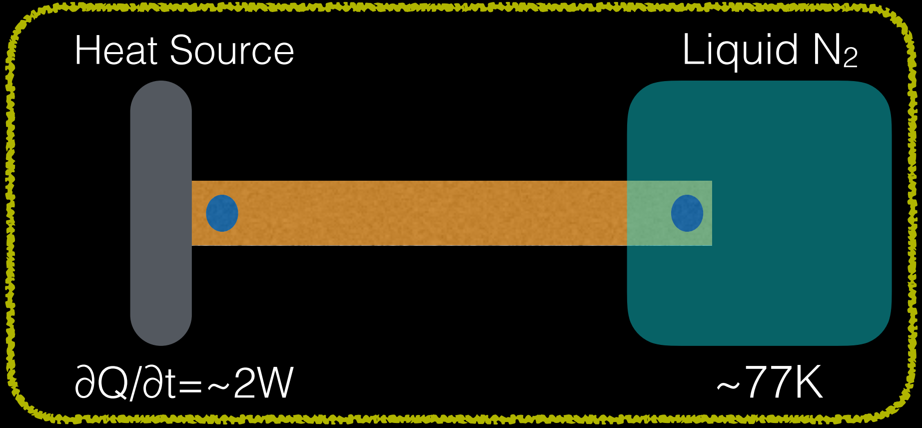


$I=0 » \Delta V=0$

Thermal Conductivity



Thermal Conductivity

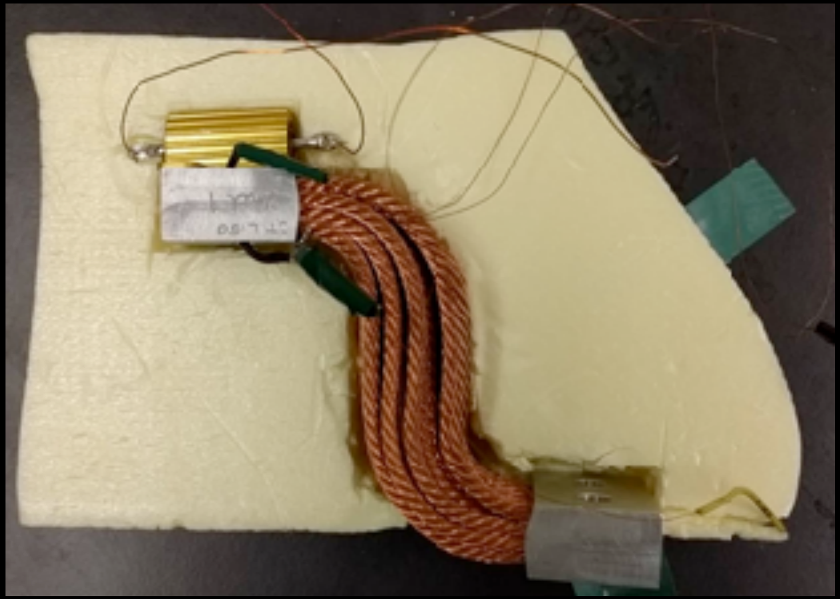


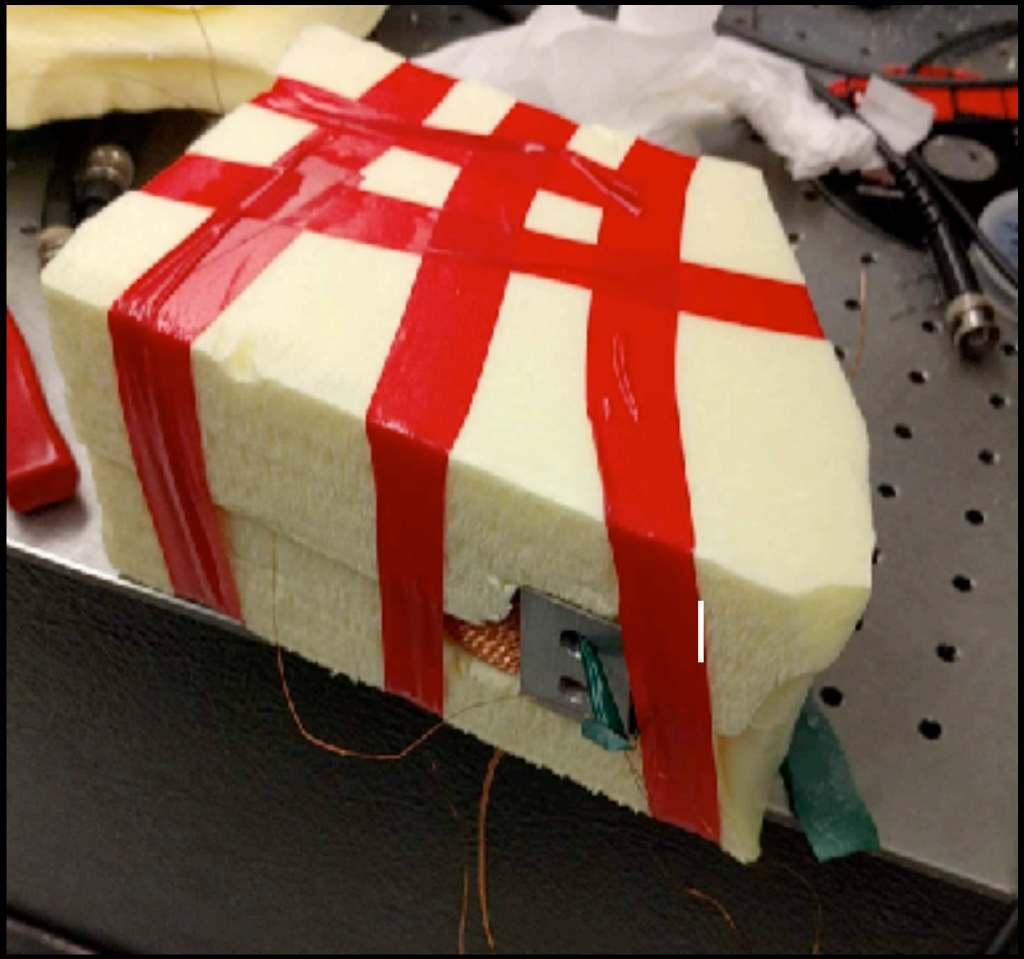
Heat Source

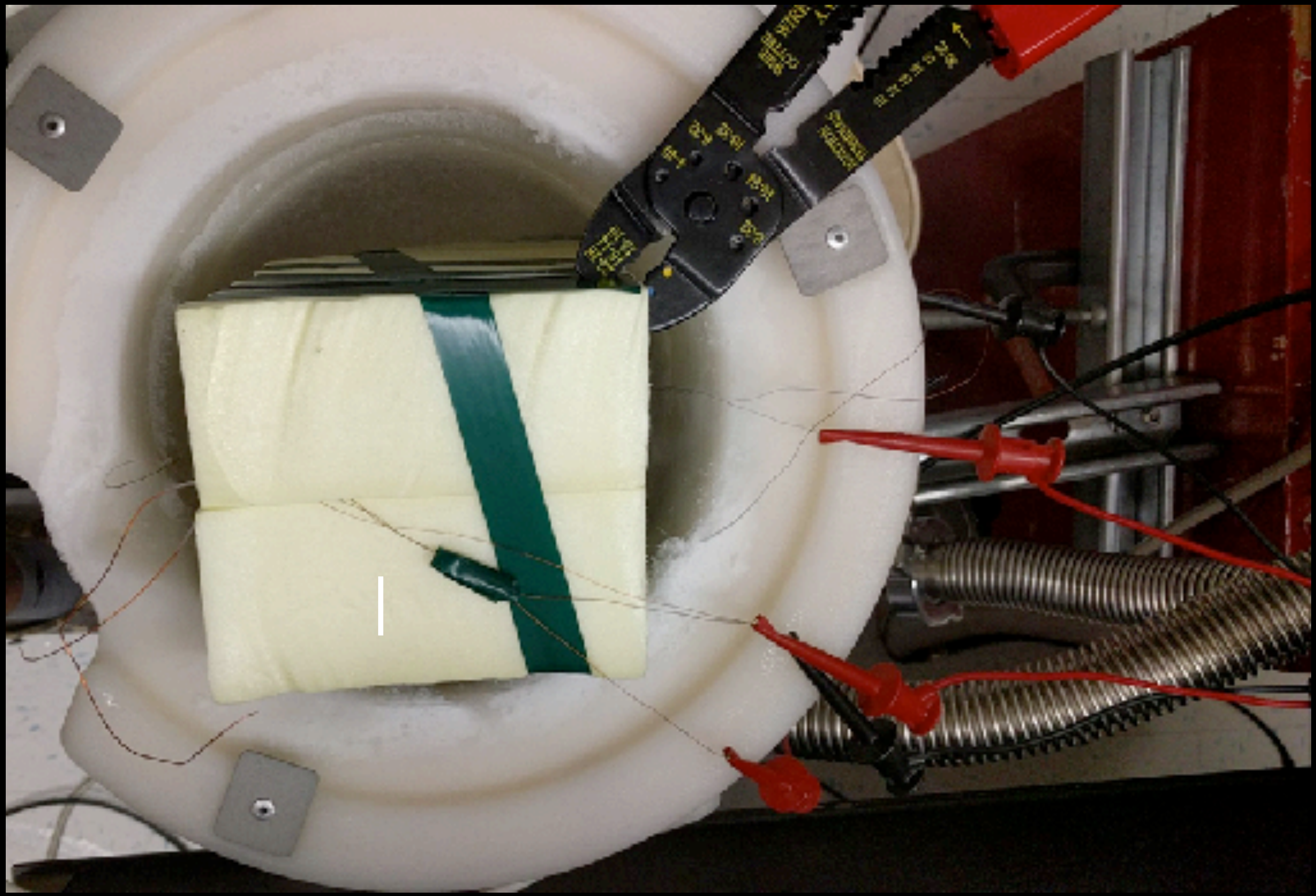
Liquid N_2

$\partial Q/\partial t = \sim 2W$

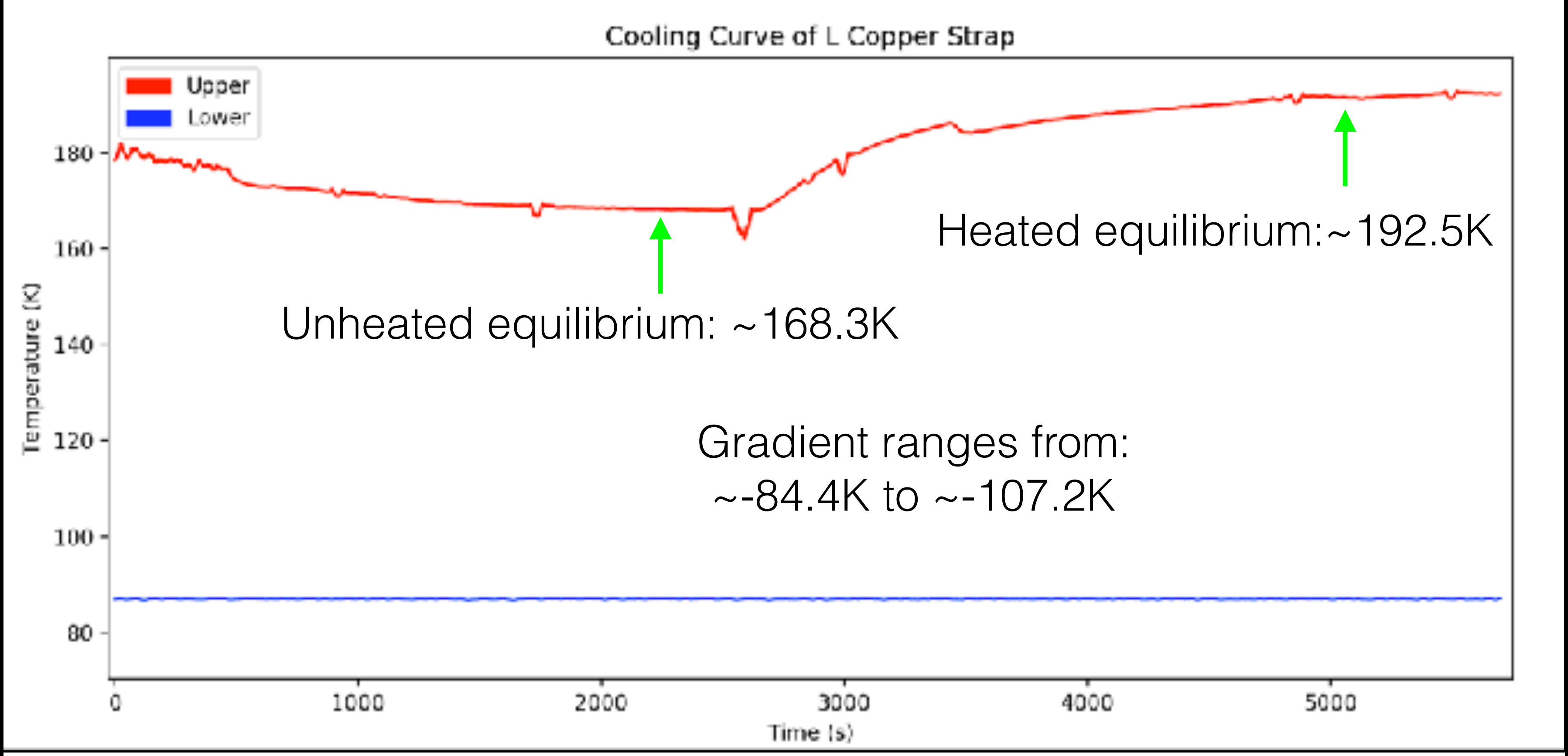
~77K







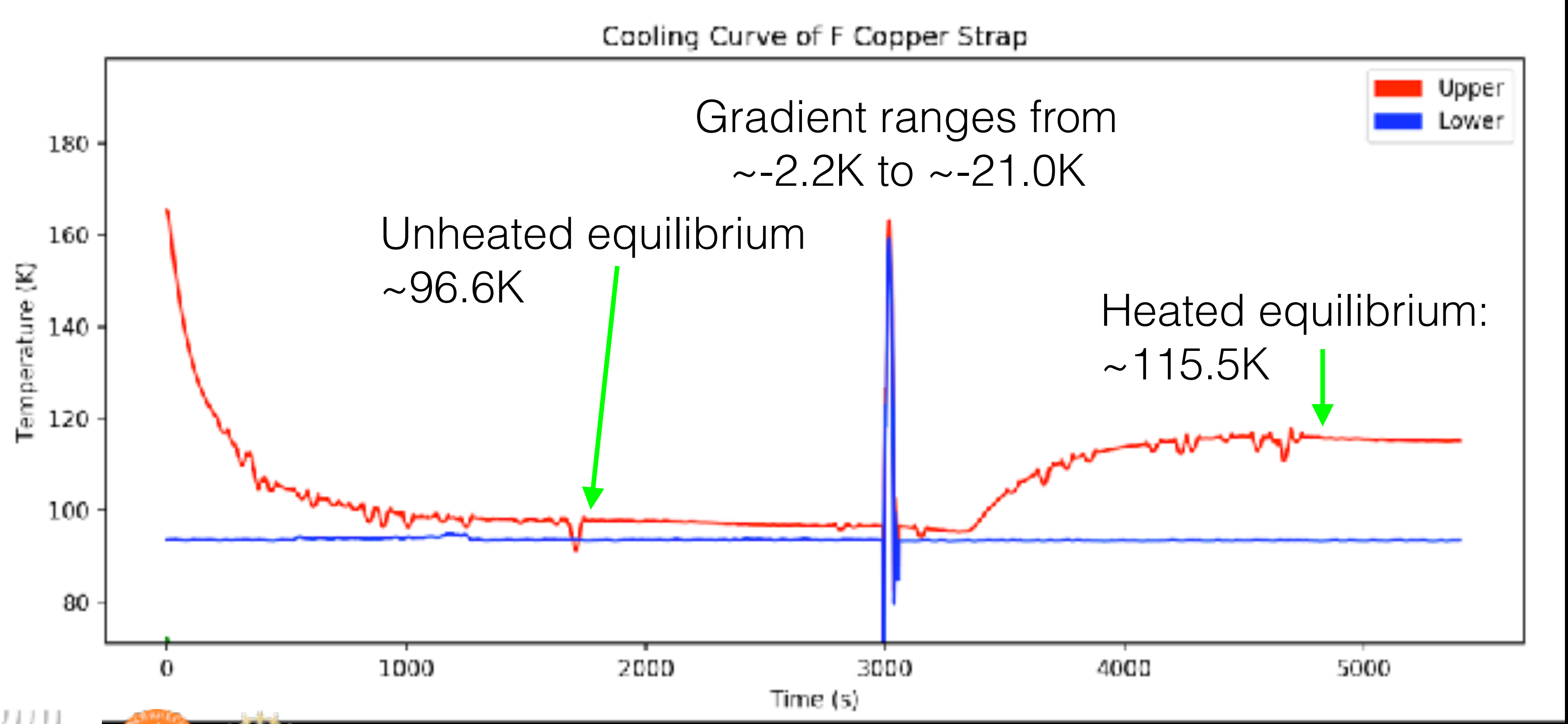
Thermal Conductivity



Heated equilibrium:~192.5K

Unheated equilibrium: ~168.3K

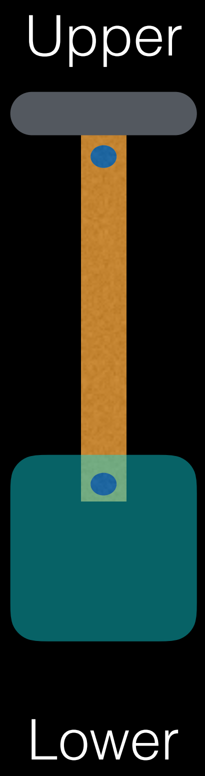
Gradient ranges from:
~-84.4K to ~-107.2K



Gradient ranges from
~-2.2K to ~-21.0K

Unheated equilibrium
~96.6K

Heated equilibrium:
~115.5K



Computed conductivities:
L Strap: 284.76 +/- 15 W/m*K
S Strap: 260.60 +/- 15 W/m*K
F Strap: 34.88 +/- ? W/m*K

The F strap maintains lower temperatures, but L/S have higher conductivity

What is LIGO Voyager?

Voyager intends on implementing these upgrades into the current aLIGO infrastructure.

A few questions

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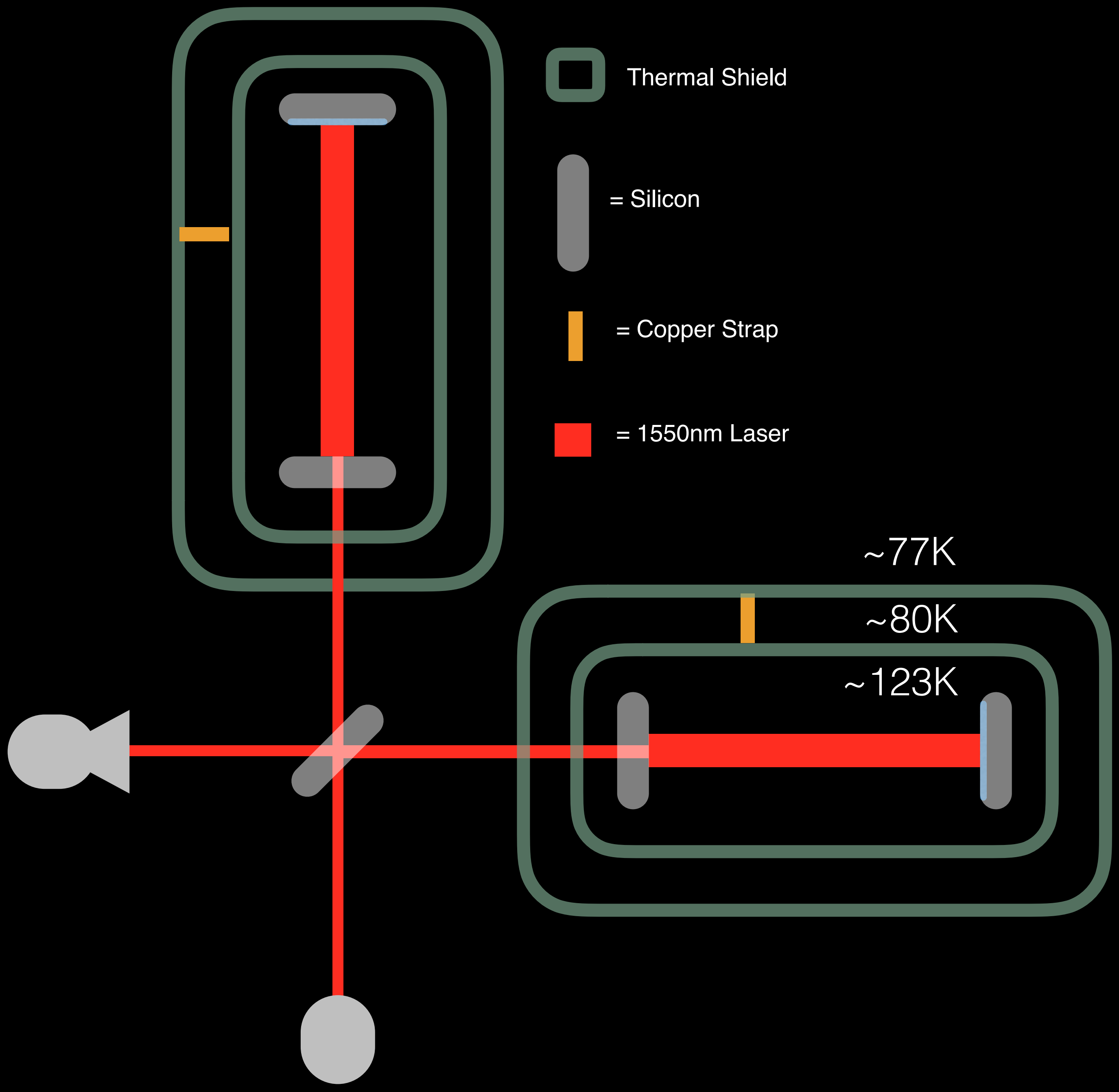
- How can we determine the loss angle of the coatings?
- ~~How do we keep the test masses cold without mechanically coupling them to the environment?~~



Simplified Voyager Upgrade Schematic

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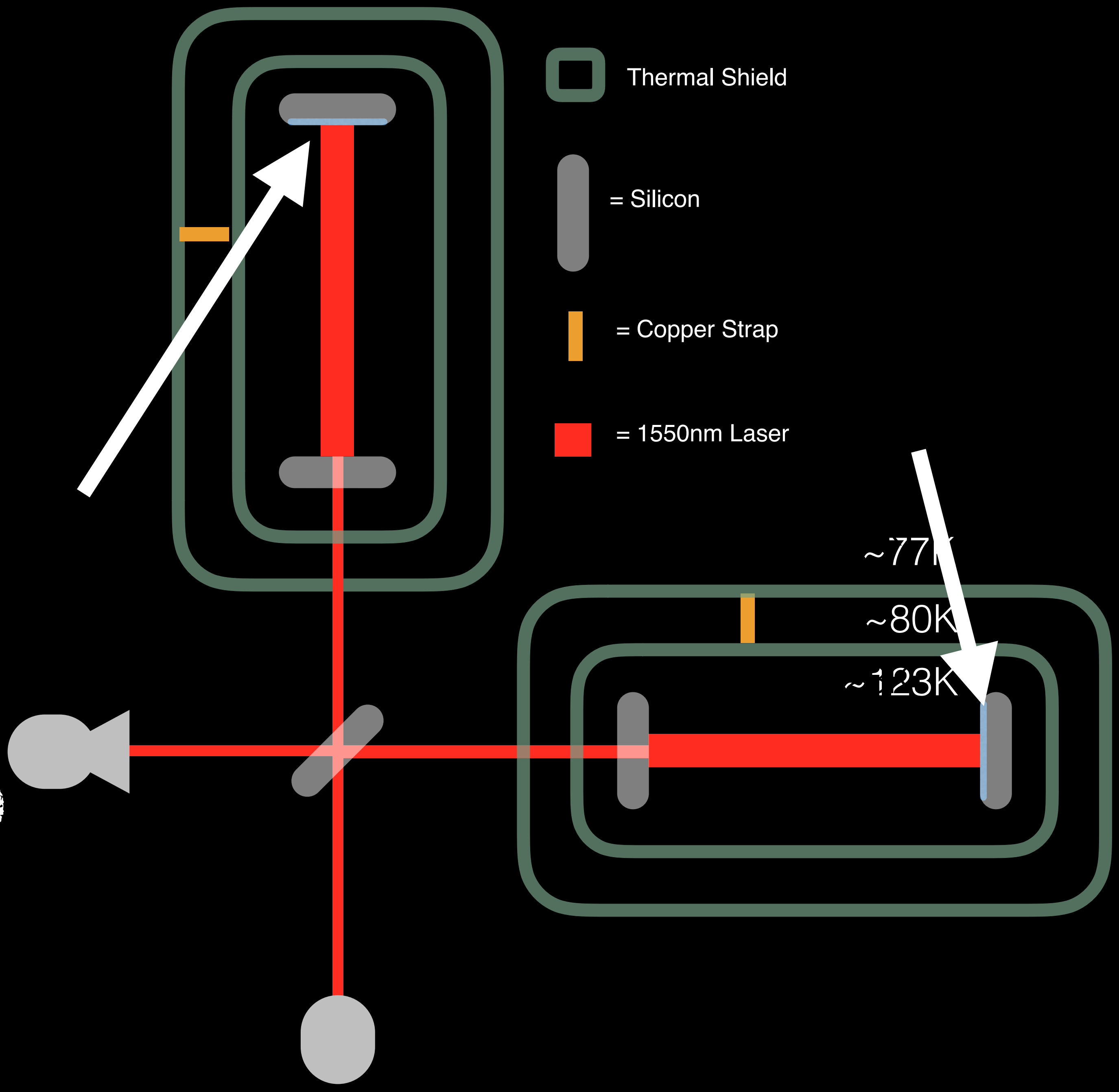


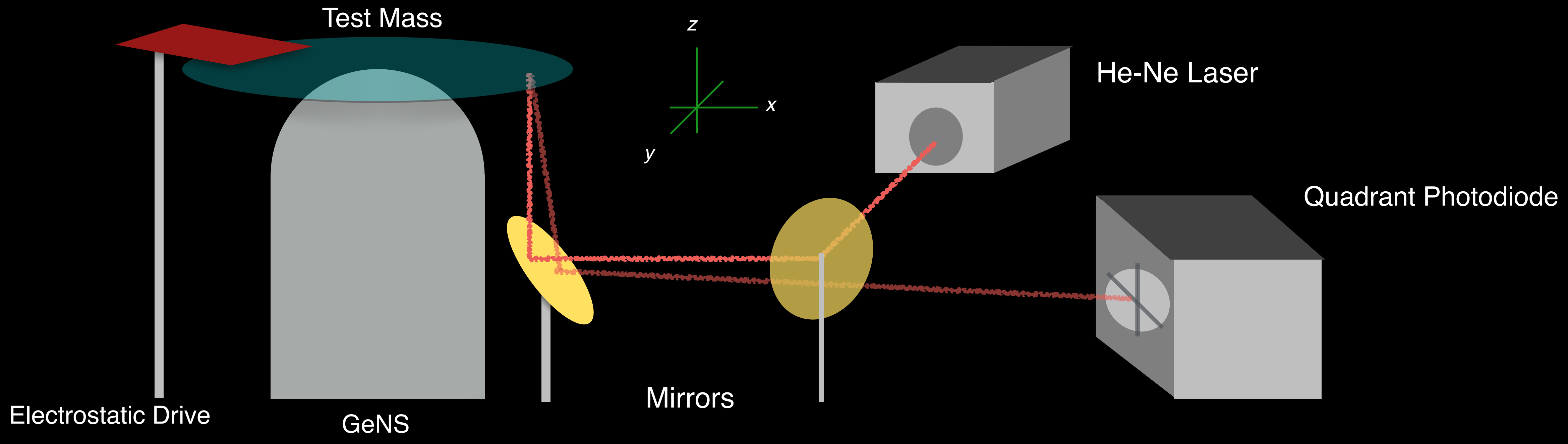
Simplified Voyager Upgrade Schematic

Coatings Loss

- The loss angle is the fraction of energy decayed during its oscillation

$$Q \overset{\text{def}}{=} 2\pi \times \frac{\text{energy stored}}{\text{energy dissipated per cycle}}$$

- A silicon disk's Q can be determined by performing a ringdown measurement
- This is done using the Gentle Nodal Suspension (GeNS)



- Mechanically coupled objects have a collectively unique Q factor

$$\phi_{\text{total}} = \phi_1 \frac{E_1}{E_{\text{total}}} + \phi_2 \frac{E_2}{E_{\text{total}}}$$

Vajente, et. al., 2017

Coatings Loss

- Mechanically coupled objects have a collectively unique Q factor

$$\phi_{total} = \phi_1 \frac{E_1}{E_{total}} + \phi_2 \frac{E_2}{E_{total}}$$

- We can determine a poor oscillator, such as a coating's, Q using this equation and the GeNS

$$\phi_2 = \phi_{total} \frac{E_{total}}{E_2} + \phi_1 \frac{E_1}{E_2}$$

-High Loss Deposition: Coating

-High Q Substrate: Silicon Disk

Coatings Loss

Parameter estimation

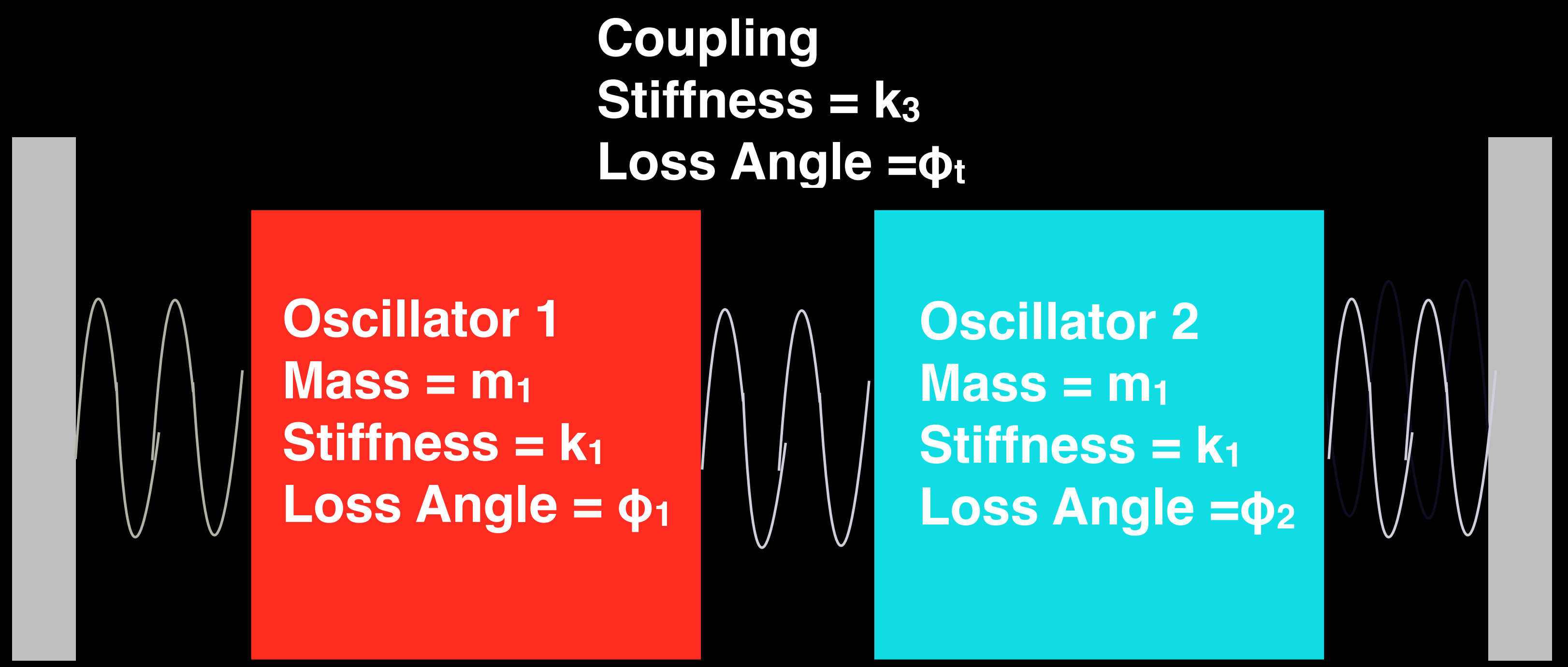
- Model disk/coating to propagate parameter uncertainties into loss angle calculation
- There are many complicated parameters:
    - Young’s Modulus, Poisson’s ratio, shear modulus
- Analyzing a simpler model will build intuition for behavior of system

$$\phi_2 = \phi_{total} \frac{E_{total}}{E_2} + \phi_1 \frac{E_1}{E_2}$$

Approximating:

$$E_1 = k_1 x_1^2 + k_3 (x_1 - x_2)^2$$
$$E_2 = k_1 x_1^2 + k_3 (x_1 - x_2)^2$$
$$E = E_1 + E_2 - + k_3 (x_1 - x_2)^2$$

Coupled Harmonic Oscillator!



Coupling
Stiffness = k_3
Loss Angle =ϕ_t

Oscillator 1
Mass = m_1
Stiffness = k_1
Loss Angle = ϕ_1

Oscillator 2
Mass = m_1
Stiffness = k_1
Loss Angle =ϕ_2

Coatings Loss

Parameter estimation

★ Analytically derive equation of motion

$$\mathcal{L} = \frac{1}{2}\left(k_1 x_1^2 + k_2 x_2^2 + k_3 (x_1 - x_2)^2 + m_1 \dot{x}_1^2 + m_2 \dot{x}_2^2\right)$$

Solve the equation of motion for the basis states

$$x_1 = a_1 \cos(\omega_1 t)$$
$$x_2 = a_2 \cos(\omega_2 t)$$

$$\omega_1 = \omega_2$$
$$\omega_1 = -\omega_2$$

Painful algebra

Solve for the ratio of amplitude values

$$a_{12} = \frac{m_2}{2 k_3}\left(\frac{k_2 + k_3}{m_2} - \frac{k_1 + k_3}{m_1} - \sqrt{\left(-\frac{4 (k_2 k_3 + k_1 (k_2 + k_3))}{m_1 m_2} + \frac{((k_2 + k_3) m_1 + (k_1 + k_3) m_2)^2}{m_1^2 m_2^2}\right)}\right)$$
$$a_{21} = \frac{m_2}{2 k_3}\left(\frac{k_1 + k_3}{m_1} - \frac{k_2 + k_3}{m_2} - \sqrt{\left(-\frac{4 (k_2 k_3 + k_1 (k_2 + k_3))}{m_1 m_2} + \frac{((k_2 + k_3) m_1 + (k_1 + k_3) m_2)^2}{m_1^2 m_2^2}\right)}\right)$$

Substitute into loss angle equation, take partial derivatives

$$\phi_2 = \phi_t \left(1 + \frac{a_{12}^2 k_1}{k_2 + k_3 (a_{12} - 1)^2}\right) - \phi_1 \frac{a_{12}^2 k_1 + k_3 (a_{12} - 1)^2}{k_2 + k_3 (a_{12} - 1)^2}$$

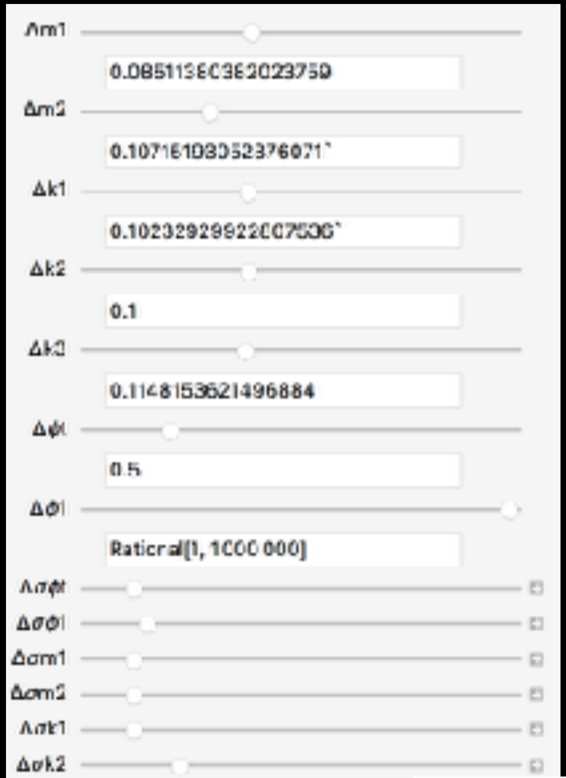
(cosines cancel out)

Loss angle uncertainty

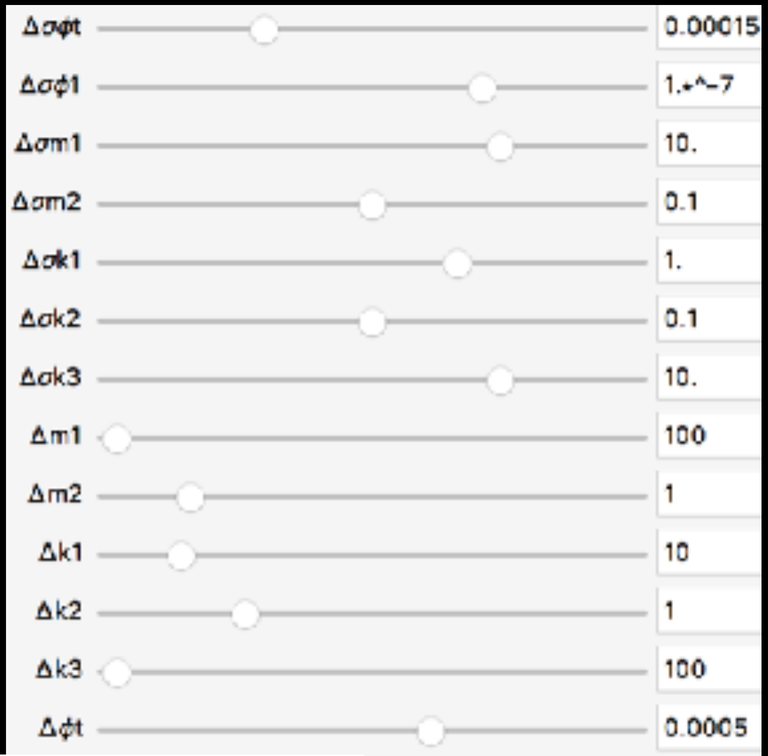
$$\delta\phi_2 = \sqrt{\sum_{\alpha = k_1}^{k_2, k_3, m_1, m_2, \phi_1, \phi_t} \left(\sigma_\alpha \frac{\partial \phi_2}{\partial \alpha}\right)^2}$$

Coatings Loss

Parameter estimation

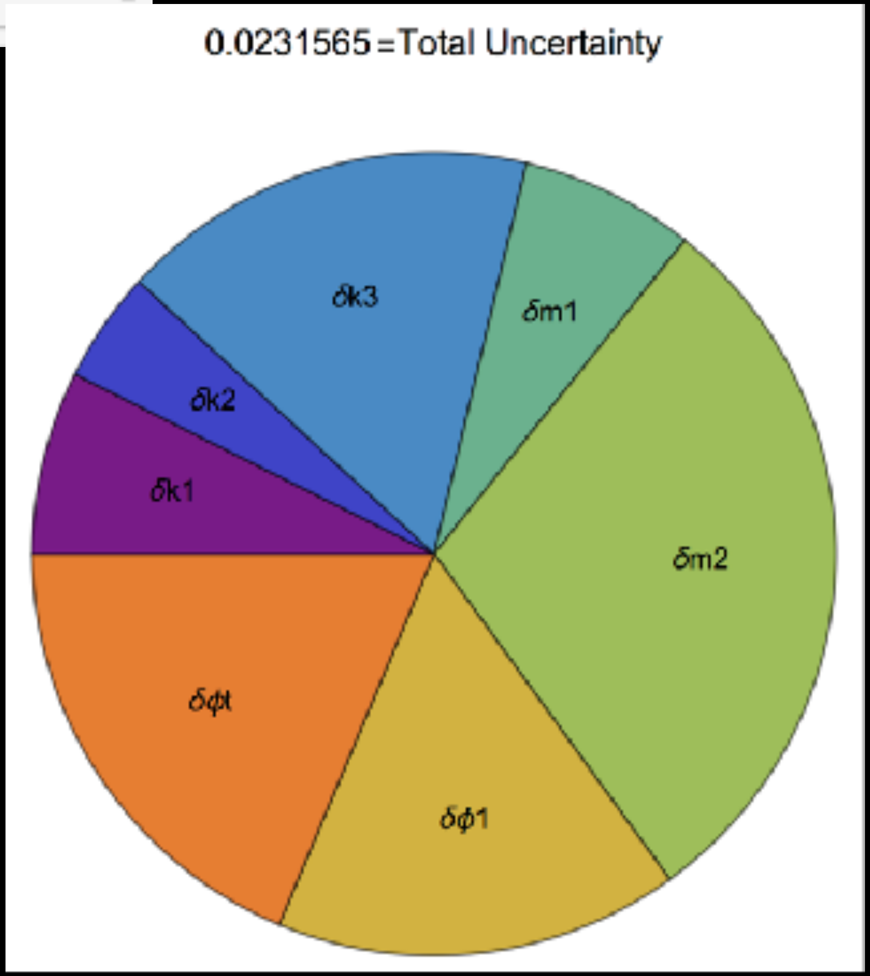


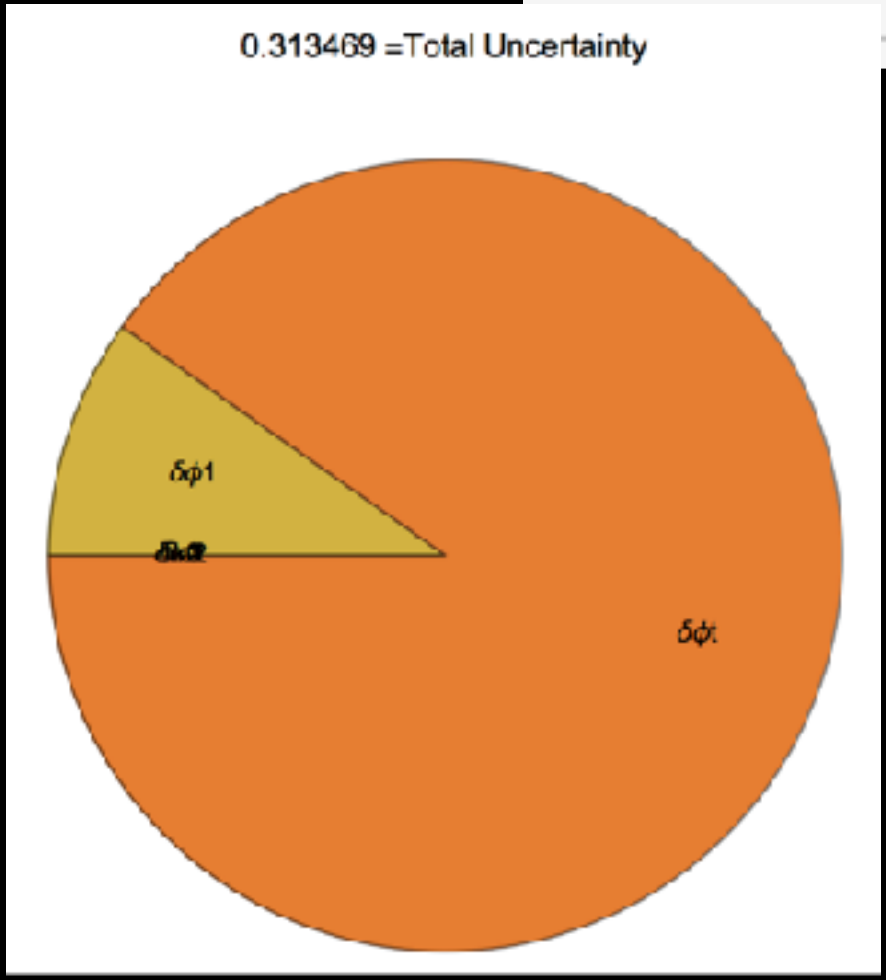
Δm1	0.08511360362023759
Δm2	0.10716108352376071
Δk1	0.10232929922007536
Δk2	0.1
Δk3	0.1148153621496884
Δϕt	0.5
Δϕ1	Rational[1, 1000000]
Δσϕt	
Δσϕ1	
Δσm1	
Δσm2	
Δσk1	
Δσk2	
Δσk3	



Δσϕt	0.00015
Δσϕ1	1.*^-7
Δσm1	10.
Δσm2	0.1
Δσk1	1.
Δσk2	0.1
Δσk3	10.
Δm1	100
Δm2	1
Δk1	10
Δk2	1
Δk3	100
Δϕt	0.0005
	Rational

Contrived



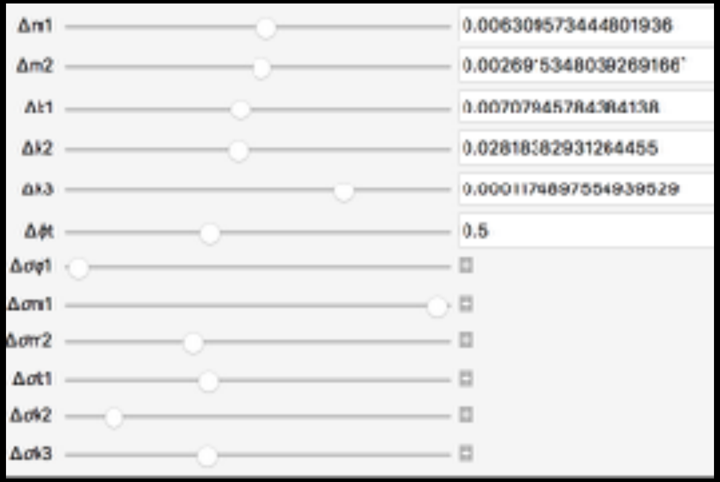


Realistic

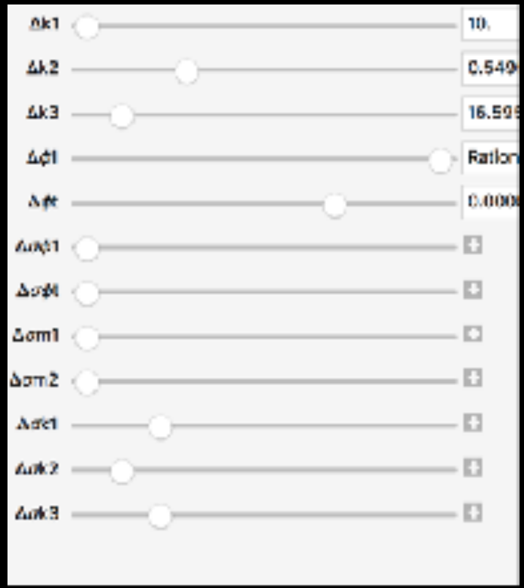
Contrived vs. realistic uncertainty output

Coatings Loss

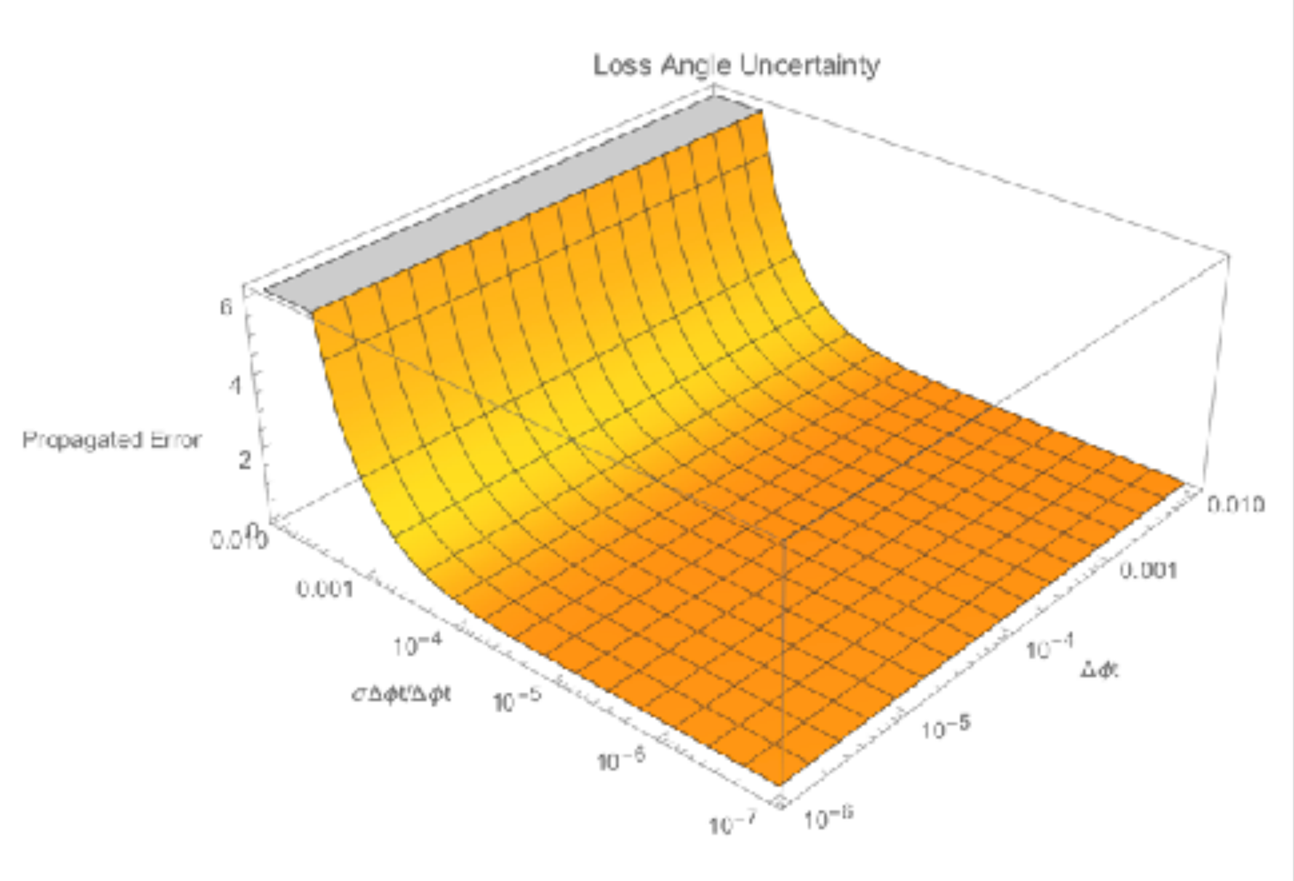
Parameter estimation

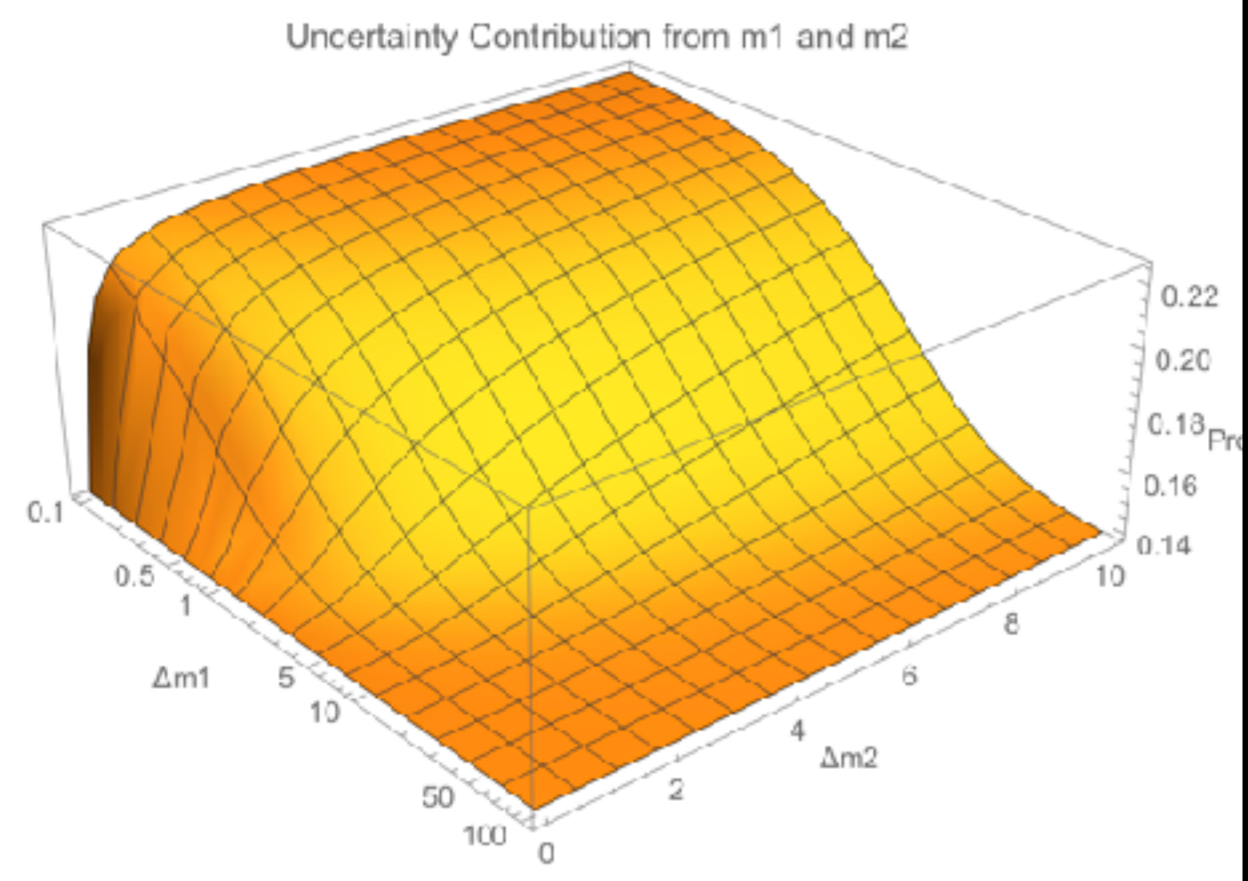


Δm1 0.006309573444801936
Δm2 0.00269'5348039269166'
Δk1 0.00707945784384138
Δk2 0.02818382931264455
Δk3 0.0001174897554939529
Δϕt 0.5
Δσϕ1
Δσm1
Δσm2
Δσk1
Δσk2
Δσk3



Δk1 10.
Δk2 0.549
Δk3 16.59
Δϕ1 Ration
Δϕt 0.000
Δσϕ1
Δσϕt
Δσm1
Δσm2
Δσk1
Δσk2
Δσk3





- Explore error propagation in coupled oscillator system
- Examine covariances, local minima in parameter space
- Generic tool for coupled oscillators

What is LIGO Voyager?

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<u>A few questions</u>

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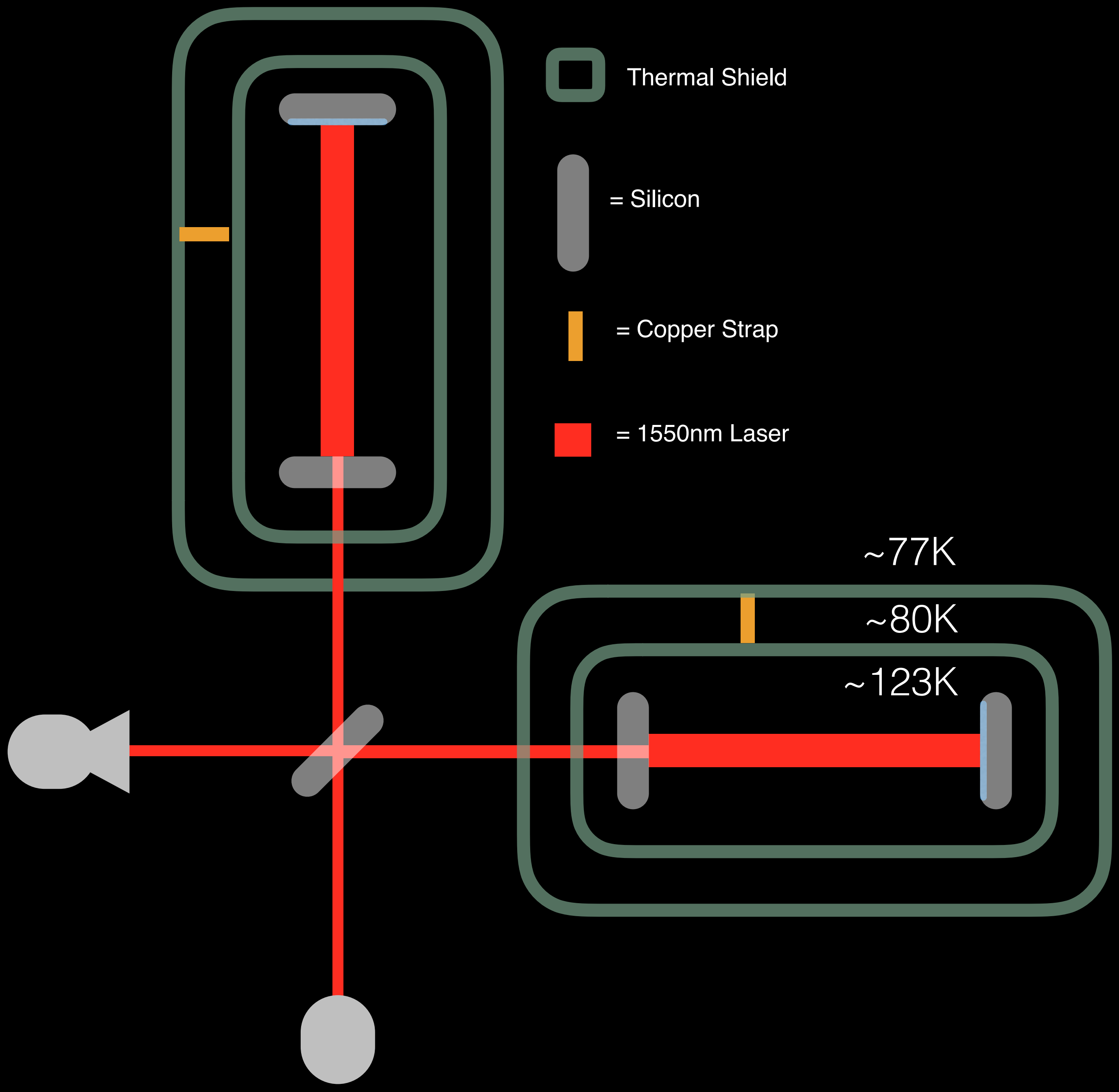
- ~~How can we determine the loss angle of the coatings?~~
- ~~How do we keep the test masses cold without mechanically coupling them to the environment?~~



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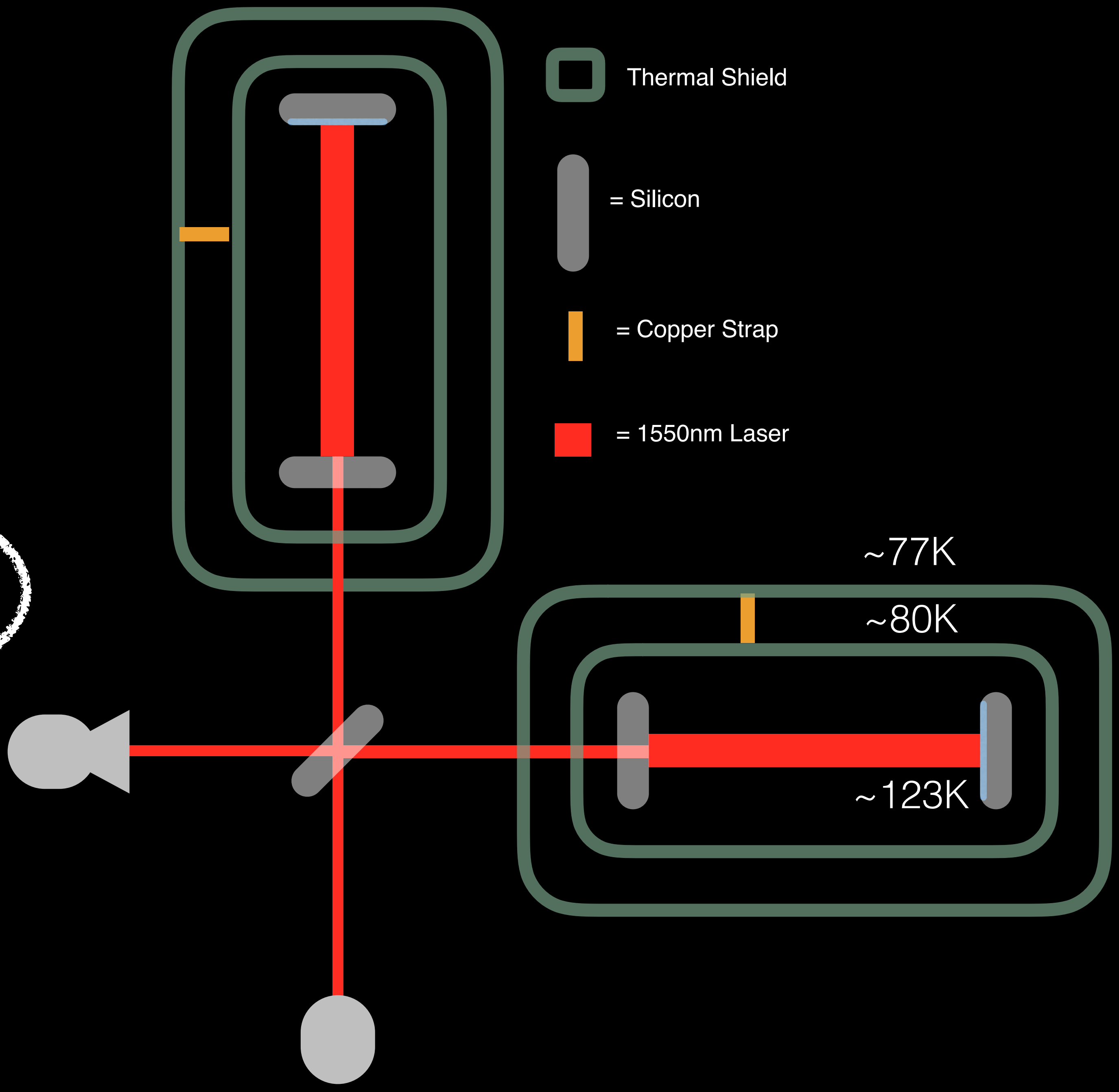
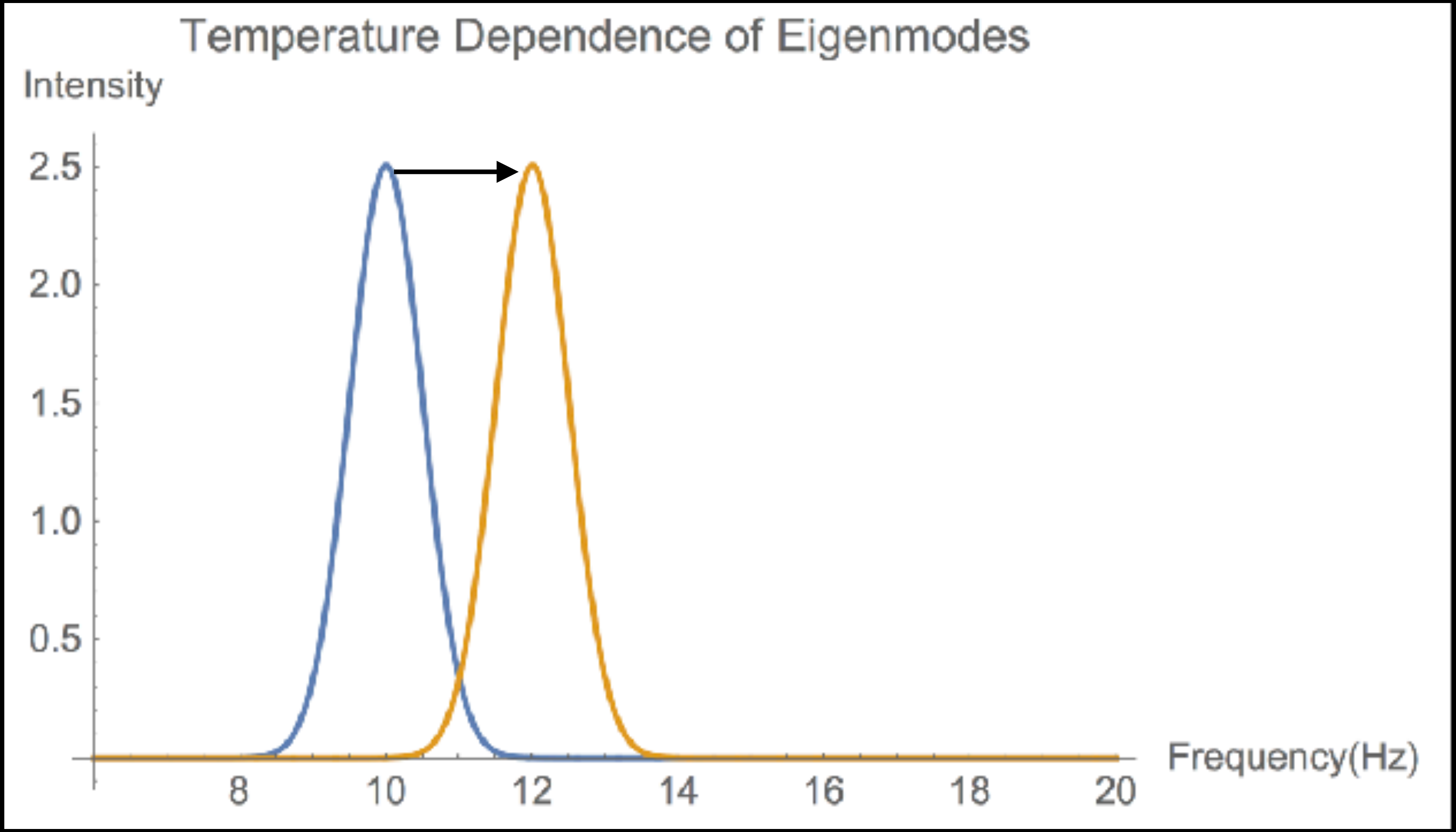


Simplified Voyager Upgrade Schematic

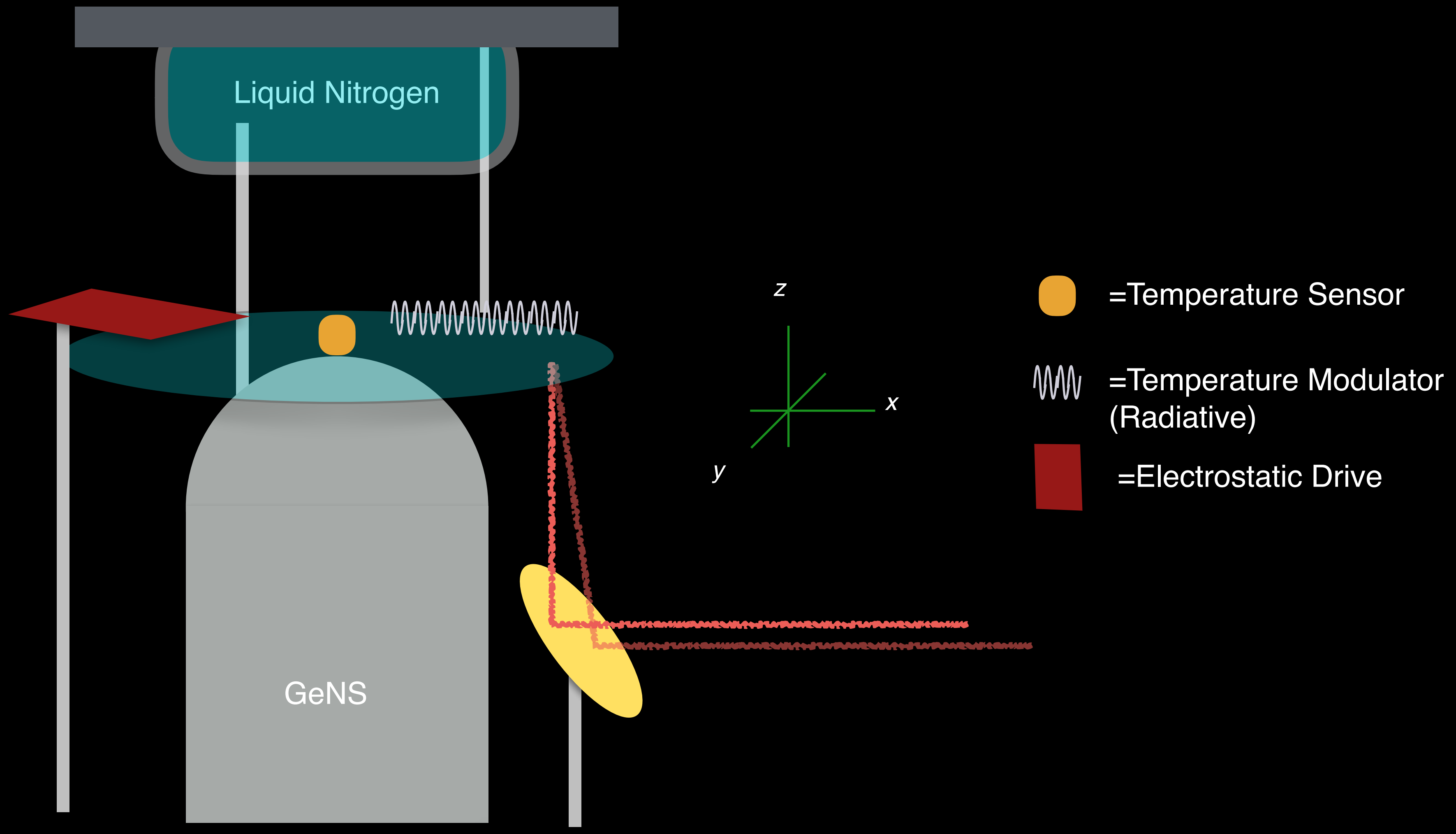
Cryogenic Temperature Determination

- Temperature must be a controlled variable for cryogenic measurements
- Conductively measuring temperature introduces loss
- Elasticity is a function of temperature, elasticity and geometry determine location of mode frequencies in parameter space
- Mode locations in parameter space can be used as an uncoupled thermometer



Cryogenic Temperature Determination

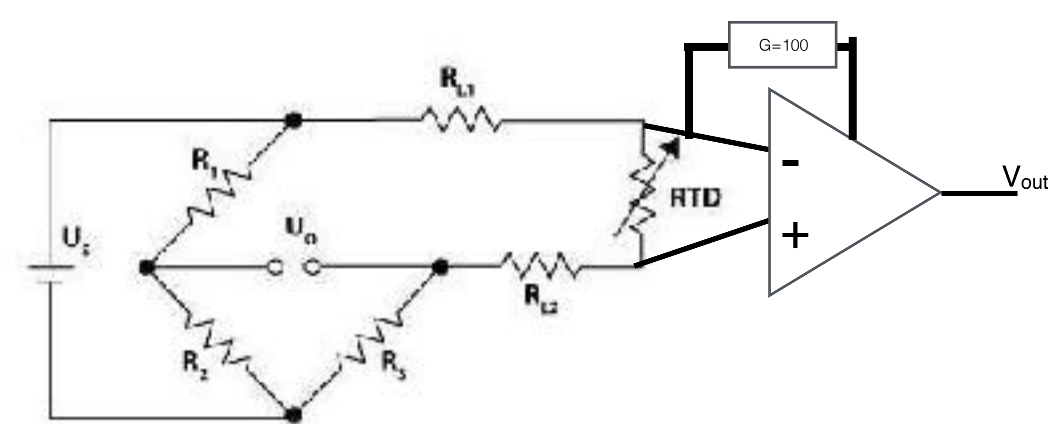
- Modified GeNS can be utilized
- Excite modes with the ESD, modulate temperature with liquid nitrogen and heater
- Monitor temperature and location of modes concurrently
- Allows us to calibrate temperature dependence of frequency shifts

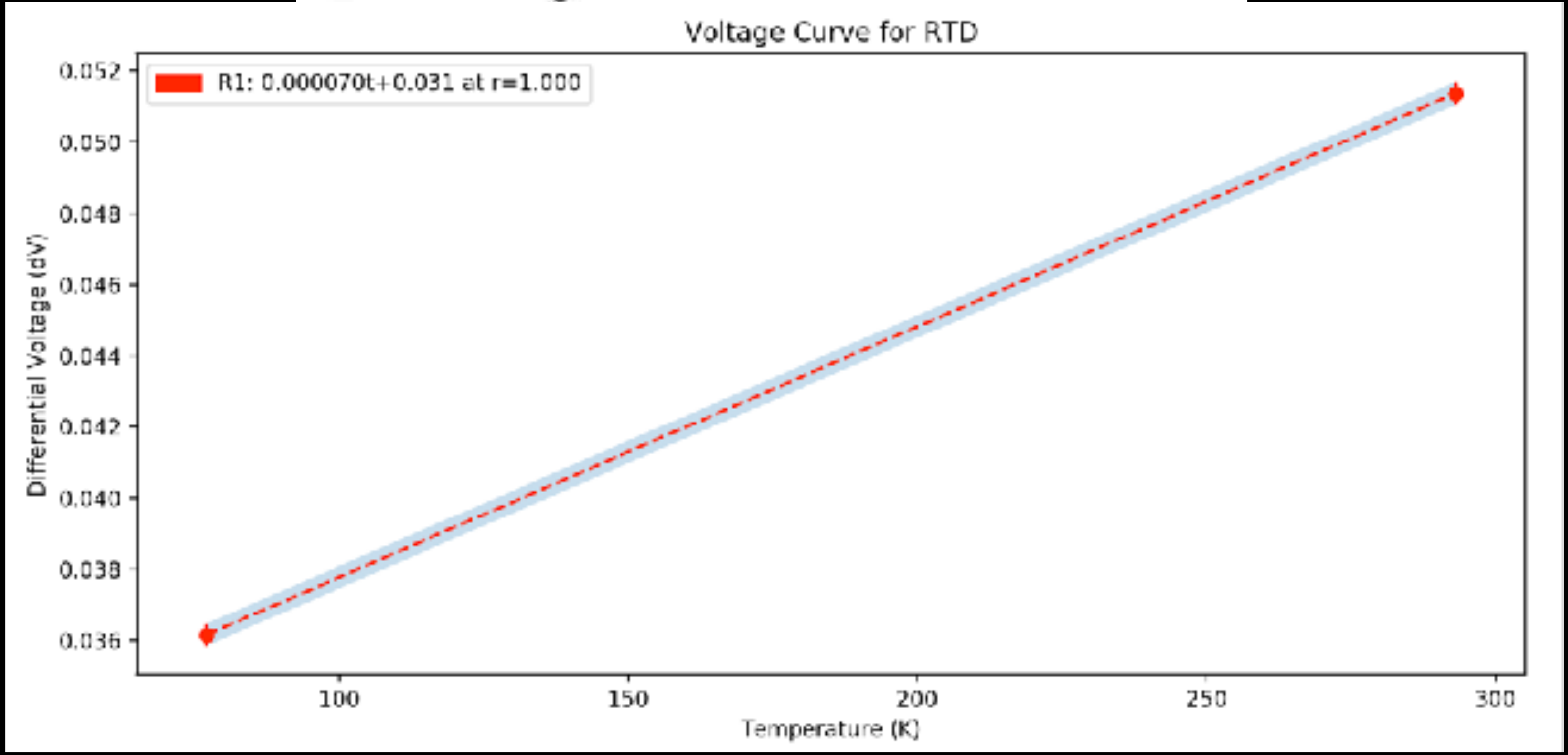


Cryogenic Temperature Determination

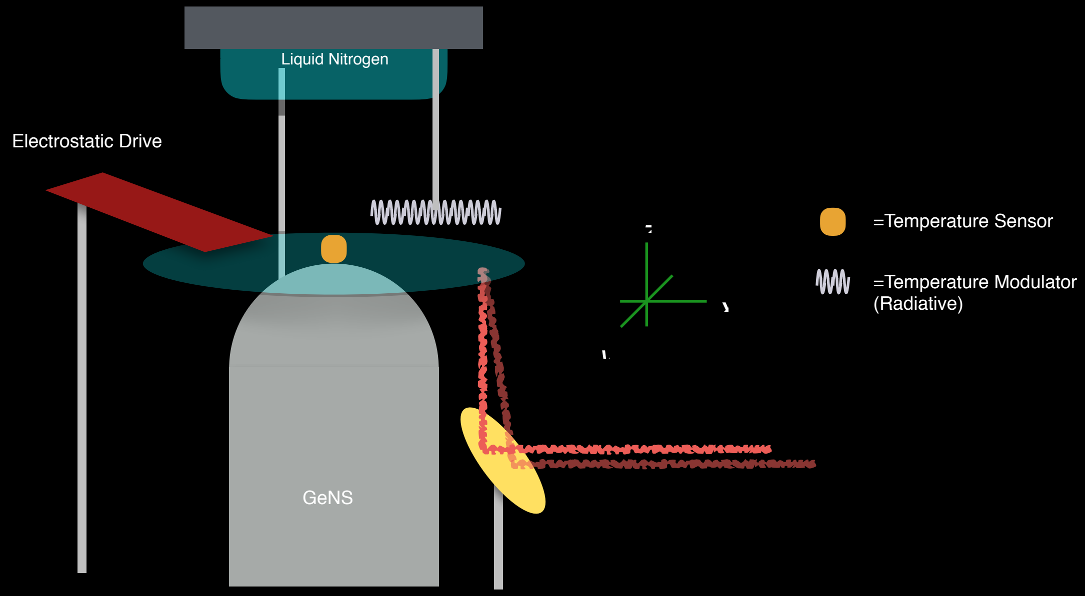
Resisted Temperature Detector (RTD) Calibration

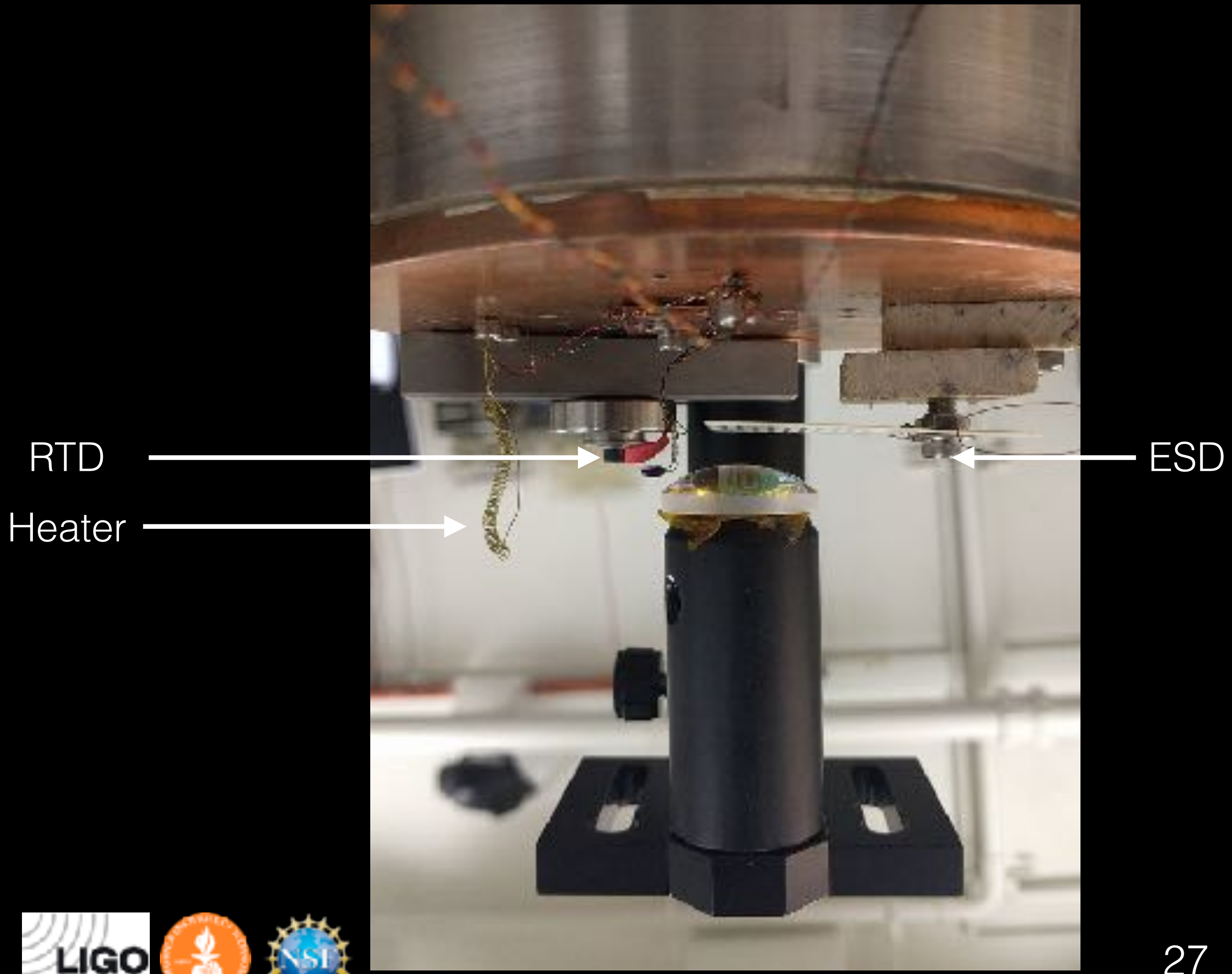
- Interested in high precision temperature measurement
- Use RTD with Wheatstone bridge to produce differential signal

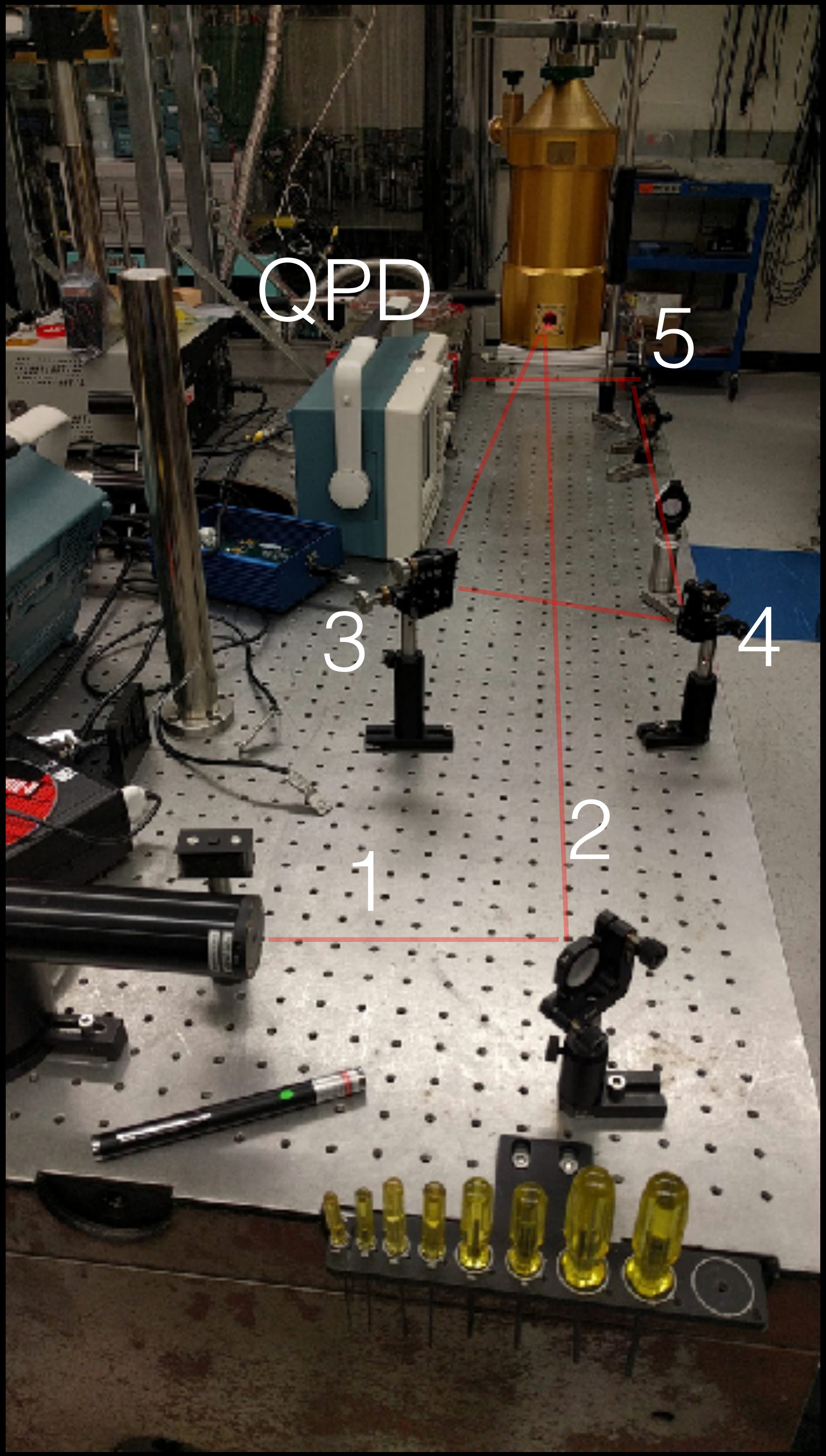




Cryogenic Temperature Determination





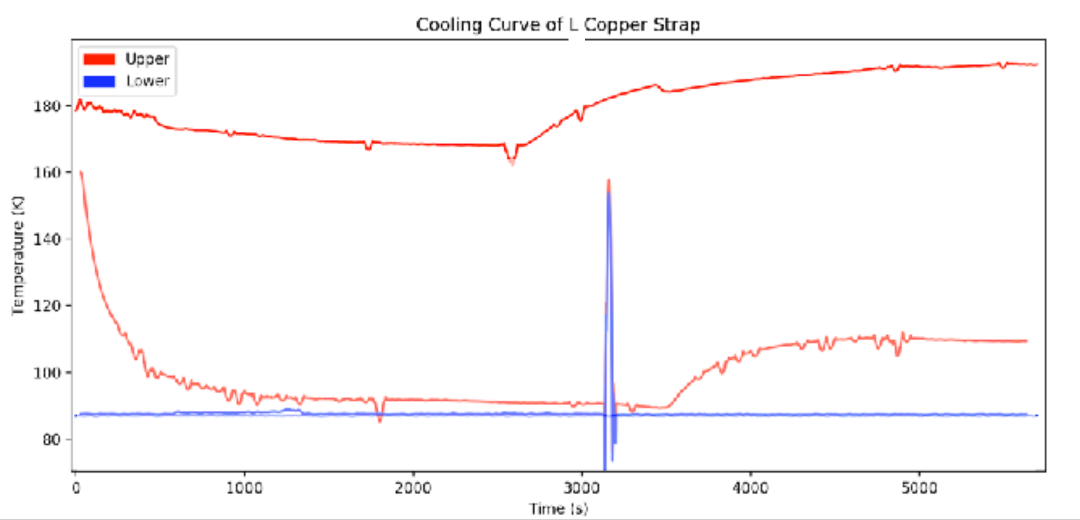


Jordan Kemp, 2017

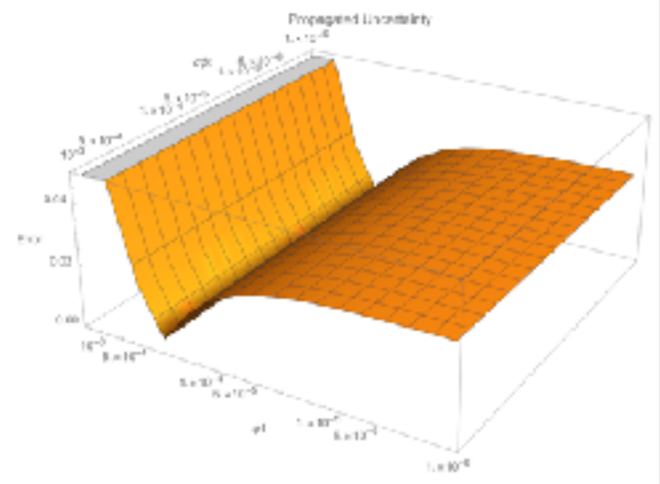
Future Experiments

- Copper Straps
    - Determine mechanical transfer function
    - Determine spring constant for accurate Voyager planning
- Coatings Loss Propagation
    - Utilize uncertainty estimations to optimize coatings Q experiments
    - Perform finite element analysis, use results to produce more precise uncertainty estimations and compare
- Temperature dependence of eigenfrequencies
    - Examine the temperature dependence of different modes
    - Utilize calibration for lossless temperature determination

Conclusion

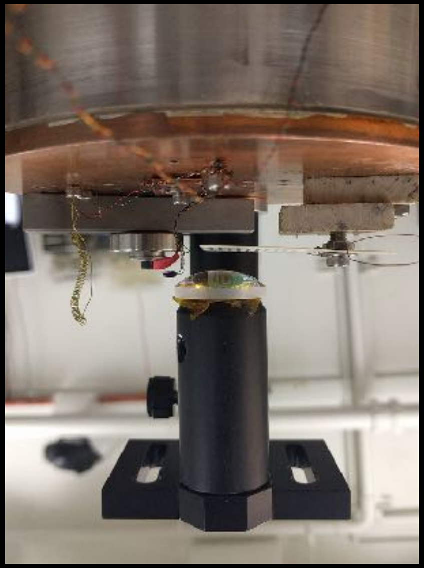


- Determined that the fancy copper straps have much higher conductivity



- Developed tools for propagating uncertainties for coupled harmonic oscillators
- Analyzed relationships between parameter uncertainties
- Concluded uncertainty in the ringdown measurement dominates the total error in the coating Q experiment.

- Set up an experiment for calibrating the temperature/eigenmode relationship in suspended disks



Acknowledgements

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