LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY
- LIGO -
CALIFORNIA INSTITUTE OF TECHNOLOGY
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Technical Note **LIGO-T1700224–v1** 2017/07/31

Testing General Relativity with Binary Black Hole Mergers

SURF 2017 Progress Report I

Radha Mastandrea, MIT
Mentor: Alan Weinstein, Caltech

California Institute of Technology
LIGO Project, MS 18-34
Pasadena, CA 91125
Phone (626) 395-2129
Fax (626) 304-9834
E-mail: info@ligo.caltech.edu

Massachusetts Institute of Technology
LIGO Project, Room NW22-295
Cambridge, MA 02139
Phone (617) 253-4824
Fax (617) 253-7014
E-mail: info@ligo.mit.edu

LIGO Hanford Observatory
Route 10, Mile Marker 2
Richland, WA 99352
Phone (509) 372-8106
Fax (509) 372-8137
E-mail: info@ligo.caltech.edu

LIGO Livingston Observatory
19100 LIGO Lane
Livingston, LA 70754
Phone (225) 686-3100
Fax (225) 686-7189
E-mail: info@ligo.caltech.edu

http://www.ligo.caltech.edu/

1 Motivation

Gravitational waves, ripples in spacetime caused by rapidly changing gravitational fields, were predicted by Einstein's theory of general relativity. Since September 2015, the Advanced LIGO experiment has detected several gravitational wave signatures originating from binary black hole (BBH) systems [1], [2].

In classifying these mergers, LIGO ran a matched filter search across detector data with template BBH waveforms generated by numerical relativity. These waveforms were based off Einstein's formulation of general relativity and modeled BBH inspiral, merger, and ringdown phases parametrized by several characteristics of BBH mergers (such as masses and spins). These template waveforms provided a good fit to the signals that LIGO was detecting, leading to the successful detection and characterization of a number of merging events.

After detection, the post-newtonian coefficients to each BBH merger were computed, showing that general relativity was consistent with the observed signals. This provided further evidence that the signals were indeed from BBH mergers.

However, post-newtonian coefficients calculated in this manner are valid only for the early inspiral phase of a merge. As the black holes in a binary system spiral closer together, v/c approaches 1, leading to a probable breakdown of the post-newtonian expansion (see Figure 1). The post-newtonian coefficients become invalid as the BBH system pushes even further into the strong-field gravity limit. It is during the merger and ringdown phase that these BBH systems become perfect candidates for testing for deviations from general relativity.

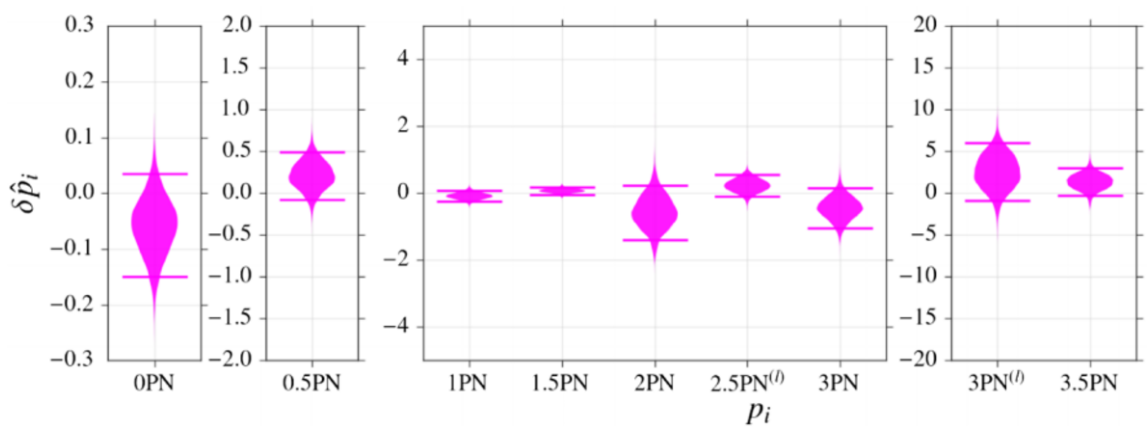

Figure 1: Deviations of some post-newtonian coefficients from their nominal values from GW150914 and GW151226 inspirals. These show general relativity is valid at least for the early inspiral phase of a merger [2].

2 Research Problem

My goal in this project is to determine the extent to which deviations from general relativity can account for the BBH signals LIGO has detected. Any deviations from general relativity will be most evident when gravity approaches the strong field limit, when $\frac{v}{c}$ approaches 1 (which occurs in the merger and ringdown phases; see Figure 2). Note that $\frac{v^2}{c^2} = \frac{4\pi^2 r^2}{P^2 c^2} =$ (using Kepler's 3rd law) $\frac{GM_{tot}}{rc^2}$, making the term $\frac{GM_{tot}}{rc^2}$ an apt one to characterize strong-field gravity. Therefore, for my analyses, I will assume that any such deviations will take the functional form $e^{\frac{\lambda GM}{rc^2}}$, and I will investigate what happens as the parameter λ deviates from 0 (the general relativistic limit).

Note that although the parameter was derived classically here (through Kepler's laws), it is a parameter measuring compactness of strong-field relativistic objects ([4], [5]) and therefore acceptable to use in testing general relativity..

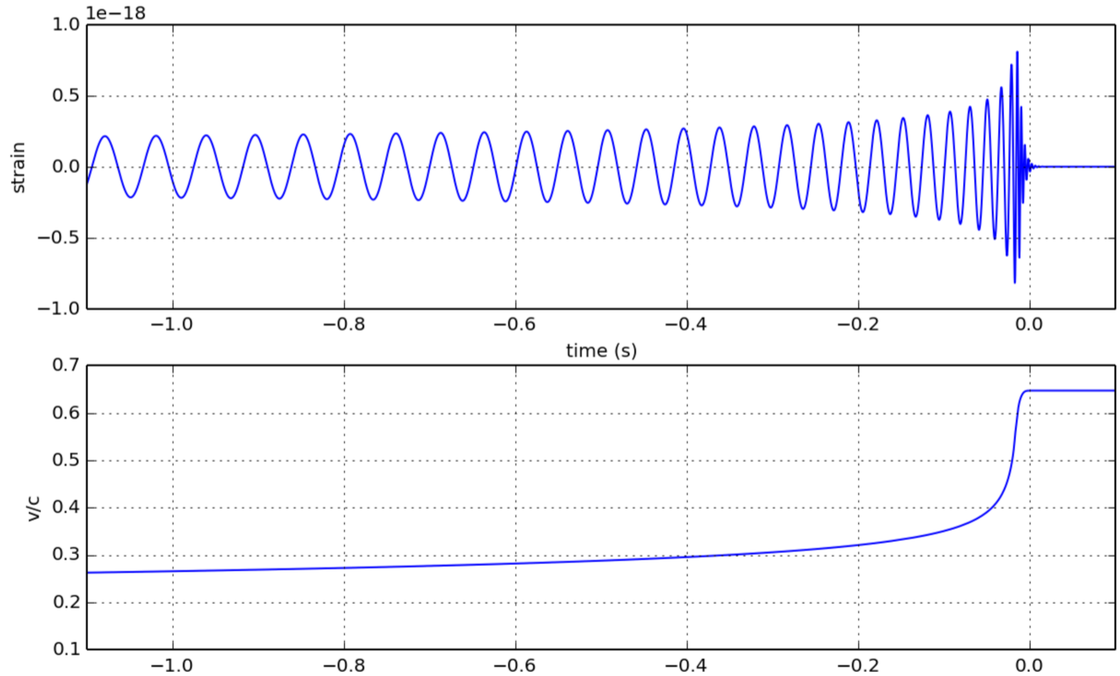

Figure 2: From the GW150914 event. Note that during the merger and ringdown phases, v/c becomes an appreciable fraction of 1 [3].

I anticipate my analyses being broken up into four stages:

Stage 1 (preparation): I will familiarize myself with the parameters used to classify BBH mergers, then simulate a number of random mergers. I will then learn Maximum Likelihood Estimation and Bayesian analysis techniques to characterize the behavior of the parameters across the entirety of the mergers. Learning these techniques, specifically Bayesian analysis, will improve the sophistication of my analyses during later stages. Time: 3 weeks.

Stage 2: for each BBH signal that LIGO has detected and characterized, I will modify the template waveform that was the best fit for that signal by $e^{\frac{\lambda GM}{rc^2}}$, taking care to vary λ from 0 in a controlled fashion. For different values of λ, I will check to what degree the modified waveform template proves a better fit to the BBH signal than the original waveforms (i.e which template, when subtracted from the detector signal, creates noise that is more Gaussian), taking care to ensure my methods and findings are statistically meaningful. The result (for each BBH signal) will be a graph modeling the signal-to- noise ratio for a range of λ's. Anticipated time: 2-3 weeks.

Stage 3: I will analyze the findings of stage 2 and use Bayesian analysis to calculate a probability distribution function for each BBH event, measuring the likelihood that a given value of λ correctly accounts for that BBH event. I will then compile all the analyzed BBH events into a single distribution function with the goal of finding the value of λ that best models general relativity deviations for all the BBH events. This graph will mark the completion of stage 3. Anticipated time: 2-3 weeks.

Stage 4: I will calculate the number of BBH detections needed to corroborate (or rule out) a deviation from general relativity at any given value of λ. Creating this graph will mark the successful completion of my analyses. Anticipated time: 2-3 weeks.

3 Work Completed

Week 1 was spent familiarizing myself with the 15 parameters characterizing BBH mergers as well as their probability distribution functions, all of which are compiled in the table below.

BBH Parameters		
Parameter	pdf	Range
Right Ascension α	uniform in α	$[0, 2\pi)$
Declination δ	uniform in $\cos\delta$	$[0, \pi)$
Inclination angle ι	uniform in $\cos\iota$	$[0, \pi)$
Phase ψ	uniform in ψ	$[0, 2\pi)$
Distance r	quadratic	$[0, 1\text{Mpc}]$
M_{tot}	Salpeter stellar mass	$M_{tot} \geq .5M_{sol}$
symmetric mass ratio η	gaussian, $\mu = .25, \sigma = .05$	$[0, .25]$
$\|s_1\|$	gaussian, $\mu = .7, \sigma = .1$	$[0, .25]$
$\|s_2\|$	gaussian, $\mu = .7, \sigma = .1$	$[0, .25]$
s_1 azimuthal angle ϕ_1	uniform in ϕ_1	$[0, 2\pi)$
s_2 azimuthal angle ϕ_2	uniform in ϕ_2	$[0, 2\pi)$
s_1 polar angle θ_1	uniform in $\cos\theta_1$	$[0, \pi)$
s_2 polar angle θ_2	uniform in $\cos\theta_2$	$[0, \pi)$
Coalesence phase κ	uniform in κ	$[0, 2\pi)$
Coalesence time t	uniform in t	$[0, 1*\text{LIGO observing run})$

Note that the spins for the two black holes are uncorrelated in magnitude and in all angles, which may or may not be a good model for actual mergers. At the time of writing, too few mergers have been detected to prove a convincing argument for any type of spin correlation where the benefits of modeling correlated spins outweigh the complexities of actually implementing these correlations.

Using `python`, I randomly simulated 10,000 BBH mergers by taking 10,000 random draws from each nontrivial (i.e. each parameter that didn't have a uniform or uniform in cos distribution) parameter's pdf and comparing it to the expected distribution.

Week 2 was spent on further analyses of the BBH simulations. Once again, I simulated

10,000 BBH events (this time simulating all 15 parameters), then used Maximum Likelihood Estimation (i.e. `scipy` curve fit) to fit each simulated parameter distribution and compare the fit to the expected pdf. The mass function proved to be a bit challenging to fit, mainly because the steep exponential represses events with high M_{tot}, so the fits for both masses consistently had the largest errors out of all 15 parameters.

I spent the next few days familiarizing myself with the general benefits of Bayesian over Maximum Likelihood analyses as well as learned `pymc`. I then simulated 20,000 random BBH mergers and used Bayesian techniques to fit all the parameters. For the trivial parameters, this analysis was costly in terms of time and effort spent learning the new software, but the effort was justified since the same techniques can be used to efficiently analyze higher-dimensional systems, which I will be doing in the last stages of my project as I test general relativity.

All images generated during weeks 2 and 3 of the project are in Appendix A. Note the differences between the Maximum Likehood fits and the Bayesian fits.

Week 3 (the short week) was spent cleaning up the code to make the images as described above. It was also spent preparing for **Week 4**'s assignment, which involves using the formula for BBH signal SNR (slightly modified from formula 4.3 in the FINDCHIRP paper, [6]) to estimate some representative SNRs for future LIGO detectors. I also began to write code that would modify some of LIGO's template waveforms by the factors of $e^{\frac{\lambda GM}{rc^2}}$ with the intention of using them to compute the SNR as a function of λ during the following week.

4 Challenges and Anticipated Issues

To this point, I have encountered no challenges beyond issues with debugging code (since all of my work has been computer-based), and none of them were unsolved after conferring with another SURF student, my co-mentor, mentor, or the forums of Stack Overflow. Moving forward, I don't anticipate having problems of another nature.

References

[1] B. P. Abbott et al., *GW151226: Observation of Gravitational Waves from a 22-Solar-Mass Binary Black Hole Coalescence.* PHYSICAL REVIEW LETTERS. 116, 241103 (2016).

[2] B. P. Abbott et al., *Binary Black Hole Mergers in the First Advanced LIGO Observing Run.* PHYS. REV. X 6, 041015 (2016).

[3] https://losc.ligo.org/s/events/GW150914/LOSC_Event_tutorial_GW150914.html

[4] Blanchet, L. Living Rev. Relativ. (2014) 17: 2. doi:10.12942/lrr-2014-2

[5] Charalampos Markakis et al., *Neutron star equation of state via gravitational wave observations.* J. Phys.: Conf. Ser. 189 012024 (2009).

[6] Bruce Allen et al., *FINDCHIRP: an algorithm for detection of gravitational waves from inspiraling compact binaries*. Phys. Rev. D 85, 122006 (2012).

5 Appendix A: Images

Below is a selection of the images created during weeks 1 and 2 of my project. Each page contains images for one of the BBH parameters, with graphs showing both a Maximum Likelihood Estimation (MLE) fit and Bayesian fit. Note that there are not 15 pages of images; many BBH parameters had identical distributions (i.e. 4 parameters all had uniform distributions), so I chose to include images for a single parameter for every unique pdf. All graphs with MLE fits contain 10,000 random events; all graphs with Bayasian fits contain 20,000 random events.

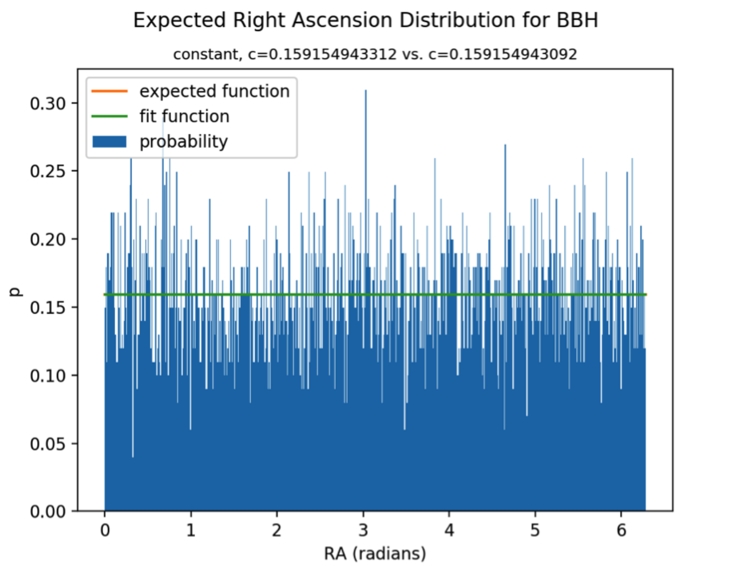

Figure 3: BBH Right Ascension; MLE fit

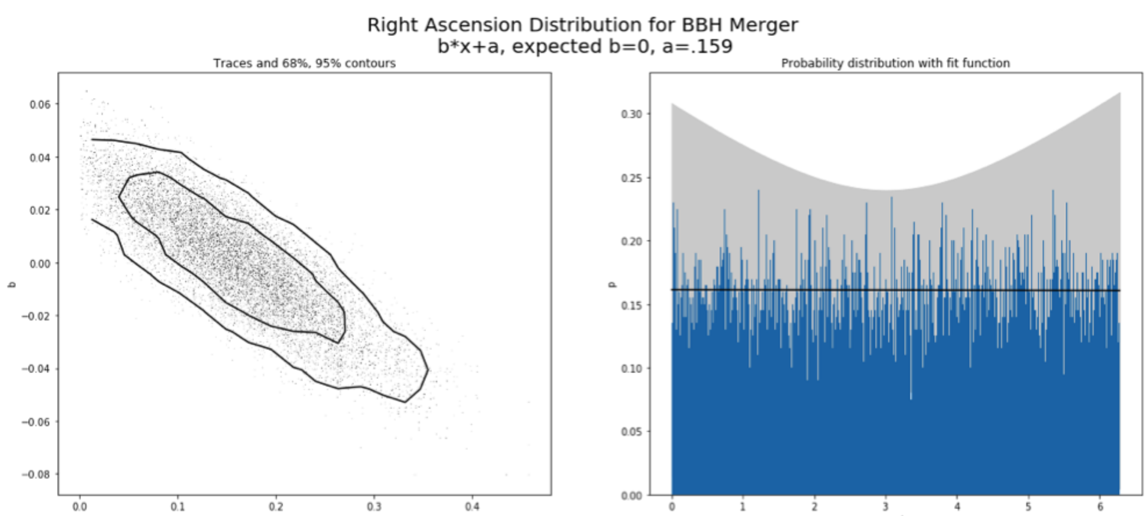

Figure 4: BBH Right Ascension; Bayesian fit

Note that the fits for Right Ascension are almost identical (and are very accurate) due to the simplicity of the probability distribution.

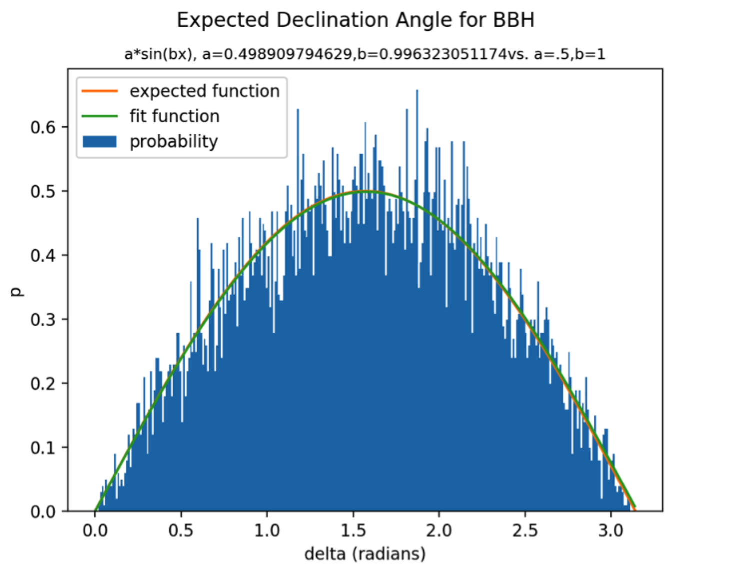

Figure 5: BBH Declination; MLE fit

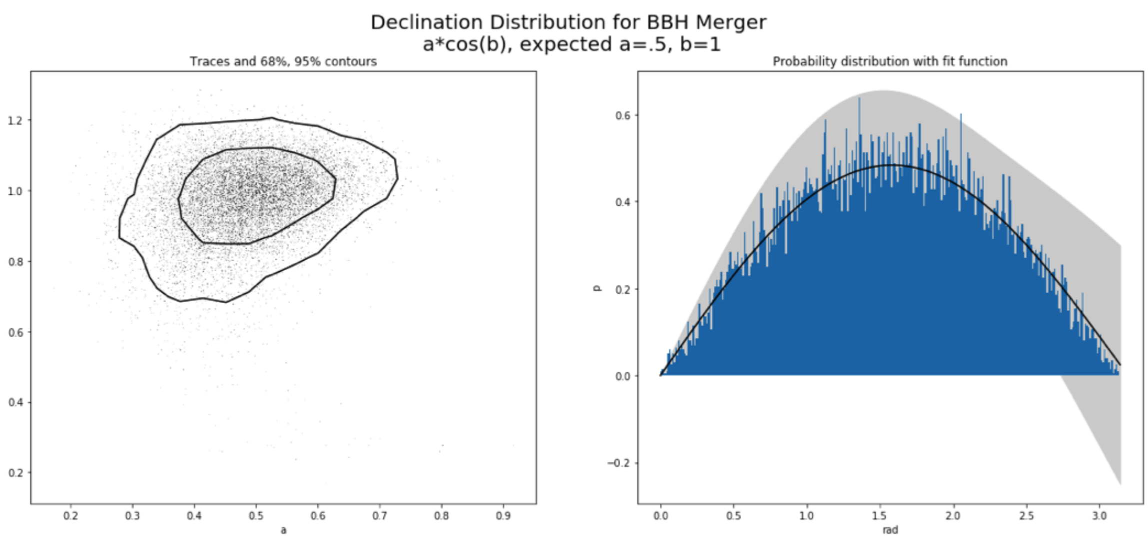

Figure 6: BBH Declination; Bayesian fit

Both the MLE and Bayesian analyses provide exellent fits, again due to the simplicity of the pdf.

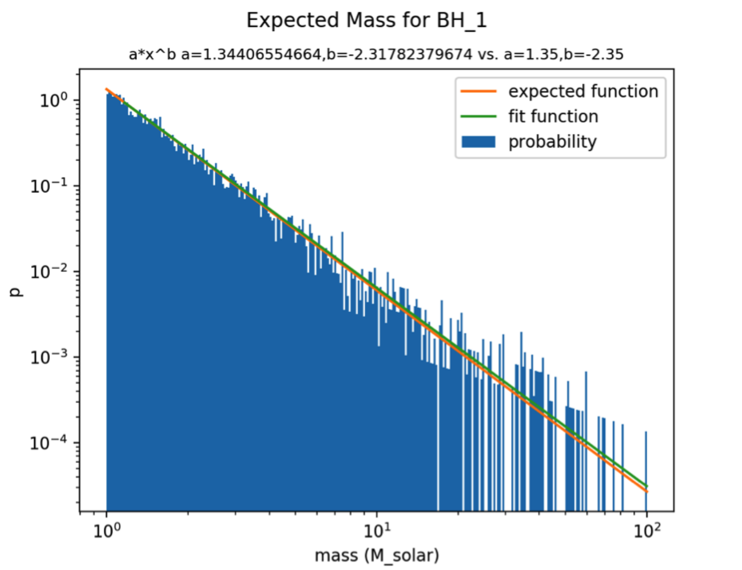

Figure 7: BBH M_1; MLE fit

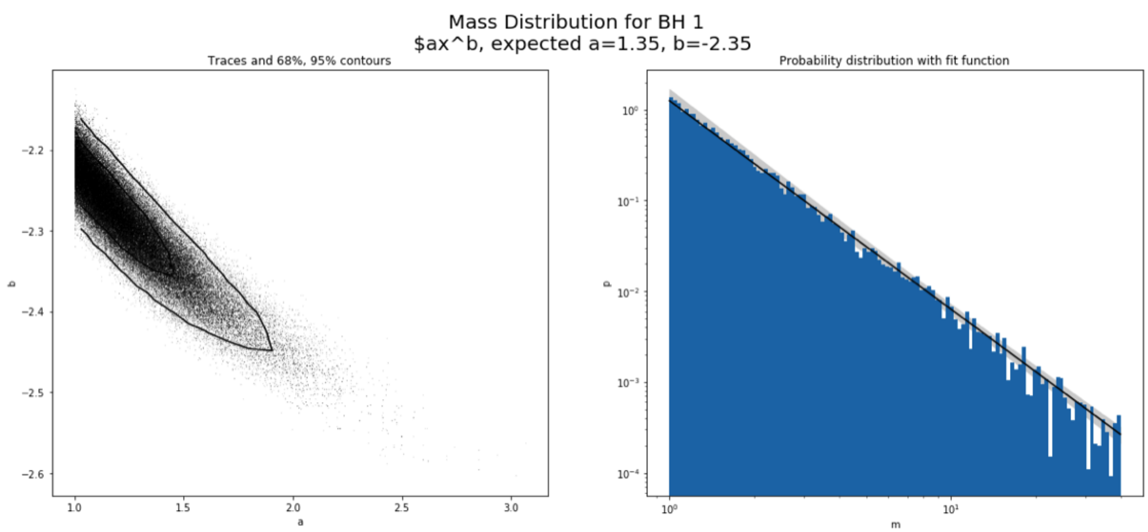

Figure 8: BBH M_1; Bayesian fit

The MLE fit is a good enough one, but it consistently underestimates the exponential parameter. The Bayesian fit is a bit less precise, but is at least consistent with true pdf.

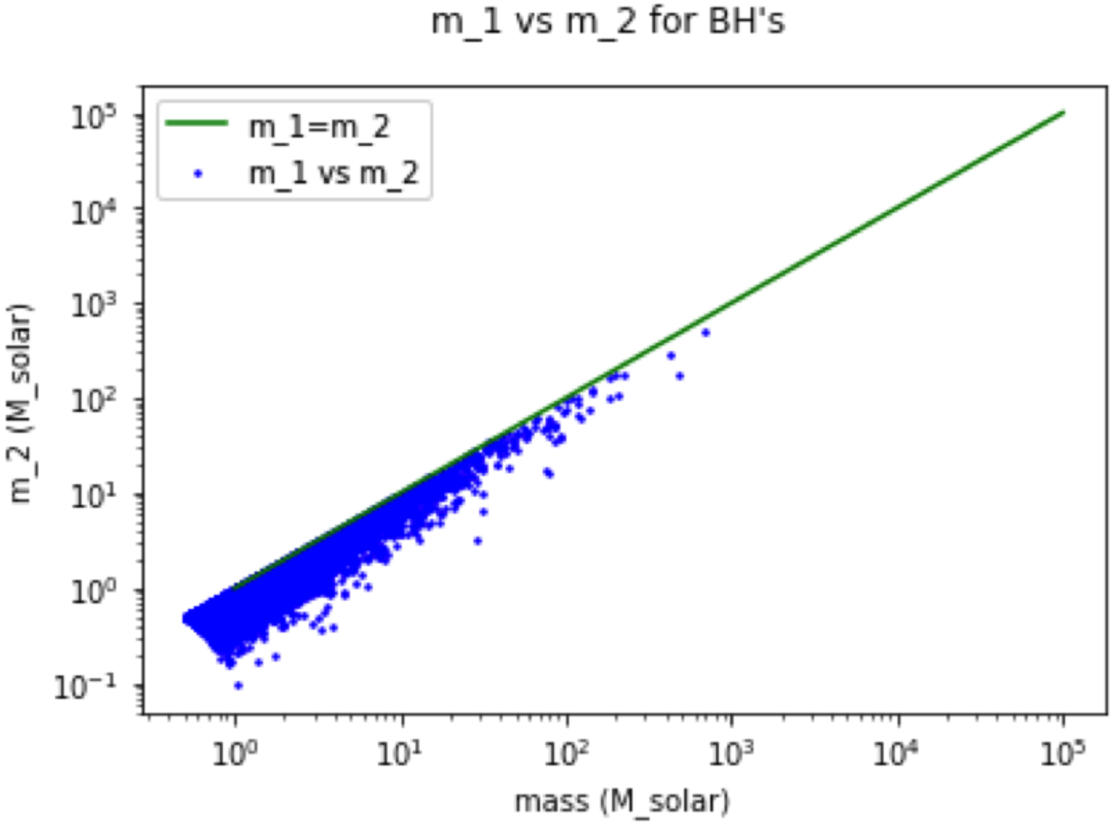

Figure 9: M_1 vs M_2

This figure is not so much a “fit” as an illustrative image showing the distribution of M_{tot} among the two black holes for 10,000 random merger events.

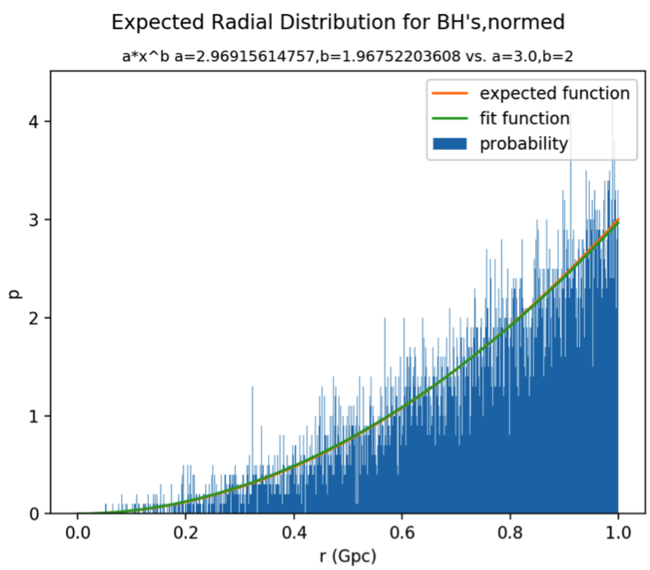

Figure 10: BBH Distance; MLE fit

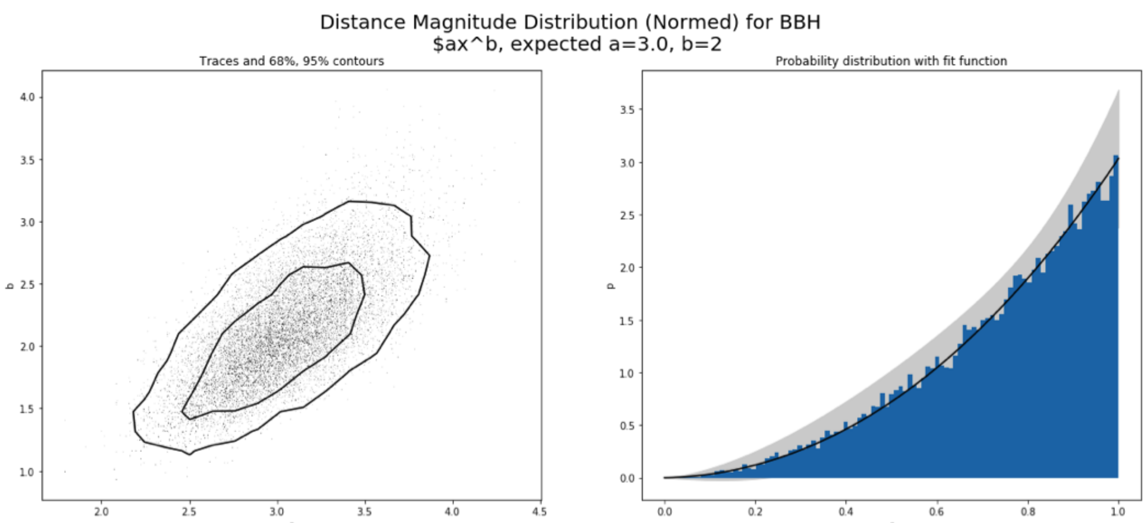

Figure 11: BBH Distance; Bayesian fit

Both fits are rather good; the Bayesian fits to power laws seem to be less precise than is desirable.

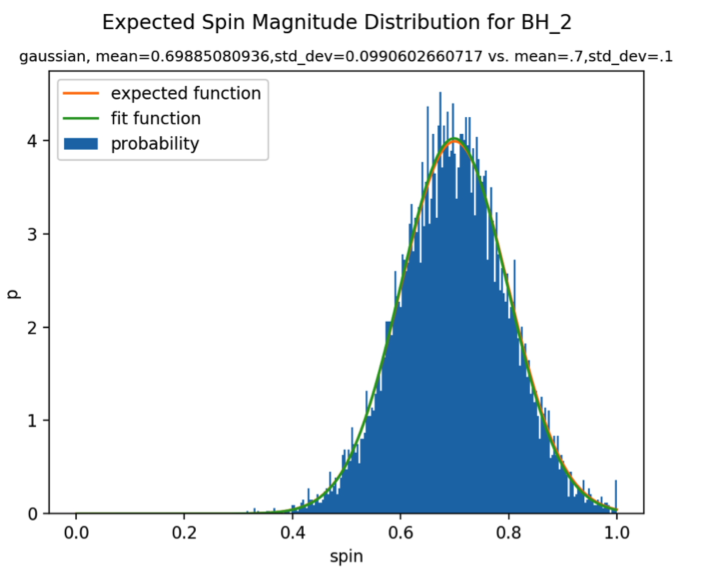

Figure 12: BH $|s_2|$; MLE fit

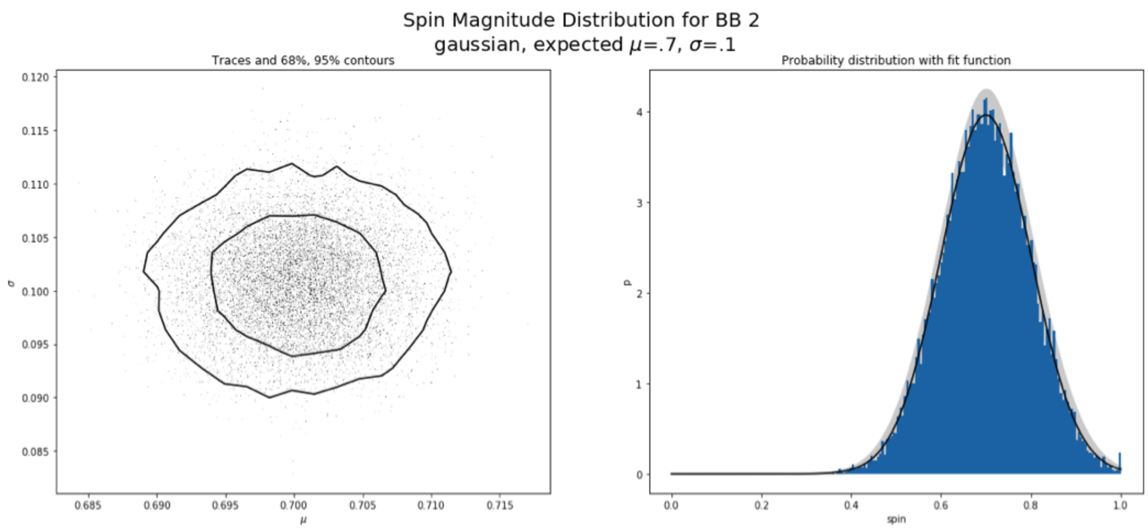

Figure 13: BH $|s_2|$; Bayesian fit

Both the MLE and Bayesian fits are true to the actual pdf and precise.

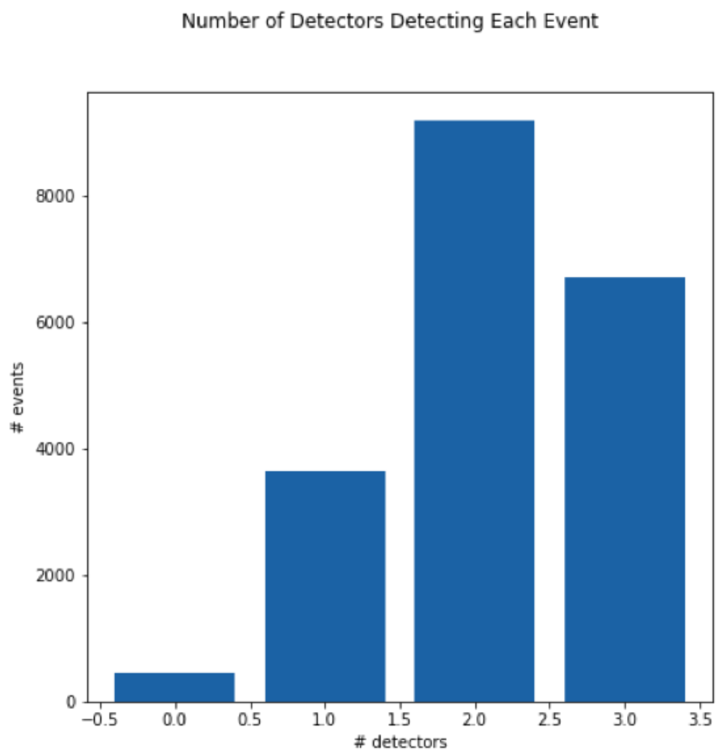

Figure 14: Over a certain time period when 10,000 random BBH gravitation wave signatures pass through the Earth, our detectors may or may not be on to detect them. This graph assumes 3 detectors, one that is on $70\% \pm 5\%$ of the time, one that is on $60\% \pm 10\%$ of the time, and one that is on $80\% \pm 5\%$ of the time. This graph does not take into account individual detector sensitivity.

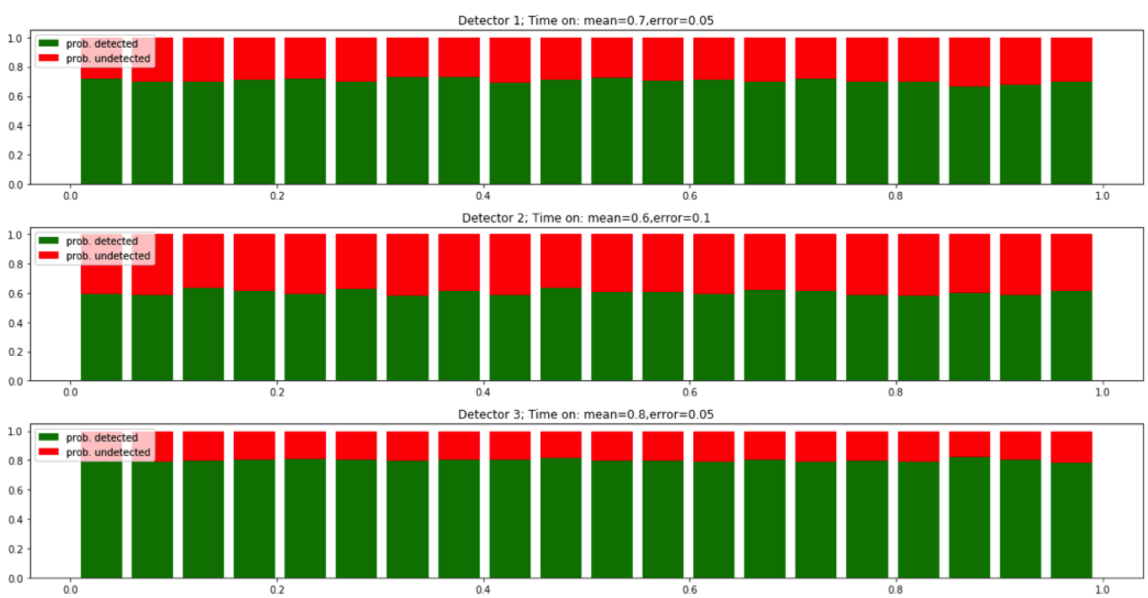

Figure 15: Using the same detectors as above: the x-axes range from 0 to 1 in units of "observing run duration" (ex. 2 weeks). Over any time interval in the period, a given detector will only be on for a certain period of time, and will therefore only be able to detect any events that pass through the Earth with a given (random) probability. This graph does not take into account individual detector sensitivity.