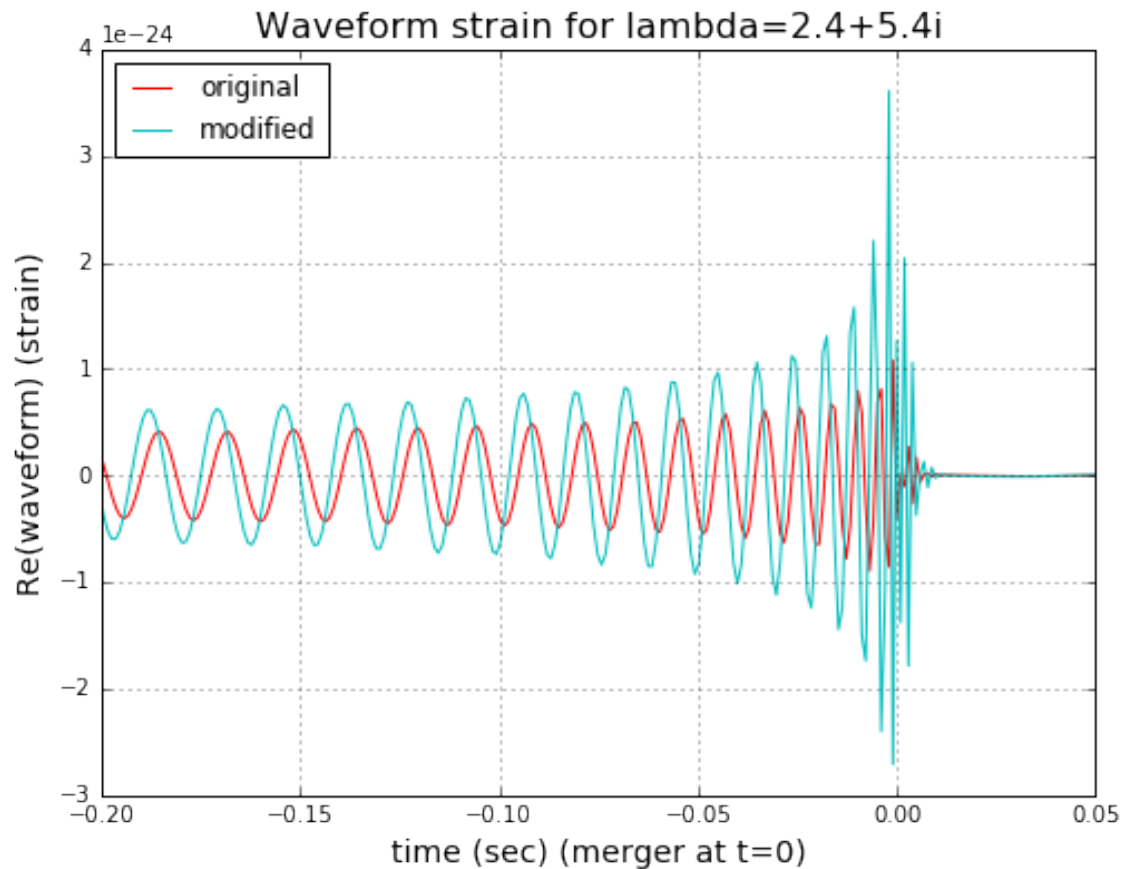
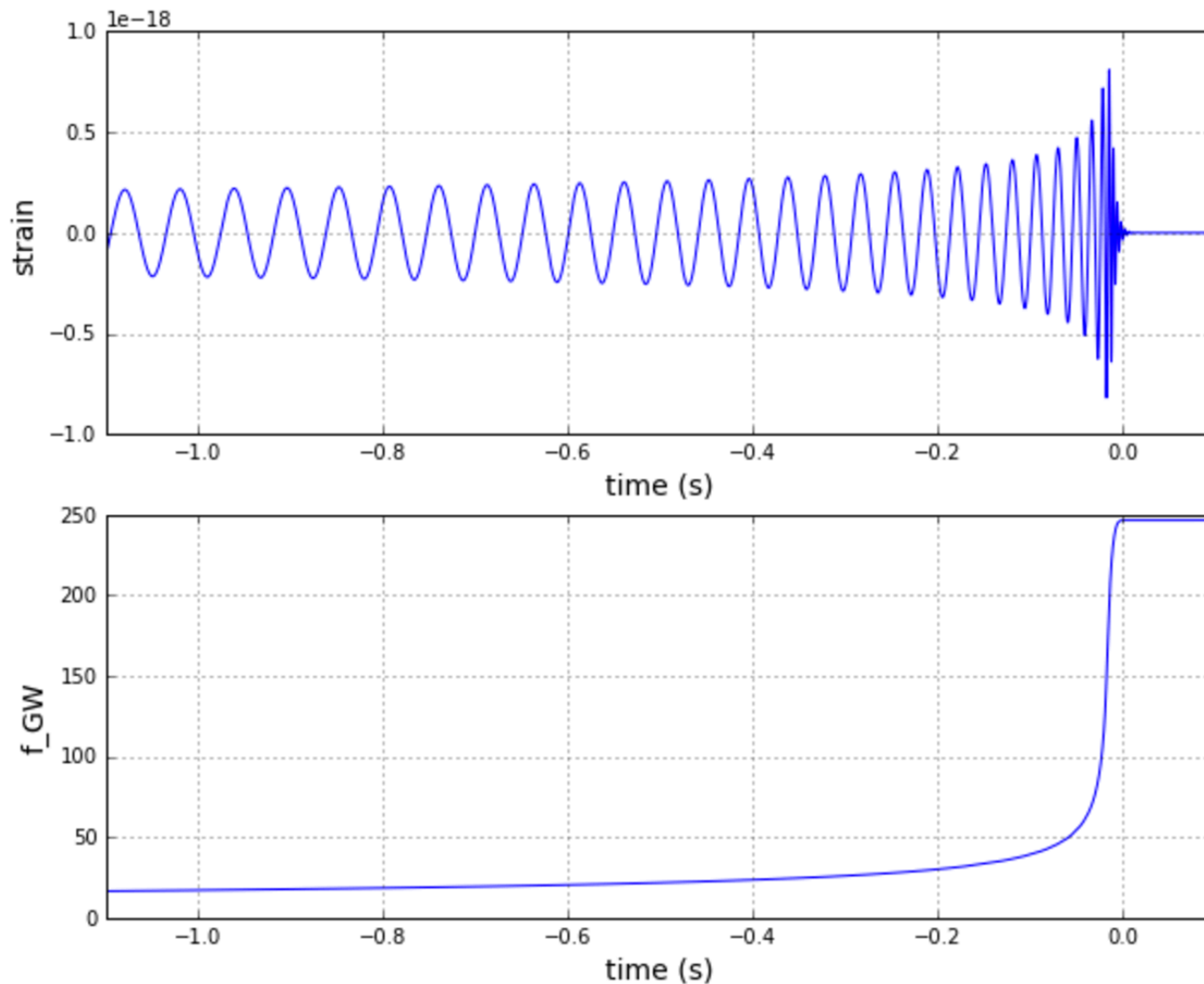


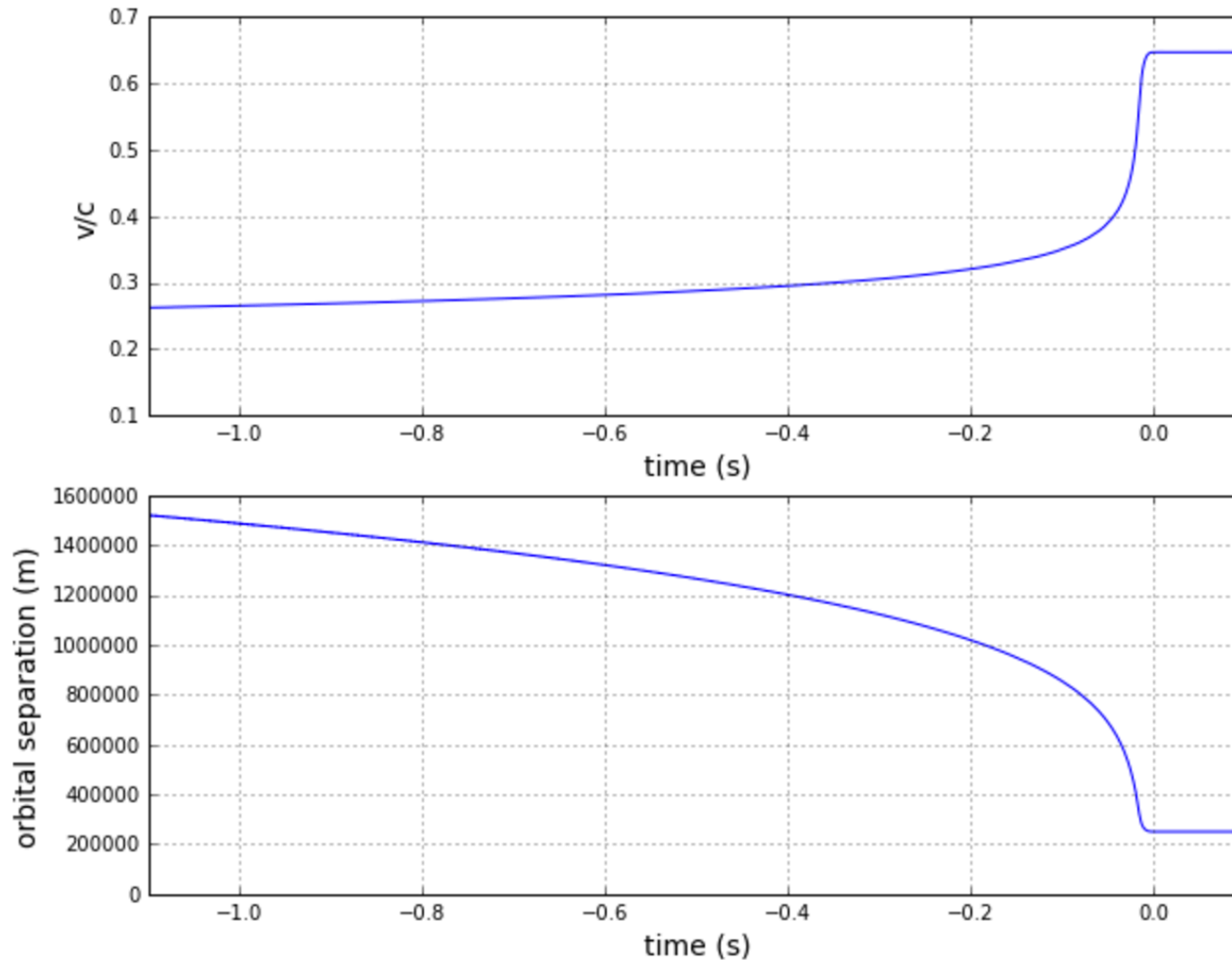
Quantifying Deviations from General Relativity



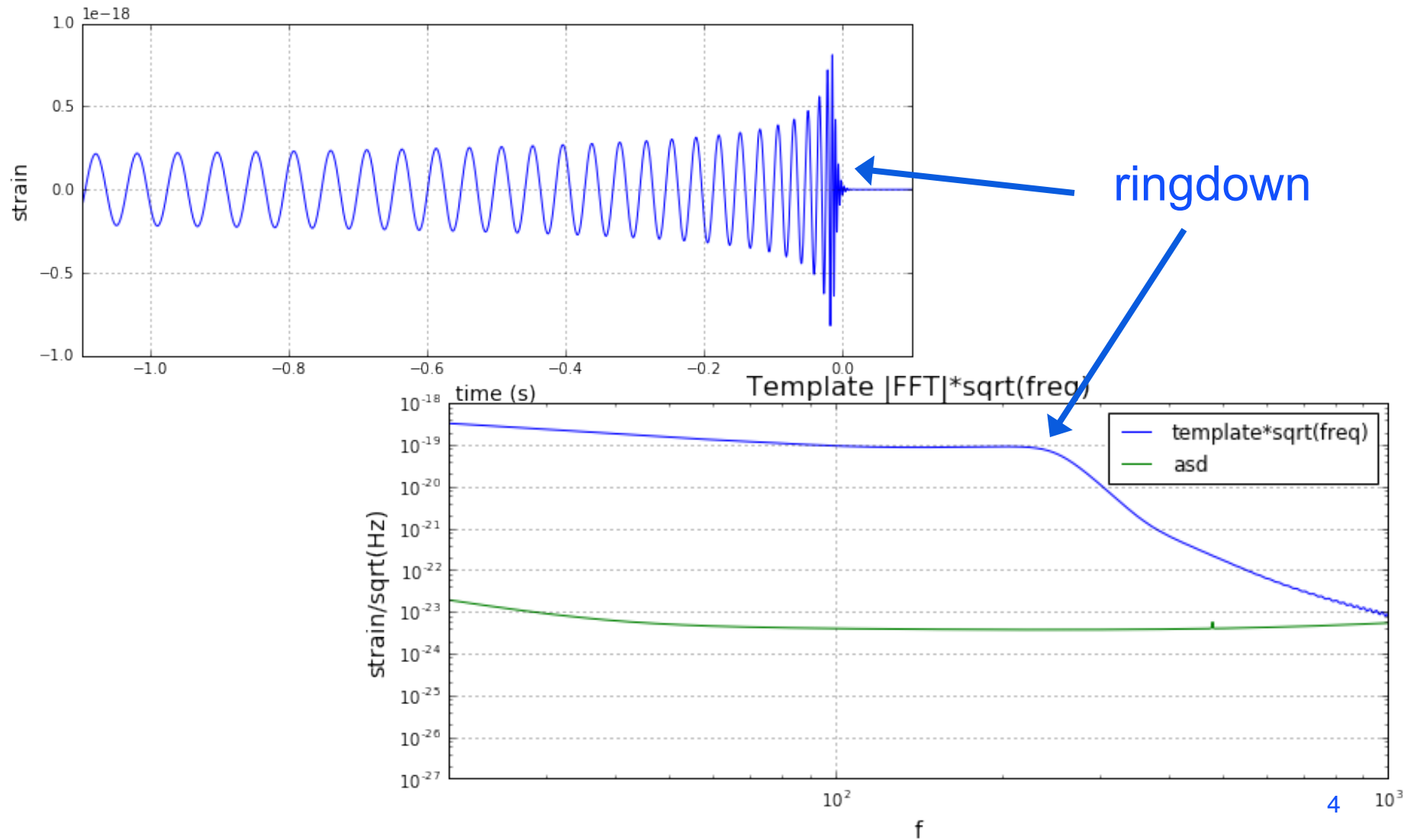
GW Waveforms: GW150914 Template



GW Parameters: GW150914 Template



Inspiral, Merger, Ringdown



Kepler's Third Law

- Strong-field gravity is characterized by v/c
- Any deviations from GR will probably have some dependence on v/c

$$\frac{v^2}{c^2} \Rightarrow \frac{4\pi^2 r(f)^2}{P^2 c^2} \Rightarrow \frac{GM_{tot}}{r(f)c^2}$$

Quasi-circular motion
Kepler's 3rd law
derived classically...but still used in GR!

$$P = \frac{1}{f_o} \qquad f_{GW} = 2f_o$$

Waveform Modulation

- So, we'll model deviations from GR as being an extra multiplicative factor

$$\tilde{h}_{non-GR}(f) = e^{\lambda \frac{GM_{tot}}{r(f)c^2}} \tilde{h}_{GR}(f) = e^{\text{Re}(\lambda) \frac{GM_{tot}}{r(f)c^2}} A(f) e^{i \left(\text{Im}(\lambda) \frac{GM_{tot}}{r(f)c^2} + \Phi(f) \right)}$$

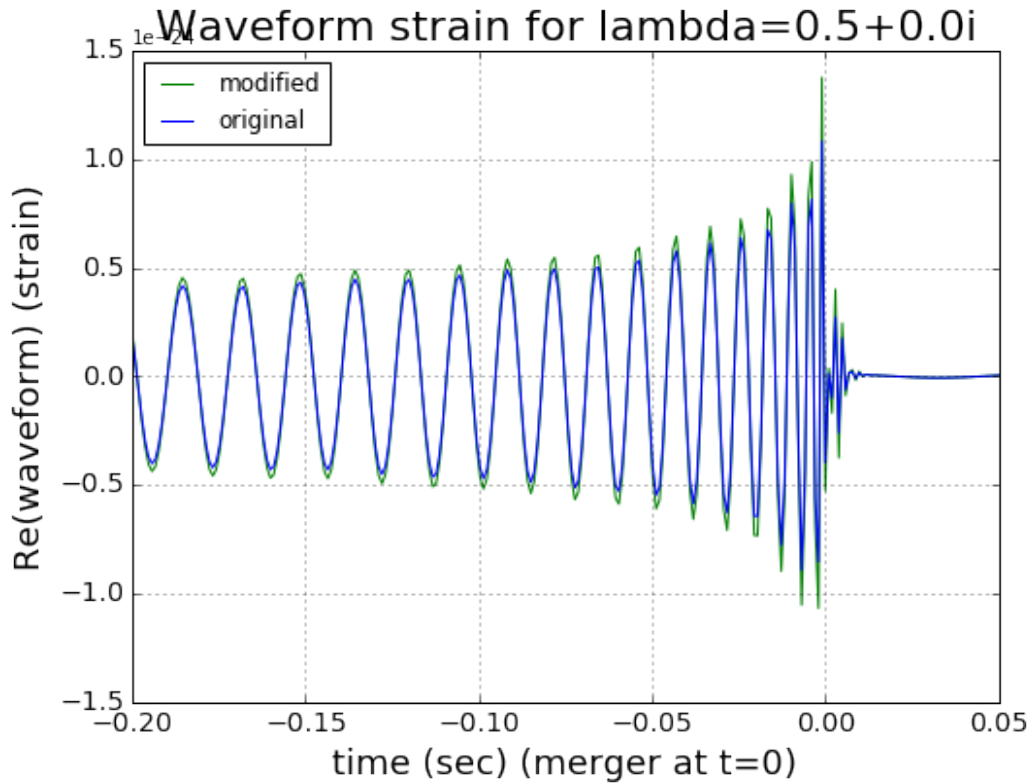
where

$$\tilde{h}_{GR}(f) = A(f) e^{i\Phi(f)}$$

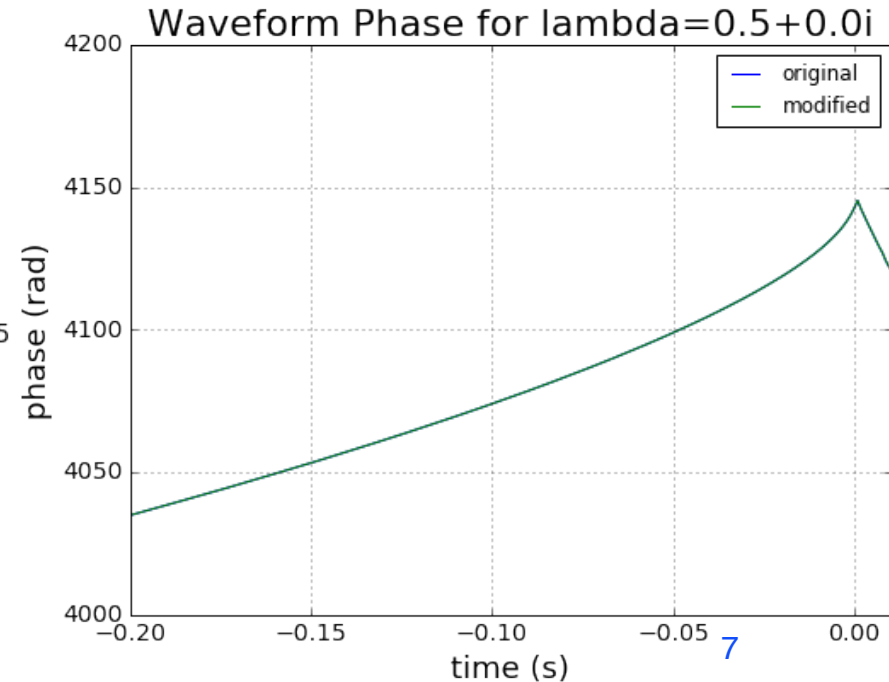
If $\lambda=0$, we're working with Einstein's GR

- λ is complex
- $\text{Re}(\lambda)$ corresponds to an amplitude modulation
- $\text{Im}(\lambda)$ corresponds to a phase modulation

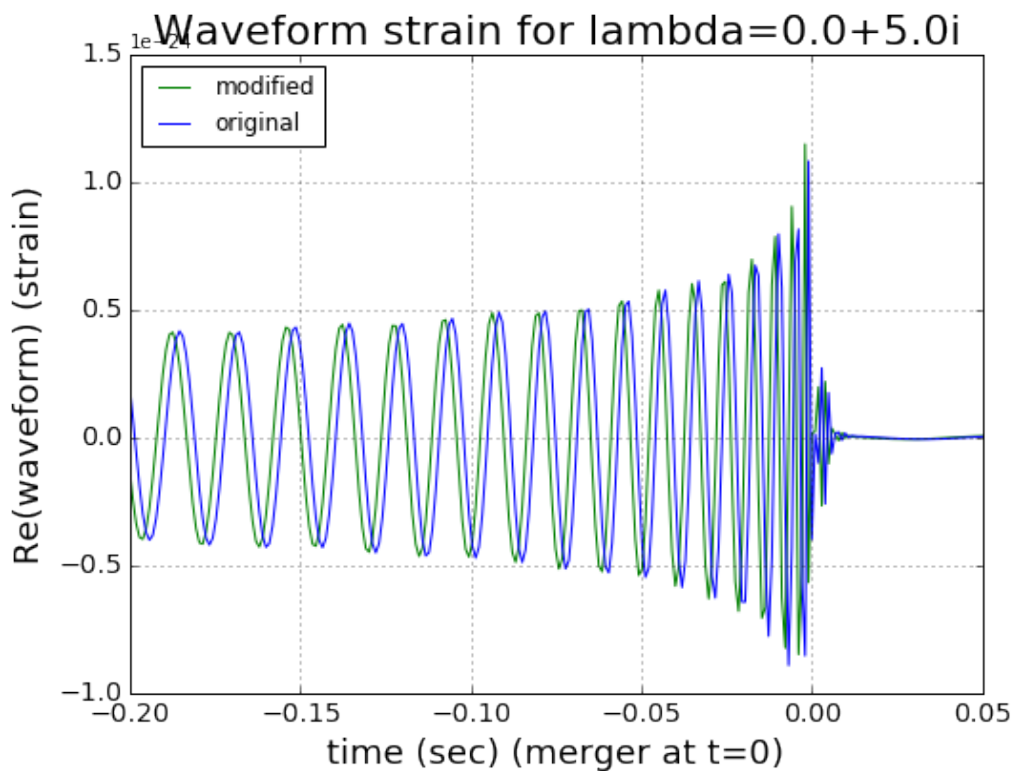
Examples: Low Mass System



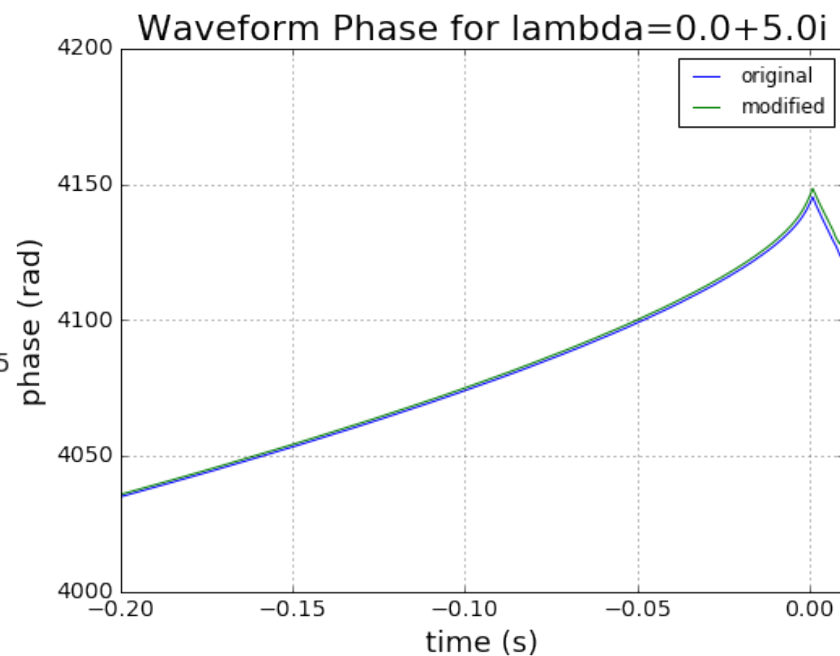
$M_1=20M_{\text{sol}}$
 $M_2=10M_{\text{sol}}$
 $D=50\text{Mpc}$



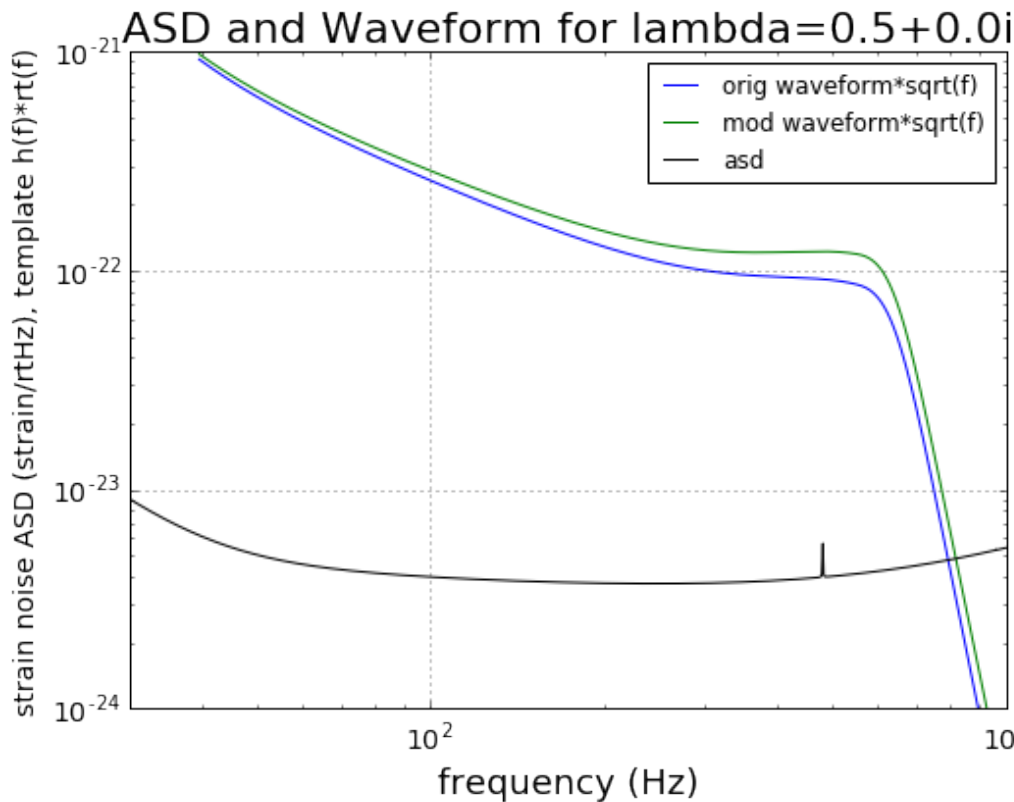
Examples: Low Mass System



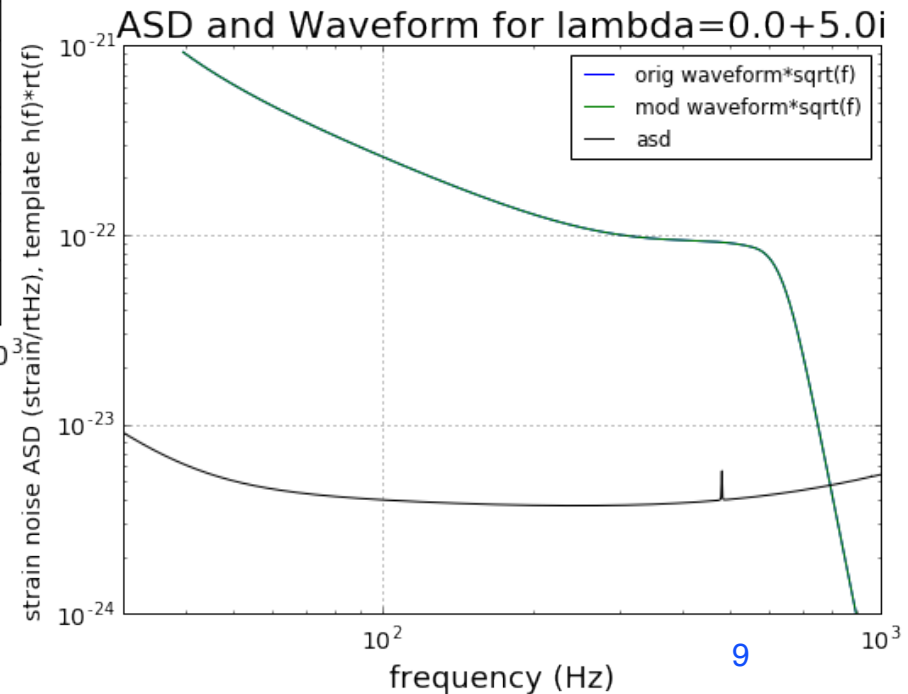
$M_1=20M_{\text{sol}}$
 $M_2=10M_{\text{sol}}$
 $D=50\text{Mpc}$



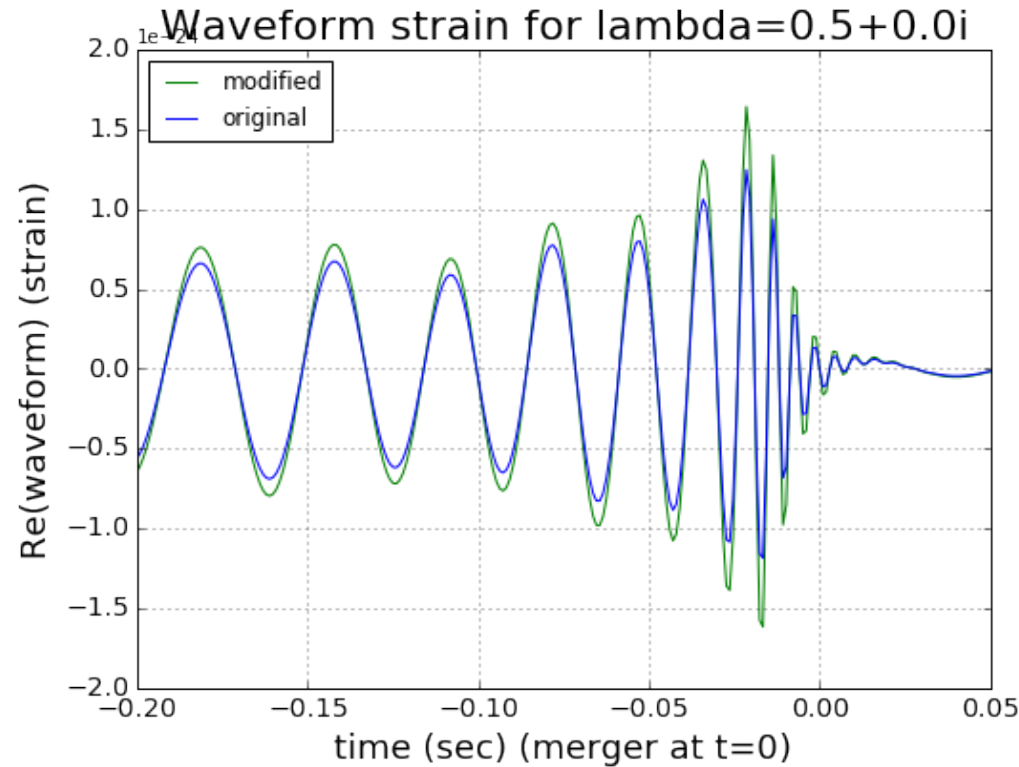
Examples: Low Mass System



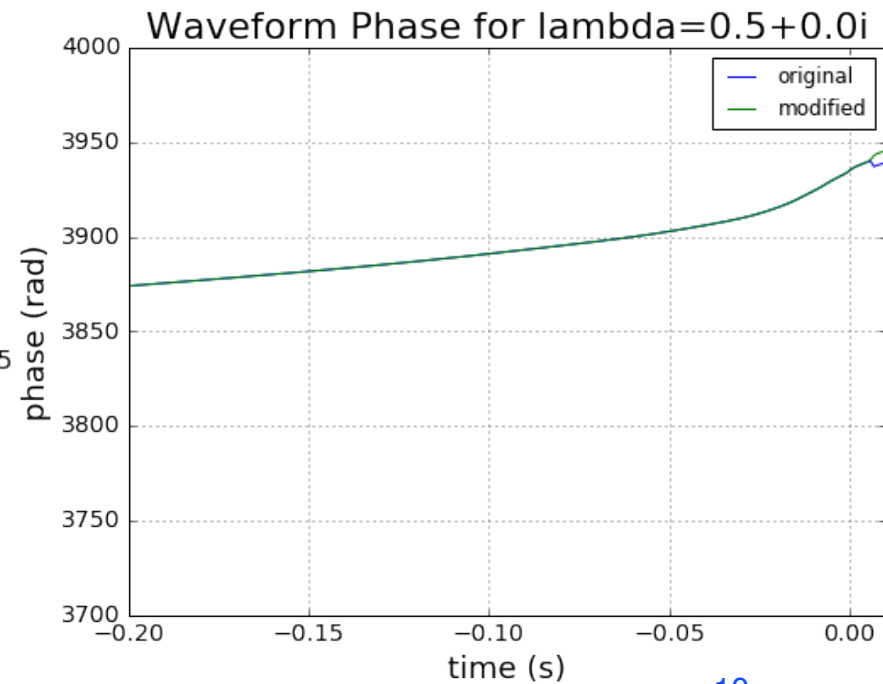
$M_1 = 20 M_{\text{sol}}$
 $M_2 = 10 M_{\text{sol}}$
 $D = 50 \text{ Mpc}$



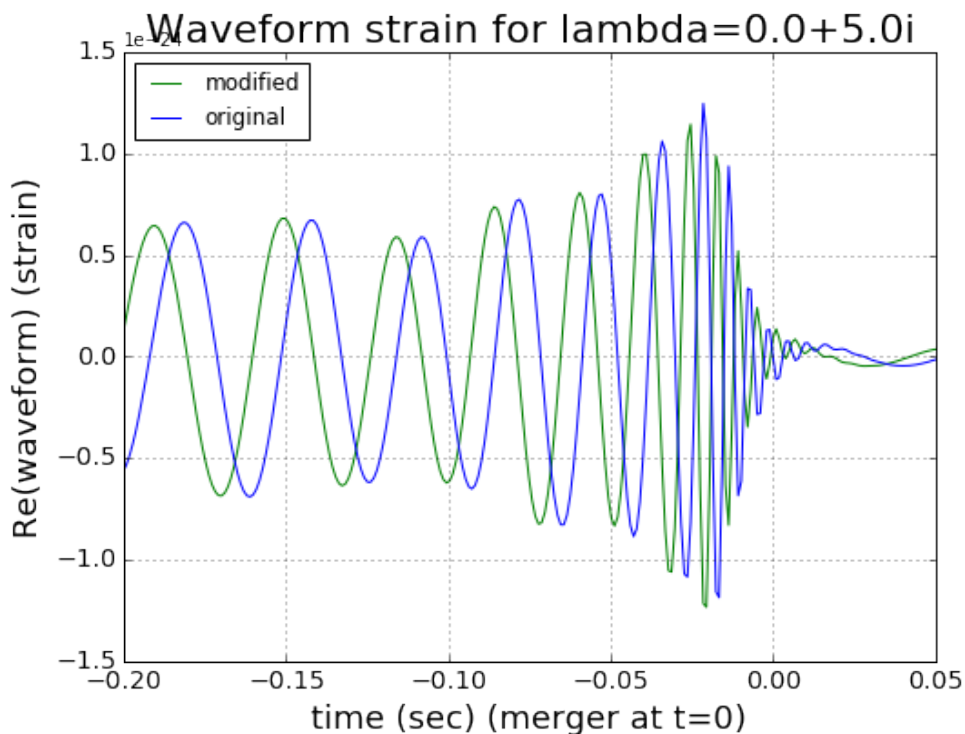
Examples: High Mass System



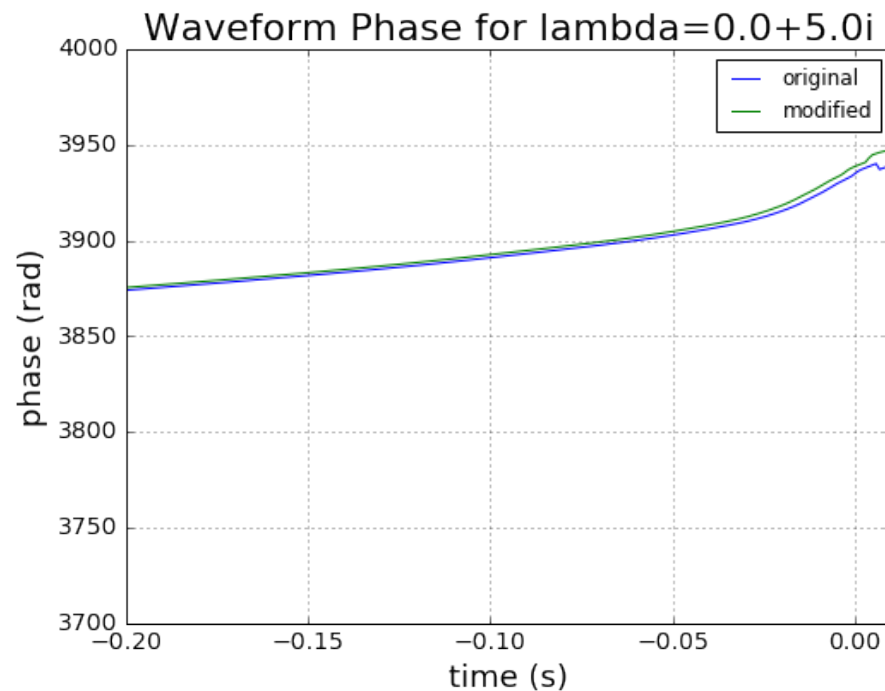
$M_1=60M_{\text{sol}}$
 $M_2=50M_{\text{sol}}$
 $D=200\text{Mpc}$



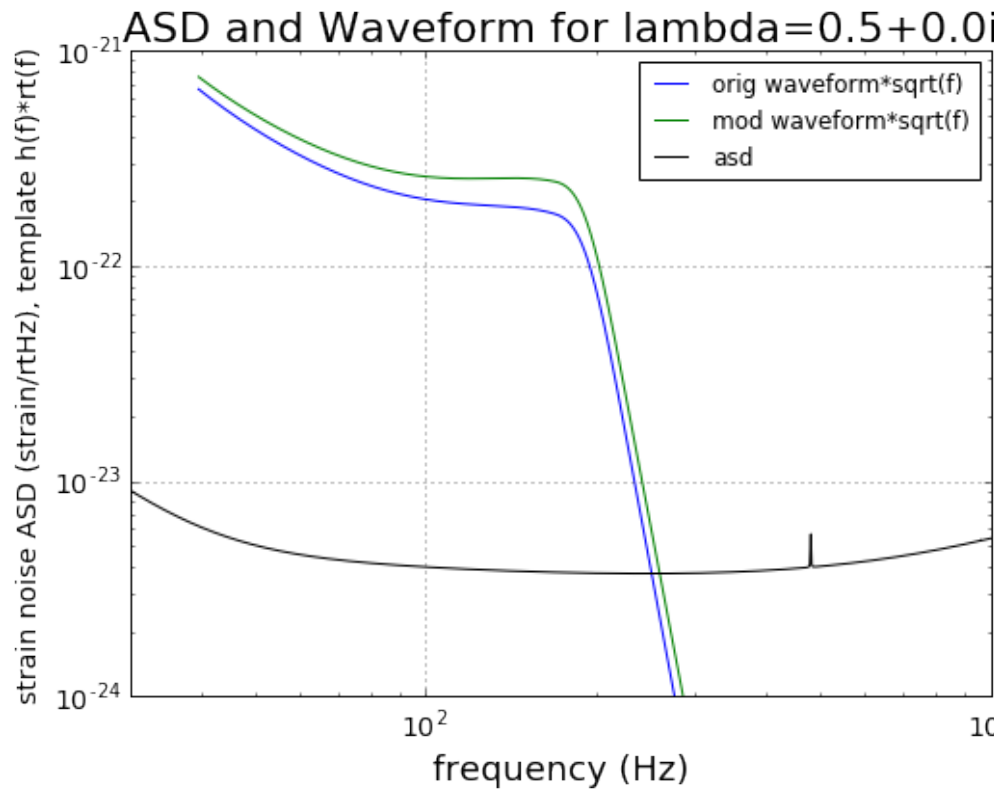
Examples: High Mass System



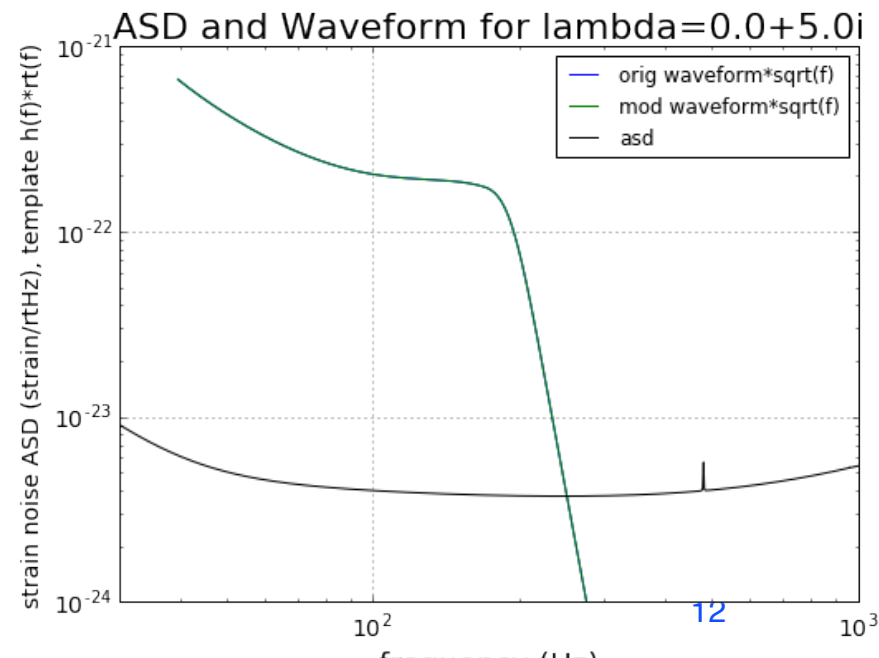
$M_1=60M_{\text{sol}}$
 $M_2=50M_{\text{sol}}$
 $D=200\text{Mpc}$



Examples: High Mass System

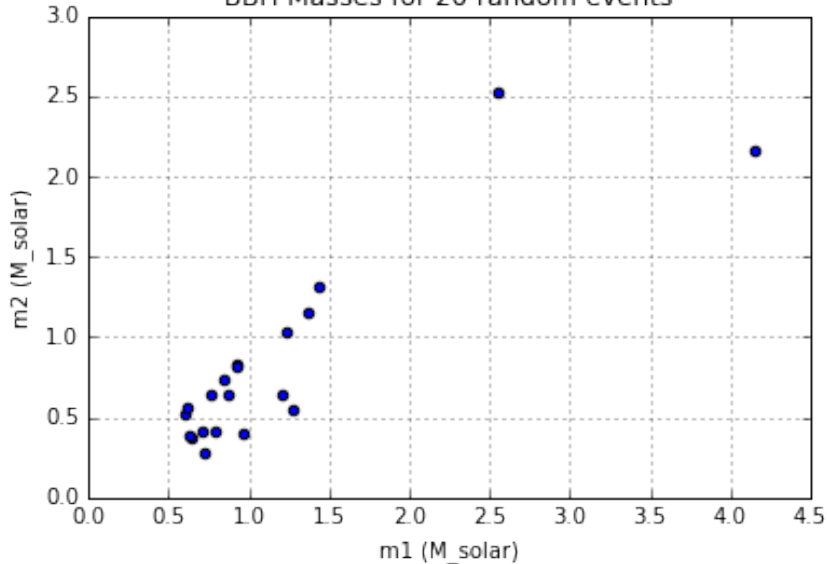


$M_1 = 60 M_{\text{sol}}$
 $M_2 = 50 M_{\text{sol}}$
 $D = 200 \text{ Mpc}$

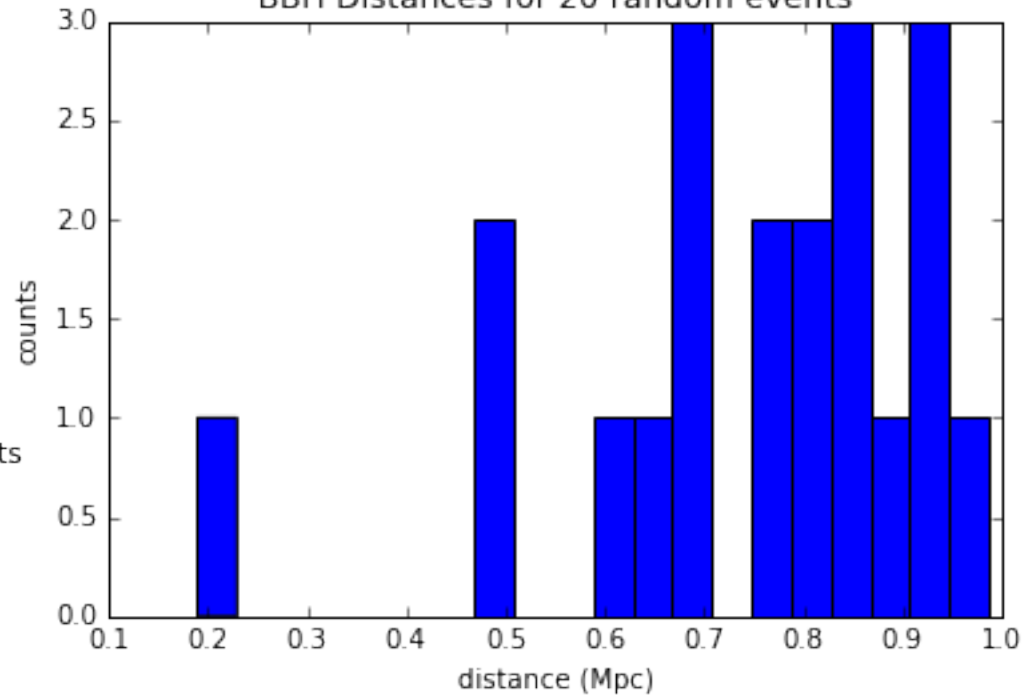


The Simulated Mergers

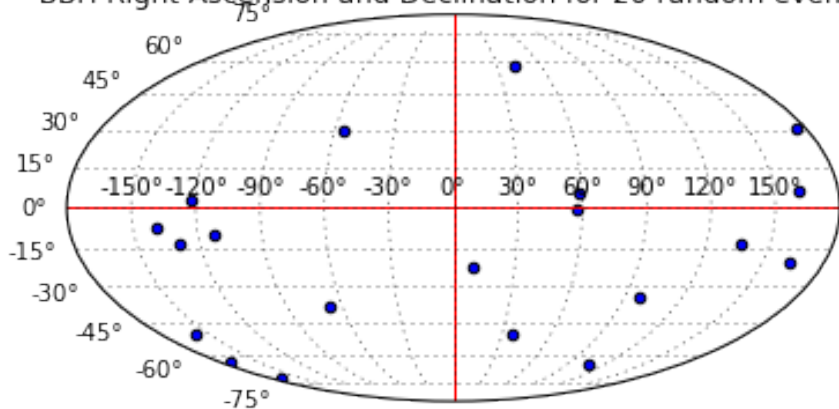
BBH Masses for 20 random events



BBH Distances for 20 random events



BBH Right Ascension and Declination for 20 random events



Bayesian Parameter Estimation

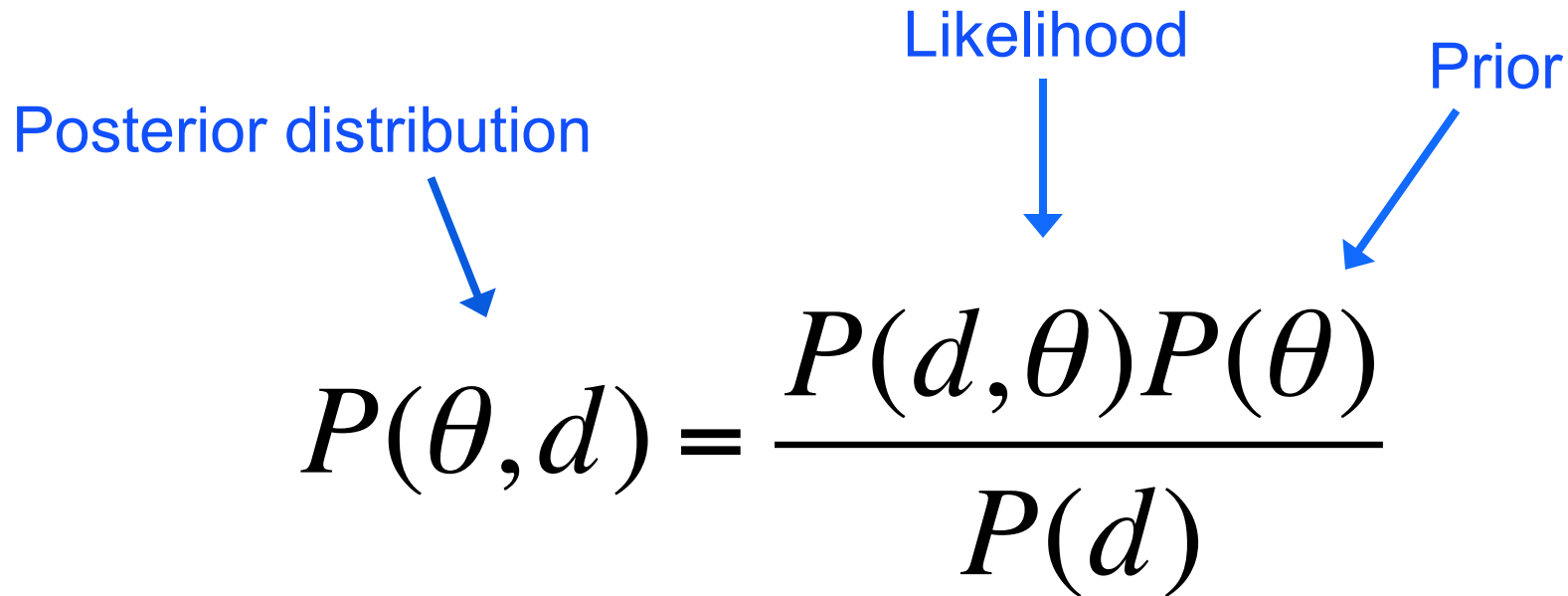
Posterior distribution

Likelihood

Prior

$$P(\theta, d) = \frac{P(d, \theta)P(\theta)}{P(d)}$$

evidence



$\theta = \lambda$

$d = \text{data}$

the evidence normalizes the pdf

Bayesian Probabilities (Testing phase)

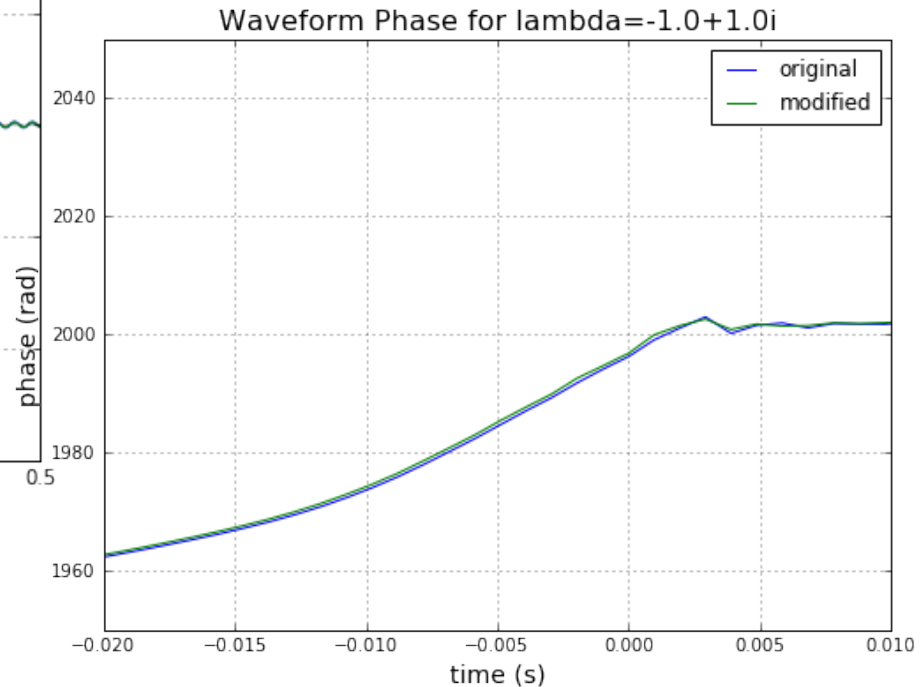
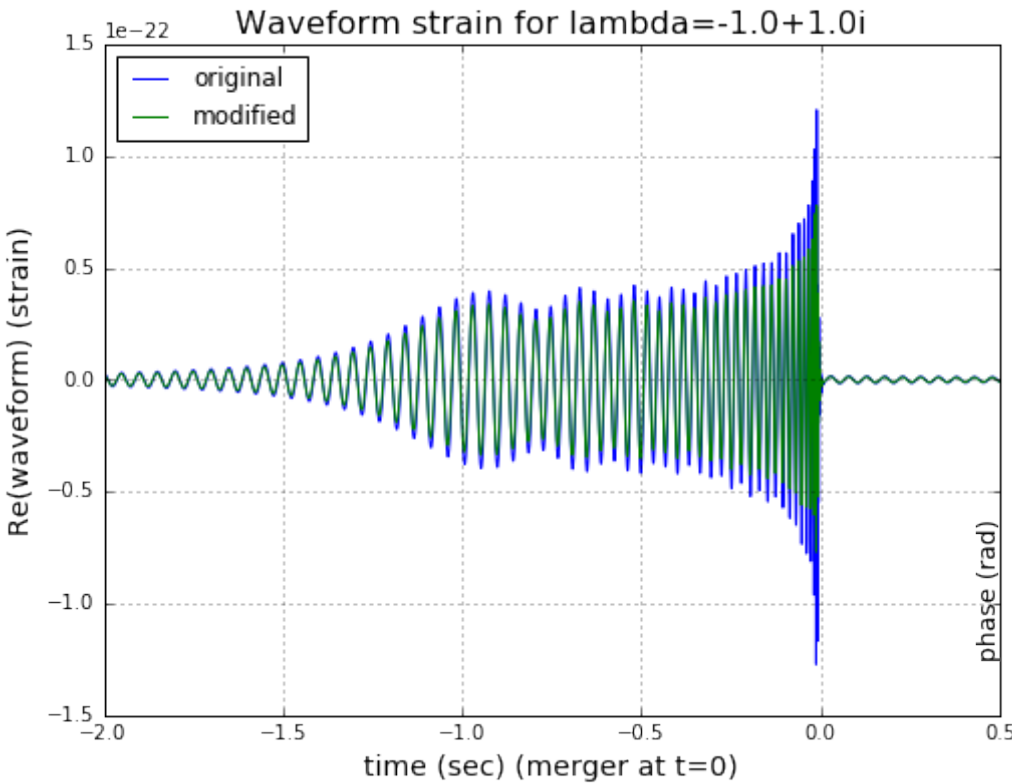
The prior:
 Jeffreys (uninformative) prior

$$P(\lambda) = \frac{1}{|\lambda|}$$

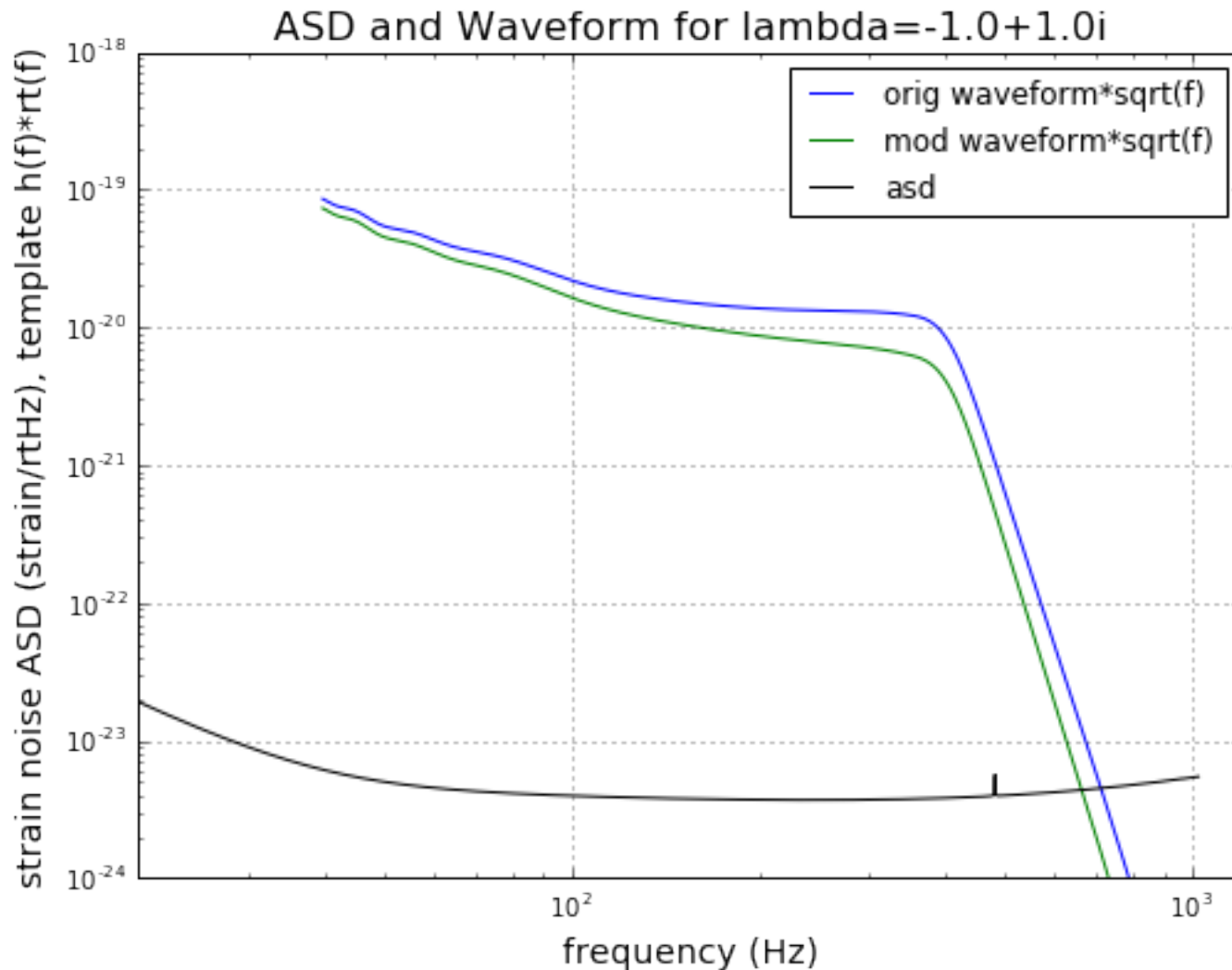
$$p(\text{Re}(\lambda), \text{Im}(\lambda))d\lambda = \frac{d\lambda}{|\lambda|} = d \ln \lambda$$

$P(d, \theta)$ The likelihood:
 normalized SNR maximized over time
 data = modified waveform (λ)
 template = modified waveform (sampling λ)

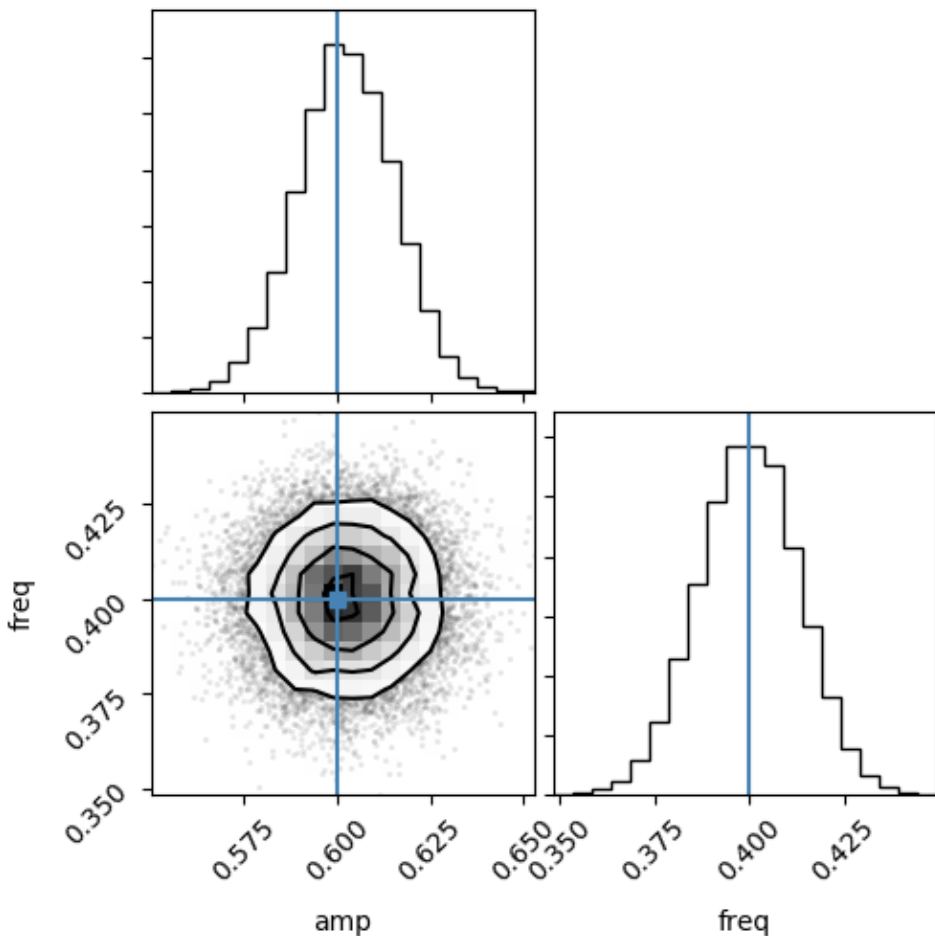
Test #1: Noiseless Waveforms



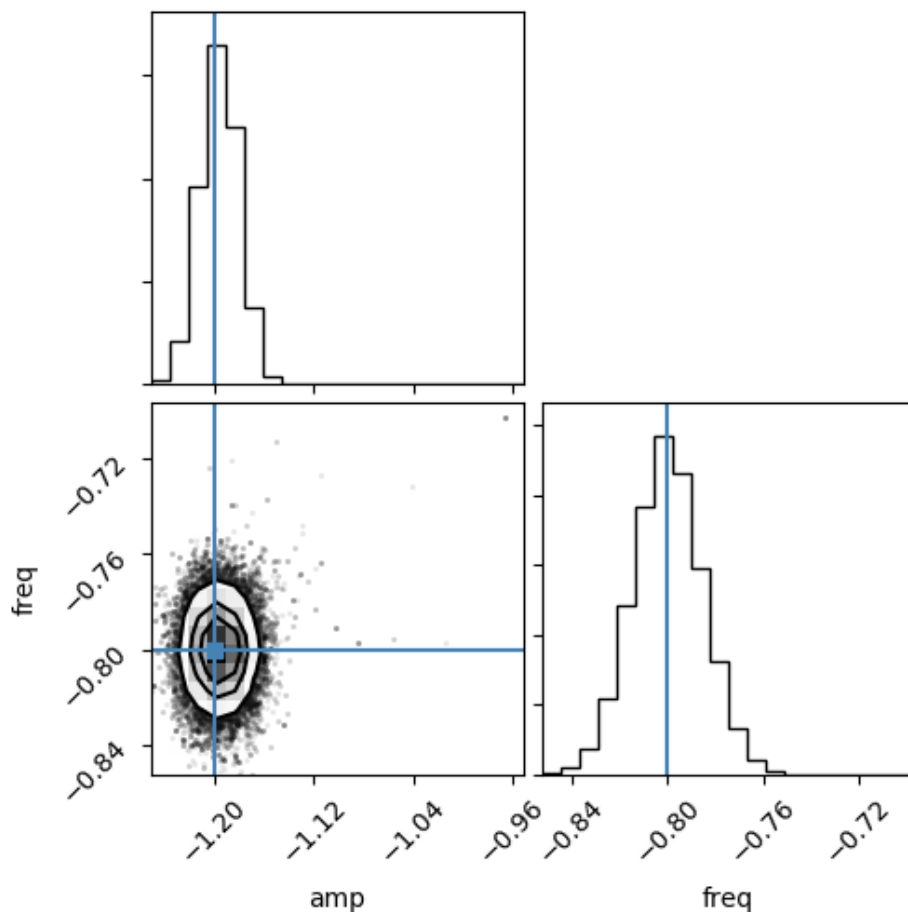
Test #1: Noiseless Waveforms



Test #1: Results

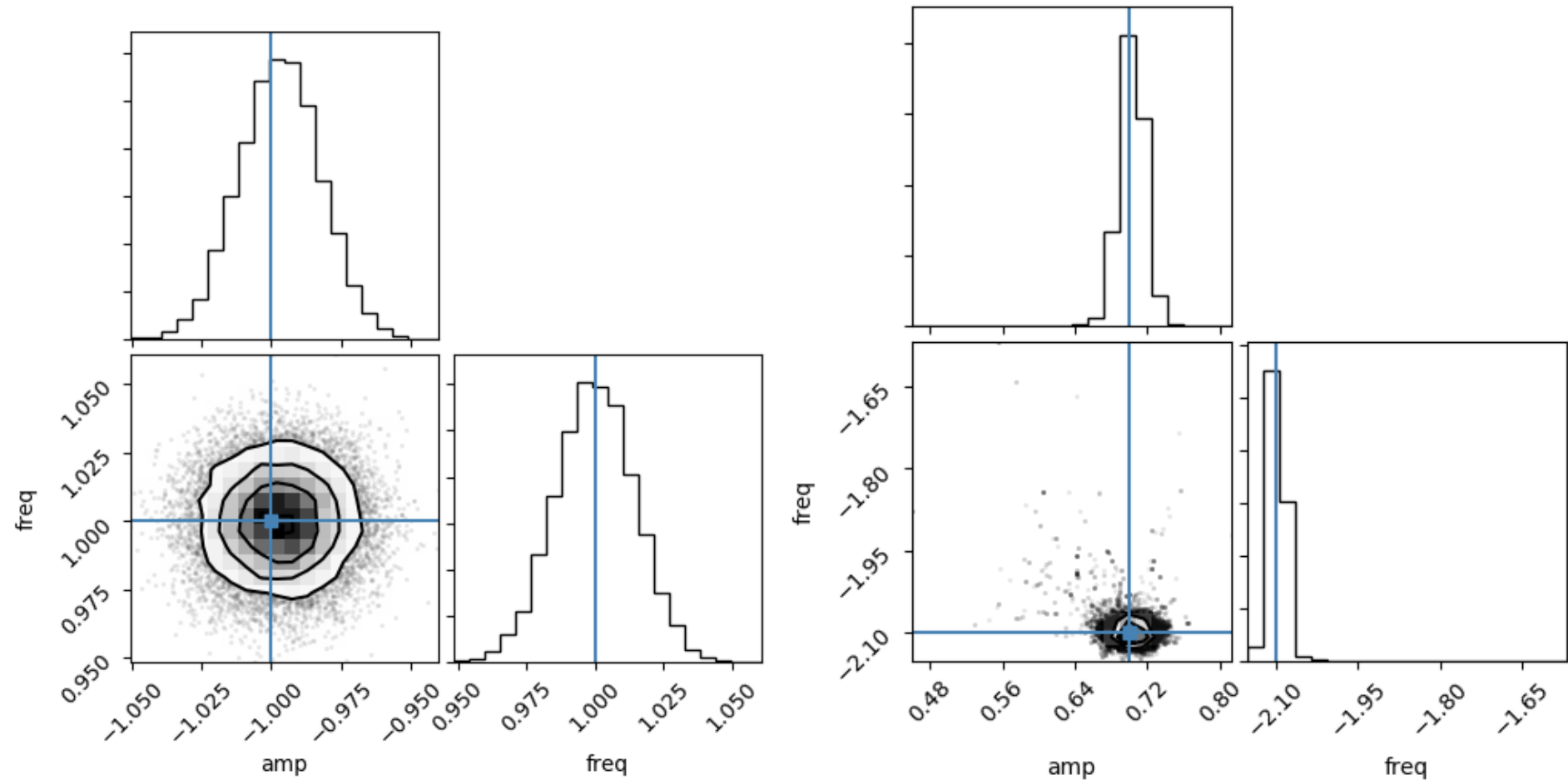


$$\lambda = .6 + .4i$$



$$\lambda = -1.2 - .8i$$

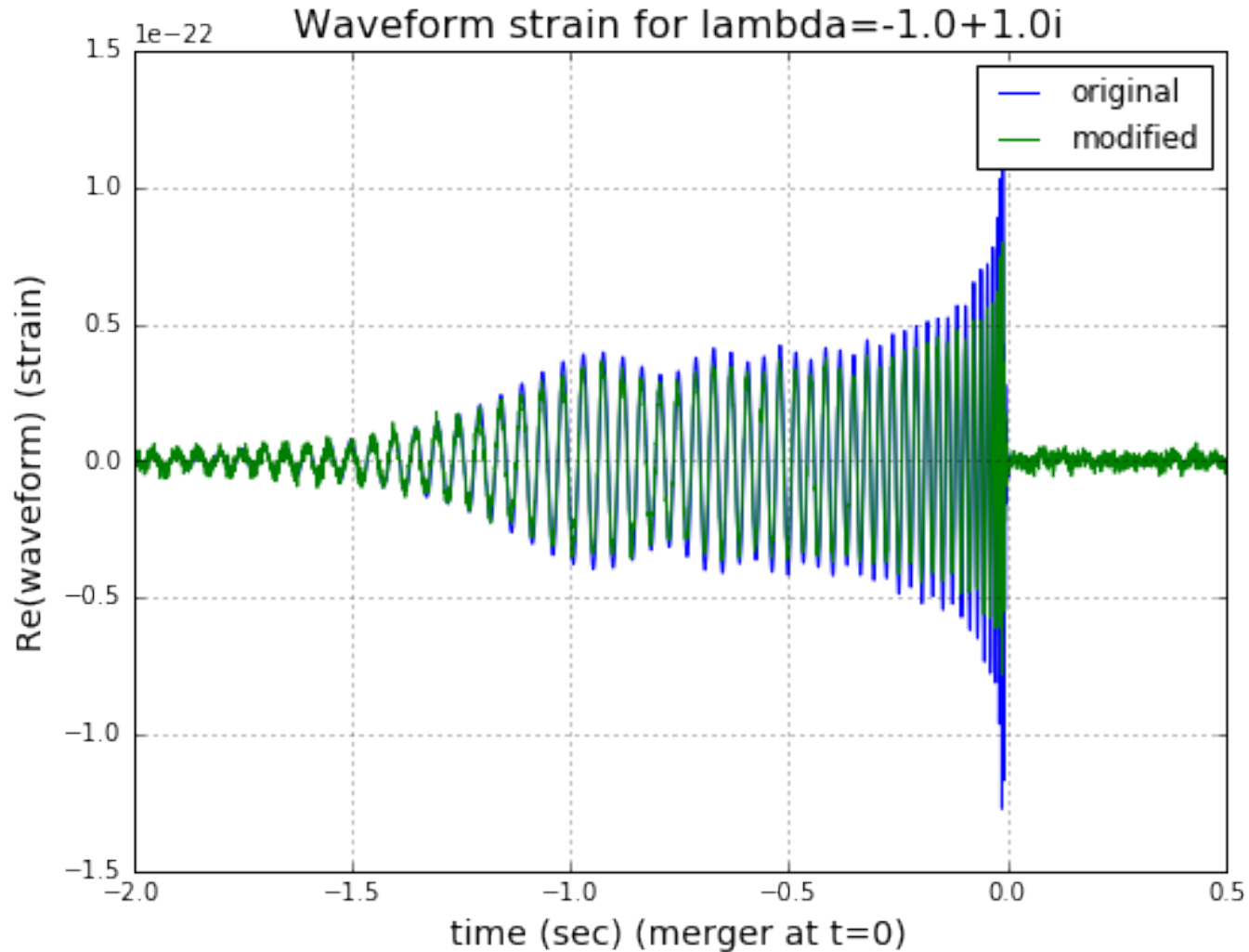
Test #1: Results



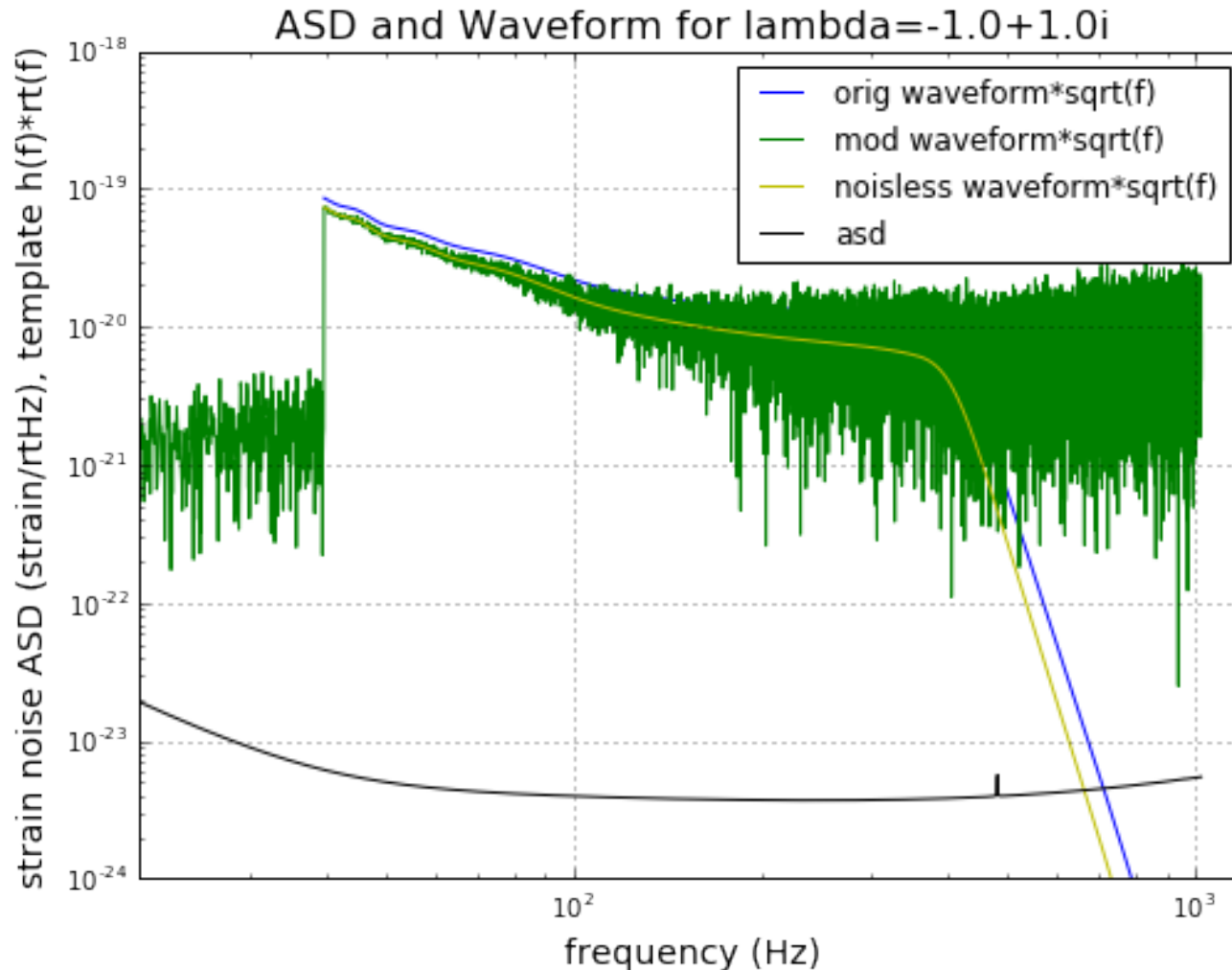
$\lambda = -1 + i$

$\lambda = .7 - 2.1i$

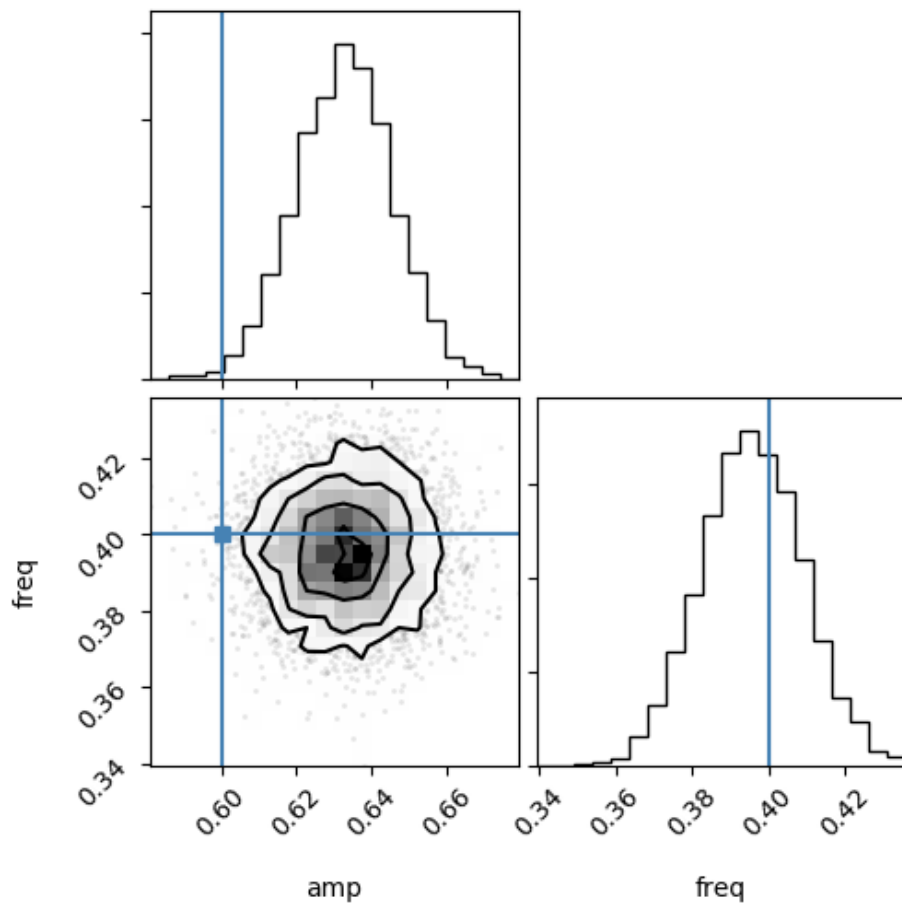
Test #2: Waveforms + WGNoise



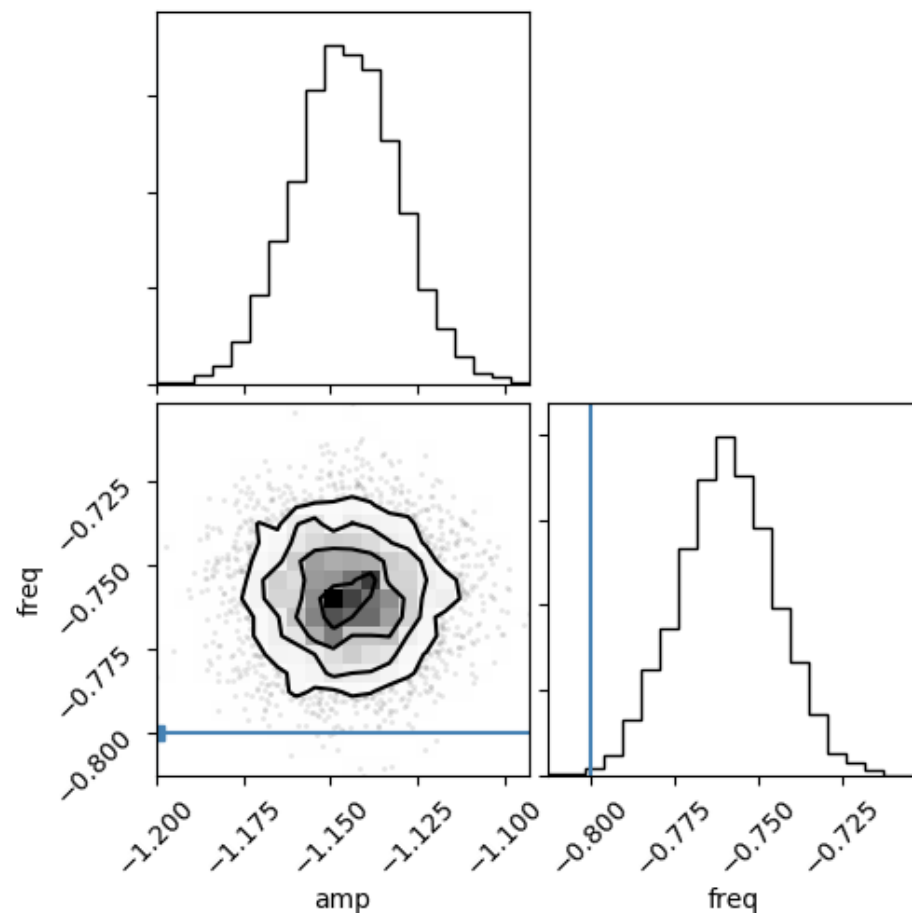
Test #2: Waveforms + WGNoise



Test #2: Results

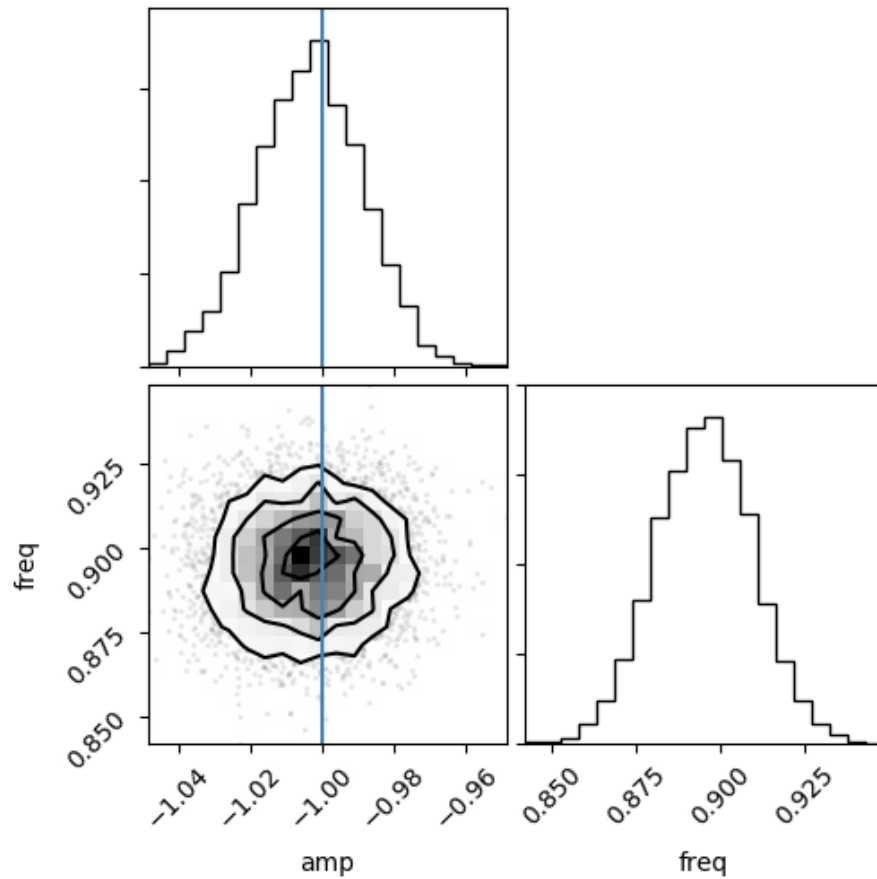


$$\lambda = .6 + .4i$$

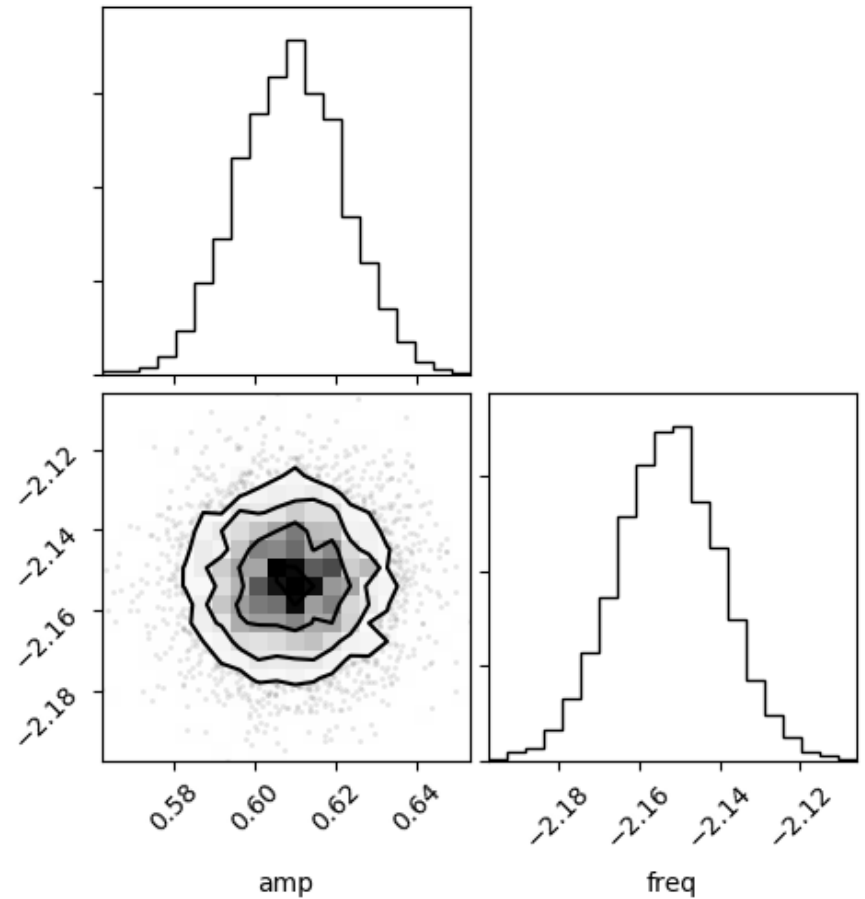


$$\lambda = -1.2 - .8i$$

Test #2: Results



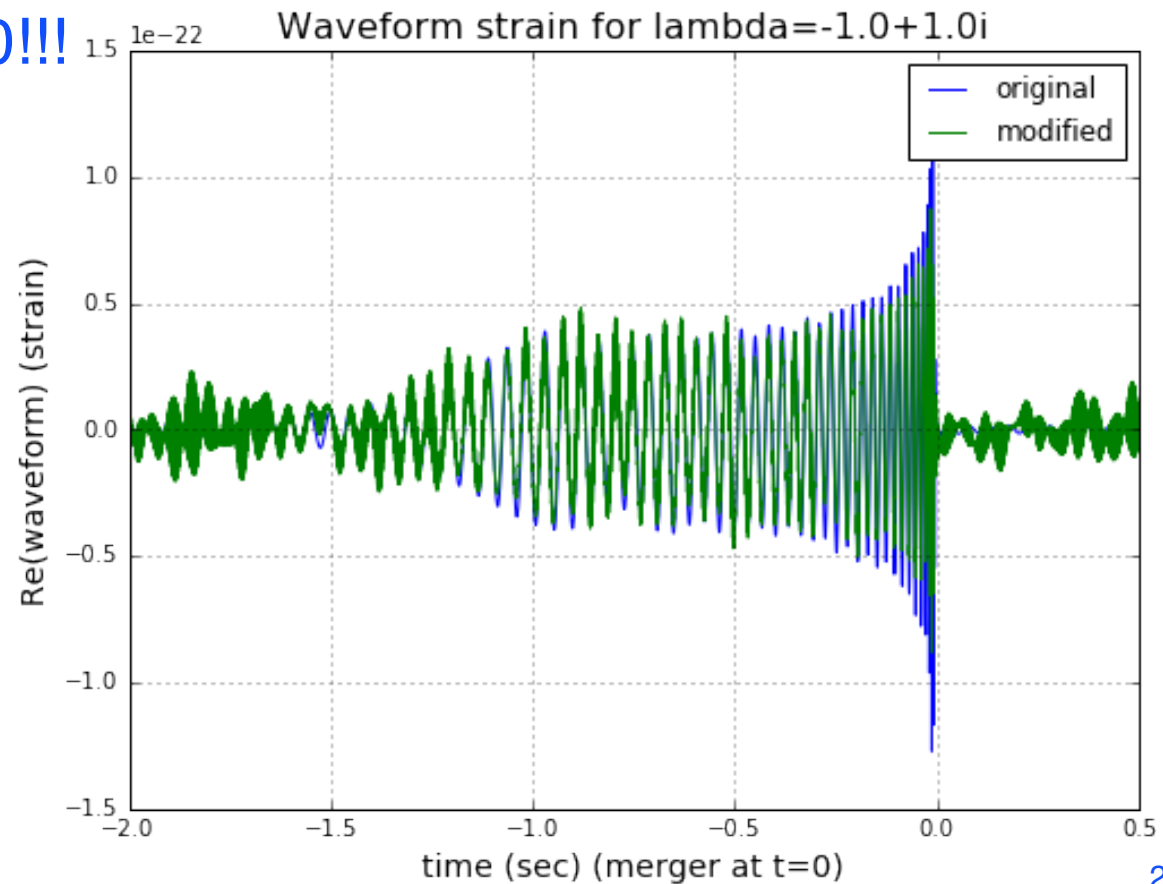
$$\lambda = -1 + i$$



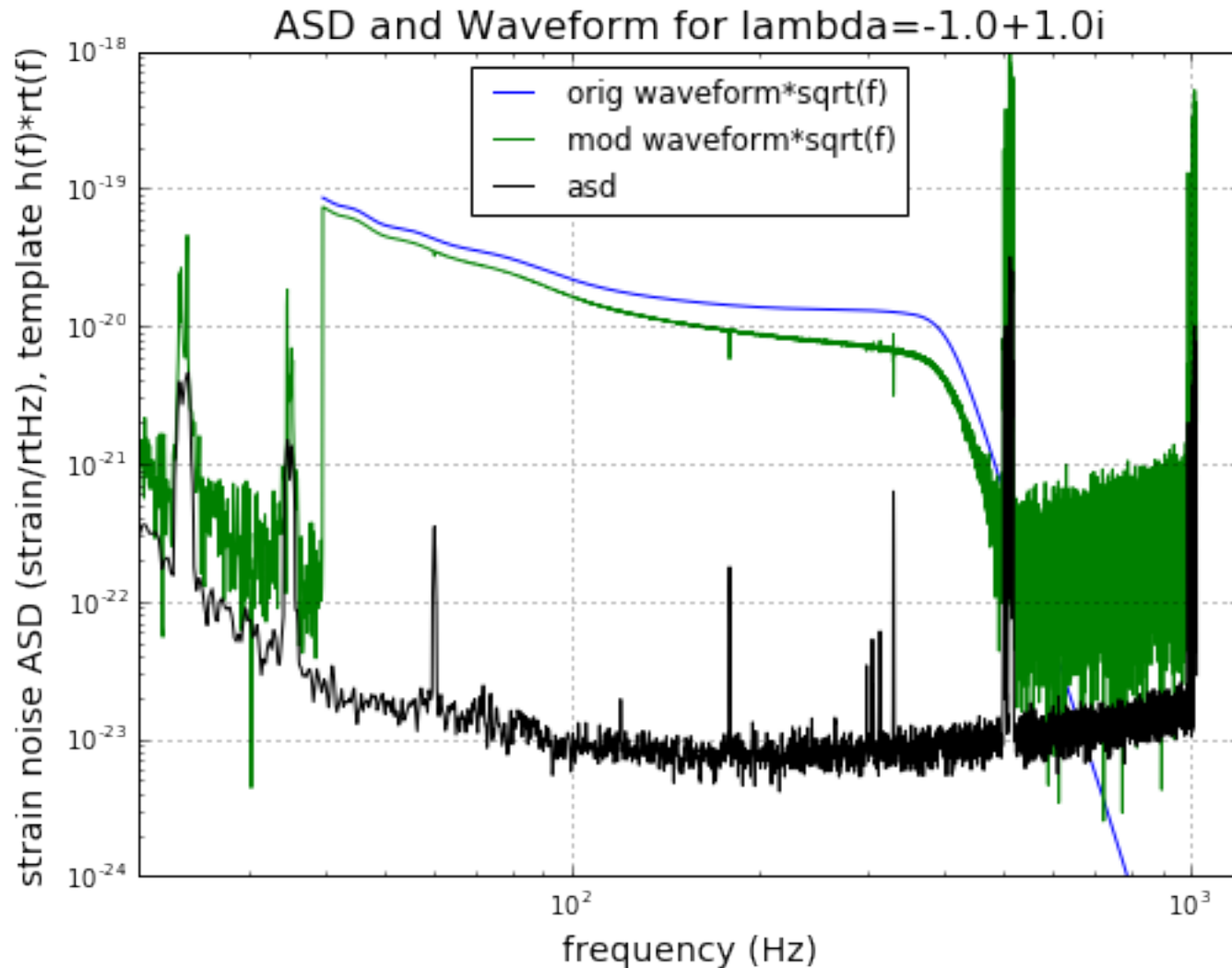
$$\lambda = .7 - 2.1i$$

Test #3: Waveforms + LIGONoise

- Noise taken from GW150914 away from the event
- SNR $\sim 1000!!!$

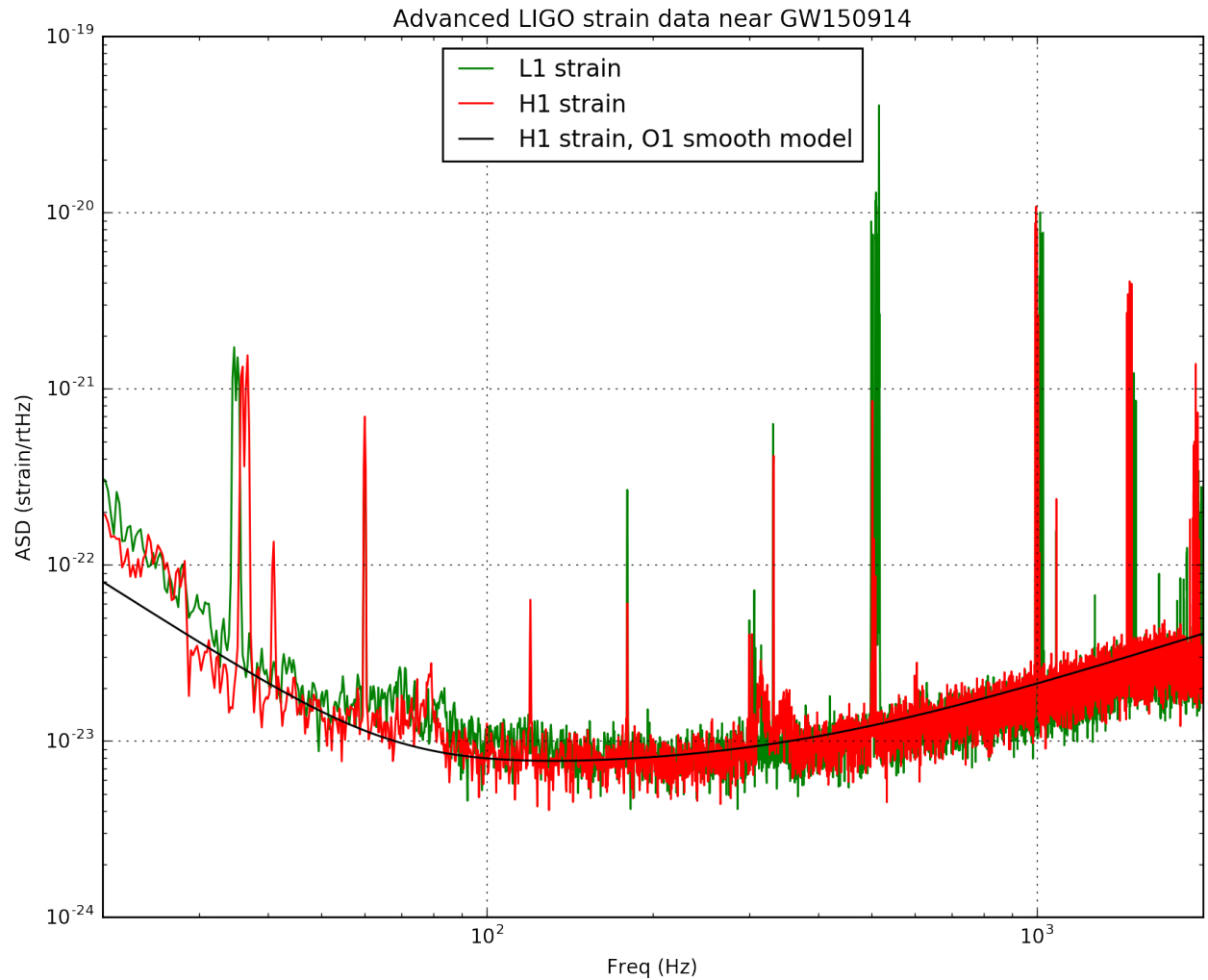


Test #3: Waveforms + LIGO Noise

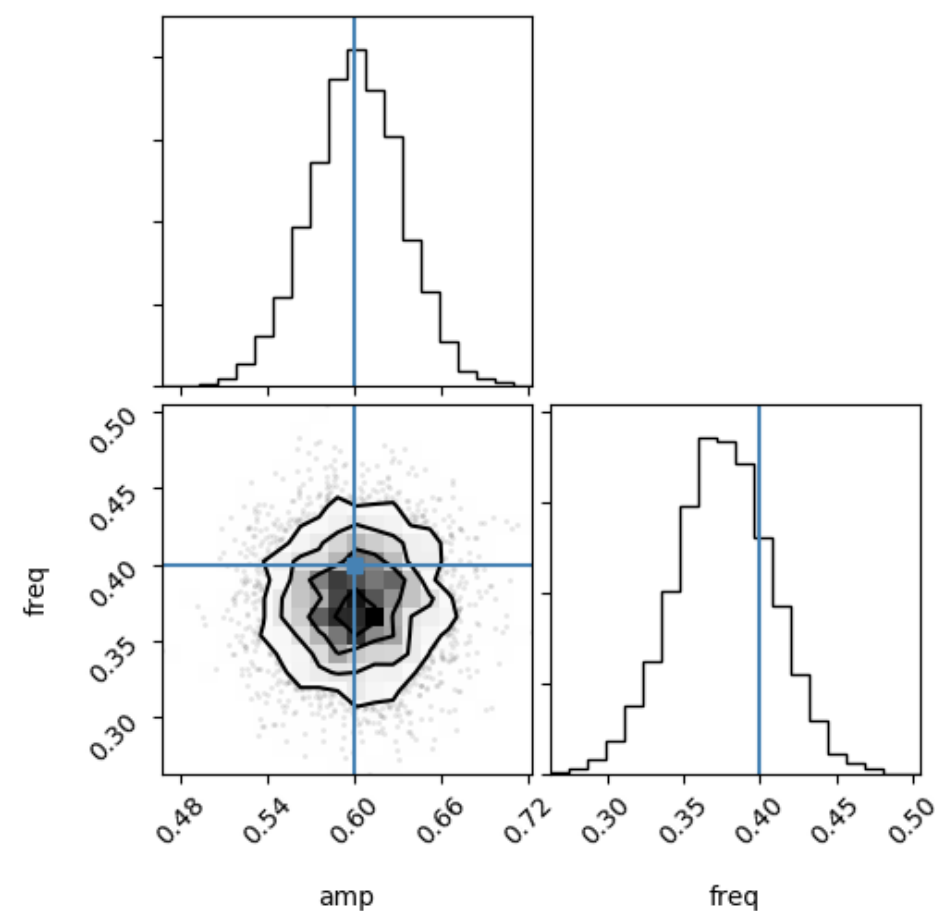


For comparison...

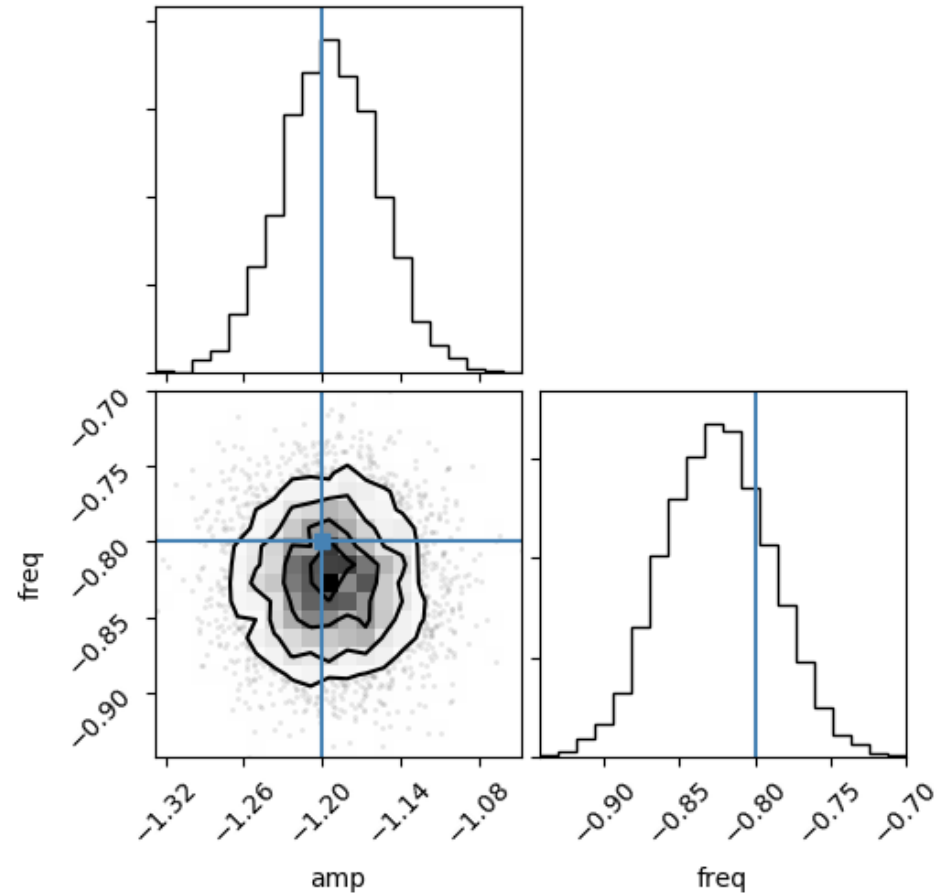
for
GW150914:



Test #3: Results

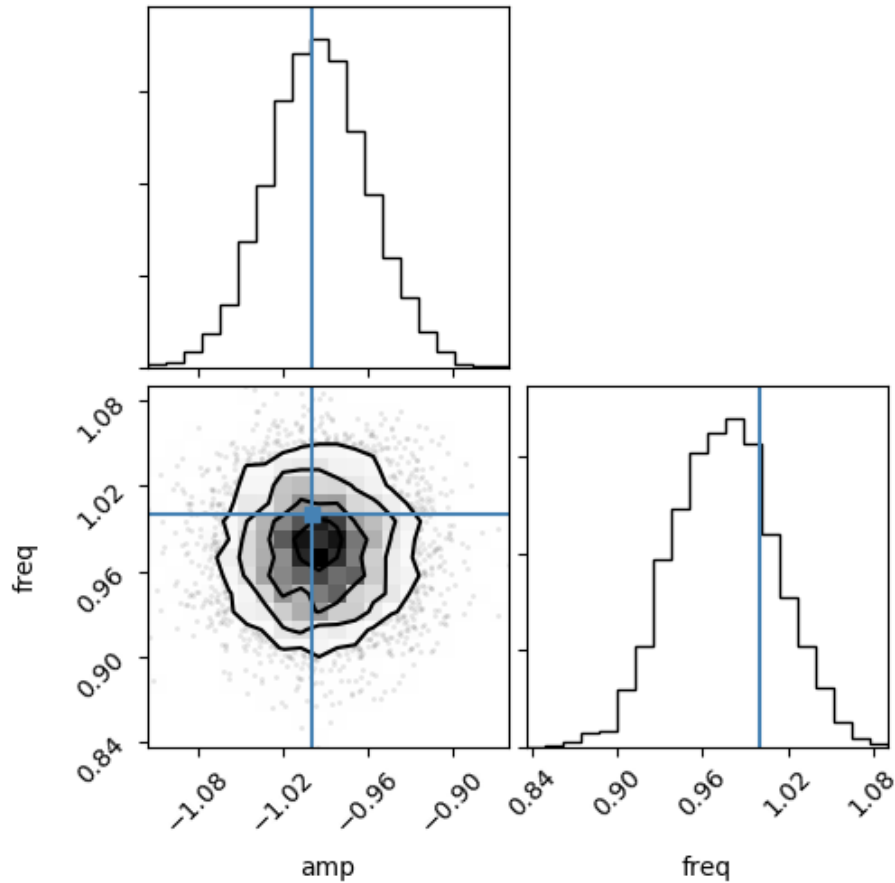


$$\lambda = .6 + .4i$$

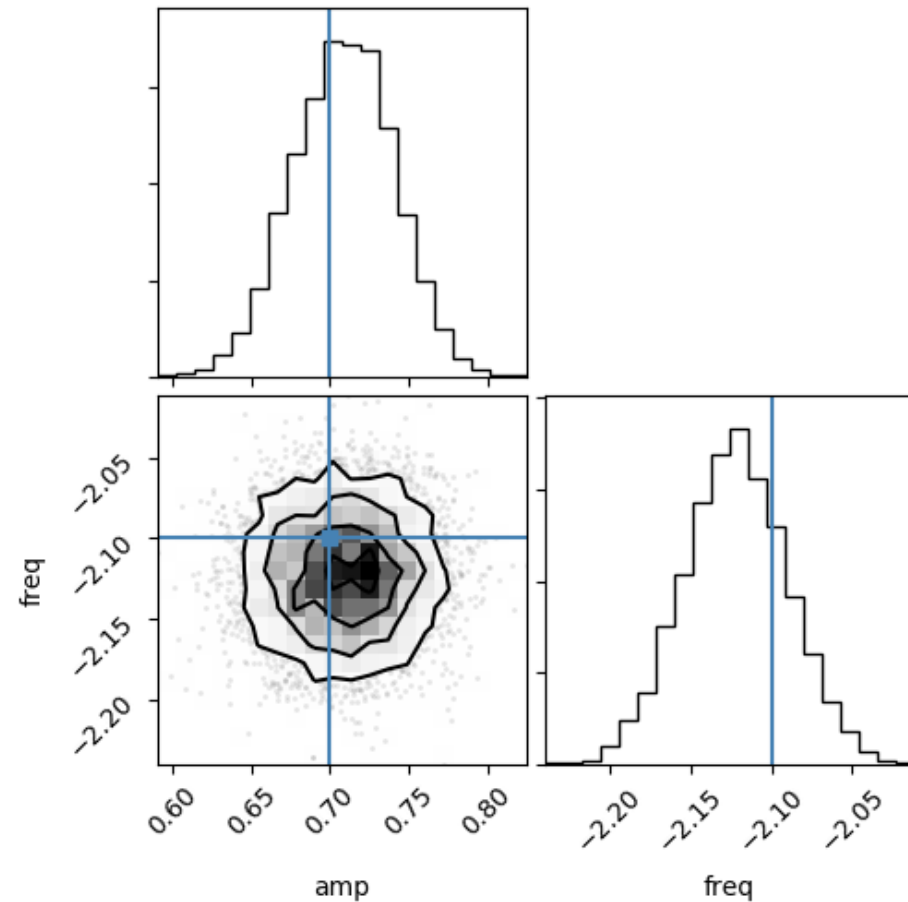


$$\lambda = -1.2 - .8i$$

Test #3: results



$$\lambda = -1 + i$$



$$\lambda = .7 - 2.1i$$

Bayesian Probabilities

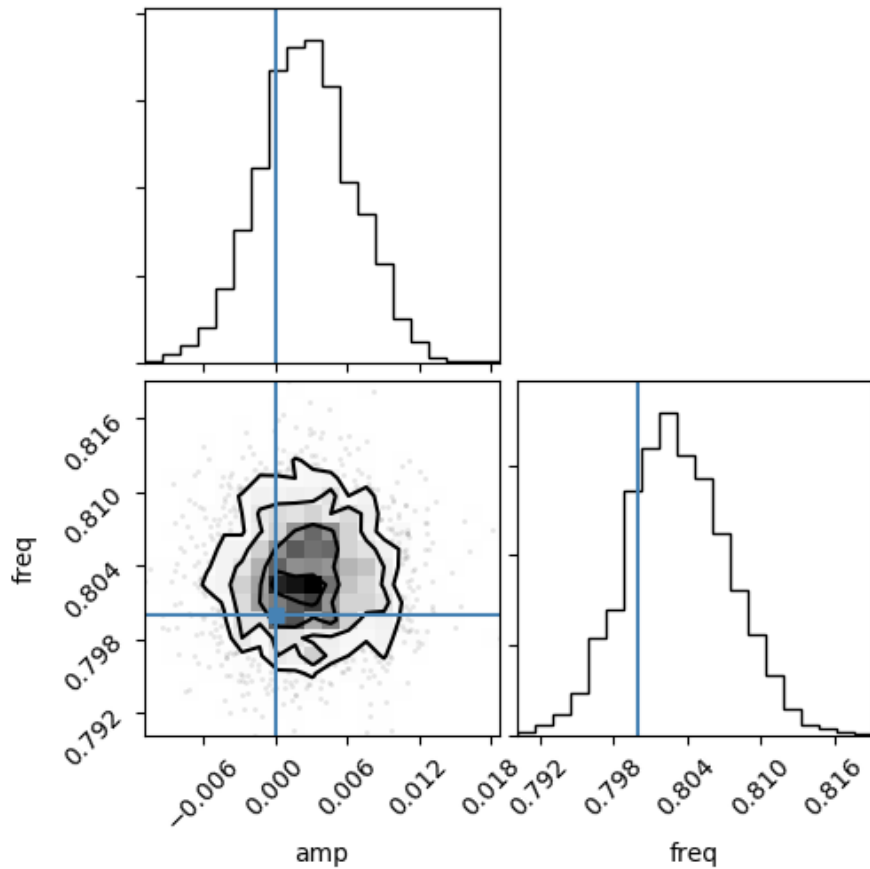
The prior:
Jeffreys (uninformative) prior

$$p(\text{Re}(\lambda), \text{Im}(\lambda))d\lambda = \frac{d\lambda}{|\lambda|} = d \ln \lambda$$

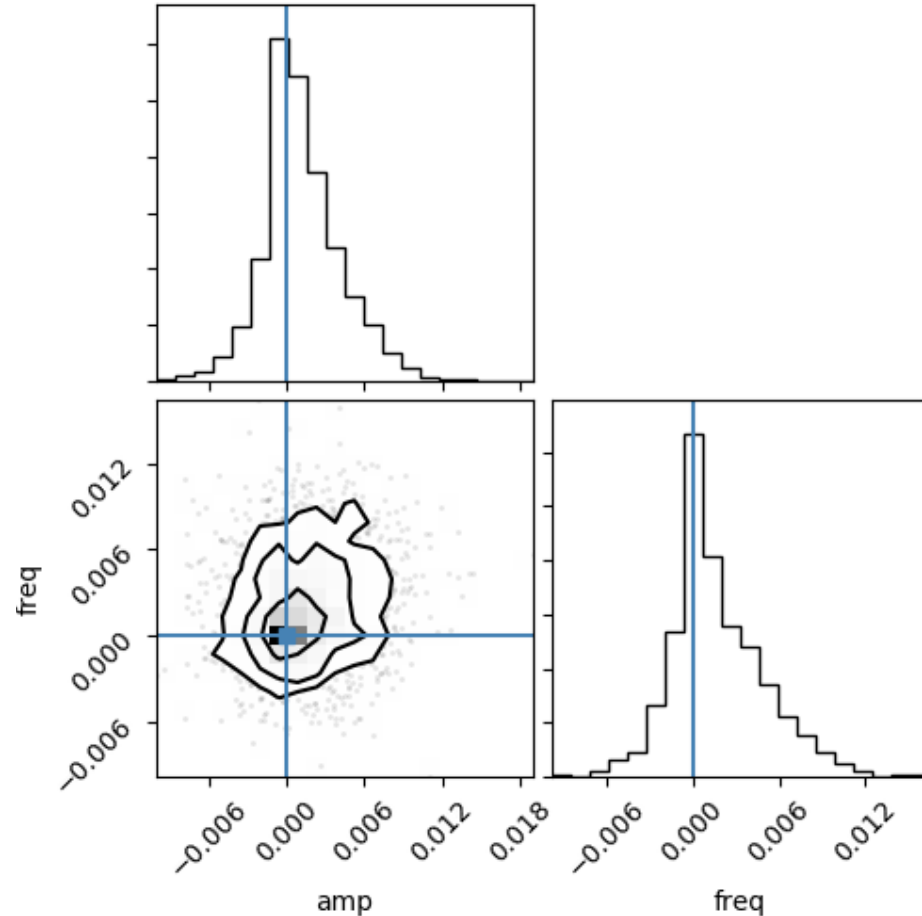
$P(d, \theta)$

The likelihood:
normalized SNR maximized
over time
20 events

A Few Results

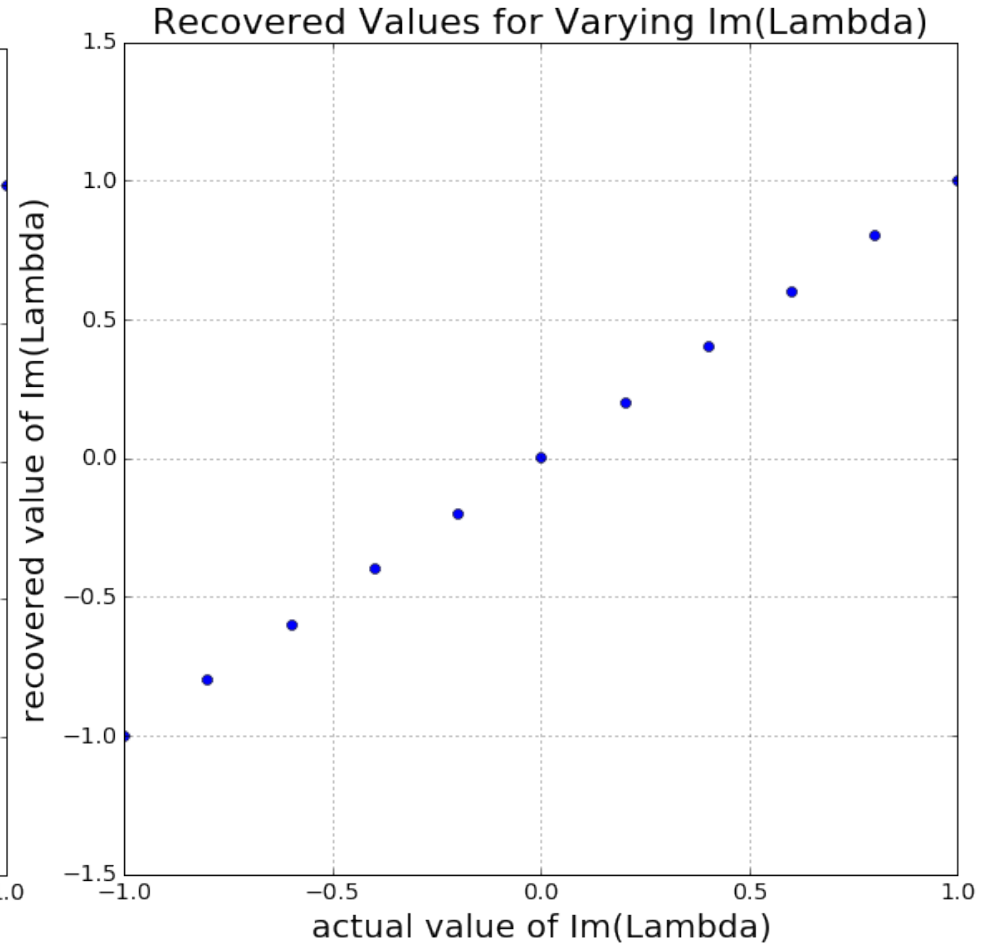
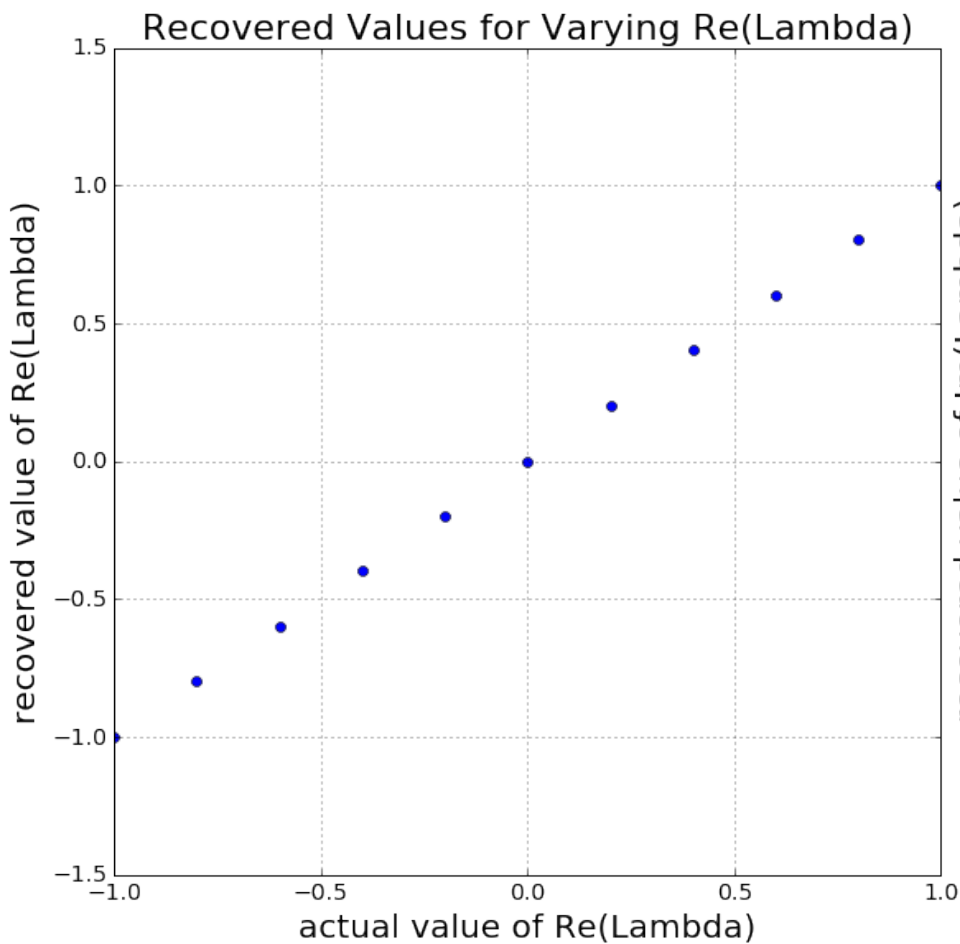


$\lambda = .8i$

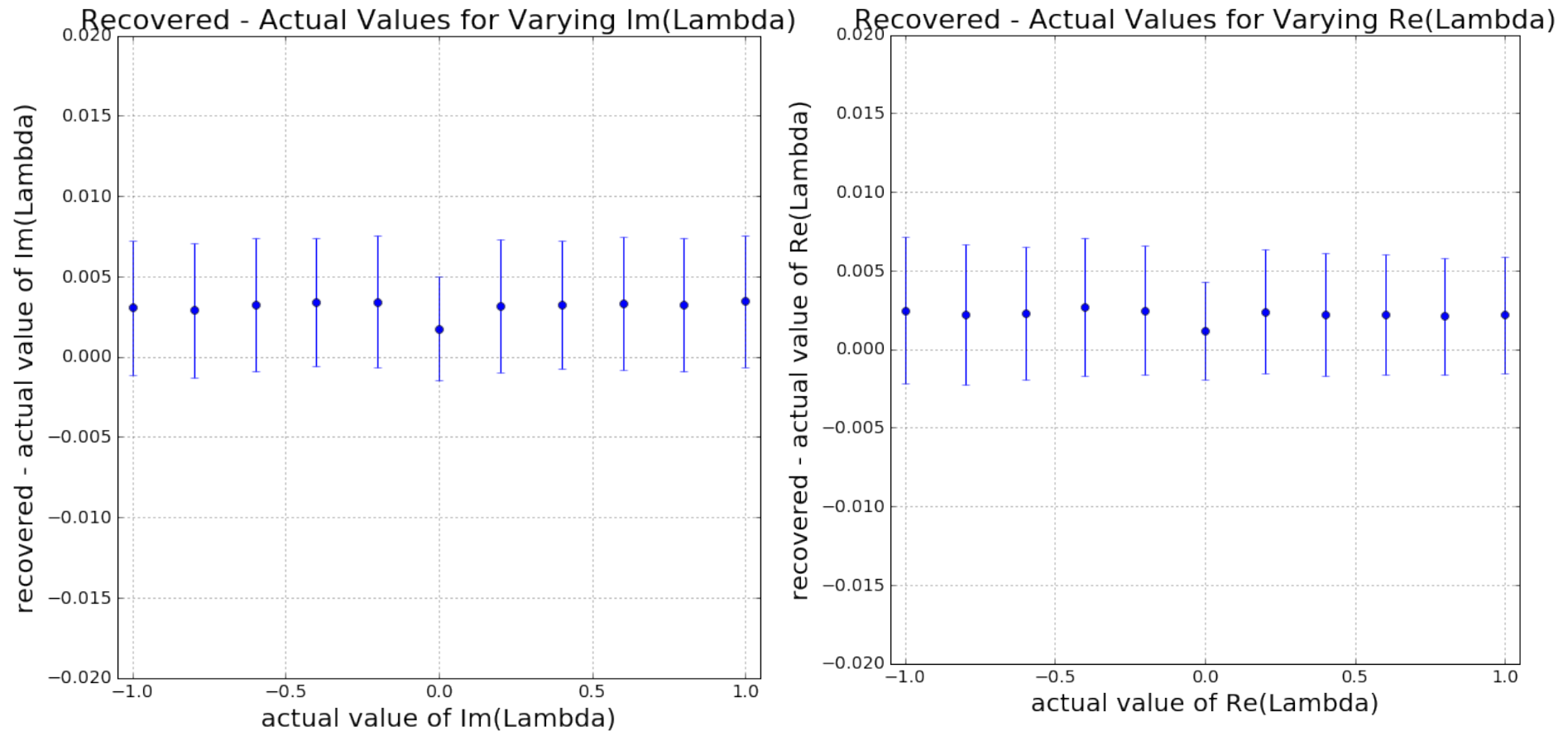


$\lambda = 0$

Recovering Lambda

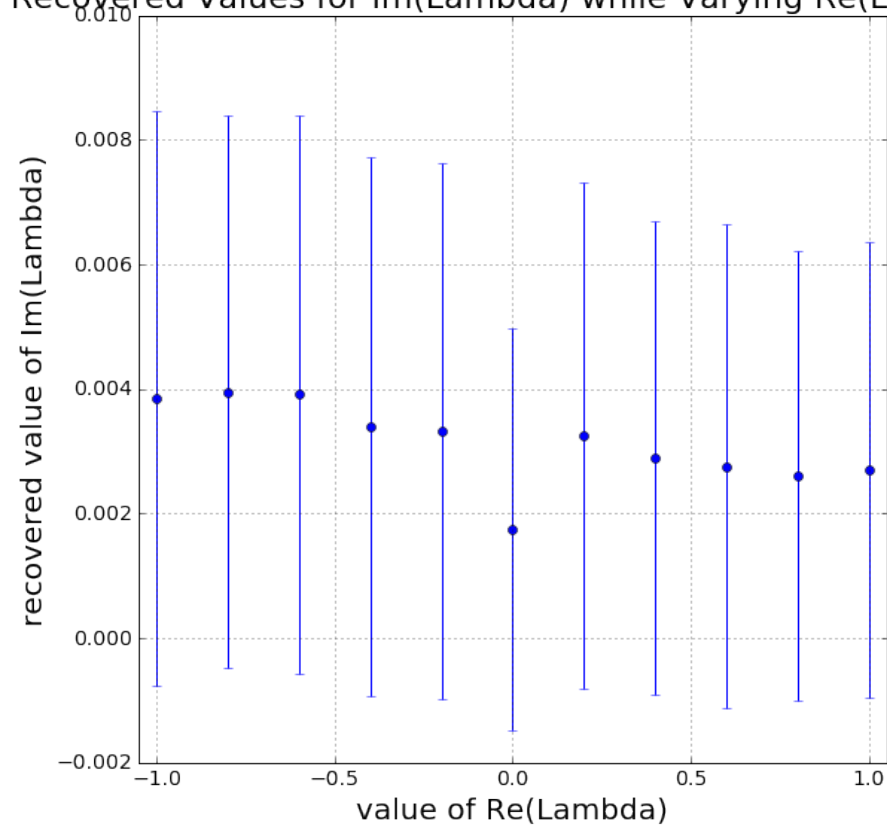


Recovering Lambda

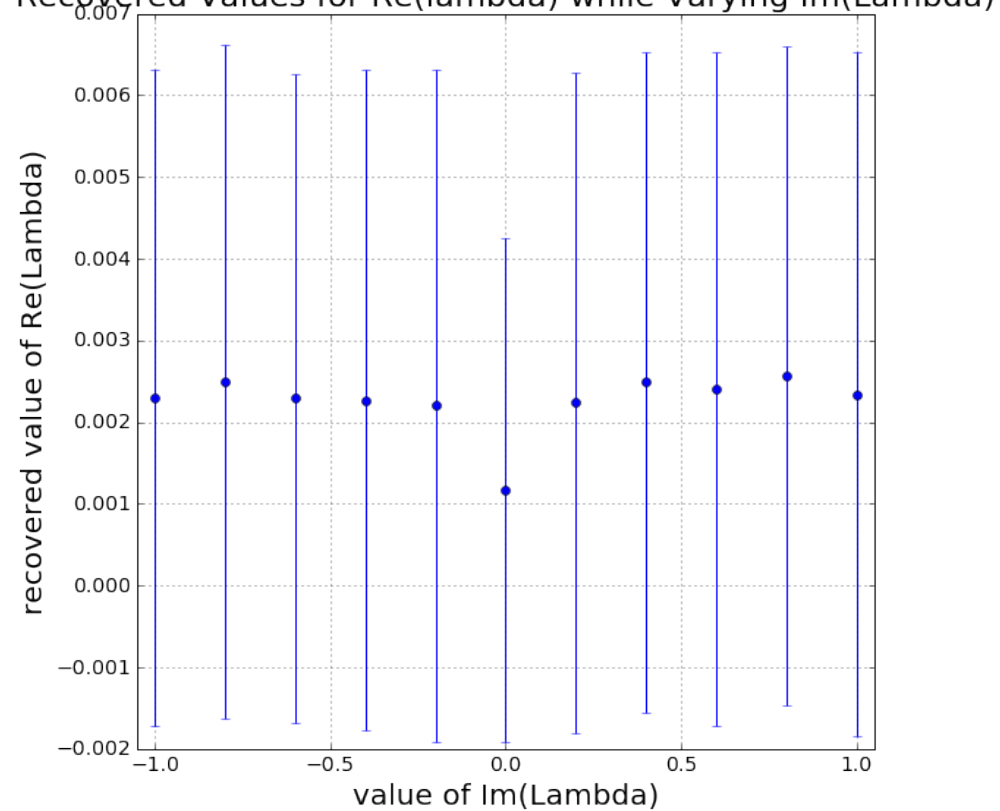


Recovering Lambda

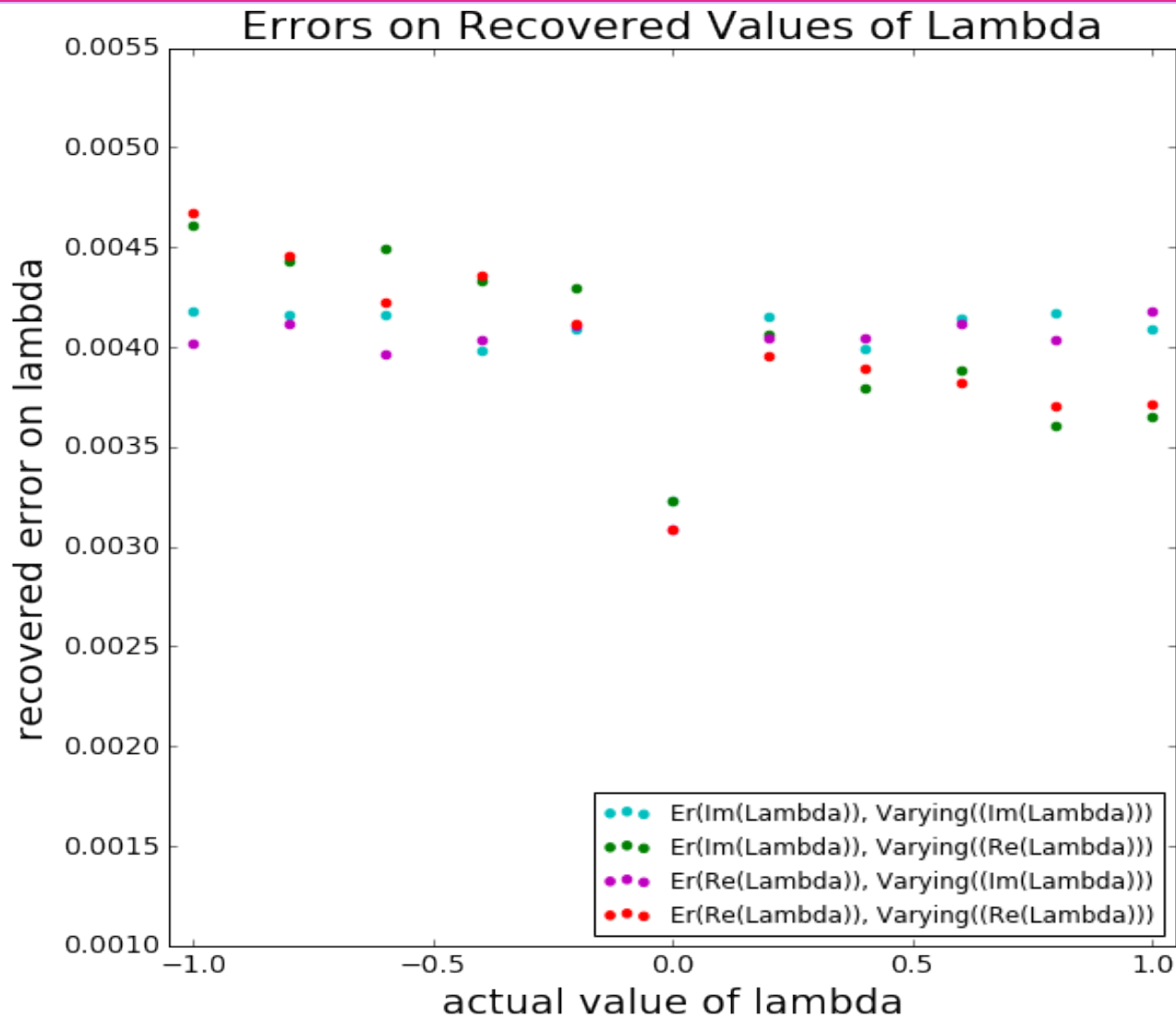
Recovered Values for $\text{Im}(\text{Lambda})$ while Varying $\text{Re}(\text{Lambda})$



Recovered Values for $\text{Re}(\text{lambda})$ while Varying $\text{Im}(\text{Lambda})$



Recovering Lambda



Summary and conclusions

- Summary
 - » Using only 20 events we can pin down a given value of λ very precisely

- Conclusions
 - » White gaussian noise is not a good model for LIGO noise
 - » There may or may not be a correlation between the real and imaginary parts of λ

Next steps

- Short term:
 - » More reasonable events and SNRs
 - » Figuring out the consistent overestimation of λ
 - » Figuring out why $\lambda = 0$ has lower errors
 - » changing both $\text{Re}(\lambda)$ and $\text{Im}(\lambda)$
- Long term:
 - » How many events to constrain λ by a certain amount
 - » Letting all 15 (17) parameters vary
 - » Running on the actual data

Acknowledgments

- NSF / SURF
- LIGO Caltech
- Rory Smith
- Rico
- Osase
- Alan