Inferring the Astrophysical Population of Black Holes Binaries from their Mass Distribution

Osase Omoruyi[1]; Alan Weinstein, Mentor[2]
[1]Yale University, [2]California Institute of Technology

Abstract

LIGO's gravitational-wave detections have not only proved the existence of black hole binaries but also confirmed the presence of stellar mass black holes larger than 20 solar masses. Our project aims to develop a system that will allow us to study the mass distribution of these binaries throughout space. Currently, LIGO has made 4 detections of binary black hole (BBH) mergers. Within the next 10 years, LIGO expects this number to rise significantly. With these future detections in mind, our project utilizes simulated data to generate a large population of BBH systems. From our general astrophysical knowledge about black holes and nature, we expect the underlying population to fall like a power-law in the total mass of the BBH system, $M^{-\alpha}$, in which α is the power-law index. Using the large sample of events our simulations provide, we seek to constrain the value of the power-law index more precisely and accurately. Successfully recovering the simulated value of α, in turn, will provide confidence in our ability to recover the actual value of α when LIGO detects enough events to form a significant population of BBH systems. Furthermore, understanding the mass distribution of BBH will allow us to make inferences about how black hole binaries have formed and evolved over time.

Introduction

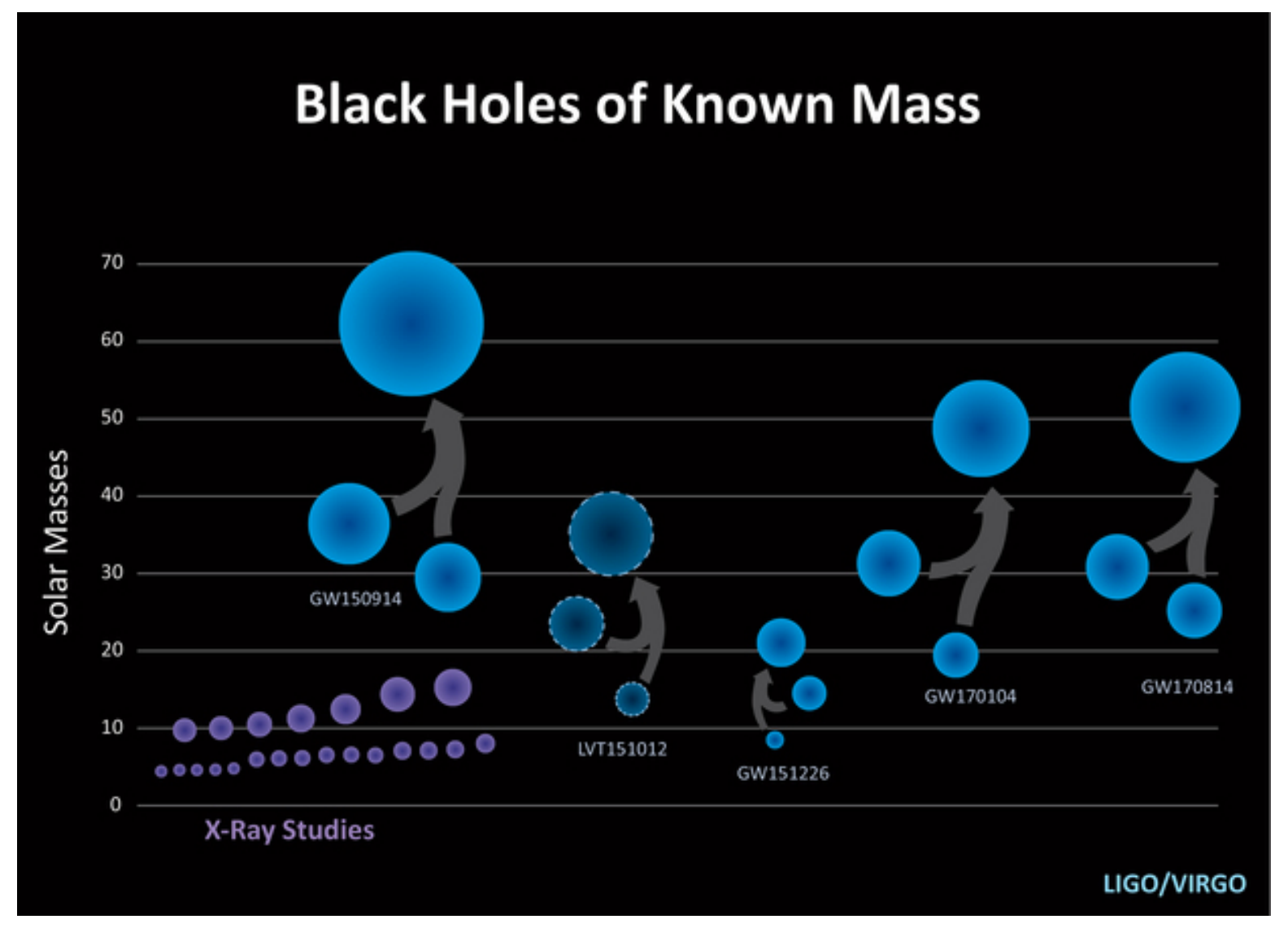

Figure 1. Known Population of Binary Black Holes (BBH). [1]

LIGO's detections of Binary Black Holes (BBH) have not only increased number of known black holes, but also extended the *mass distribution* range to nearing $100M_{sun}$. In the future, as the network of ground-based GW detectors grows, LIGO expects to have increased detector sensitivity and more observing runs.[2] As a result of this expansion and overall improvement, we expect there to be on the order of tens, hundreds, and eventually thousands of gravitational wave events, which will allow us to draw significant conclusions about the mass distribution. Constraining the mass distribution will allow astronomers to understand how BBH have formed and evolved over time. Astronomers have two main theories for how BBH form: dynamical capture or isolated binary evolution. The mass distribution will help astronomers understand which scenario dominates BBH evolution.

Figure 2. Dynamical capture.[3]

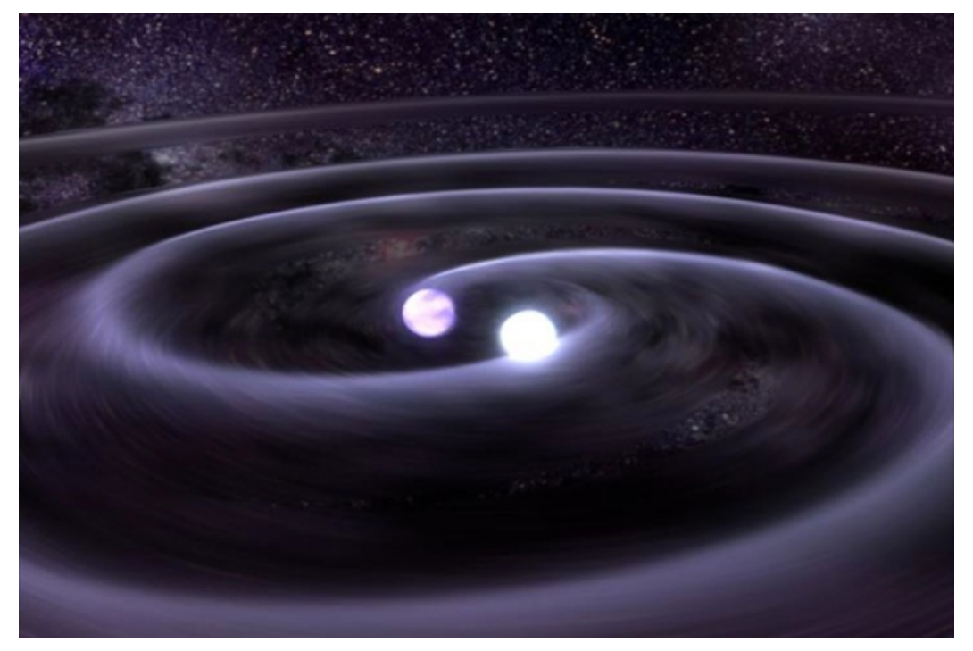

Figure 3. Isolated Binary Evolution.[4]

Method

1. Create 100,000 Simulations of BBH mergers from list of parameters that describe both the black holes and their binary system.
   a. Use Ian Salpeter's Initial Mass Function (IMF) for massive stars to postulate the distribution of BBH masses as a power law in the total mass of the black hole binary. (see Equations 1 and 2, Figure 4)
2. Use simulated parameters to generate gravitational waveforms. (see Figure 5)
3. To obtain the number of simulated events LIGO can observe, consider an event observable if and only if the event has a Signal-Noise Ratio (SNR) > 8 in BOTH Hanford and Livingston detectors. (see Figure 6)
   a. To increase efficiency, simulate each event out to its horizon distance – the distance at which a perfectly oriented binary has the optimal SNR of 8. (see Figure 7)
4. To recover the rate density (mergers/yr/Mpc³), relate the rate to the number of observed number of events and their observed volume of detectability. (see Equation 3, Figures 8, 9)
5. Constrain the power-law index α using Bayesian Parameter Estimation, assuming Poisson errors on the number of observed events. (see Figure 10)

Table 1. List of parameters describing the black holes and the binary system used to create simulations.

Parameters Describing the Binary		Parameters Describing the Black Holes	
Parameter	Distribution of BBH	Parameter	Distribution of BBH
Right Ascension (α)	Uniform	Total Mass (M)	Power Law
Declination (δ)	Uniform in cos δ	Symmetric Mass Ratio [$\eta(m_1,m_2)$]	Gaussian
Luminosity Distance (d_L)	Volumetric	Spin Magnitude (a_1,a_2)	Gaussian
Orbital inclination (ι)	Uniform in cos ι	Spin Azimuthal Angle (ϕ_1, ϕ_2)	Uniform in [0,2π]
Time of Coalescence (t_c)	Uniform	Spin Polar Angle (μ_1, μ_2)	Uniform in cos μ
Phase of Coalescence (φ_c)	Uniform in [0,2π]		
Polarization angle (Ψ)	Uniform in [0, π/4]		

$$N = \int_{M_l}^{M^u} \xi_0 [M / M_\oplus]^\alpha \, dM$$

$$\alpha = -2.35, \ \xi_0 = \text{constant}$$

Equation 1. Salpeter IMF.

$$R = cM_{total}^{\alpha}$$

Equation 2. Rate equation.

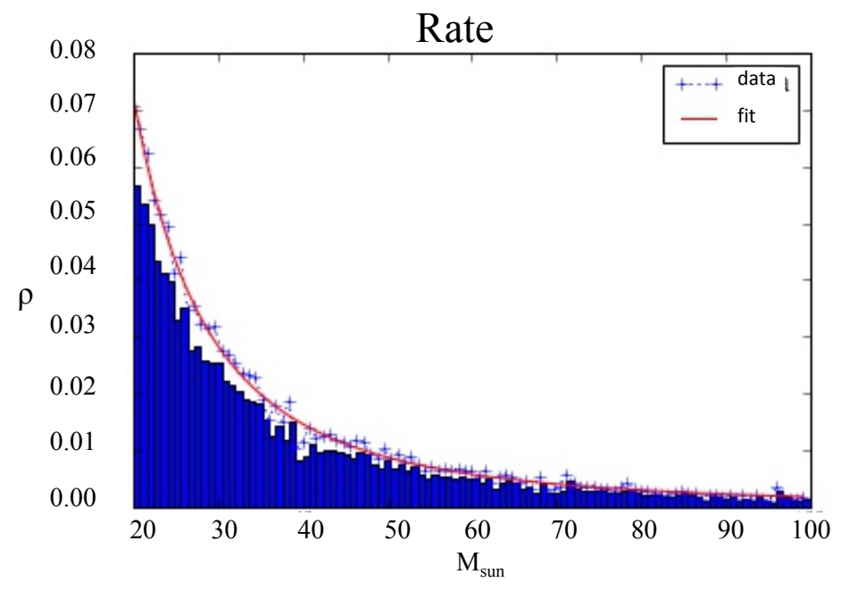

Figure 4. Rate = cM^α
$\alpha = -2.30 \pm 0.03$, $c = 71.79 \pm 5.58$

$$N_{universe} = RV_{universe}T$$
$$N_{observed} = RV_{observed}T$$
$$V_{universe} = \frac{4}{3}\pi r^3_{universe}$$
$$V_{observed} = f_{SNR>8}V_{universe}$$
$$\boxed{N_{observed} = RV_{observed}T}$$

Equation 3. Relate Observed Number of Events to Natural Rate Density.

Discussion

- We recovered the simulated natural rate density within reasonable statistical error of the actual value we simulated.
  - If the mass distribution of the future events LIGO detects is distributed in the total mass, we know how to recover it! Our simulations are also flexible enough to recover the rate if the power law is governed by more massive black hole in the binary.
- A more thorough version of this project would entail calculating the rate density for all kinds of models: i.e. α is another value other than 2.35, or the rate density is a broken power law with several different power indices.

Results

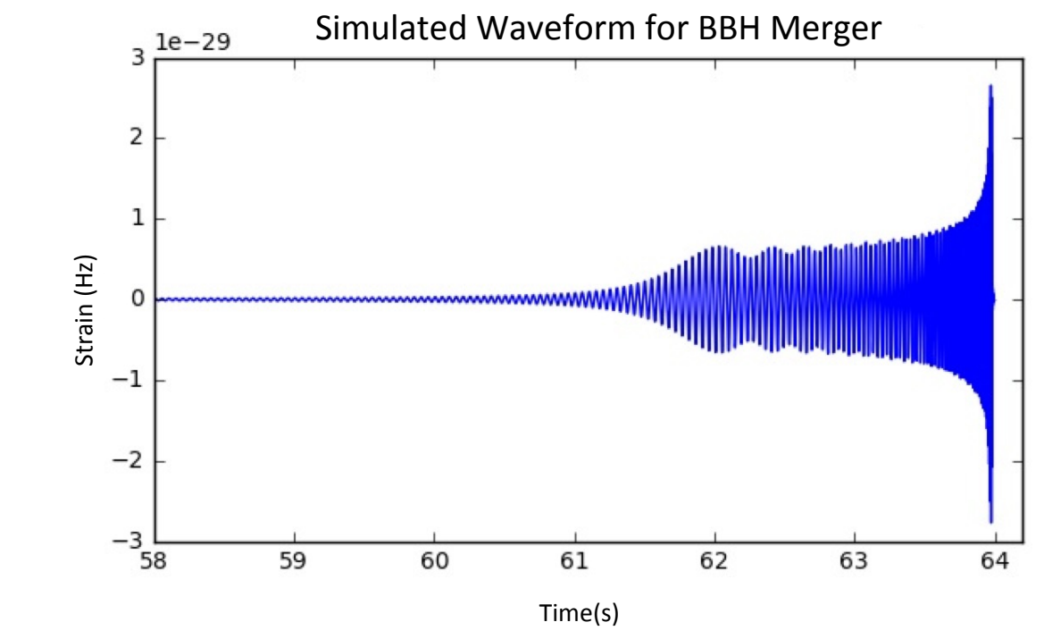

Figure 5. Sample of Simulated gravitational waveform for BBH merger.

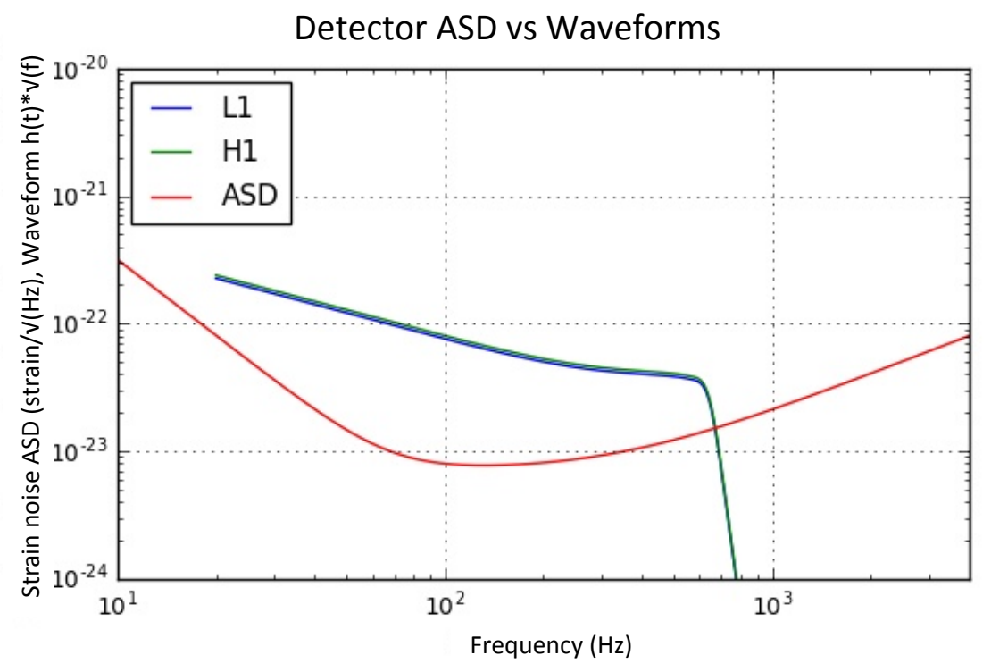

Figure 6. ASD curve of Hanford and Livingston Detectors vs the Frequency of the Sample Simulated Gravitational Waveform (begins at 20 Hz). Waveforms with frequencies above the ASD curve are detectable.

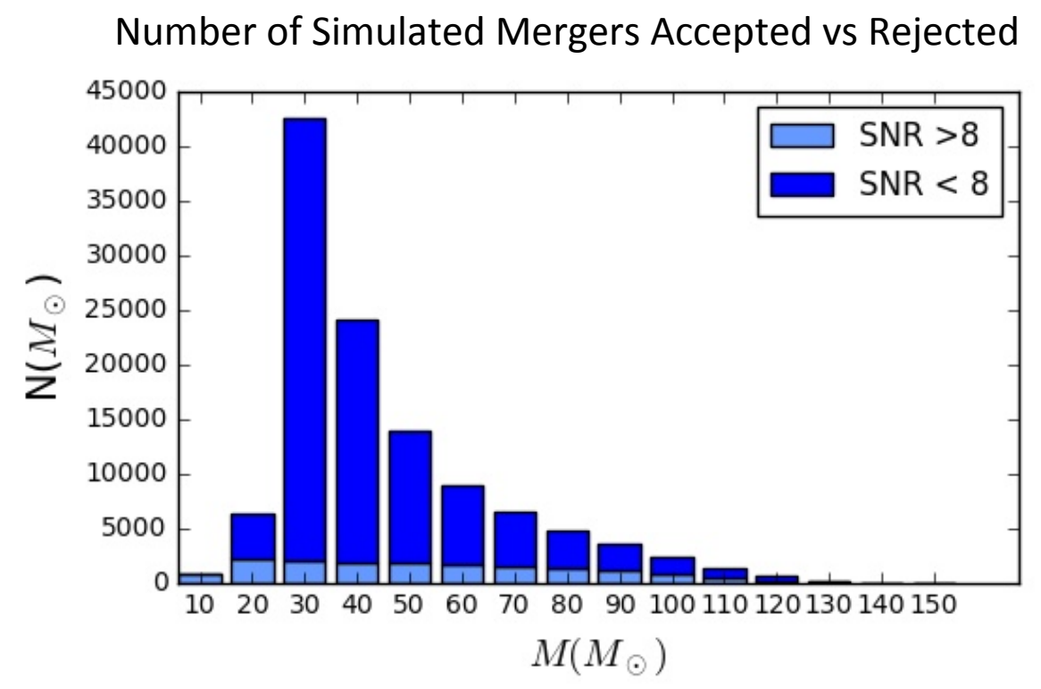

Figure 7. Observed events with SNR > 8 in both Hanford and Livingston detectors

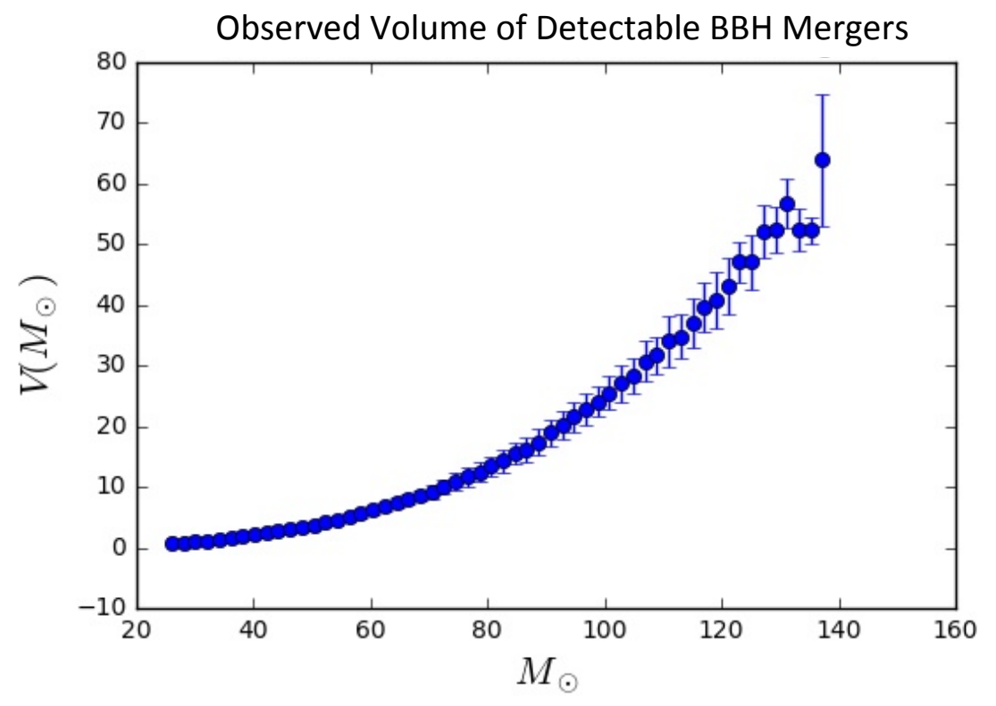

Figure 8. Observed volume of detectable events

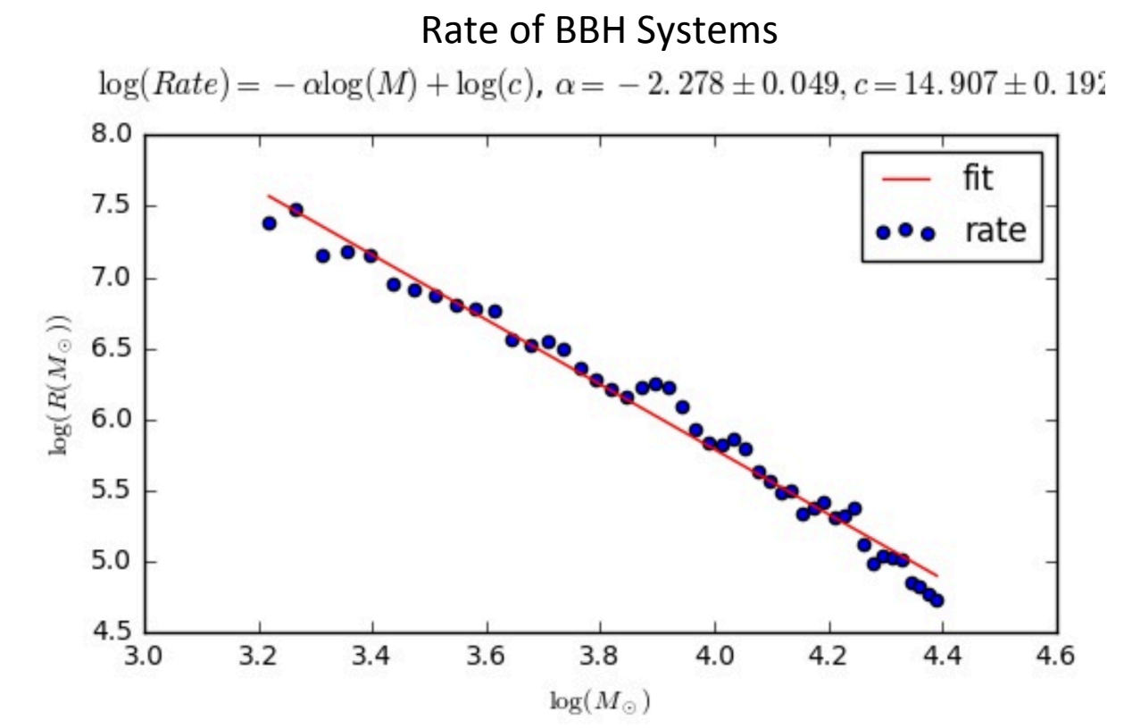

Figure 9. Natural Log of Rate of BBH Mergers (mergers/yr/Mpc³) that meet SNR requirement fit to a line y = αx + b. The slope, α, is -2.278 ± 0.049. Poisson Errors not shown.

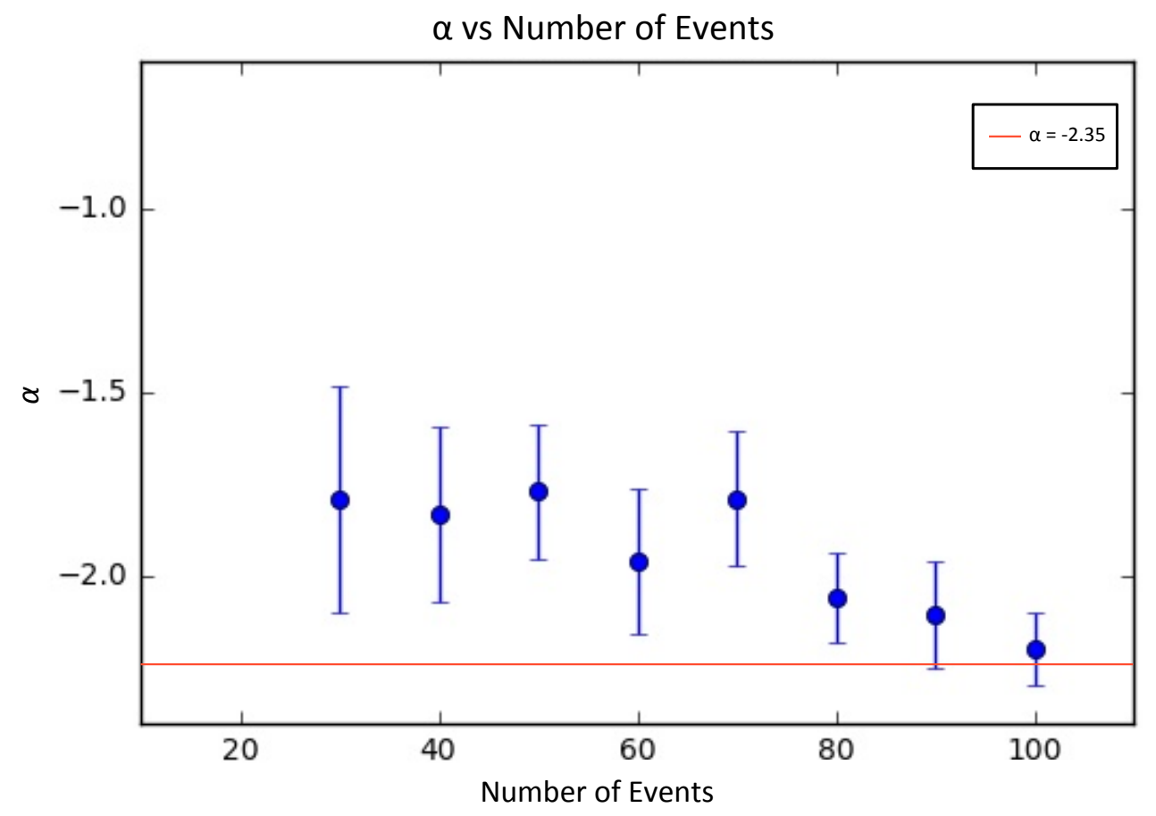

Figure 11. α vs the number of events. The error bars on α decrease as the number of events increases. We will be able to get a close estimate of the rate with 100 events, rather than 100,000.

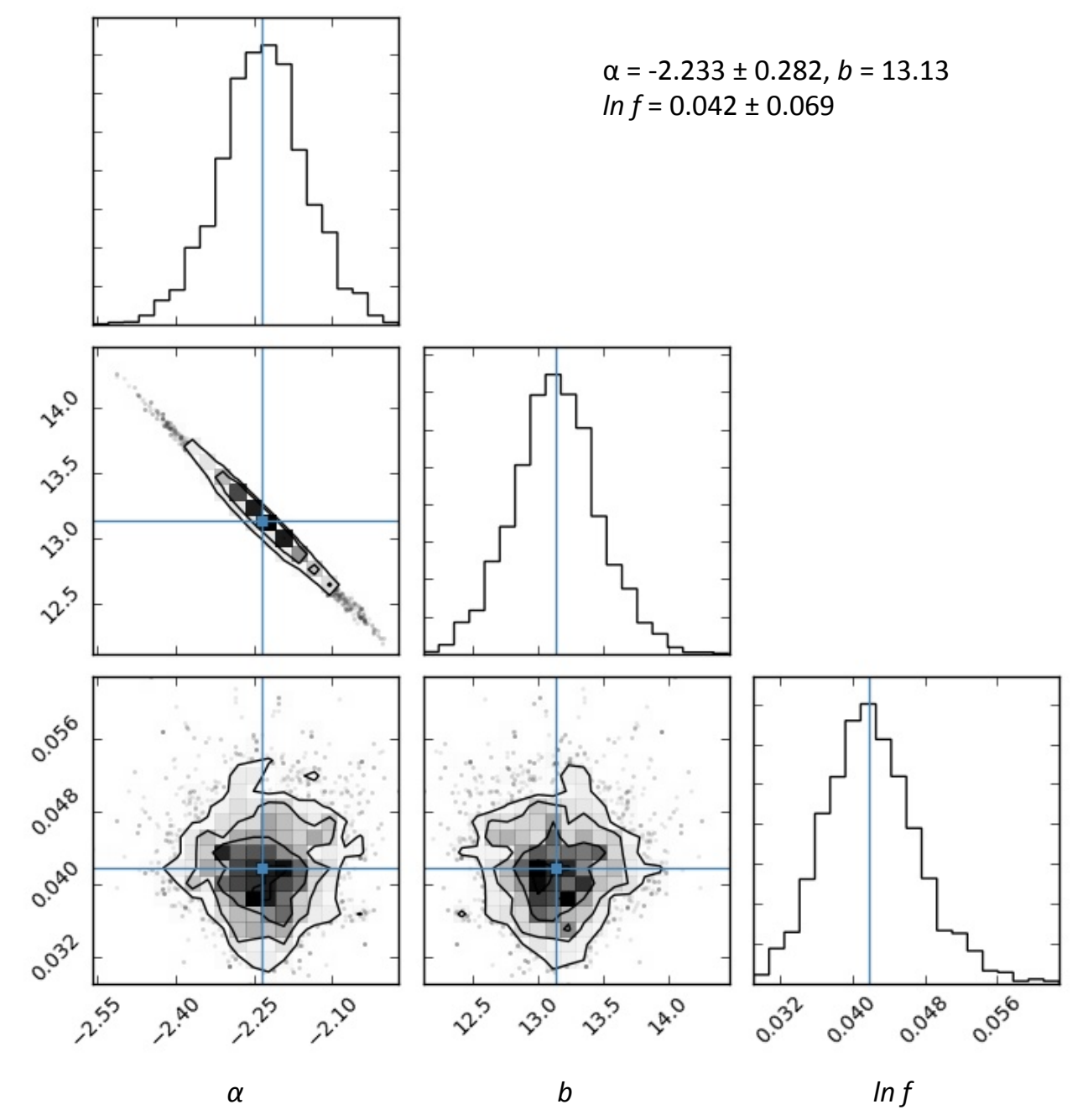

Figure 10. Corner plot showing the Posterior probability Distribution of α, *b* and *ln f*. α is -2.233 ± 0.282, which is within error of our simulated value of α, -2.35.

Summary

- The mass distribution of BBH can be a very useful tool in understanding how BBH formed and evolved over time.
- Within the next 10-20 years, LIGO expects to detect enough events to begin showing a conclusive mass distribution
- Using simulated events, we can determine methods for retrieving the actual rate density of BBH from observed events.
  - Our method of modeling the rate density in the total mass of the BBH system works!
- More work can and is being done to test multiple models of the mass distribution of BBH.

Contact

Osase Omoruyi
Yale University
Email: osase.omoruyi@gmail.com
Website: afroastrophysics.wordpress.com
Phone: 361-249-2192

References

1. Image credit: LIGO/Caltech/Sonoma State (Aurore Simonnet)
2. S. Vitale and M. Evans. Parameter estimation for binary black holes with networks of third-generation gravitational-wave detectors. Phys. Rev. Lett. , 95(6):064052, March 2017.
3. Image credit: LIGO/Caltech/Sonoma State (Aurore Simonnet)
4. Image credit: NASA/ Tod Strohmayer (GSFC)/Dana Berry (Chandra X-Ray Observatory)

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