Inferring the Astrophysical Population of Black Hole Binaries from their Mass Distribution

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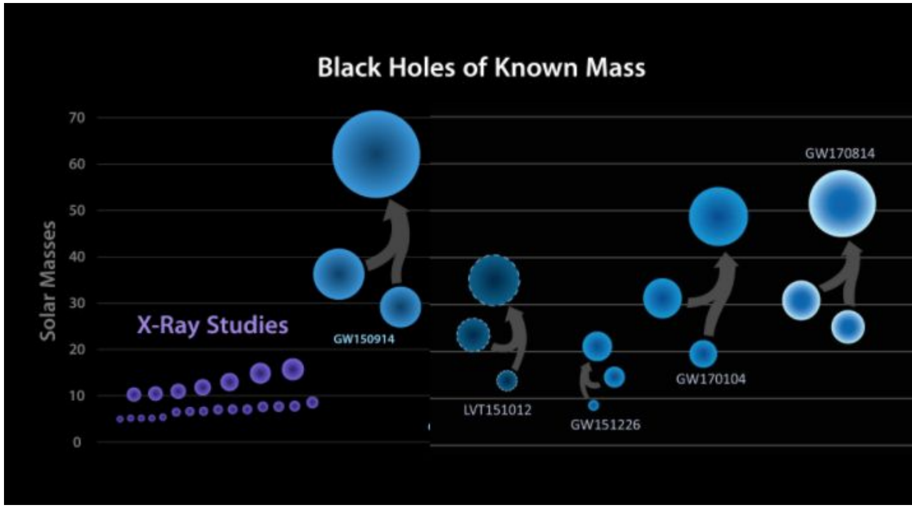
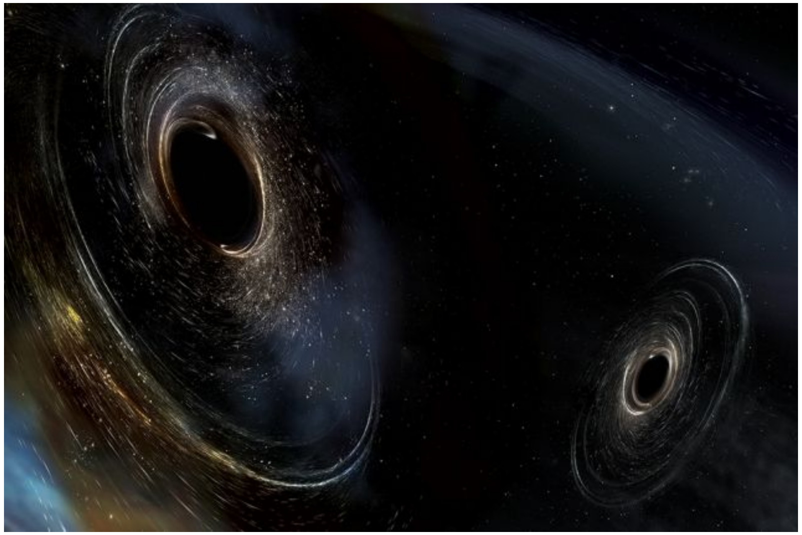
Motivation: A Population of Black Hole Binaries Exists!

- As astronomers we believe gravitational waves are great, but the black holes they've revealed are even better.
- LIGO's detections confirmed the presence of black holes larger than 20 solar masses, giving astronomers more information about the underlying population of binary black holes
  - We want to know the mass distribution
- The mass distribution can help us understand how binary black holes formed and evolved over time



Motivation: How are Black Hole Binaries formed?

Dynamical Capture



- Black hole captures another black hole
- Characteristic misaligned spins

vs.

Isolated Binary Evolution

- Formed from binary star system
- Each star must withstand being blown away by supernovae

Understanding the mass distribution of BBH may help us determine which scenario dominates the formation and evolution of BBH!

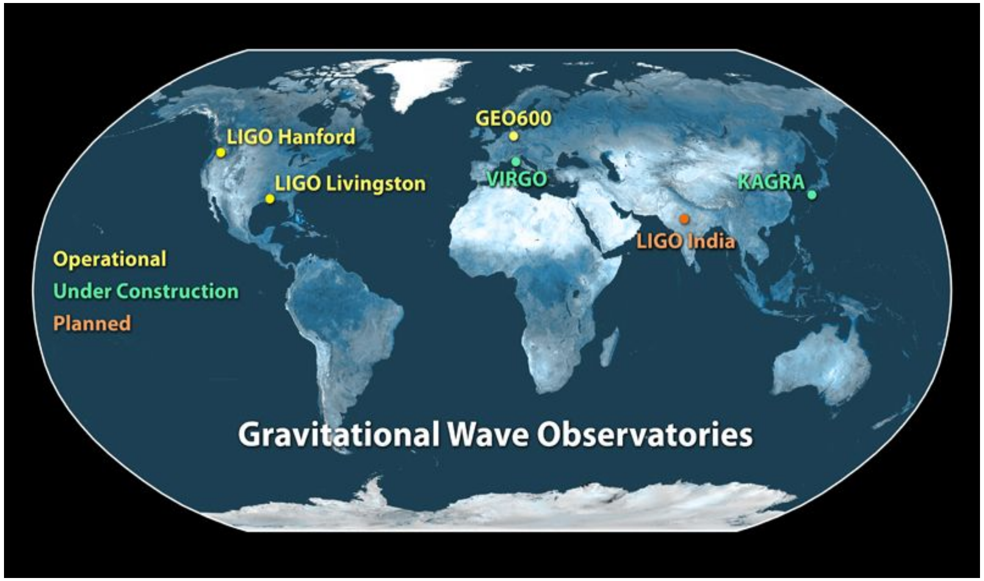
How can we figure out the actual mass distribution of BBH from the few events we have?

We can infer the rate of BBH mergers and the mass distribution from the events we have but we cannot make absolute conclusions.

Motivation: LIGO + 10-20 Years

More detectors + Increased Sensitivity + Extended Observing Time = More Events

More Events = More Information About the Population



We can model the future using simulations!

Method: Simulating Binary Black Hole Mergers

What makes a Black Hole Binary… a Black Hole Binary?

Parameters Describing the Binary

Parameters Describing the Black Holes within the Binary

Parameter	Symbol	BBH Distribution
Right Ascension	α	Uniform
Declination	δ	Uniform in cosδ
Luminosity Distance	d_L	Volumetric
Orbital Inclination	ι	Uniform in $\cos\iota$
Time of Coalescence	t_c	Uniform
Phase of Coalescence	φ_c	Uniform in [0,2]pi

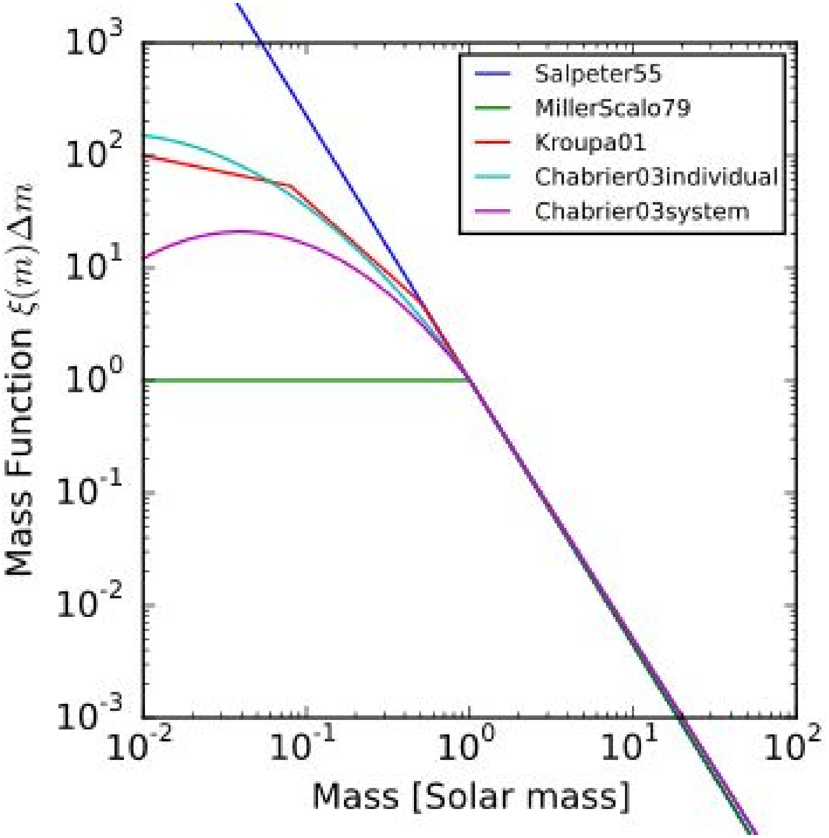
Parameter	Symbol	BBH Distribution
Total Mass	M	Power Law
Symmetric Mass Ratio	$\eta\ (m_1, m_2)$	Gaussian
Spin Magnitude	$a_1,\ a_2$	Gaussian
Spin Azimuthal Angle	$\phi_{a1},\ \phi_{a2}$	Uniform in $[0, 2\pi]$
Spin Polar Angle	$\mu_{a1},\ \mu_{a2}$	Uniform in $\cos\mu$

Method: Simulating the Mass Distribution

- Become a reasonable God—model the rate density using the Initial Mass Function
- The Initial Mass Function describes the mass distribution for an initial population of stars
- We can use the IMF to make a plausible simulation of the mass distribution of BBH because black holes are formed from stars
- Use Edwin Salpeter's IMF

$$\xi(M)\Delta M = \xi_0 (M/M_\odot)^{-2.35} (\Delta/M_\odot)$$

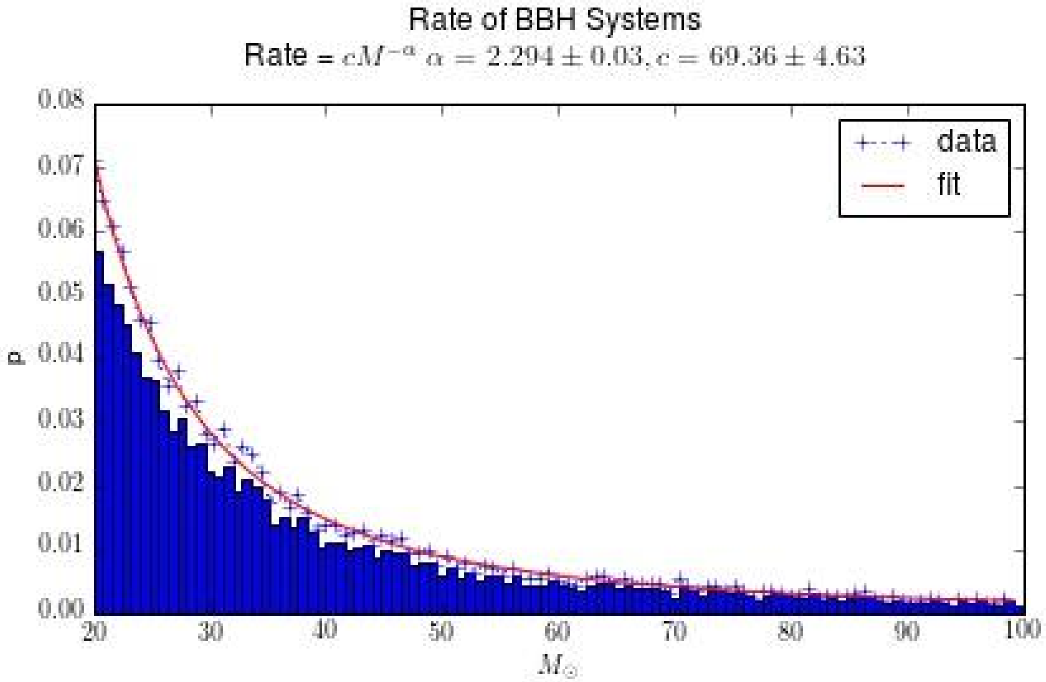
$$N = \int_{M_l}^{M_u} \xi_0 \left[(M/M_{sun})^{-2.35} \right] dM$$



Method: Simulating the Mass Distribution

- Using Salpeter's function, we postulate the rate of the BBH is distributed as a power law in the total mass of the black hole binary

$$R = cM^{-\alpha}$$



Goal: Recover this rate given our simulated observations of BBH mergers

Method: Observing Simulated Events

- Using simulated parameters, created thousands of simulated gravitational waveforms

- However, not all events are detectable

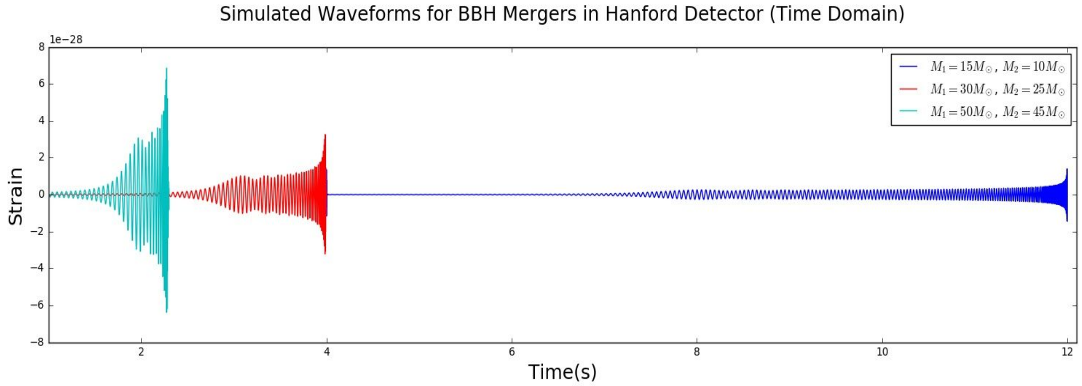


Figure 1. Simulated Waveforms for BBH mergers of 25, 55 and 95 M_{sun} in total mass. Massive systems have shorter wavelengths.

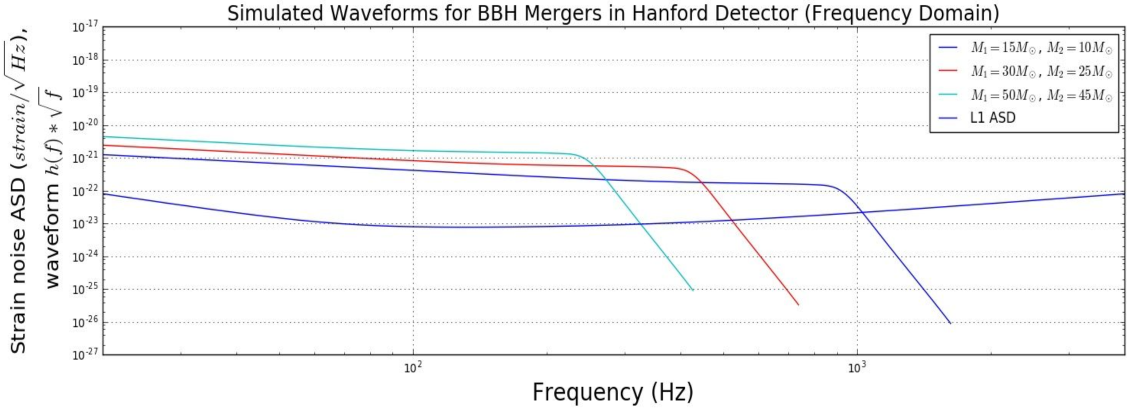
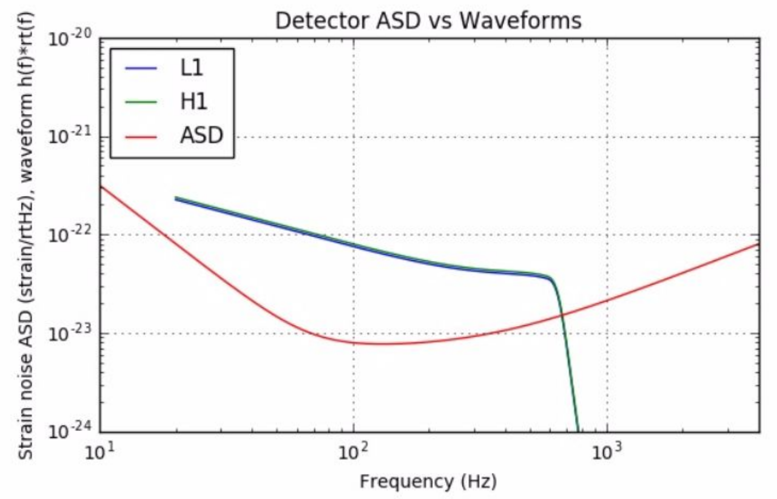
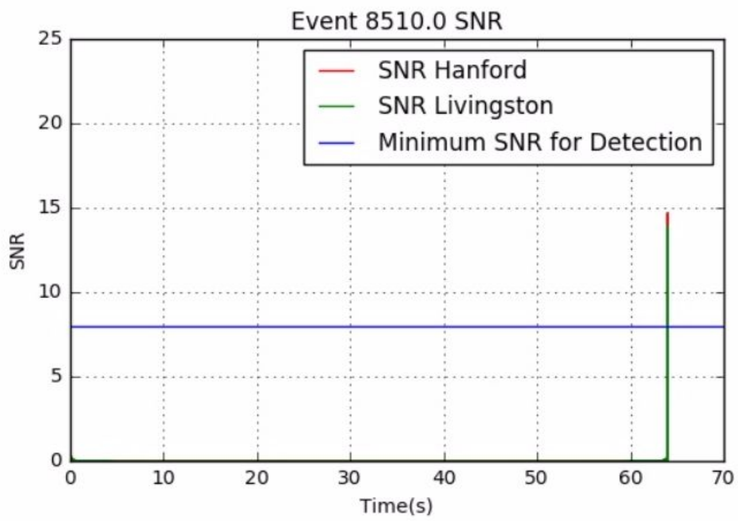


Figure 2. Observed Frequencies of BBH Events vs LIGO's noise curve (ASD)

Method: Observing Simulated Events

- We consider an event to be observable if SNR > 8 in **BOTH** detectors
- To increase efficiency, we simulate each event out to its **horizon distance**
  - The horizon distance is the distance at which a perfectly oriented binary has the optimal SNR of 8





Method: Determining the Mass Distribution from Observations

From the IMF, we know that the Rate Density is dominated by the power-law index α. By constraining α, we will constrain the mass distribution.

$$R(m_{total}) = Cm_{total}^{-\alpha}$$

$$R_{true} = \int dm_{total} R(m_{total}) = c \int_{m_{min}}^{m_{max}} m_{total}^{-\alpha} dm_{total}$$

$$\text{Normalization factor: } I_\alpha = \int_{m_{min}}^{m_{max}} m_{total}^{-\alpha} dm_{total}$$

$$R(m_{total}) = \frac{R}{I_\alpha} m_{total}^{-\alpha}, \frac{R}{I_\alpha} = c$$

Rate Parameters: c and α

Relate total number of events (N) to natural rate density (R)

$$N_u(m_{total}) = R(m_{total}) V_u T$$

$$N_o(m_{total}) = R(m_{total}) V_o(m_{total}) T$$

$$V_u = \frac{4\pi d_u^3}{3}, d_u = 10Gpc$$

$$V_o = f_{SNR>8} * V_u$$

Corrected observed volume from the increased fraction of observable events ($f_{SNR>8}$) due to generating events out to their horizon distance

Relates **observed** number of events (N) to natural rate density (R)

$$N_o(m_{total}) = R(m_{total}) V_o(m_{total}) T$$

Method: Constraining the Power-law Index α using Bayesian Parameter Estimation

Recall: $R(m_{total}) = \frac{R}{I_\alpha} m_{total}^{-\alpha}$

Goal: Constrain α and R

- Taking the log of the rate returns a straight line with formula:

$$y = mx + b$$

$$\log(Rate) = -\alpha \log(M) + \log(c)$$

with f as the error on the $\log(Rate)$.

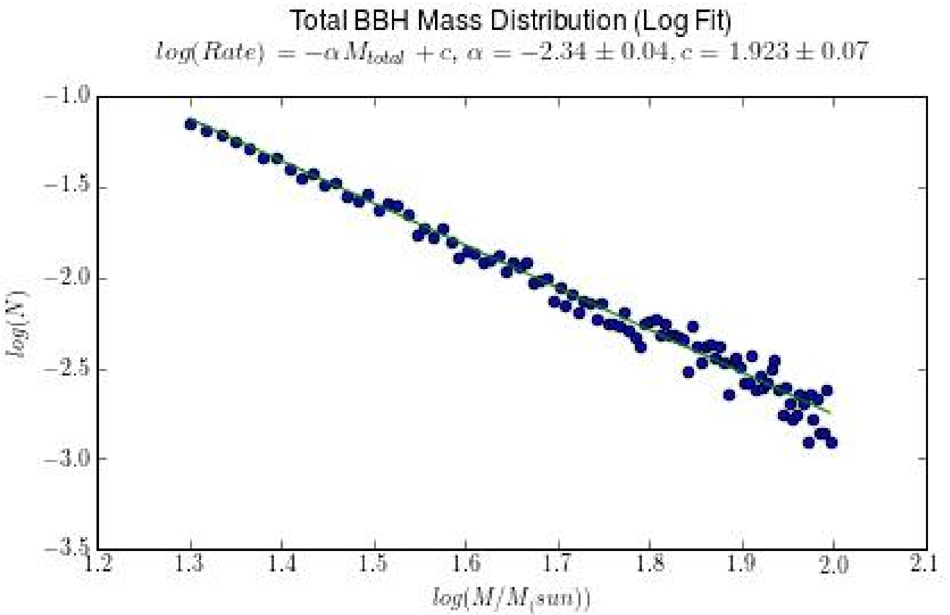


Figure 3. log(Rate)

Method: Constraining the Power-law Index α using Bayesian Parameter Estimation

Bayes' Theorem:

Posterior probability

Likelihood

Prior probability

$$p(A|B) = \frac{p(B|A)\,p(A)}{p(B)}$$

Our Function:

$$y = mx + b$$

Log Likelihood Function:

$$\ln p(y|x, \sigma, m, b, f) = -\frac{1}{2}\sum_n \left[\frac{(y_n - mx_n - b)^2}{s_n^2} + ln(2\pi s_n^2)\right]$$

where

$$s_n^2 = \sigma_n^2 + f^2(mx_n + b)^2$$

Uninformative Priors:

$$p(m) = \begin{cases} 1/5, if -5 < m < 0 \\ 0, otherwise \end{cases}$$

$$p(b) = \begin{cases} 1/20, if 0 < b < 20 \\ 0, otherwise \end{cases}$$

$$p(lnf) = \begin{cases} 1/10, if -5 < lnf < 5 \\ 0, otherwise \end{cases}$$

Results: Simulated Observations

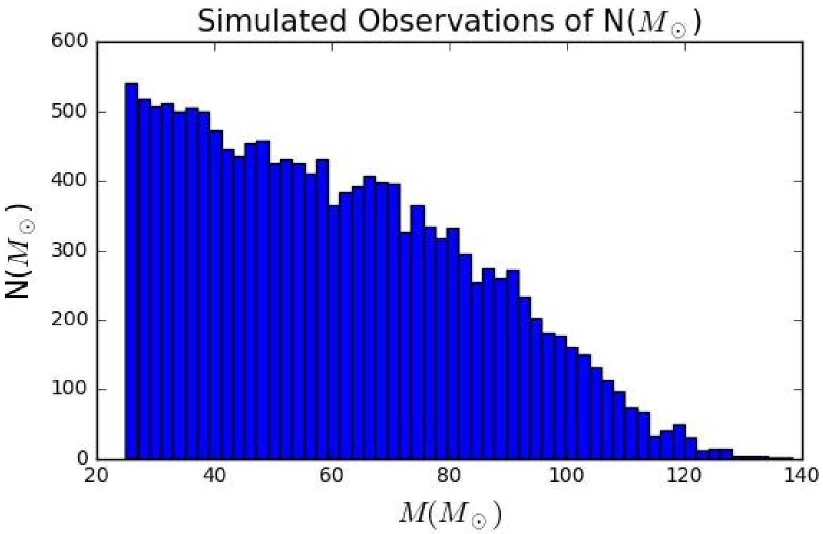


Figure 4. Observed Events with SNR > 8

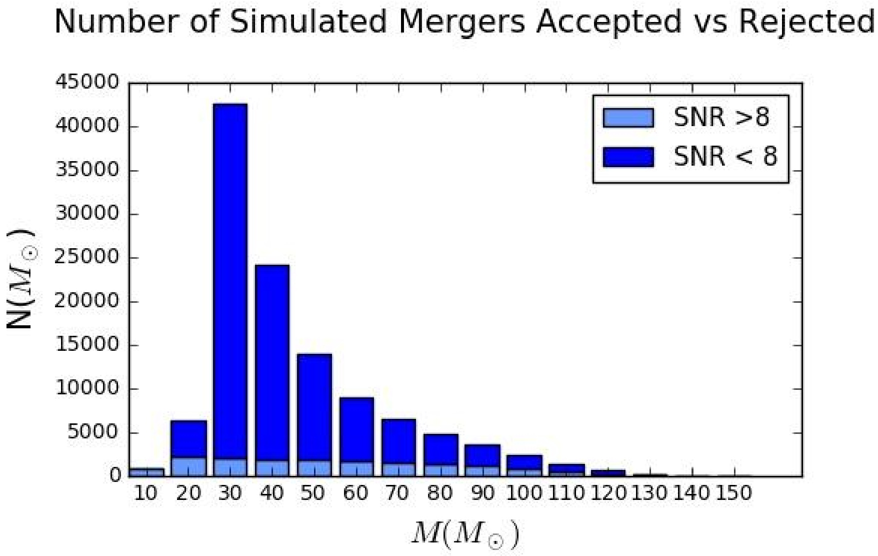
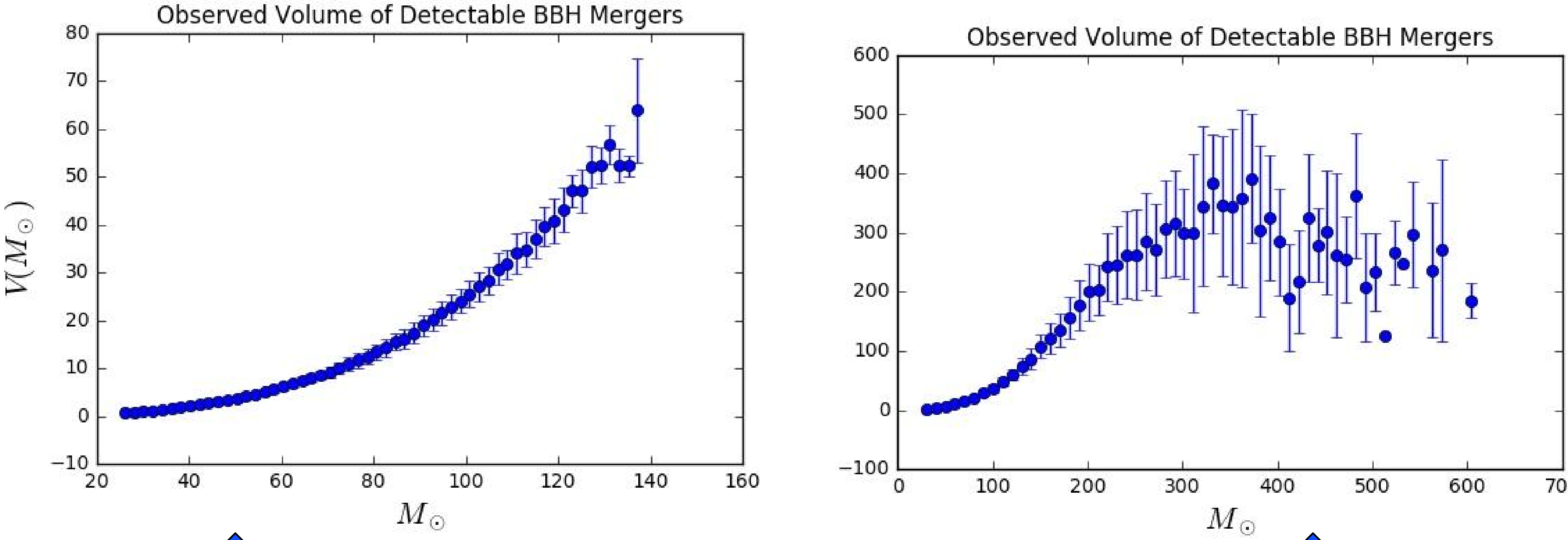


Figure 5. Events with SNR > 8, distance < horizon distance, distance > horizon distance

20% of simulated events were considered observable when events were capped at the horizon distance.

Results: Observed V(M)


Observed Volume of Detectable BBH Mergers

Observed Volume of Detectable BBH Mergers

Figure 6 and 7. Volume as a function of Mass

The volume of detectability increases as the mass increases

The volume decreases for extremely high mass systems because high mass systems emit frequencies outside of LIGO’s detection band

Results: Observed R(M)

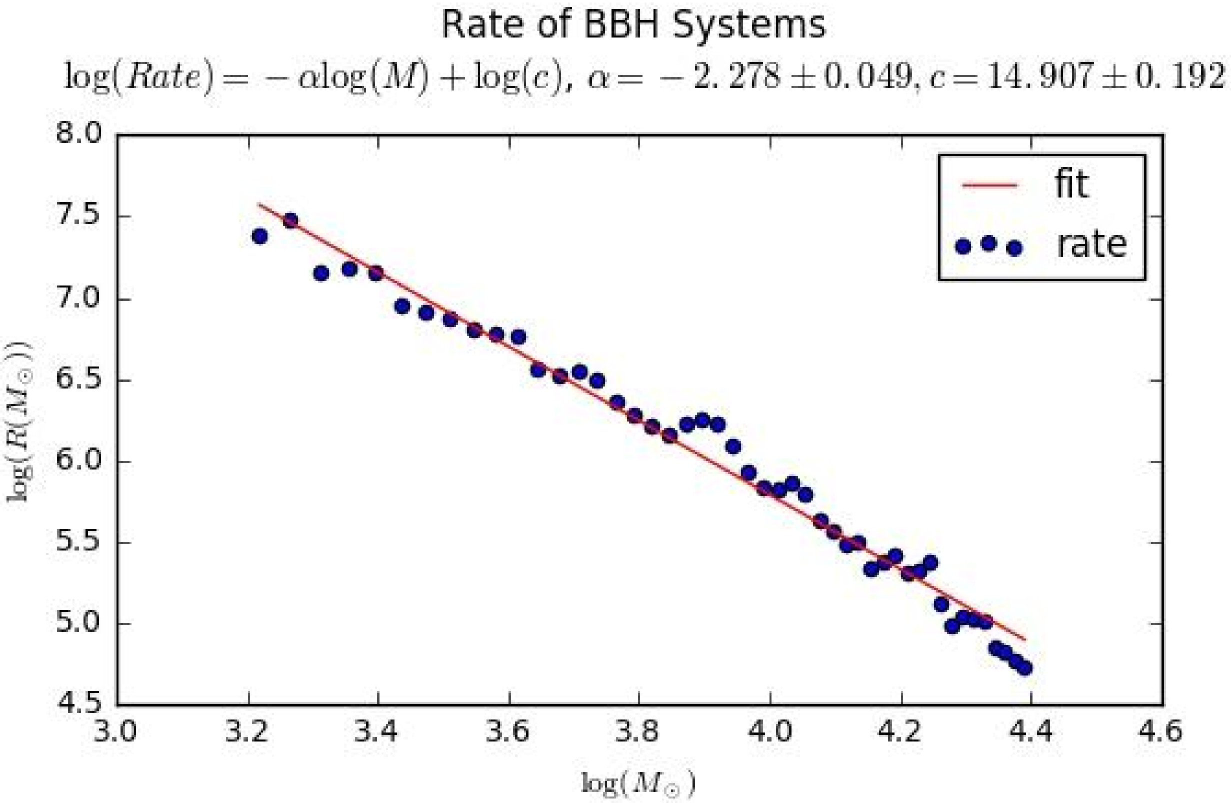


Figure 8. Observed Events with SNR > 8

Results: Bayesian Parameter Estimation of R(M)

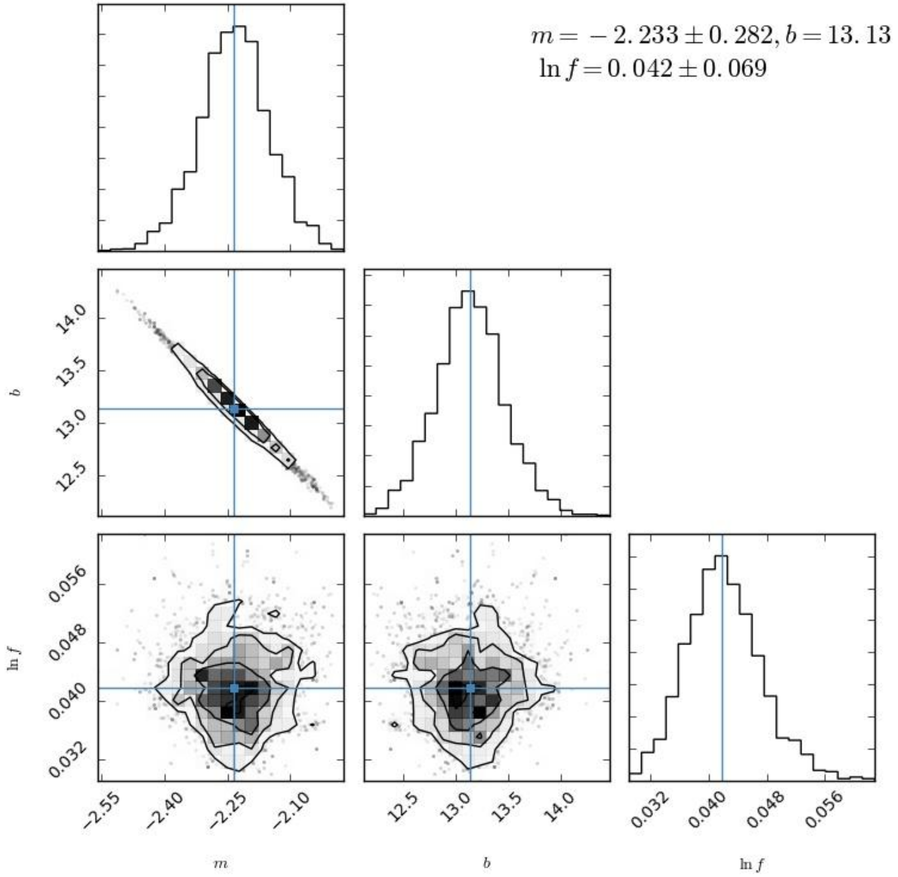

$m = -2.233 \pm 0.282, b = 13.13$
$\ln f = 0.042 \pm 0.069$

Figure 9. Corner Plot of the Posterior Probability Distribution of α

Results: Retrieving α Using Realistic Number of Events

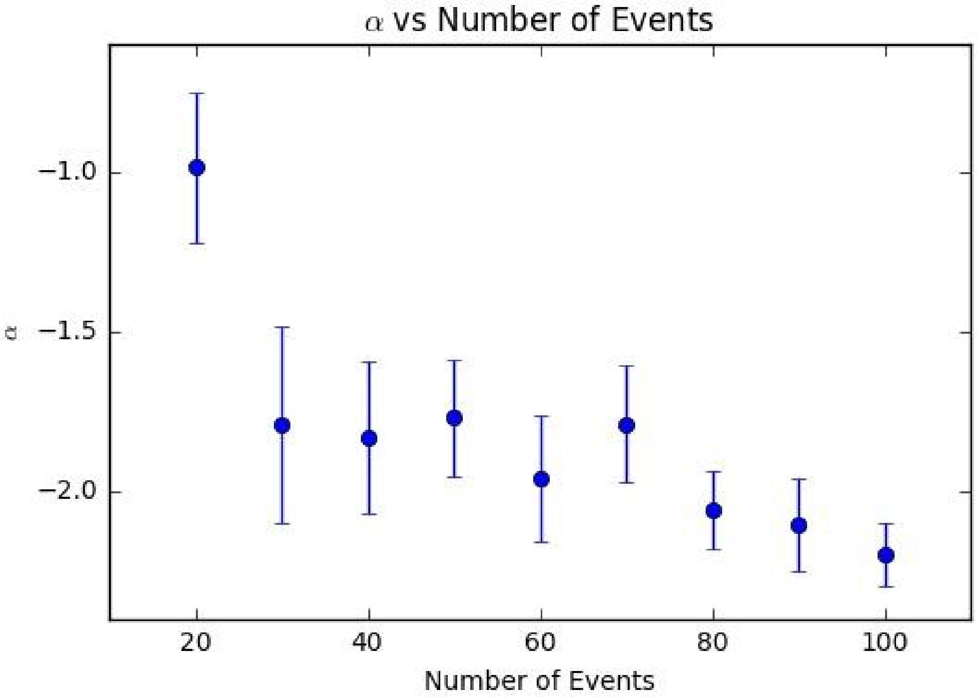


Figure 10. Number of events vs α

What Now?

- We were able to recover the simulated natural rate density within reasonable error of the actual value we simulated
  - Therefore, if the mass distribution of the future events LIGO detects is distributed in the total mass, we know how to recover it!

Something to keep in mind: A more thorough version of this project would entail calculating the rate density for all kinds of models: ie alpha is another value other than -2.35, or the rate density is a "broken" power law with several different power indexes

Summary

- The mass distribution of BBH can be a very useful tool in understanding how BBH formed and evolved over time
- Within the next 10-20 years, we expect LIGO to detect enough events to begin showing a conclusive mass distribution
- Using simulated events, we can determine methods for retrieving the actual rate density of BBH from observed events.
  - Our method of modeling the rate density in the total mass of the binary black hole system, works!
- More work can and is being done to test multiple models of the mass distribution of binary black holes.

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References

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THANK YOU!