# Inferring the Astrophysical Population of Black Hole Binaries from their Mass Distribution

Osase Omoruyi, Yale University

Alan J Weinstein, Caltech

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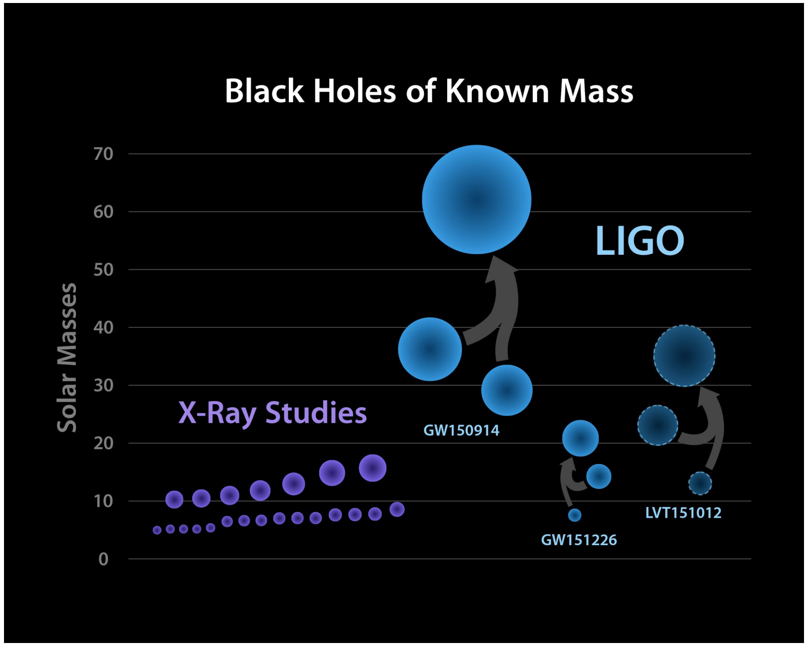
# Motivation: A Population of Black Hole Binaries Exists!

- As astronomers we believe gravitational waves are great, but the black holes they've revealed are even better.
- LIGO's detections confirmed the presence of black holes larger than 20 solar masses, proving that an underlying mass distribution of binary black holes exists
  - » We want to know this distribution!



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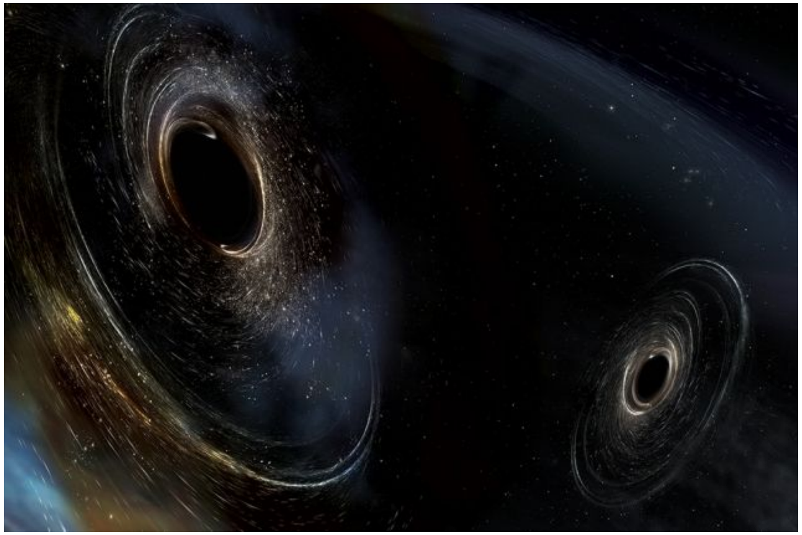
# Motivation: How are Black Hole Binaries formed?

**Dynamical Capture**



**vs.**

**Isolated Binary Evolution**

- BH captures another BH
- Characteristic misaligned spins

- Formed from binary star system
- Each star must withstand being blown away by supernovae

Understanding the mass distribution of BBH will allow us to determine which of these evolution theories is most plausible, or if we need an entirely new one!

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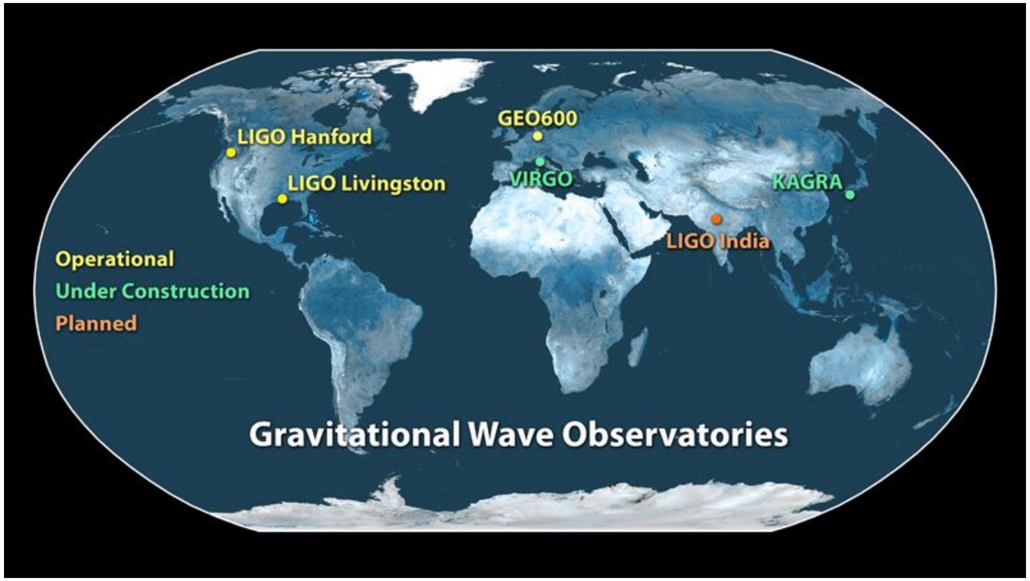
## How can we figure out the actual mass distribution of BBH from the few events we have?

We can infer the rate of BBH mergers and the mass distribution from the events we have but we cannot make absolute conclusions.

# Motivation: LIGO + 10/20 Years

More detectors + Increased Sensitivity + Extended Observing Time = More Events!

More Events = Stronger Population




We can model the future using simulations!

## Method: Simulating Binary Black Hole Mergers

What makes a Black Hole Binary… a Black Hole Binary?

Parameters Describing the Binary

Parameters Describing the Black Holes within the Binary

# Method: Simulating Binary Black Hole Mergers

## Parameters Describing the Binary

| Parameter | Symbol | BBH Distribution |
|---|---|---|
| Right Ascension | $\alpha$ | Uniform |
| Declination | $\delta$ | Unifrom in cos$\delta$ |
| Luminosity Distance | $d_L$ | Radial |
| Orbital Inclination | $\iota$ | Uniform |
| Time of Coalescence | $t_c$ | Uniform |
| Phase of Coalescence | $\varphi_c$ | Uniform |

# Method: Simulating Binary Black Hole Mergers

## Parameters Describing the Black Holes Within the Binary

| Parameter | Symbol | BBH Distribution |
|---|---|---|
| Total Mass | M | Exponential |
| Symmetric Mass Ratio | $\eta\ (m_1, m_2)$ | Exponential |
| Spin Magnitude | $a_1,\ a_2$ | Gaussian |
| Spin Azimuthal Angle | $\phi_{a1},\ \phi_{a2}$ | Uniform |
| Spin Polar Angle | $\mu_{a1},\ \mu_{a2}$ | Uniform |

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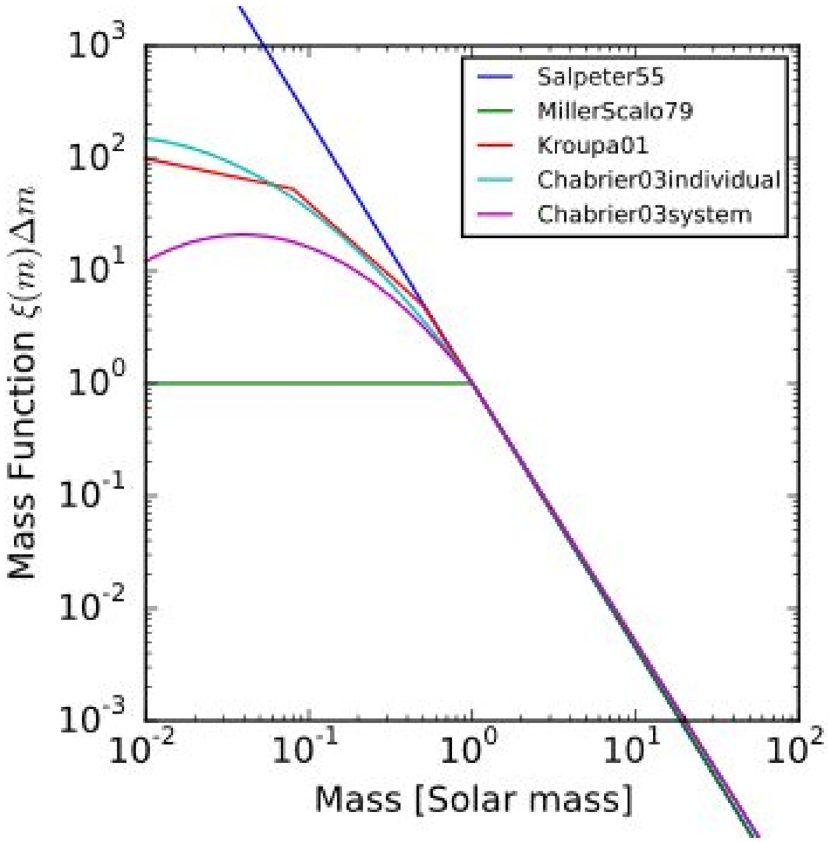
# Method: Simulating the Mass Distribution

- Become a reasonable God—model the rate density using the Initial Mass Function
- The Initial Mass Function describes the mass distribution for an initial population of stars
- We can use the IMF to make a plausible simulation of the mass distribution of BBH because black holes are formed from stars
- Use Edwin Sapleter's IMF

$$\xi(M)\Delta M = \xi_0 (M/M_\odot)^{-2.35}(\Delta/M_\odot)$$

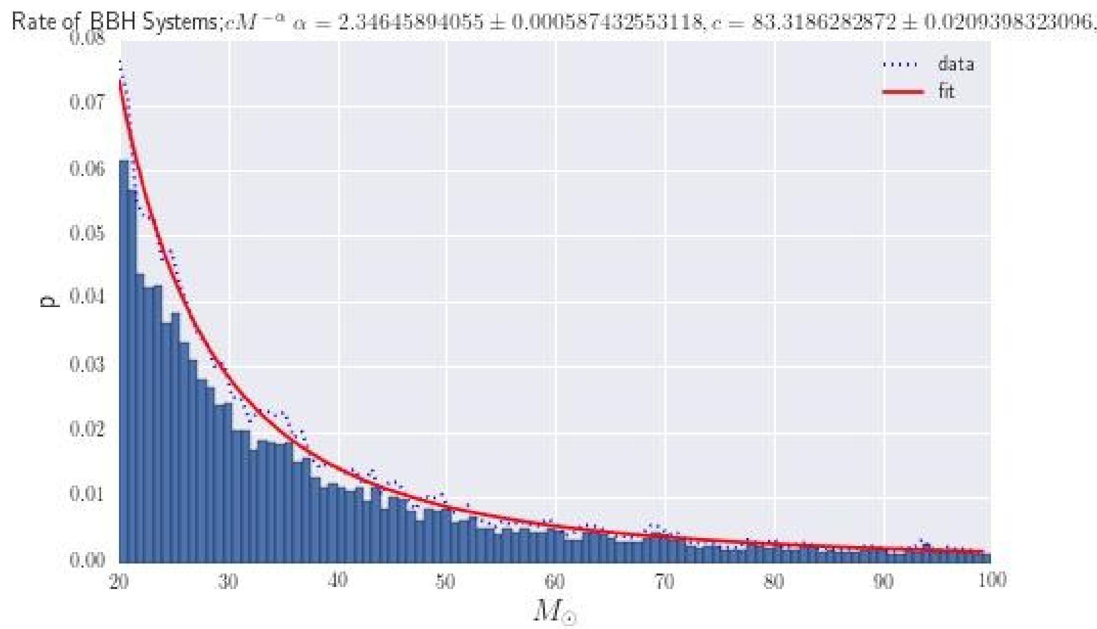
$$N = \int_{M_l}^{M_u} \xi_0 \left[(M/M_{sun})^{-2.35}\right] dM$$



# Method: Simulating the Mass Distribution

- Using Sapleter's function, we postulate the rate of the BBH is distributed as a power law in the total mass of the black hole binary

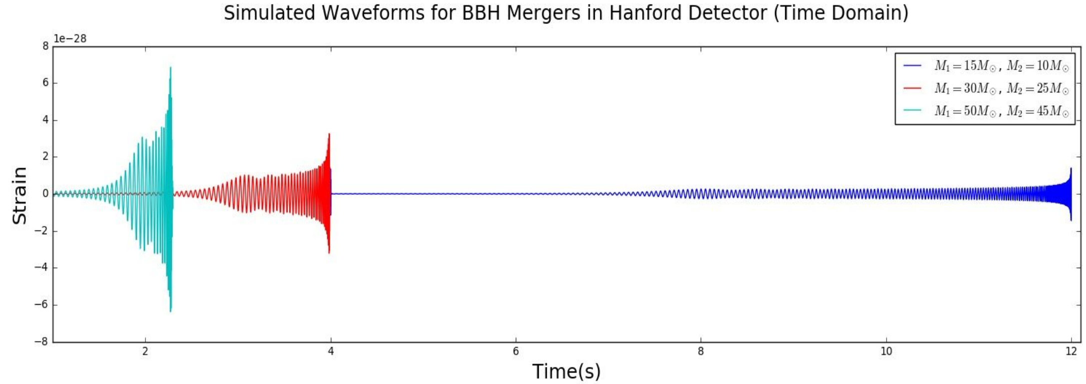
$$R = cM^{-\alpha}$$



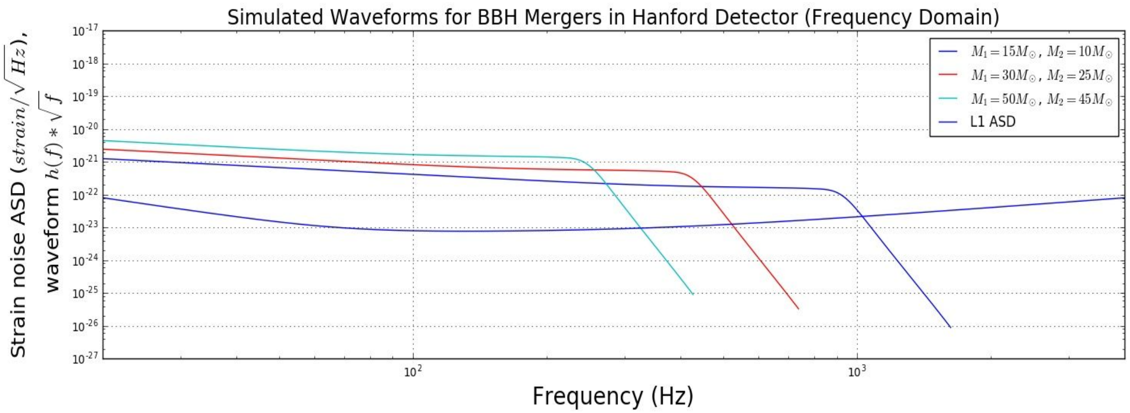

**Goal:** Recover this rate given our simulated observations of BBH mergers

# Method: Observing Simulated Events

- Using simulated parameters, create thousands of simulated gravitational waveforms

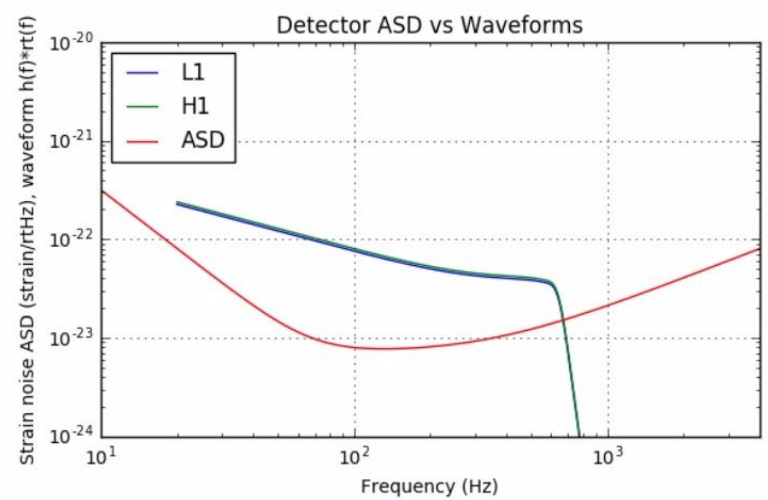


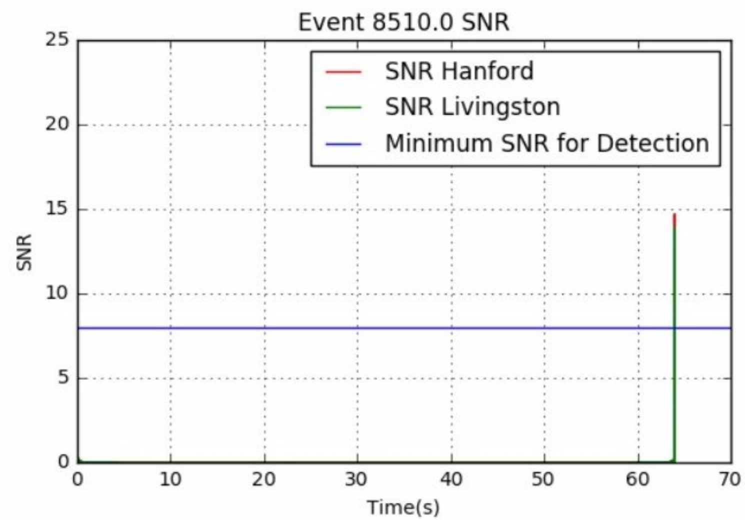

- However, not all events are detectable



# Method: Observing Simulated Events

- We consider an event to be observable if SNR > 8 in BOTH detectors
- To increase efficiency, we simulate each event out to its **horizon distance**
  - » The horizon distance is the distance at which a perfectly oriented binary has the optimal  SNR of 8



# Method: Determining the Mass Distribution from Observations

From the IMF, we know that the Rate Density is dominated by the power-law index α. By constraining α, we will know how the mass is distributed.

$$R(m_{total}) = Cm_{total}^{-\alpha}$$

$$R_{true} = \int dm_{total} R(m_{total}) = c \int_{m_{min}}^{m_{max}} m_{total}^{-\alpha} dm_{total}$$

$$\text{Normalization factor: } I_\alpha = \int_{m_{min}}^{m_{max}} m_{total}^{-\alpha} dm_{total}$$

$$R(m_{total}) = \frac{R}{I_\alpha} m_{total}^{-\alpha}, \frac{R}{I_\alpha} = c$$

$$N_u(m_{total}) = R(m_{total}) V_u T$$

$$N_o(m_{total}) = R(m_{total}) V_o(m_{total}) T$$

$$V_u = \frac{4\pi d_u^3}{3}, d_u = 10Gpc$$

$$V_o = f_{SNR>8} * V_u$$

$$N_o(m_{total}) = R(m_{total}) V_o(m_{total}) T$$

# Method: Constraining the Power-law Index α using Bayesian Parameter Estimation

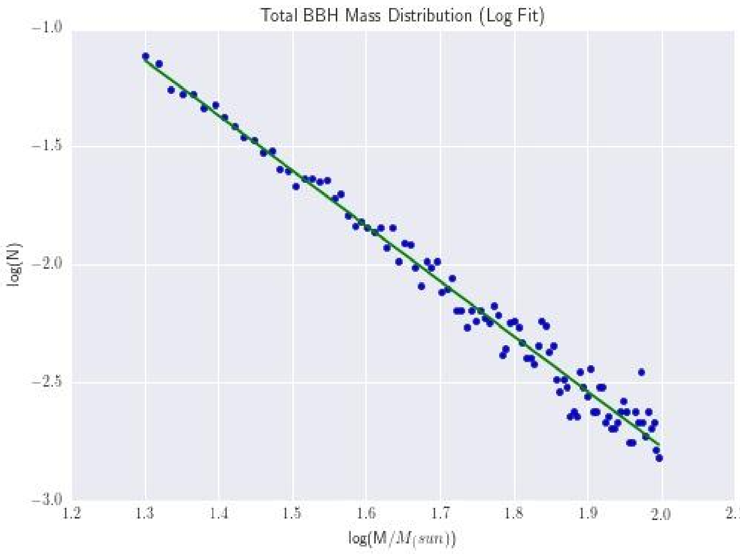
**Recall:** $R(m_{total}) = \frac{R}{I_\alpha} m_{total}^{-\alpha}$

**Goal:** Constrain α and R

- Taking the log of the rate returns a straight line with formula:

$$y = mx + b$$

$$\log(Rate) = -\alpha \log(M) + \log(c)$$



# Method: Constraining the Power-law Index $\alpha$ using Bayesian Parameter Estimation

**Bayes' Theorem:**

Posterior probability — Likelihood — Prior probability

$$p(A|B) = \frac{p(B|A)\,p(A)}{p(B)}$$

**Our Function:**

$$y = mx + b$$

**Log Likelihood Function:**

$$\ln p(y|x,\sigma,m,b,f) = -\frac{1}{2}\sum_n \left[\frac{(y_n - mx_n - b)^2}{s_n^2} + ln(2\pi s_n^2)\right]$$

where

$$s_n^2 = \sigma_n^2 + f^2(mx_n + b)^2$$

**Uninformative Priors:**

$$p(m) = \begin{cases} 1/5, if -5 < m < 0 \\ 0, otherwise \end{cases}$$

$$p(b) = \begin{cases} 1/20, if 0 < b < 20 \\ 0, otherwise \end{cases}$$

$$p(lnf) = \begin{cases} 1/10, if -5 < lnf < 5 \\ 0, otherwise \end{cases}$$

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# Results: Simulated Observations

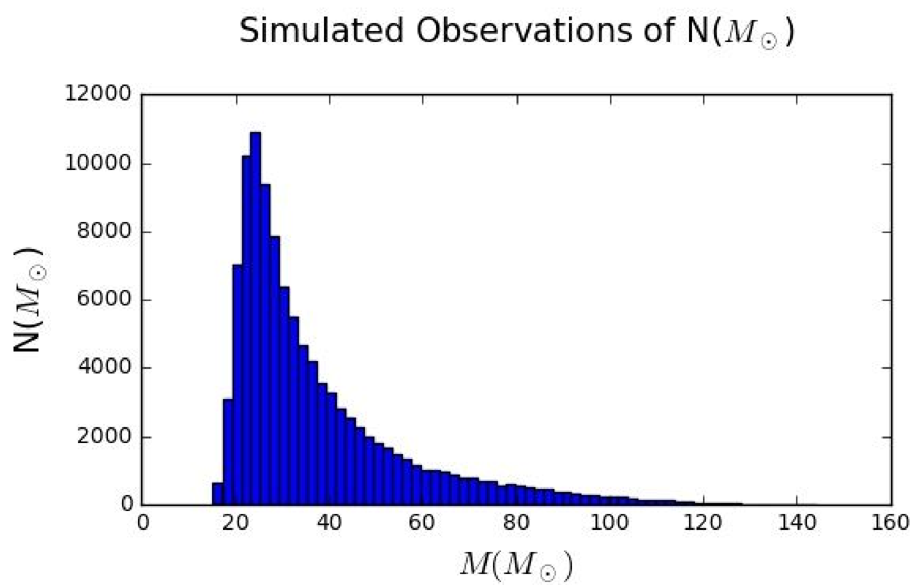



Figure 1. Observed Events with SNR > 8

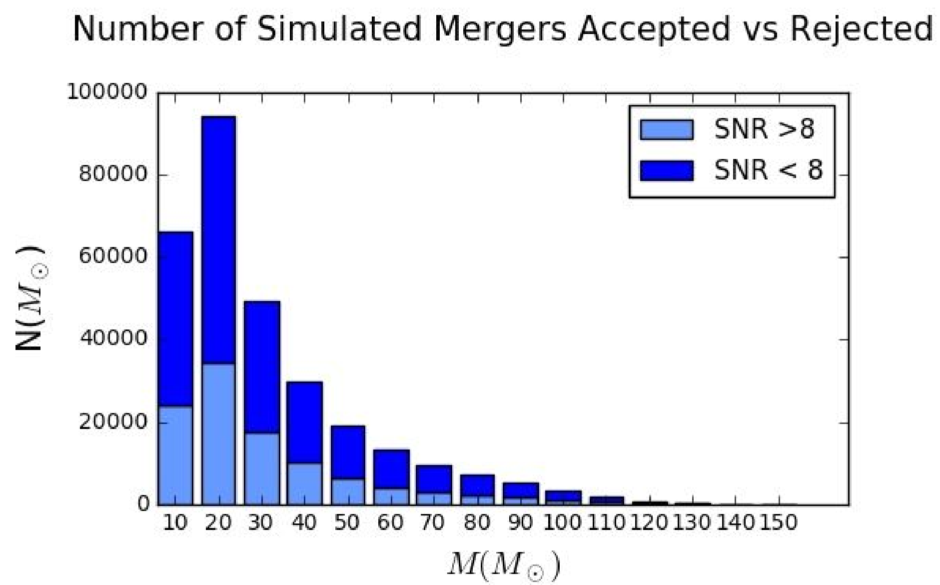
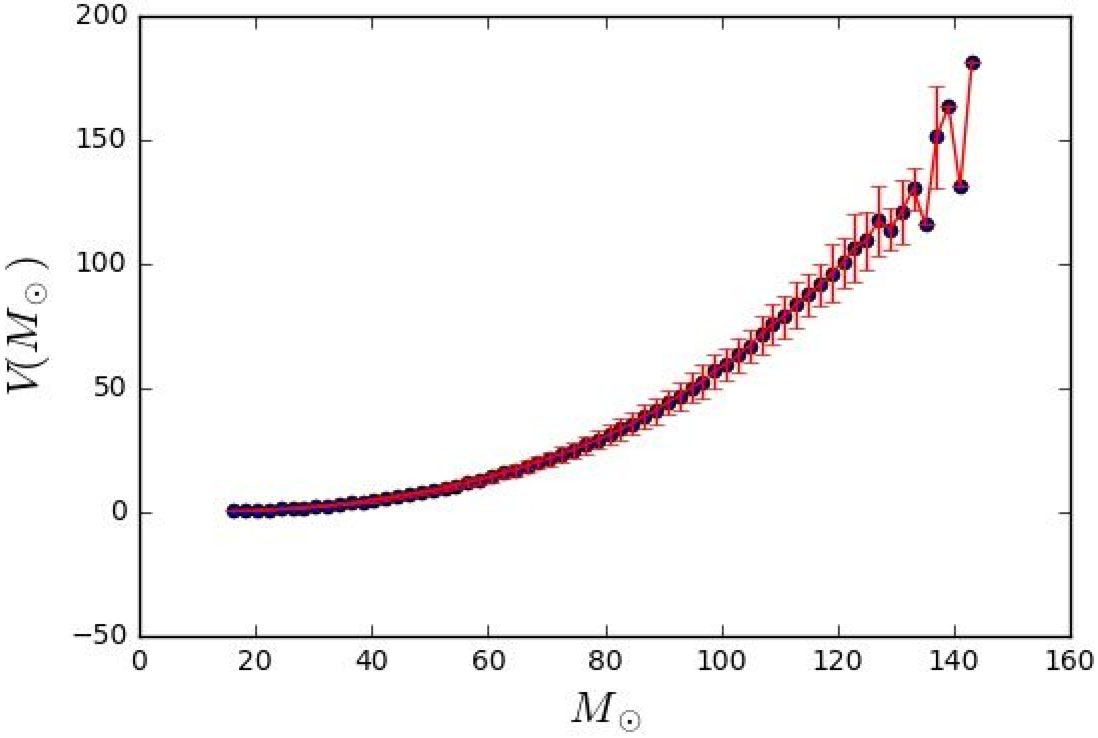
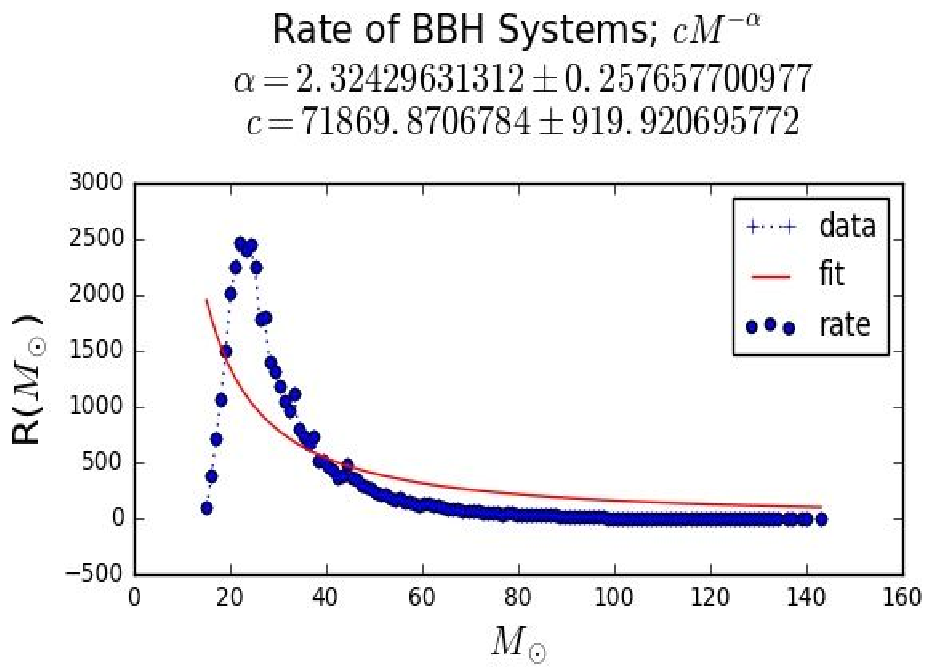


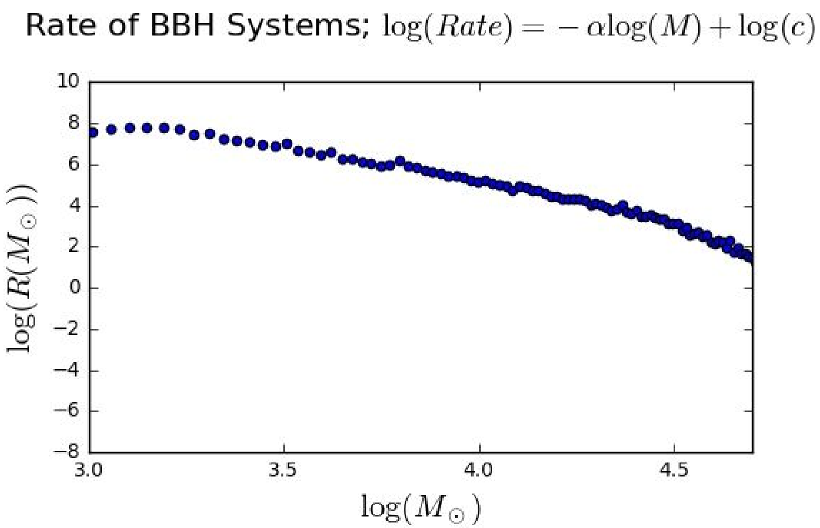

Figure 2. Events with SNR > 8, distance < horizon distance, distance > horizon distance

# Results: Observed V(M)

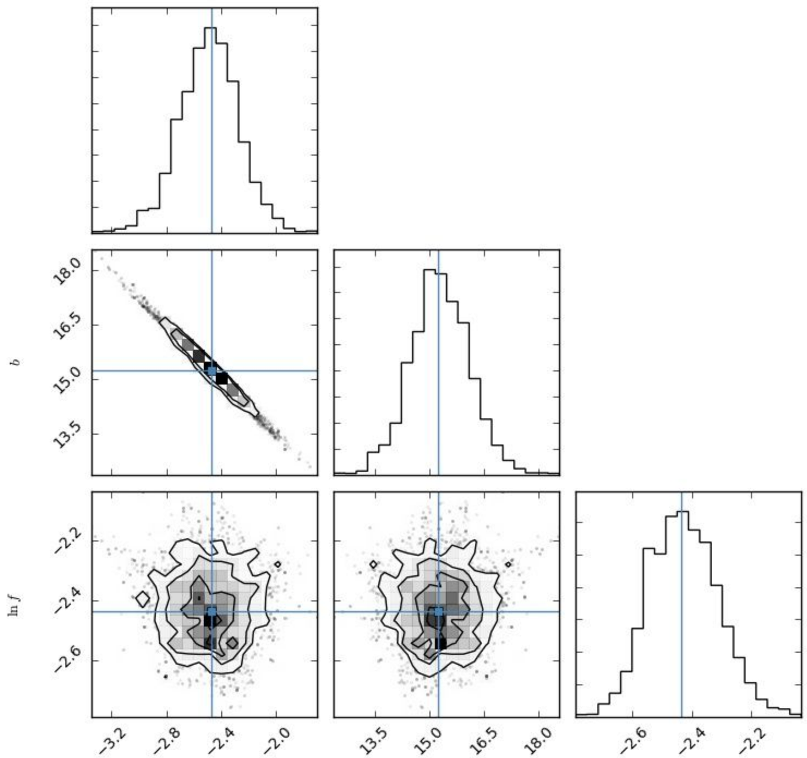


# Results: Observed R(M)



# Results: Bayesian Parameter Estimation of R(M)




Corner Plot of the Probability Distribution Function

# Conclusions

- Given a Rate density of -2.35,

# Future Work

- Constrain alpha according to a more realistic number events
  - How well can we measure alpha given 50 events? 100 events? 1000 events?
-

**Something to keep in mind:** A more thorough version of this project would entail calculating the rate density for all kinds of models: ie alpha is another value other than -2.35, or the rate density is not a power law at all.

# Summary

- The mass distribution of BBH can be a very useful tool in understanding how BBH formed and evolved over time
- Within the next 10/20 years, we expect LIGO to detect enough events to begin showing a conclusive mass distribution
- Using simulated events, we can determine methods for retrieving the actual rate density of BBH from observed events
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# Acknowledgements


- Mentor: Alan Weinstein
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- Stack Overflow

# References