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Time domain implementation of DCPD cross correlation

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Contents

1 Overview

This document presents a time domain implementation of the cross correlated DCPD spectrum which suppresses sensing noises such as photon shot noise. The idea is to implement an additional signal stream in the front end digital system and produce a pair of artificial channels such that the cross correlated power spectrum of them then yields such sensing noise free spectrum. As opposed to the offline implementation based on a frequency domain consideration, this scheme offers two advantages: (1) calibration of the resulting spectrum is automatically achieved as it makes use of the existing calibration front end system and (2) it allows easy access to the cross correlated spectrum in the control room because it can be handled by DTT.

2 Review

The main advantage of the cross correlated DCPD spectrum is that it can suppress photon shot noise and other sensing noises that are incoherent between the two OMC DCPDs. This allows us to study the structure of underlying noises which are otherwise hidden by sensing noises. In the rest of this section, we review the idea of the cross correlated DCPD spectrum and give a brief overview of the offline implementation that we have done for O1 [1, 2].

2.1 DARM control loop

Figure 1 shows a block diagram of the control of the DARM (Differential arm) degree of freedom. We assume that the two DCPDs have an identical response of $H/2$ [V/m] for simplicity. Note that the interferometer response (e.g. cavity pole) is included within H. The signals from each DCPD (d_{A}, d_{B}) satisfy the following pair of equations,

$$\begin{aligned} d_{\mathrm{A}} &= -\left(d_{\mathrm{A}} + d_{\mathrm{B}}\right)\frac{AFH}{2} + \frac{H}{2}x + n_{\mathrm{A}}, \\ d_{\mathrm{B}} &= -\left(d_{\mathrm{A}} + d_{\mathrm{B}}\right)\frac{AFH}{2} + \frac{H}{2}x + n_{\mathrm{B}}, \end{aligned} \tag{1}$$

where A and F are the actuator response [V/m] and control filter response [V/V], respectively. And where x, n_{A} and n_{B} are interferometer noise [m], and sensing noise of DCPD A

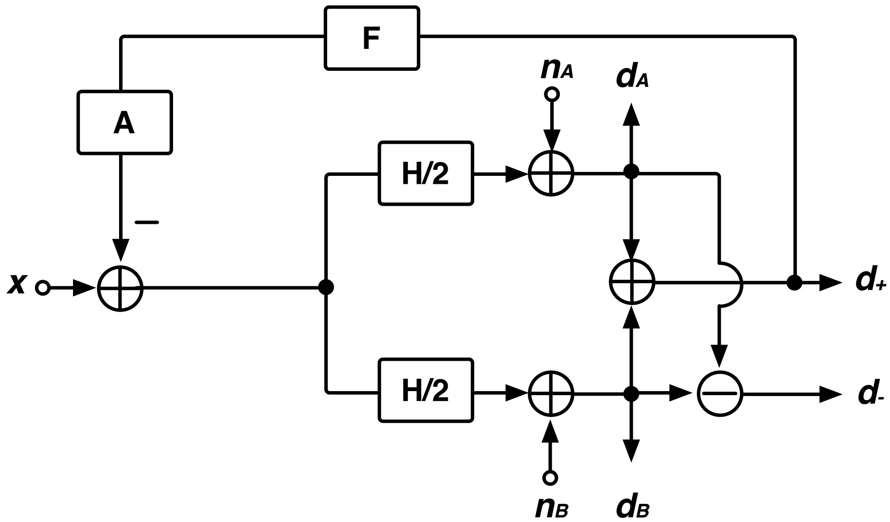

Figure 1: simplified DARM loop block diagram.

[V] and that of DCPD B [V], respectively.

Summing and subtracting each other of equations (1), one can obtain

$$\begin{aligned} d_+ &= \frac{H}{1+G}\left(x + \frac{n_\mathrm{A} + n_\mathrm{B}}{H}\right), \\ d_- &= n_\mathrm{A} - n_\mathrm{B}, \end{aligned} \tag{2}$$

where G is the open loop transfer function of the DARM control defined by $G \equiv AFH$ and where d_+ and d_- are the sum and null signals of the DCPDs, respectively. Their definitions are given by

$$\begin{aligned} d_+ &\equiv d_\mathrm{A} + d_\mathrm{B}, \\ d_- &\equiv d_\mathrm{A} - d_\mathrm{B}. \end{aligned} \tag{3}$$

As shown in equations (2), the sum signal is used to control DARM and is therefore affected by the control loop whereas the null is not.

2.2 Cross correlated DCPD spectrum

In general, one can obtain the cross correlated DCPD spectrum by performing a few steps of signal processing as follows.

1. Take out the effect of the loop suppression from the sum signal.
2. Produce two channels such that noise in one of the channels consists of only n_{A} and noise in the other channel consists of only n_{B} by linearly combining the unsuppressed sum and null signals.
3. Take cross correlated power spectrum of the two channels.

From equations (2) and (3), the in-loop DCPD signals or (d_{A}, d_{B}) can be expressed as

$$\begin{aligned} d_{\mathrm{A}} &= \frac{1}{2\,(1+G)}\left[Hx + (2+G)\,n_{\mathrm{A}} - G n_{\mathrm{B}}\right], \\ d_{\mathrm{B}} &= \frac{1}{2\,(1+G)}\left[Hx - G n_{\mathrm{A}} + (2+G)\,n_{\mathrm{B}}\right]. \end{aligned} \tag{4}$$

It is clear that the signal from one of the DCPDs, for example d_{A}, contains not only its sensing noise n_{A} but also sensing noise from the other DCPD n_{B}. This is due to the loop action which feeds n_{B} (and n_{A}) back in to the interferometer. If one takes a cross correlated power spectrum of the two in-loop DCPDs $\langle d_{\mathrm{A}}^* d_{\mathrm{B}} \rangle$, it will not remove sensing noises because it leaves terms proportional to $|n_{\mathrm{A}}|^2$ and $|n_{\mathrm{B}}|^2$. A good strategy to circumvent this situation is to remove the loop suppression in a post process and remove n_{B} from d_{A} and remove n_{A} from d_{B}.

Now, let us remove the effect of the control loop by applying $(1+G)$ to the sum signal,

$$\begin{aligned} d'_{+} &= d_{+}\,(1+G)\,, \\ &= Hx + n_{\mathrm{A}} + n_{\mathrm{B}}. \end{aligned} \tag{5}$$

Next, linearly combining d'_{+} and d_{-}, one can obtain

$$\begin{aligned} d'_{\mathrm{A}} &\equiv d'_{+} + d_{-} = Hx + 2n_{\mathrm{A}}, \\ d'_{\mathrm{B}} &\equiv d'_{+} - d_{-} = Hx + 2n_{\mathrm{B}}. \end{aligned} \tag{6}$$

Finally, taking the cross correlated spectrum of the above two signals, one can obtain

$$\frac{\left\langle \left(d'_{\mathrm{A}}\right)^* d'_{\mathrm{B}} \right\rangle}{|H|^2} = \left\langle\, |x|^2 \,\right\rangle, \tag{7}$$

where we have assumed n_A, n_B and x are independent of each other

$$\langle x^* n_\mathrm{A} \rangle = 0, \quad \langle x^* n_\mathrm{B} \rangle = 0, \quad \langle n_\mathrm{A}^* n_\mathrm{B} \rangle = 0. \tag{8}$$

Equation (7) is the cross correlated spectrum that we would like to obtain in the actual experiments. It preserves interferometer noise x while sensing noises n_A and n_B are removed.

2.3 Offline implementation

As explained in [3], one way to obtain the cross correlated spectrum is to do an offline signal processing in frequency domain.

Equations (6) can be expressed using the in-loop DCPD signals d_A and d_B as

$$\begin{aligned} d'_\mathrm{A} &= (1+G)\,(d_\mathrm{A}+d_\mathrm{B}) + d_\mathrm{A} - d_\mathrm{B}, \\ d'_\mathrm{B} &= (1+G)\,(d_\mathrm{A}+d_\mathrm{B}) - (d_\mathrm{A} - d_\mathrm{B})\,. \end{aligned} \tag{9}$$

Performing the cross correlation of d'_A and d'_B using the equations above, one can get

$$\left\langle \left(d'_\mathrm{A}\right)^* d'_\mathrm{B} \right\rangle = \left(|G|^2 + 2G\right) \left\langle |d_\mathrm{A}|^2 \right\rangle + \left(|G|^2 + 2G^*\right) \left\langle |d_\mathrm{B}|^2 \right\rangle + \left|G^2\right| d_\mathrm{AB}^* + |2+G|^2\, d_\mathrm{AB}, \tag{10}$$

where d_AB is the cross spectrum of d_A and d_B, defined by $d_\mathrm{AB} = \langle d_\mathrm{A}^* d_\mathrm{B} \rangle$.

Therefore, one can compute the cross correlated DCPD spectrum by using the above equation. This in reality requires a measurement of the power spectrum of each DCPD ($|d_\mathrm{A}|^2$, $|d_\mathrm{B}|^2$) and the cross correlated spectrum d_AB in frequency domain. Moreover, G must be known in frequency domain.

3 The online implementation

As shown in equation (10), the frequency domain approach involves not only measurement of a set of DCPD spectra but also knowledge about the DARM control loop. While this is not an issue in theory, in practice obtaining the information about the DARM control loop may not be an easy task for those who are not familiar with the interferometer commissioning.

An improvement for this situation can be achieved if we implement the time domain approach in which the effect of the DARM control loop is removed in the front end digital system.

3.1 The idea

Conveniently, the calibration front end system (a.k.a. CAL-CS) already provides time series of calibrated sum signals in which the effect of the control loop is removed. This signal can be expressed as

$$\begin{aligned} S^{(\mathrm{cal})} &= \frac{1+G}{H} d_+, \\ &= x + \frac{1}{H}\left(n_{\mathrm{A}} + n_{\mathrm{B}}\right). \end{aligned} \tag{11}$$

This calibrated signal is also known as ΔL_{ext}. Now, if we filter the null signal with a filter response of $1/H$ and add and subtract it to the above, one can obtain

$$\begin{aligned} S_{\mathrm{A}}^{(\mathrm{cal})} &\equiv S^{(\mathrm{cal})} + \frac{d_-}{H} = x + \frac{2n_{\mathrm{A}}}{H}, \\ S_{\mathrm{B}}^{(\mathrm{cal})} &\equiv S^{(\mathrm{cal})} - \frac{d_-}{H} = x + \frac{2n_{\mathrm{B}}}{H}. \end{aligned} \tag{12}$$

Taking cross spectrum of these two new channels, one can obtain the desired cross correlated spectrum,

$$\left\langle \left(S_{\mathrm{A}}^{(\mathrm{cal})}\right)^* S_{\mathrm{B}}^{(\mathrm{cal})} \right\rangle = \left\langle |x|^2 \right\rangle. \tag{13}$$

The main trick of this scheme is to make use of the existing calibration system which conveniently removes the control loop effect from the sum signal.

4 Implementation

4.1 Setup

Figure 2 shows a possible implementation of the time domain scheme. In order to calibrate the null signal in the same way as the sum signal, we need to place two additional filters, N and H^{-1} where N is the OMC normalization filter (which is often just a scaler factor).

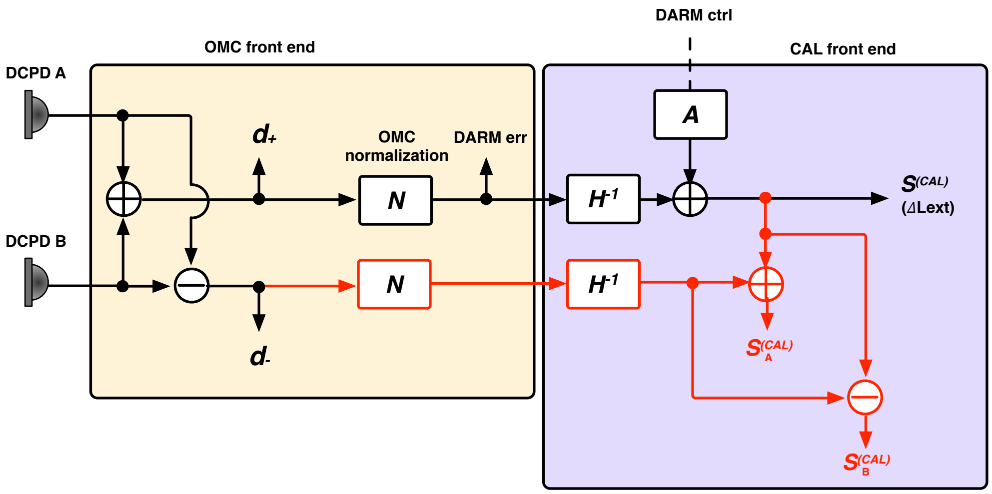

Figure 2: A possible implementation of the time domain scheme using the existing front end systems. It requires the implementation of the additional signal processing components shown in red.

4.2 Required change in the existing digital system

Here is a list of required changes for implementing the time domain scheme.

- A new filter N in the OMC digital front end model for the null signal.
- Signal transfer from the OMC model to CAL-CS model for the null signal.
- A new filter H^{1-} in the CAL-CS model.
- Sum and subtraction operations to produce the two linearly combined channels.
- Two new channels to record $S_{\mathrm{A}}^{(\mathrm{cal})}$ and $S_{\mathrm{B}}^{(\mathrm{cal})}$.

5 Calibration error

As shown in equations (6), the cross correlation technique relies on accurate subtraction of sensing noises. If there are calibration errors in the estimated loop components (specifically errors in A and H), they will spoil the perfect subtraction of sensing noises and consequently leave the terms proportional to n_{A} and n_{B} in each of the decomposed signals d'_{A} and d'_{B}.

They contaminate the resulting cross correlated spectrum by leaving the terms proportional to $|n_\mathrm{A}|^2$ and $|n_\mathrm{B}|^2$ and thus result in less reduction of sensing noises.

In this section, we consider two types of calibration errors and discuss their effects on the reduction of sensing noises.

5.1 Calibration errors in control loop elements

Assume the filters in the calibration pipelines are inaccurate by a small amount, so that

$$\begin{aligned} A^{(\mathrm{cal})} &= A + \Delta A, \\ H^{(\mathrm{cal})} &= H + \Delta H, \end{aligned} \tag{14}$$

where the variables with superscript 'cal' represent those actually installed in the calibration front end system, and where the variables with Δ upfront represents errors in the calibration filters.

In this case, the calibrated DARM signal can be calculated as

$$\begin{aligned} x^{(\mathrm{cal})} &= S^{(\mathrm{sum})} \frac{1+G^{(\mathrm{cal})}}{H^{(\mathrm{cal})}}, \\ &= \left[1 - \frac{1}{1+G}\left(\frac{\Delta H}{H}\right) + \frac{G}{1+G}\left(\frac{\Delta A}{A}\right)\right]\left(x + \frac{n_\mathrm{A}+n_\mathrm{B}}{H}\right). \end{aligned} \tag{15}$$

Computing the decomposed signals (12), one can obtain

$$\begin{aligned} S_\mathrm{A}^{(\mathrm{cal})} &= (1+r)\, x + (2+p)\, \frac{n_\mathrm{A}}{H} + q\frac{n_\mathrm{B}}{H}, \\ S_\mathrm{B}^{(\mathrm{cal})} &= (1+r)\, x + q\frac{n_\mathrm{A}}{H} + (2+p)\, \frac{n_\mathrm{B}}{H} \end{aligned} \tag{16}$$

where

$$\begin{aligned} p &= -\frac{2+G}{1+G}\left(\frac{\Delta H}{H}\right) + \frac{G}{1+G}\left(\frac{\Delta A}{A}\right), \\ q &= \frac{G}{1+G}\left(\frac{\Delta H}{H}\right) + \frac{G}{1+G}\left(\frac{\Delta A}{A}\right), \\ r &= -\frac{1}{1+G}\left(\frac{\Delta H}{H}\right) + \frac{G}{1+G}\left(\frac{\Delta A}{A}\right). \end{aligned} \tag{17}$$

Plugging these in to the cross correlated spectrum, one gets

$$\left\langle \left(S_{\mathrm{A}}^{(\mathrm{cal})}\right)^{*} S_{\mathrm{B}}^{(\mathrm{cal})} \right\rangle = (1 + r + r^{*}) \left\langle |x|^{2} \right\rangle + 2q \frac{\left\langle |n_{\mathrm{A}}|^{2} \right\rangle}{|H|^{2}} + +2q^{*} \frac{\left\langle |n_{\mathrm{B}}|^{2} \right\rangle}{|H|^{2}} + \mathcal{O}\left(\Delta^{2}\right). \tag{18}$$

Therefore, calibration errors let sensing noises n_{A} and n_{B} remain after the cross correlation.

The coefficient of the residuals q has the dependency on G in such a way that the effect of calibration errors is maxmimized in the control bandwidth because $G/(1+G) \sim 1$ while it becomes less significant at frequencies out of the control bandwidth because $|G/(1+G)| \ll 1$. Assuming a conservative calibration error of 10% in the control frequency band, we can consider q to be on the order of 10% as well. Thus, the suppression factor of sensing noise in amplitude spectral density (i.e. square root of the cross power spectrum) can be written as

$$\text{suppression} \sim \sqrt{2\,|q|} \sim 0.45 \times \sqrt{\frac{\Delta\text{Calib.}}{10\%}}. \tag{19}$$

While this is true for frequencies in the control bandwidth, the suppression improves as the frequency goes further away from the control frequency band due to the term $G/(1+G)$ in q. If one can reach a calibration error of 1%, the suppression will be 0.14 in the control band.

5.2 Imbalance between two DCPDs

Another type of error is imbalance between the two DCPDs. This will cause a similar issue to that we discussed in the previous subsection. Now we consider a pair of imbalanced DCPDs so that DCPD A has a response of $(H + \Delta H)/2$ and DCPD B has $(H - \Delta H)/2$. In this case the sum and null signals can be written as

$$\begin{aligned} d_{+} &= \frac{H}{1+G}\left[x + \frac{n_{\mathrm{A}} + n_{\mathrm{B}}}{H}\right], \\ d_{-} &= -G\left(\frac{\Delta H}{H}\right) d_{+} + \Delta H x + n_{\mathrm{A}} - n_{\mathrm{B}}. \end{aligned} \tag{20}$$

observe that the sum signal is not affected by the imbalance. Performing a bit of linear algebra, one can write the two channels as

$$\begin{aligned} S_{\mathrm{A}}^{(\mathrm{cal})} &= \left(1 - u + \frac{\Delta H}{H}\right) x + (2-u)\frac{n_{\mathrm{A}}}{H} - u\frac{n_{\mathrm{B}}}{H}, \\ S_{\mathrm{B}}^{(\mathrm{cal})} &= \left(1 - u + \frac{\Delta H}{H}\right) x + u\frac{n_{\mathrm{A}}}{H} + (2+u)\frac{n_{\mathrm{B}}}{H}. \end{aligned} \tag{21}$$

where u is the coefficient for the residual, defined as

$$u \equiv \frac{G}{1+G}\left(\frac{\Delta H}{H}\right) \tag{22}$$

Therefore the cross correlated spectrum can be

$$\begin{aligned} \left\langle \left(S_{\mathrm{A}}^{(\mathrm{cal})}\right)^* S_{\mathrm{B}}^{(\mathrm{cal})} \right\rangle = &\left(1 + u - u^* + \left(\frac{\Delta H}{H}\right)^* - \left(\frac{\Delta H}{H}\right)\right) \langle |x|^2 \rangle \\ &+ 2u\frac{\langle |n_{\mathrm{A}}|^2 \rangle}{|H|^2} - 2u^*\frac{\langle |n_{\mathrm{B}}|^2 \rangle}{|H|^2} + \mathcal{O}\left(\Delta^2\right). \end{aligned} \tag{23}$$

Therefore, similarly to the calibration errors, imbalance of the DCPDs can limit the suppression performance. However, imbalance is typically well compensated in the digital system to a precision of sub percent [4]. Therefore the effect on the suppression is not as significant as that for calibration errors,

$$\text{suppression} \sim \sqrt{2\,|u|} \sim 0.045 \times \sqrt{\frac{\text{Imbalance.}}{0.1\%}}. \tag{24}$$

6 Summary

A new scheme to implement the DCPD cross correlation spectrum is presented. It makes use of the existing calibration front end system, offering an easy access to the cross spectrum for those who are not familiar with the control system.

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