

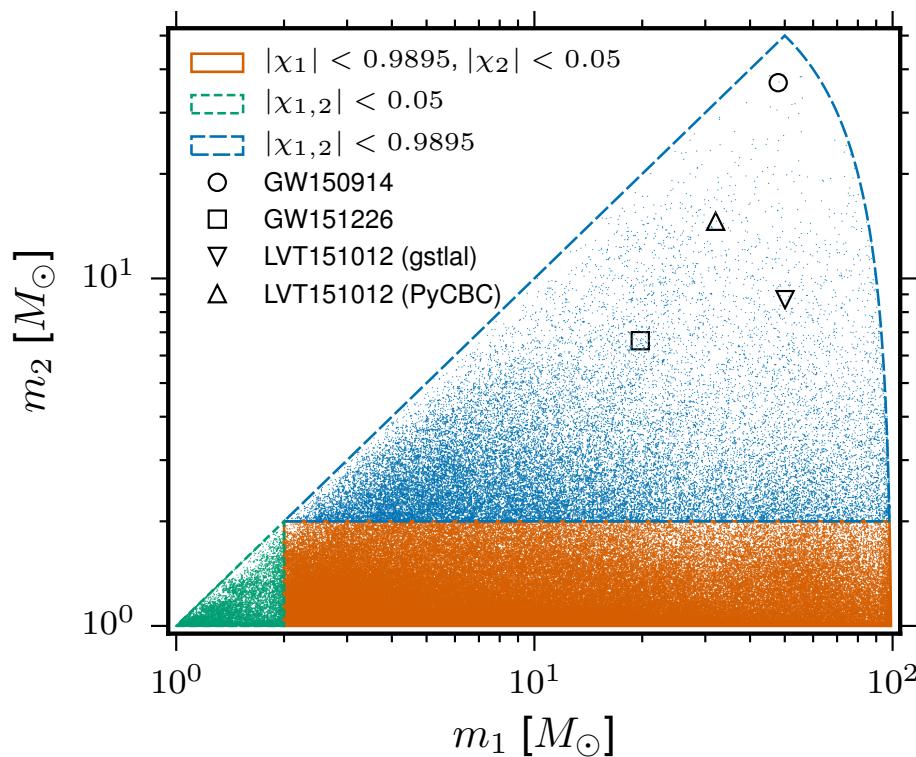
Frequency Domain Binary Black Hole Gravitational Waveforms with Higher Multipoles

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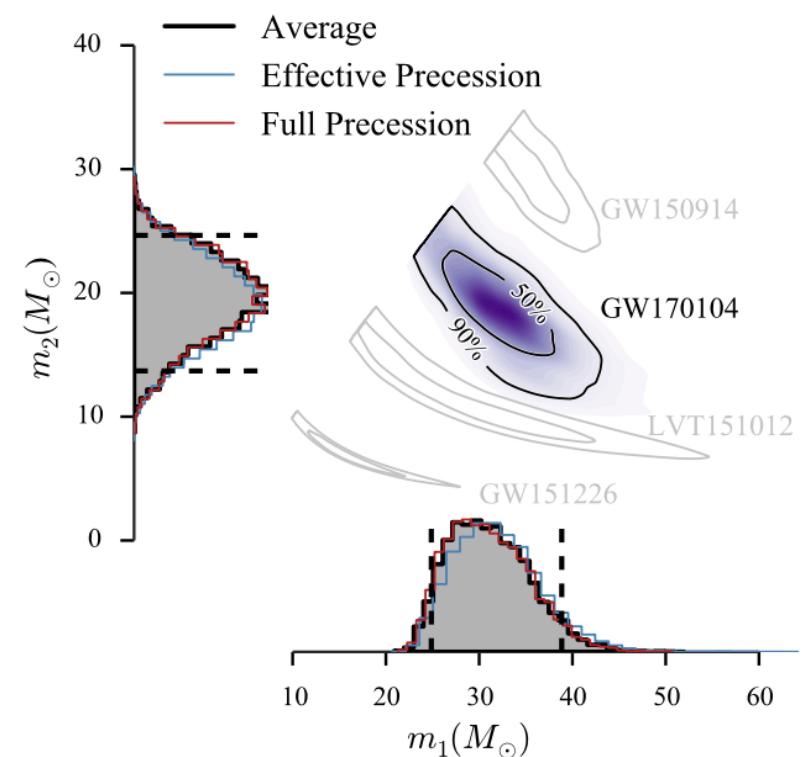
[Amaldi 12 Pasadena – July 10, 2017](#)



The Need for Gravitational Waveform Models



[Abbott *et al.*, PRX 6, 041015 (2016)]



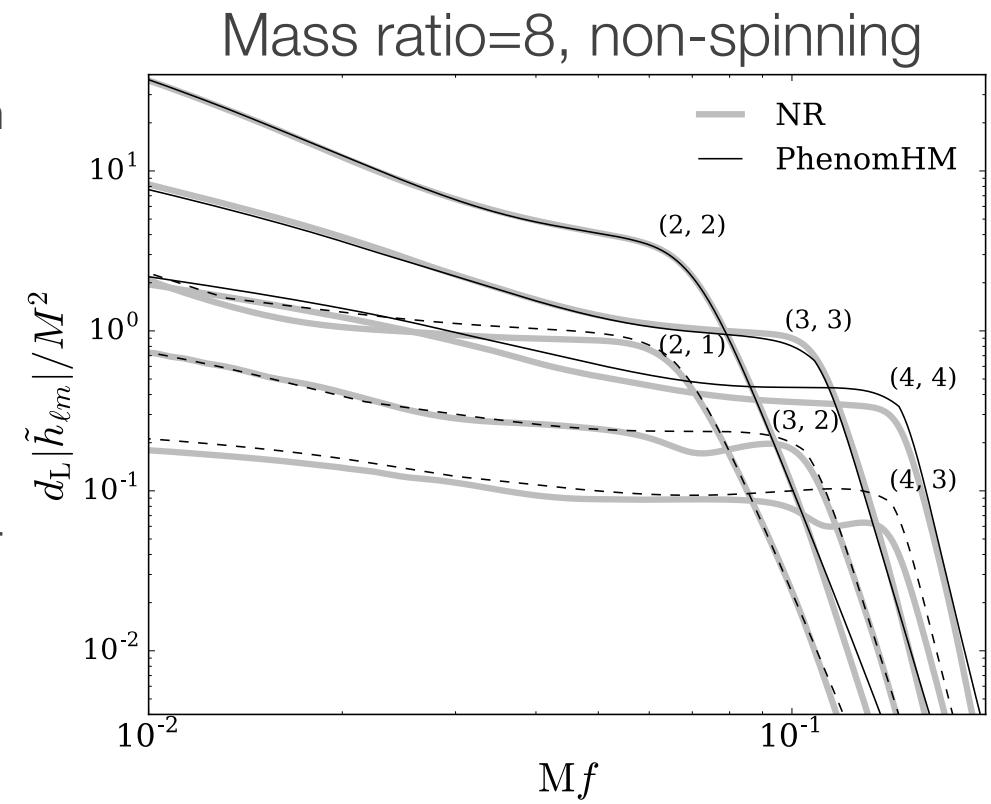
[Abbott *et al.*, PRL 118, 221101 (2016)]

The Need for Higher-Multipole Models

- Currently models include only the dominant multipoles ($\ell = |m| = 2$) of the signal
 - Modelling subdominant multipoles can:
 - increase detectable volume (for $m_1/m_2 \gtrsim 4$)
 - improve measurement accuracy
 - avoid large biases in measurements
 - Current models that include higher multipoles:
 - do not apply to spinning binary black hole (BBH) systems
 - are limited to corners of the parameter space
- [L. Pekowsky *et al.*, PRD 87, 084008 (2013)]
[Brown *et al.*, PRD 87, 082004 (2013)]
[Capano *et al.*, PRD 89, 102003 (2014)]
[Varma *et al.*, PRD 90, 124004 (2014)]
[Bustillo *et al.*, PRD D93, 084019 (2016)]
[Bustillo *et al.*, PRD 95, 104038 (2017)]
[Varma *et al.*, arXiv:1612.05608]
- [Pan *et al.*, PRD 84, 124052 (2011)]
[Blackman *et al.*, arXiv:1705.07089]

First Spinning Higher-Multipole Gravitational Waveform Model

- Takes an accurate model for the dominant multipole and **maps** it into each of the other multipoles
- Mapping based on results from Post-Newtonian (PN) and perturbation theory
- This approach can be applied to any non-precessing frequency-domain model
- PhenomD → PhenomHM



[Document Control Center: P1700203]

Construction: Basic Picture

- GW strain decomposed into spherical harmonics with spin weight -2

$$h(t, \vec{\lambda}, \iota, \phi_0) = \sum_{\ell \geq 2} \sum_{-\ell \leq m \leq \ell} h_{\ell m}(t, \vec{\lambda})^{-2} Y_{\ell m}(\iota, \phi_0)$$

- Appropriately scale and stretch the dominant multipole to reproduce each subdominant multipole – operate separately on **frequency** domain values, and **phase** and **amplitude** functions:

$$\begin{aligned} \tilde{h}_{\ell m}(f) &= A_{\ell m}(f) \times \exp \{i \varphi_{\ell m}(f)\} \\ &\approx \beta_{\ell m}(f) A^{22}(f_{\ell m}^A) \times \exp \{i [\kappa_{\ell m} \varphi_{22}(f_{\ell m}^\varphi) + \Delta_{\ell m}]\} \end{aligned}$$

Construction: Foundations

$$f_{\ell m}(f) = \begin{cases} \frac{2}{m}f, & f \leq f_0 \\ \frac{f_{22}^{\text{RD}} - 2f_0/m}{f_{\ell m}^{\text{RD}} - f_0} (f - f_0) + \frac{2f_0}{m}, & f_0 < f \leq f_{\ell m}^{\text{RD}} \\ f - f_{\ell m}^{\text{RD}} + f_{22}^{\text{RD}}, & f > f_{\ell m}^{\text{RD}}. \end{cases}$$

**Post-Newtonian time-domain scaling
+ stationary phase approximation**

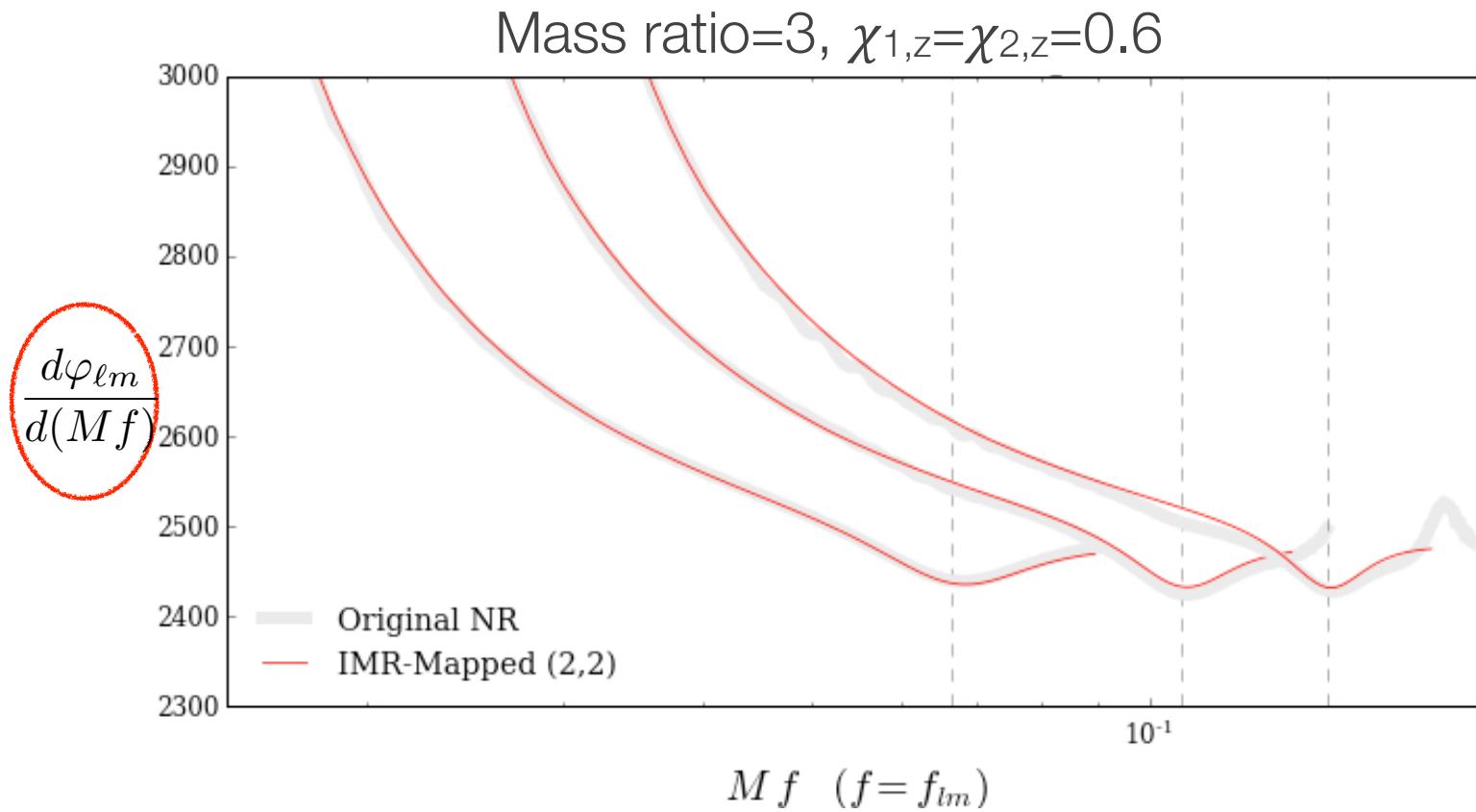
Bridge: linear interpolation

Quasi-Normal Mode theory

- Freedom to optimize the agreement with NR simulations by allowing different values of f_0 for amplitude and phase – PhenomD:

$$f_0^A = 0.018 f_{\ell m}^{\text{RD}} / f_{22}^{\text{RD}}, \quad f_0^\varphi = 0.014 f_{\ell m}^{\text{RD}} / f_{22}^{\text{RD}}, \quad f_{\ell m}^{\text{RD}} = \omega_{\ell m 0} / 2\pi$$

Construction: Foundations



Construction: Foundations

- Integrate once, then use PN theory and require continuity to obtain the additional, multipole-dependent phase offsets

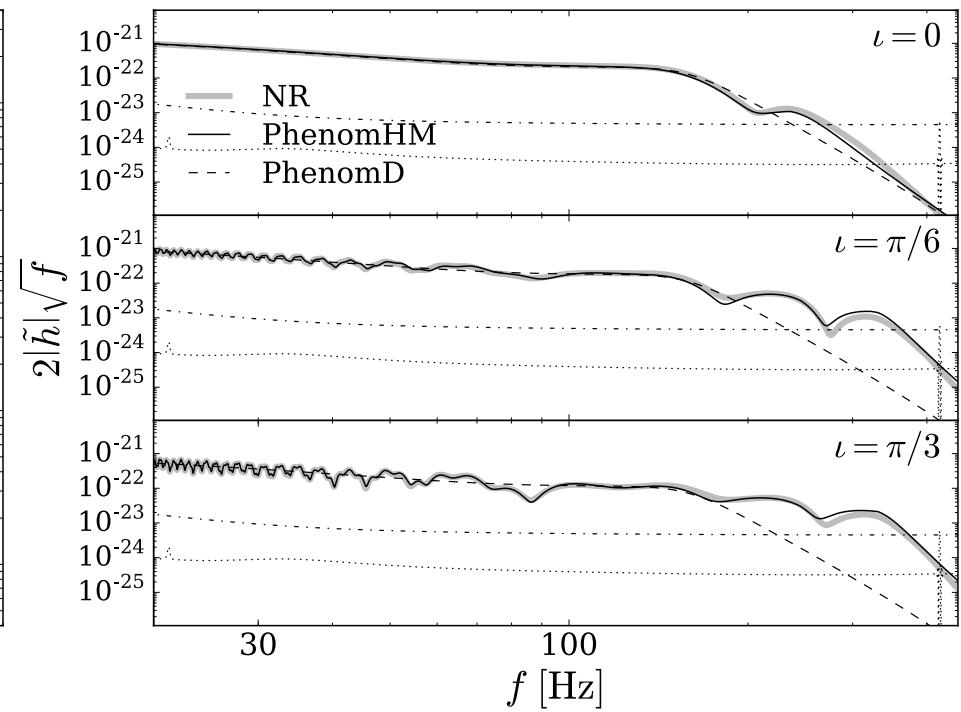
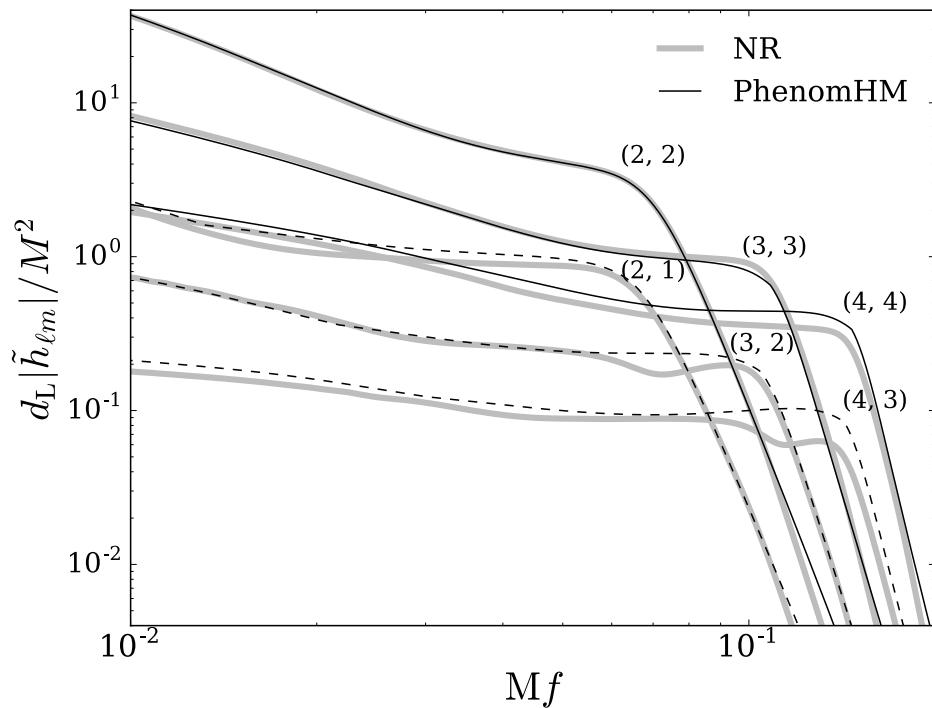
$$\kappa_{\ell m} = \frac{1}{f'_{\ell m}(f)}, \quad (\text{piecewise constant})$$
$$\Delta_{\ell m} = \begin{cases} \frac{\pi}{2} [3\ell + \text{mod}(\ell + m, 2)] - \pi, & f \leq f_0^\varphi \\ \varphi_{\ell m}(f_0^\varphi) - \kappa_{\ell m} \varphi_{22}[f_{\ell m}^\varphi(f_0^\varphi)], & f_0^\varphi < f \leq f_{\ell m}^{\text{RD}} \\ \varphi_{\ell m}(f_{\ell m}^{\text{RD}}) - \varphi_{22}[f_{\ell m}^\varphi(f_{\ell m}^{\text{RD}})], & f > f_{\ell m}^{\text{RD}}. \end{cases}$$

- Completes $\tilde{h}_{\ell m}(f) \approx \beta_{\ell m}(f) A^{22}(f_{\ell m}^{\text{A}}) \times \exp \{ i [\kappa_{\ell m} \varphi_{22}(f_{\ell m}^\varphi) + \Delta_{\ell m}] \}$

(the stationary phase approximation predicts the amplitude scalings to leading-order in f)

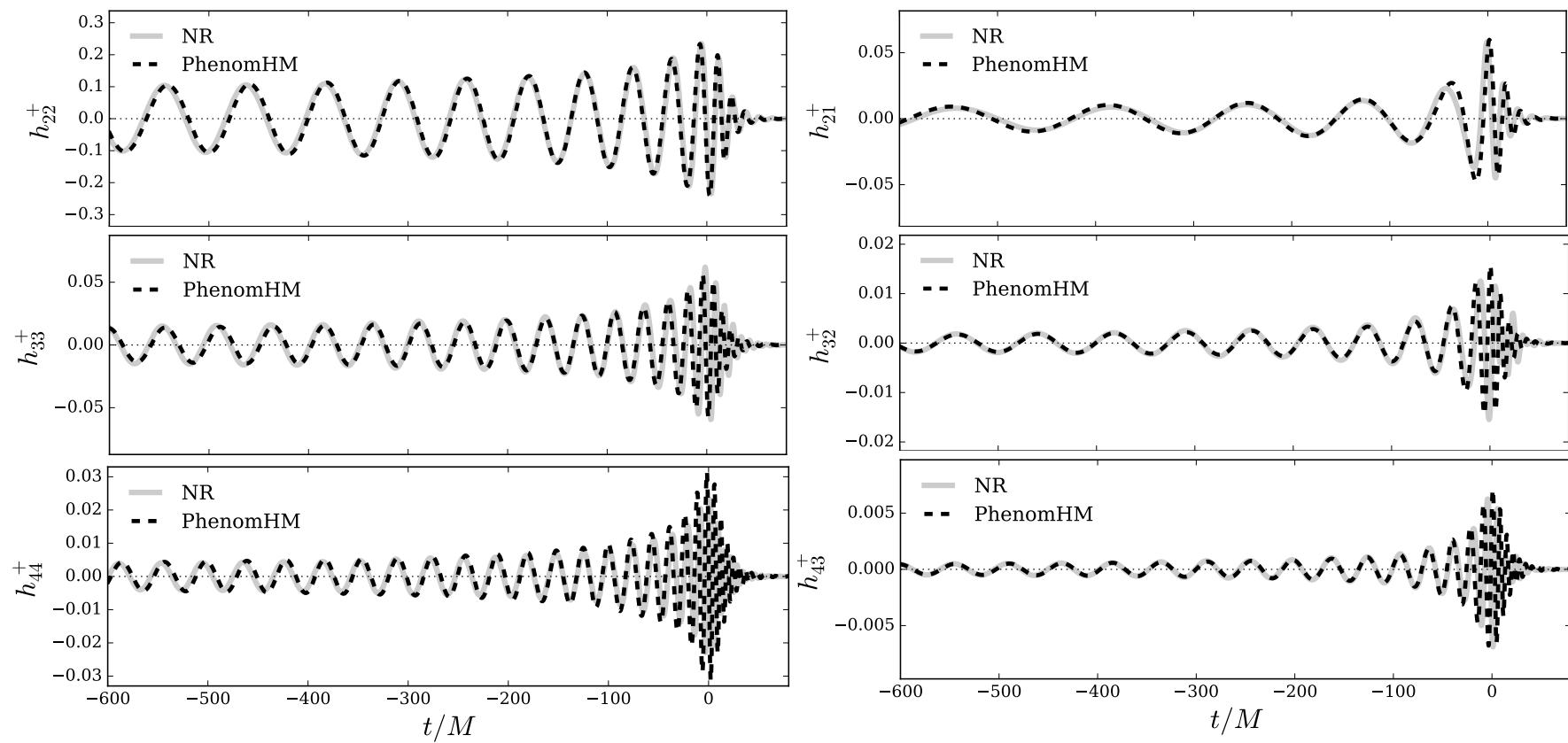
PhenomHM Validation: Frequency Domain

Mass ratio=8, non-spinning (50 solar masses, 500Mpc)

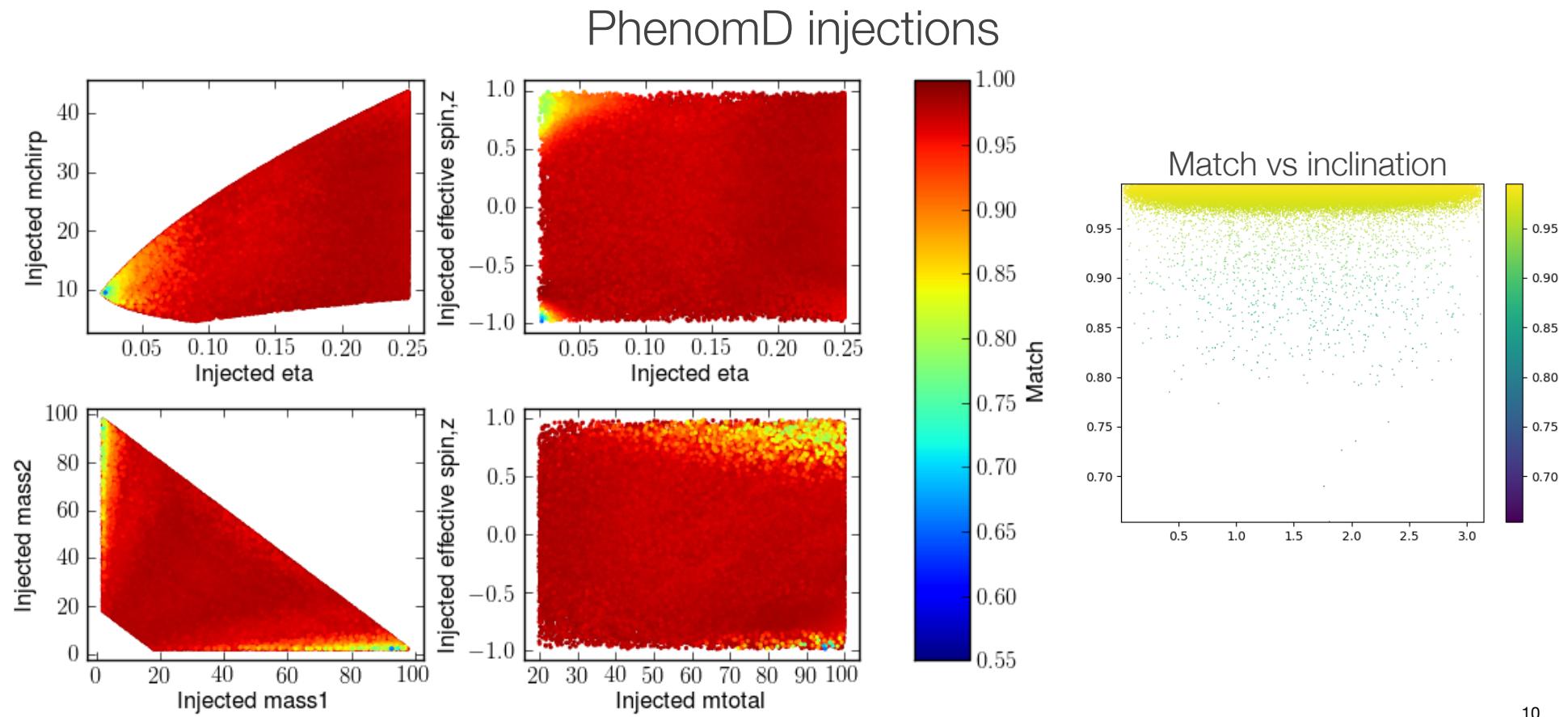


PhenomHM Validation: Time Domain

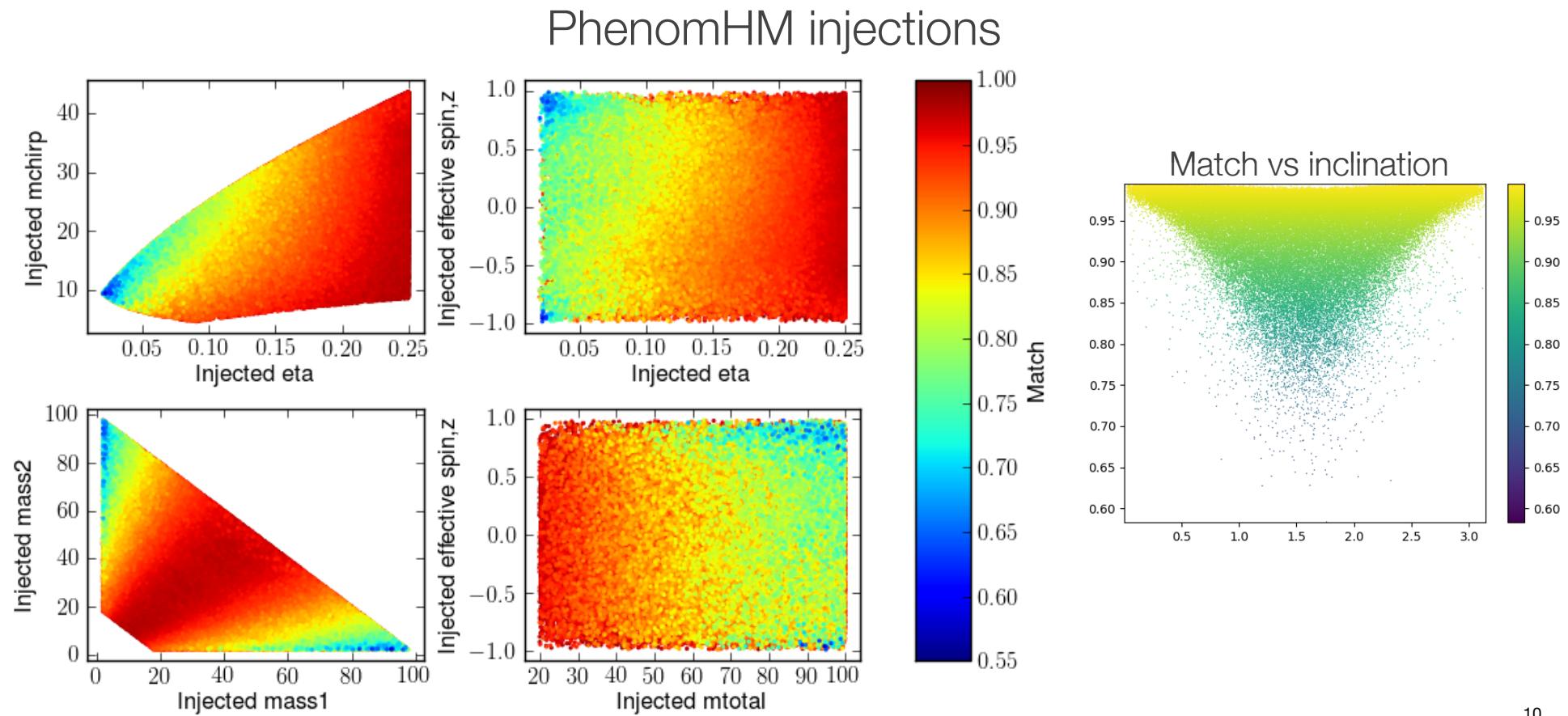
Mass ratio=4, $\chi_{1,z}=\chi_{2,z}=0.85$



Impact on Searches

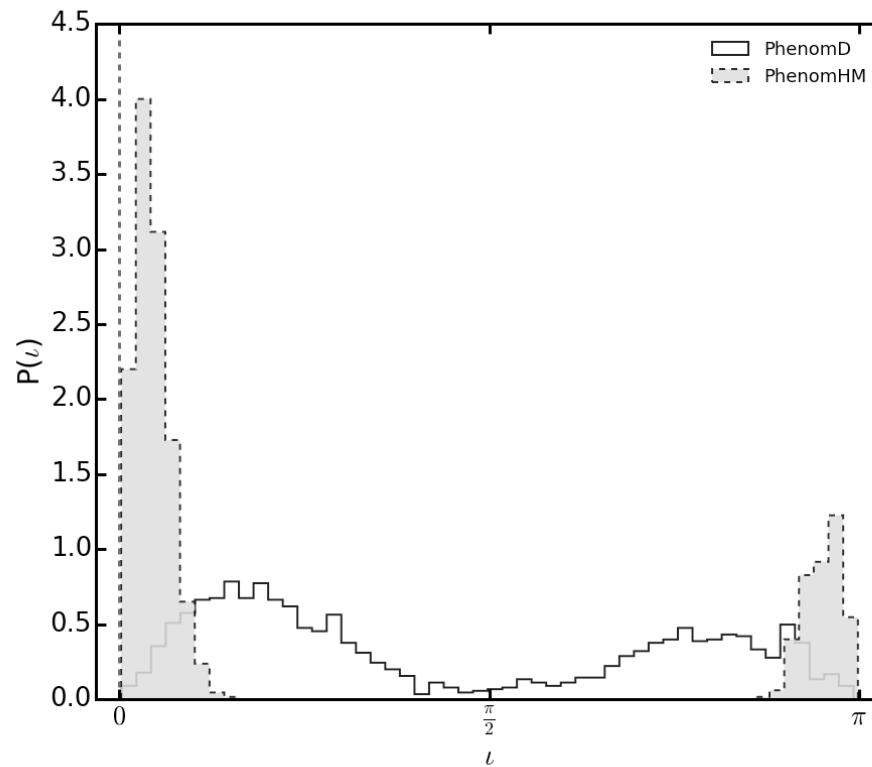


Impact on Searches



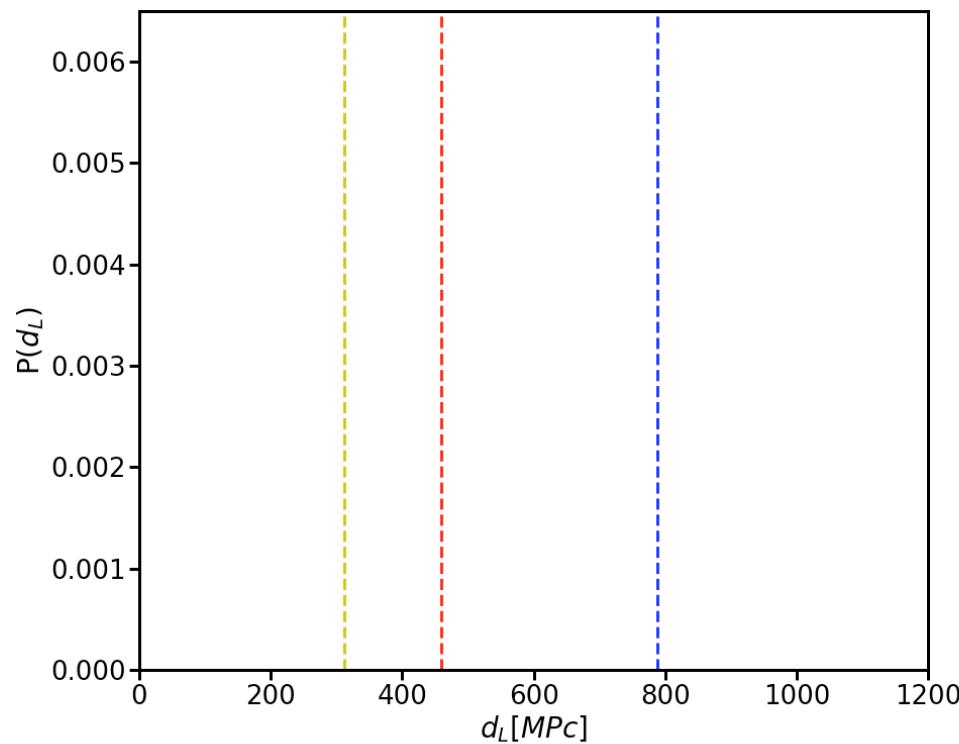
Impact on Parameter Estimation: Extrinsic Parameters

Mass ratio=4, $\chi_{1,z}=0.5$, $\chi_{2,z}=0$, 90 solar masses, SNR=25

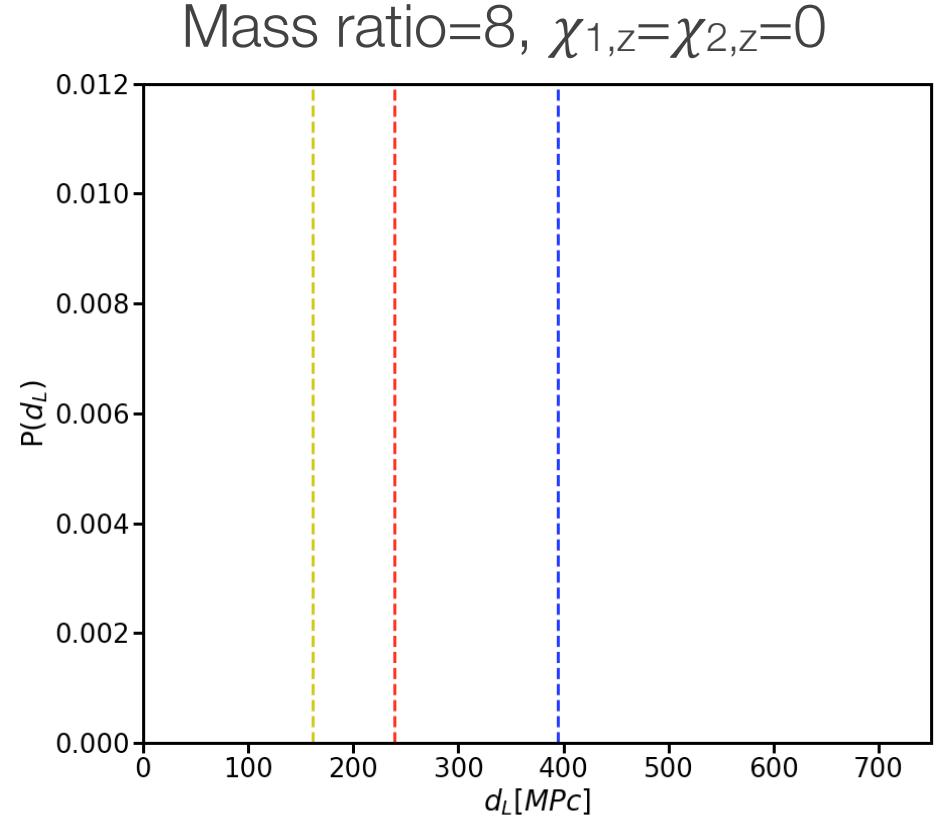


Impact on Parameter Estimation: Extrinsic Parameters

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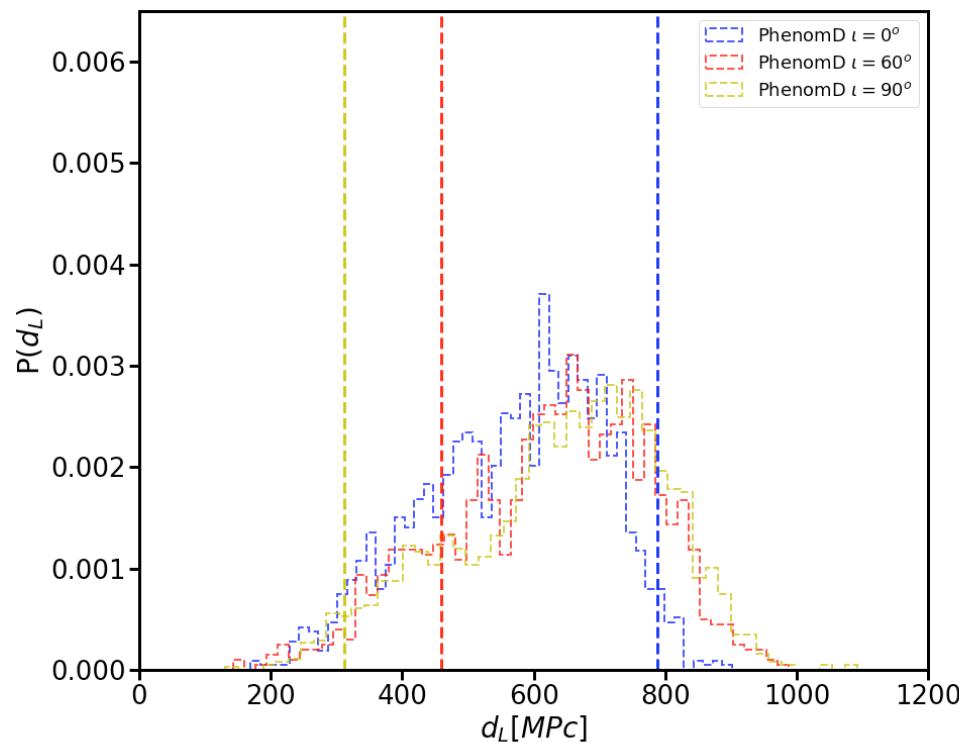


Mass ratio=8, $\chi_{1,z}=\chi_{2,z}=0$

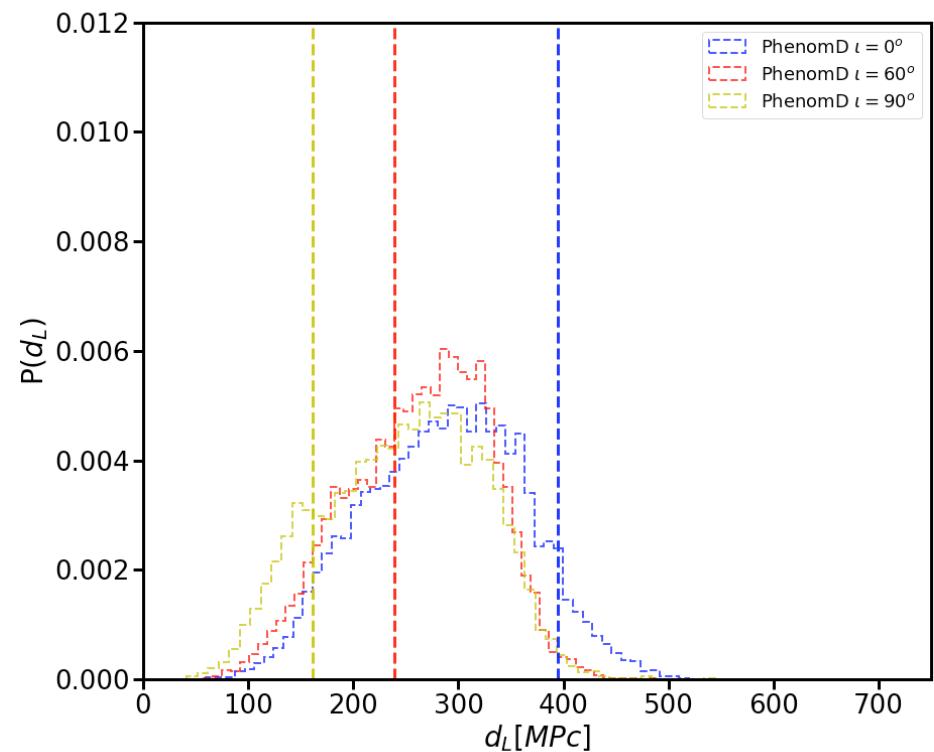


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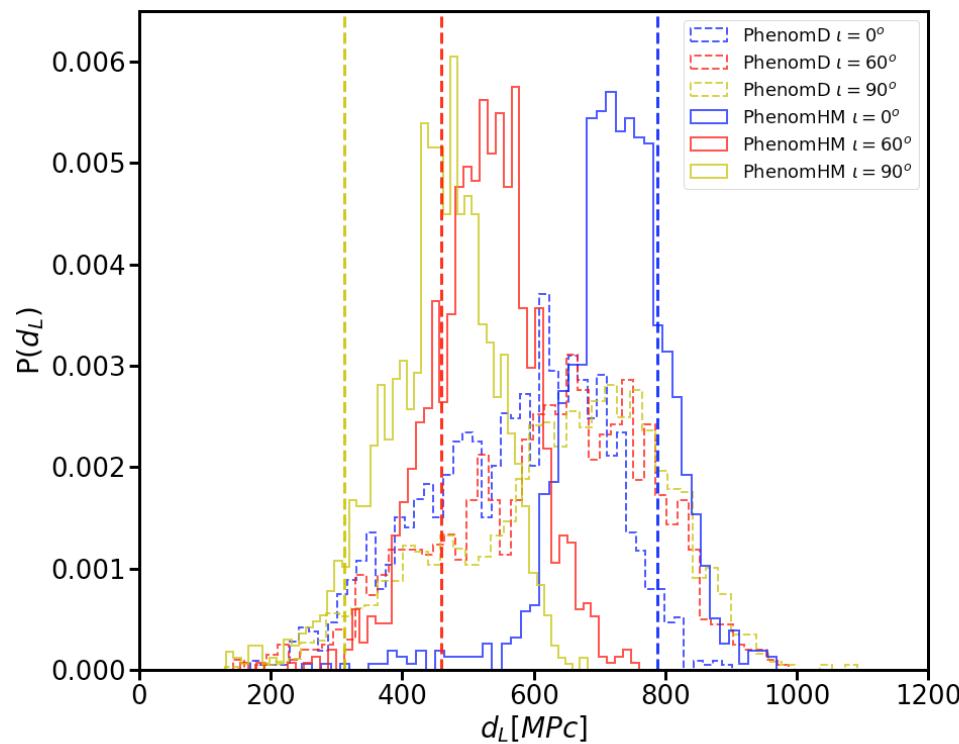


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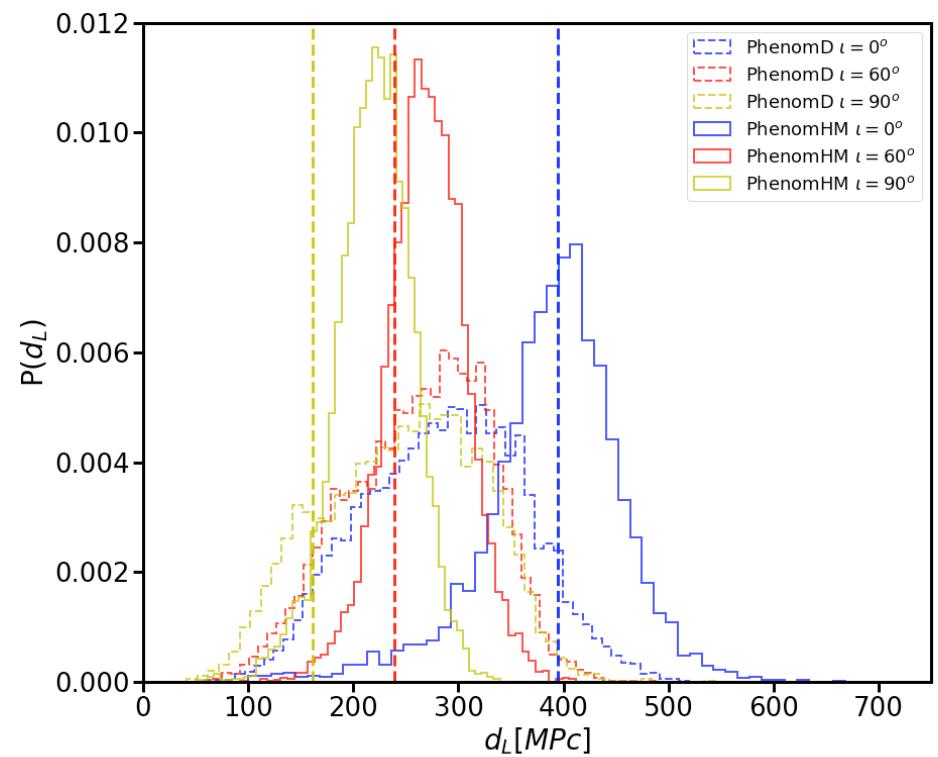


Impact on Parameter Estimation: Extrinsic Parameters

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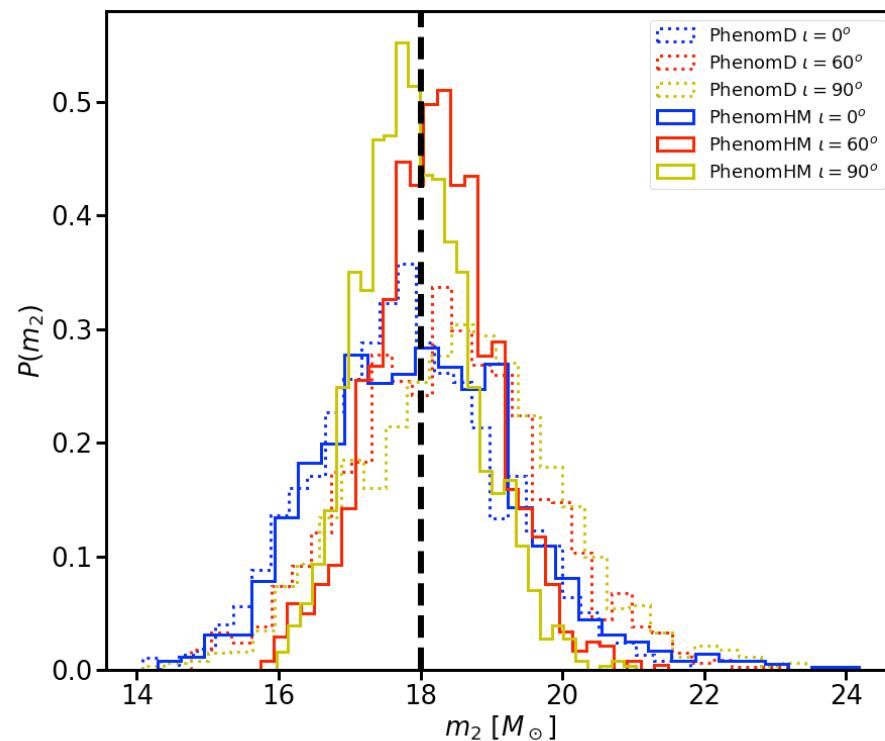


Mass ratio=8, $\chi_{1,z}=\chi_{2,z}=0$

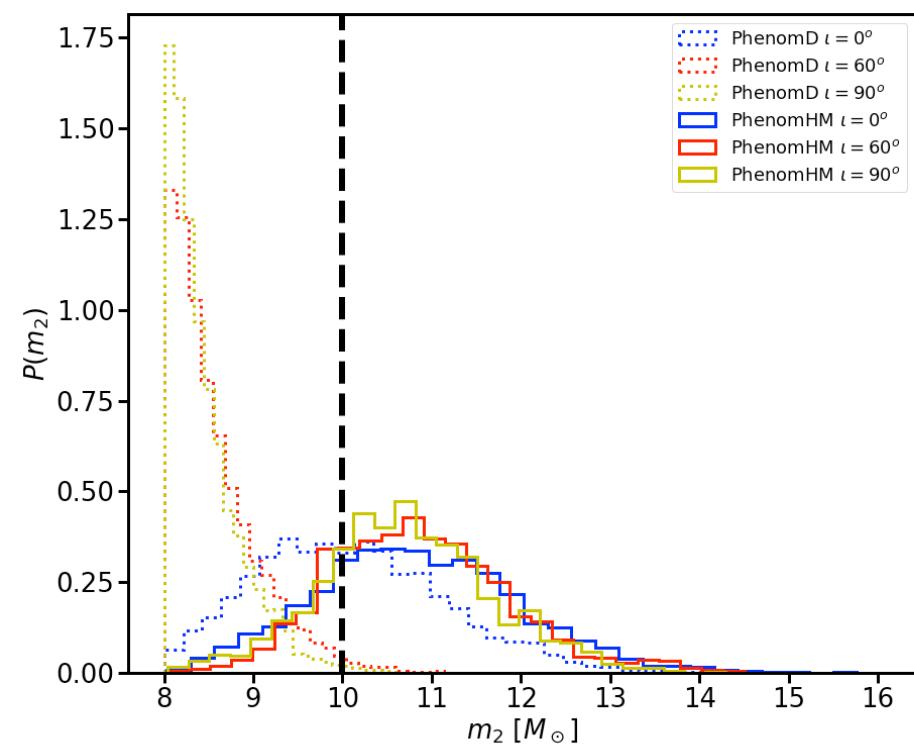


Impact on Parameter Estimation: Masses

Mass ratio=4, $\chi_{1,z}=0.5$, $\chi_{2,z}=0$

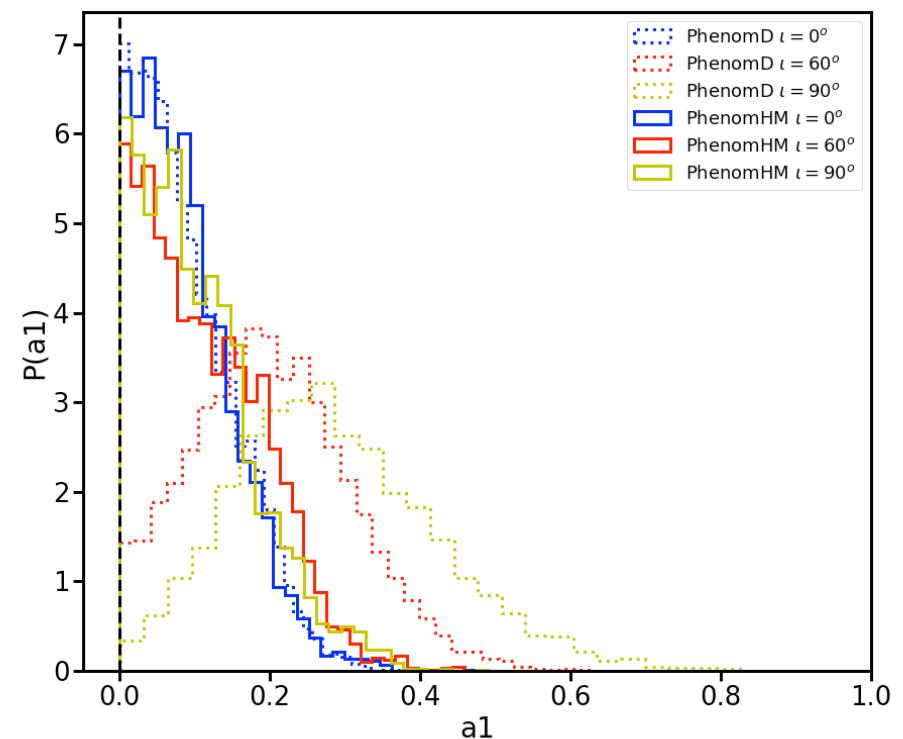
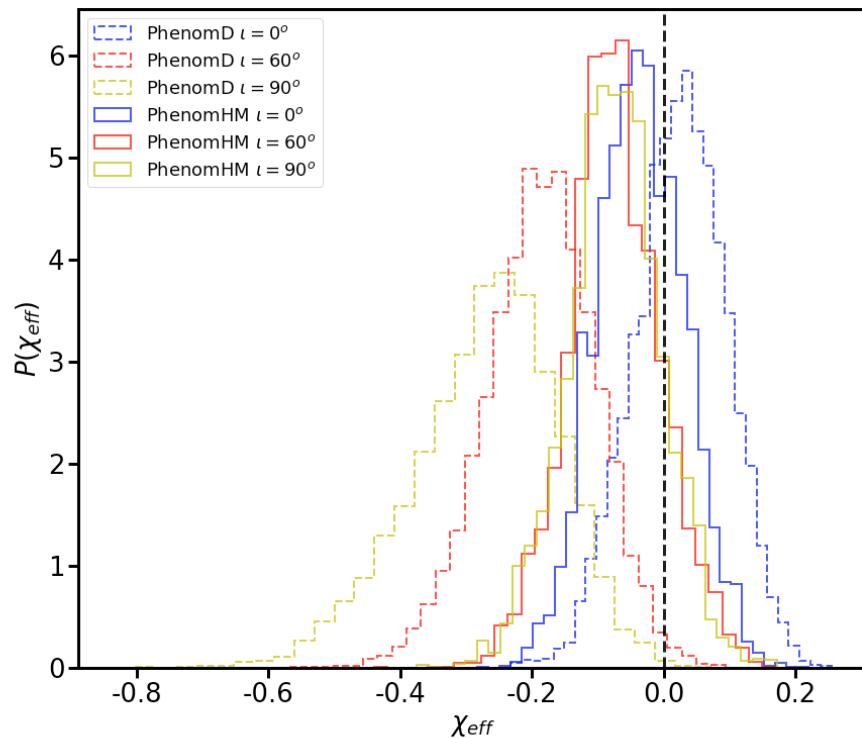


Mass ratio=8, $\chi_{1,z}=\chi_{2,z}=0$



Impact on Parameter Estimation: Spins

Mass ratio=8, $\chi_{1,z}=\chi_{2,z}=0$



Summary

- A simple, flexible method to include the subdominant multipole contributions to binary black hole gravitational waveforms: benefits for searches and parameter estimation
- Amplitude and phase of the starting frequency-domain model are appropriately stretched and rescaled (guidance from post-Newtonian and perturbation theory)
- No additional tuning to numerical-relativity simulations!
- PhenomD → PhenomHM: **first higher-multipole spinning binary black-hole model**
 1. More accurate in all comparisons to numerical-relativity data
 2. Typically leads to improved measurements of the binary properties
- Approach can be extended to precessing systems: will enable studies of the impact of higher multipoles on gravitational-wave astronomy, and tests of fundamental physics