Density Newtonian Noise Calculations

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1 Relevant Results:

Let us start by giving a small summary of the important dependencies for the displacement ASD generated by Newtonian Noise, as encountered by our simplified model.

- The ASD is directly proportional to the absorbed power P_{input}.
- The time that bubbles remain inside the pipe does not affect the ASD level.
- The ASD is proportional to $\left(\frac{dN_{bub}}{dt}\right)^{-\frac{1}{2}}$ which is reminiscent of the dependence for shot noise.
- The velocity of the fluid flow seems to only be relevant on the 'fading out' of the bubbles outside the region of interest.
- Assuming an inverse square law for the fade-out function for the bubbles, the ASD is directly proportional to v_{flow}.
- Interpolating different regimes can give you a hard time on the frequency domain.

2 Preliminary Assumptions:

2.1 Multipole expansion vs. monopole term

To model the density fluctuation noise we will abstract the pipe system to a single point mass located one meter away from the test mass, which will also be modeled by a point mass for the sake of order of magnitude estimates.

These assumptions are justified by the following two facts:

First, the force gradient due to mass fluctuations is proportional to the axial acceleration (as opposed to its gradient). The greatest axial acceleration on the test mass is produced by masses on its symmetry axis.

Second, the monopole term for the axial force (which is the point mass to point mass contribution) satisfies $1.1F_{monopole} > F$. This is shown in figure 1.

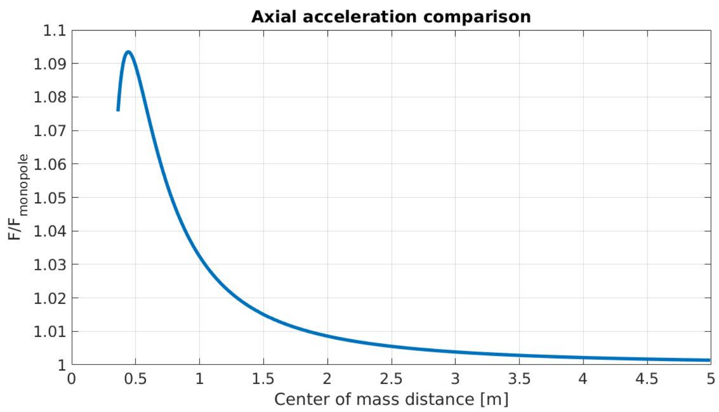

Figure 1: Comparison between the full multipole expansion for the axial force and the monopole term for masses in the symmetry axis of the test mass.

It can be seen that the corrections add up to less than 10% of the monopole term and so for the mass fluctuations we will use only this one.

2.2 Pipe mass fluctuation

We will model just a relevant section of the pipe, with the bubbles inside it pushing the liquid outside the region of interest too fast for the timescales considered in the calculation. So that effectively a snapshot of the relevant section of the pipe looks like the ones shown in figure 2, where the liquid is displaced and the gas bubbles retain their density.

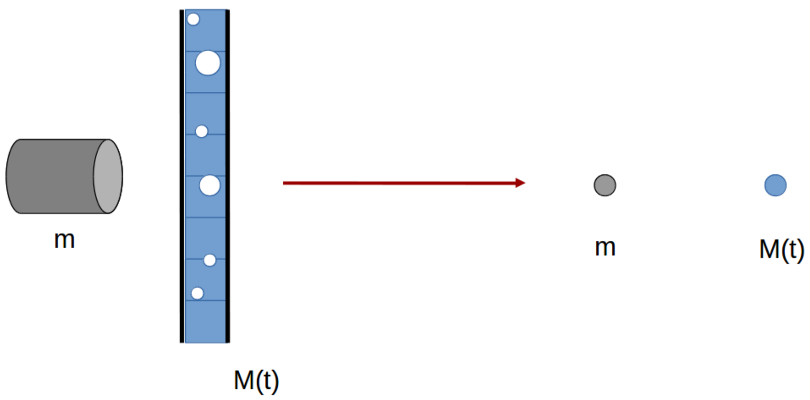

Figure 2: A depiction of the simplified model of the pipe. The mass on the relevant section of the pipe depends on the size and number of bubbles inside it

In the case of multiple bubbles on the relevant pipe section, each bubble will be set to grow at the same rate.

3 Noise Estimation Equation:

The acceleration produced by a point mass M onto another point mass separated by a distance r is given by:

$$a = \frac{GM}{r^2}; \tag{1}$$

Where $G = 6.674[Nm^2/Kg^2]$ is the gravitational coupling constant.

Suppose now that the mass M fluctuates about a mean value M_0 such that $M(t) = M_0 + \Delta M(t)$. Then we can model the differential acceleration as:

$$\Delta a = \frac{G}{r^2}\Delta M; \tag{2}$$

Corresponding to the mass fluctuations of this point mass.

Given the linear relationship between a and ΔM, to model the ASD of the displacement associated with a we only need to scale the Amplitude of ΔM according to:

$$|\Delta x(\omega)| = \frac{\Delta a}{\omega^2} = \frac{G}{\omega^2 r^2}|\Delta M(\omega)| \tag{3}$$

4 Mass Fluctuation model

The rate of mass evaporation is given by:

$$\frac{dm}{dt} = \frac{P}{l_{vap}} \tag{4}$$

Where P is the power dissipated by the cooling system of the outer shield and l_{vap} is the latent heat of vaoprization of nitrogen. This means that at any point that bubbles are not leaving the pipe, the volume occupied by them increases by:

$$\frac{dV_{bub}}{dt} = \frac{P}{l_{vap}\rho_{gas}} \tag{5}$$

Meaning that the amount of mass that gets pushed out of the pipe due to the expansion of the bubbles is given (in this simple model) by:

$$\frac{dM_{liq}}{dt} = -\rho_{liq}\frac{P}{l_{vap}\rho_{gas}} \tag{6}$$

Finally, the change of mass in the pipe is due to both the growth of the bubble (gaseous mass) and the liquid that gets pushed out.

$$\frac{dM}{dt} = \frac{dm}{dt} + \frac{dM_{liq}}{dt} = -\left(\frac{\rho_{liq}}{\rho_{gas}} - 1\right)\frac{P}{l_{vap}} \tag{7}$$

This implies that whenever bubbles are not entering/leaving the pipe,the mass of the pipe decreases as:

$$M(t) = M_0 - \left(\frac{\rho_{liq}}{\rho_{gas}} - 1\right)\frac{P}{l_{vap}}t \tag{8}$$

4.1 Instant Leaving Model:

The first leaving model assumes three basic things:

- The bubbles can appear uniformly on the pipe and do not interact with each other or the walls of the pipe.
- The bubbles share the total growth from the power consumption evenly, meaning that if there are N bubbles at any given time, the power used to grow one of them goes as P/N.
- The bubbles leave the pipe abruptly after reaching the end of it.

With these assumptions, each time a bubble leaves, we get a jump on $M(t)$. To model the Newtonian Noise, we will use the following numbers:

- $\rho_{liq} = 808.6 Kg/m^3$, density of liquid nitrogen at boiling point and 1 atm.
- $\rho_{gas} = 4.6 Kg/m^3$, density of gaseous nitrogen at boiling point and 1 atm.
- $l_{vap} = 2 \times 10^5 J/Kg$, the latent heat of vaporization at 1 atm and regular boiling point ($77K$).
- $P \approx 600 J/s$ the estimated power drained by the outer shield.

For fixed physical parameters $(P, \rho_{liq}, \rho_{gas}, l_{vap})$ the dominant factor of the ASD curve turns out to be the bubbling rate of the pipe, defined as the number of bubbles generated per second on the pipe ($\frac{dNbub}{dt}$). Larger bubbling rate implies more bubbles in the pipe at any given time and consequently more frequent but smaller jumps when bubbles leave the relevant area.

The effect of the bubbling rate on the ASD can be seen on the figure 3below:

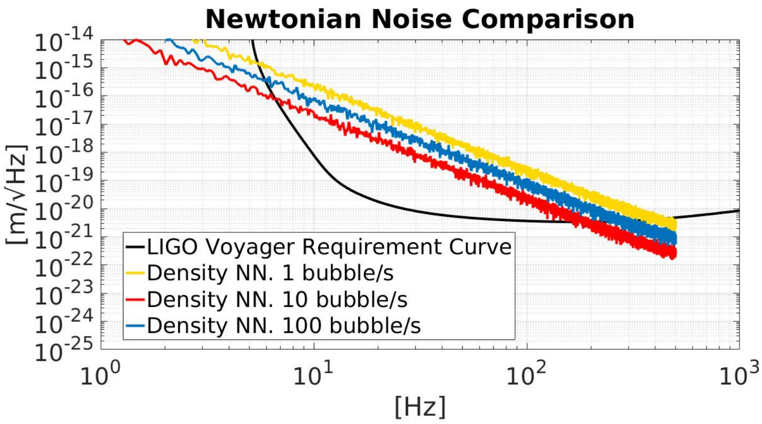

Figure 3: Displacement ASD generated by different instances of the Newtonian Noise model. For a fixed travel time on the pipe of 10 seconds various bubbling rates are compared.

It can be inferred directly form figure 3 that the amplitude of the mass fluctuations goes as $|M(\omega)| \propto \left(\frac{dN_{bub}}{dt}\right)^{-\frac{1}{2}}$. This is reminiscent of the way that

Shot noise behaves and it might not be a mere coincidence (We should explore this more).

On the other hand, for a fixed bubbling rate, the time spent by the bubbles on the pipe does not affect the ASD of the mass fluctuation. This can be seen on figure 4

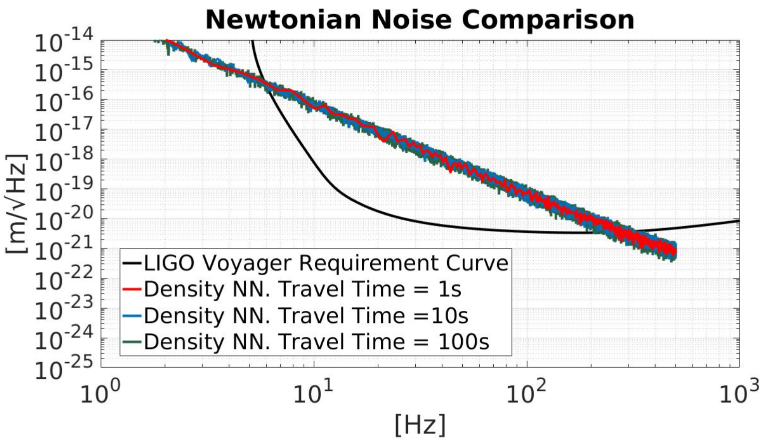

Figure 4: Displacement ASD generated by different instances of the Newtonian Noise model. For a fixed bubbling rate of 10 bubbles per second different travel times are compared.

This model, while very simple, with the dissapearances of bubbles gives us insight on the general behavior of the ASD curve. Moreover, the dependencies turn out to hold even for slightly more complicated models that smoothly fade bubbles out of the area of influence.

4.2 Fade Models:

We next turn to investigate what happens if we add a fade out function to the bubbles. So that instead of dissappearing abruptly from the pipe, their contribution to the relevant mass slowly fades out. We test three simple fade models before turning to a more physical one, just for the sake of investigating their influence on the ASD.

The first fading function is the instant fade, which is exactly what we had before. Bubbles abruptly dissapear after reaching the end of the relevant area.

The second one is a linear fade. Which turns the mass of the bubble off linearly up to a fading time.

The third one is a smooth fade, which is a refined version of the linear fade with zero first and second derivative at the fading ends.

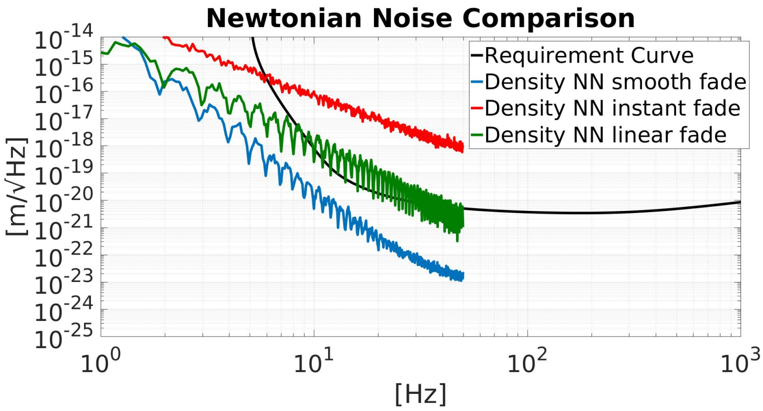

Figure 5: Displacement ASD generated by different instances of the Newtonian Noise model. For a fixed bubbling rate of 10 bubbles per second different fade-out smoothings are compared. For the non-instant fades we set a fading time of 1 second. (Distances are 1m)

As shown in figure 5, the fade out smoothing can have a strong impact on the predicted ASD for the model and should be addressed as an important feature of the model. However, to get a stronger sense of the right fade out function that our system might have, we need to make an explicit choice for the geometry of our piping system.

4.3 Inverse-Square fade

One argument that can be made about the fading out of the bubbles is that their influence goes away as the inverse of their distance to the test mass. So if R_0 gives the original distance that the point mass modeling the relevant section of the pipe, the influence of a bubble goes as:

$$F_{bubble} = F_0 \left(\frac{R_0}{R} \right)^2 = F_0 \left(\frac{1}{1 + \frac{\bar{v}}{R_0} t} \right)^2 \tag{9}$$

Where the last equality was obtained assuming that the bubble moves away (on the test mass's axis) with a constant speed. In practice, this speed can be approximated by the speed of the fluid flow on the pipe.

With these assumptions (Plus $R_0 = 1\text{m}$), we get the displacement ASD shown in the Figure 6.

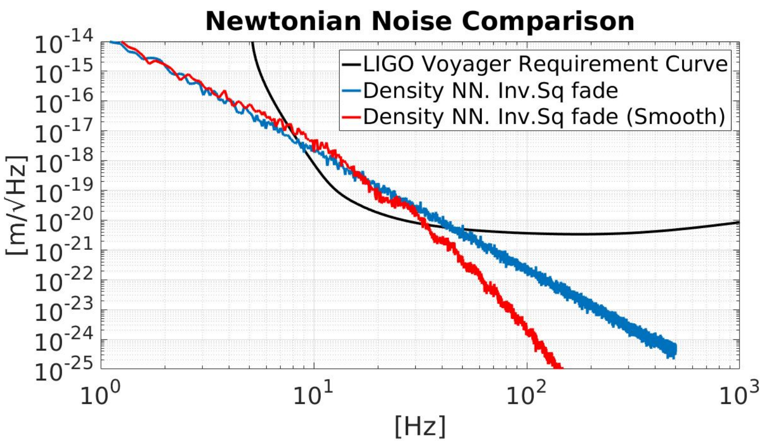

Figure 6: Displacement ASD for the mass fluctuation, in both cases the speed of the flow is set to 1 m/s.

The fading functions for both curves are shown in Figure 7. This tells us that eliminating the sharp corners on the transition of the bubbles has a string impact on the high frequency part of the displacement ASD.

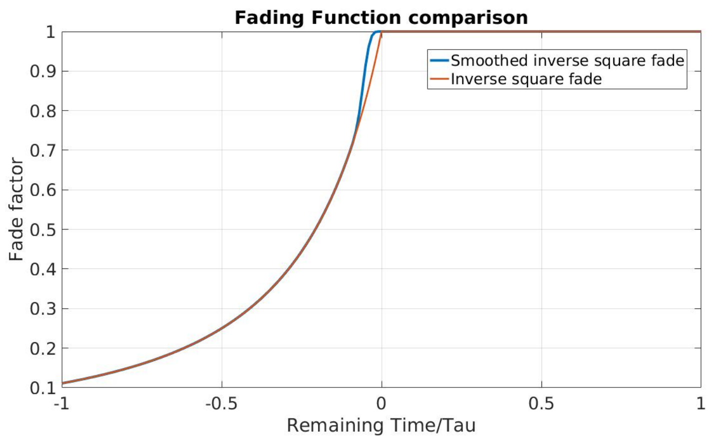

Figure 7: The smoothed and unsmoothed transitions for the bubbles used for the simulations on Figure 6.

In any case we can try to describe the effect that the speed change will have on the ASD, results are shown on Figure 8.

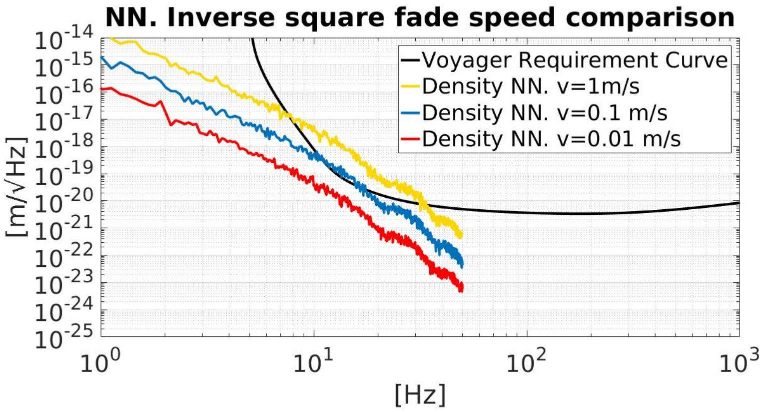

Figure 8: Comparison of different flow velocities on the ASD. In accordance with the equation 9.

It can be inspected directly from Figure 8 that the ASD is linear with respect to the fluid flow's speed.