Finite Element Analysis of the Third Generation Mirror Suspension Systems for Voyager

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Abstract

The Advanced Laser Interferometer Gravitational-Wave Observatory (aLIGO) first detected gravitational waves (GW) from a binary black hole merger on September 14, 2015 at both the Hanford and Livingston aLIGO sites. The transition from initial LIGO to aLIGO enhanced sensitivity across the frequency band of the detectors. One such change was increasing a one stage mirror suspension system to a multilevel system in order to improve isolation to seismic ground motion and thermal noise. The goal to improve aLIGO is to understand the sources of noise that decreases the sensitivity of the detectors. Thermal noise in the coatings and suspensions of the test masses is one of the main sources of noise that limits detector sensitivity within the suspension systems. Thermal noise results from the thermal energy of atoms and molecules in the mirrors and their suspensions (Saulson, 1990). Using the program known as ANSYS, Finite Element Analysis (FEA) is performed on different models of the mirror suspension systems. FEA allows the analysis of realistic features of the suspensions and using Levin's formalism can be used to calculate thermal noise in the detectors.

1 Introduction to the Laser Interferometer Gravitational-Wave Observatory Advancement

In 1916, Albert Einstein predicted the existence of gravitational waves from the field equations of general relativity. Gravitational waves travel at the speed of light as ripples in the curvature of spacetime (Einstein and Rosen, 1937). Gravitational waves are a strain in space-time caused by accelerating masses. These can be thought of as waves of distorted space being radiated by the source. The U.S. Laser Interferometer Gravitational-Wave Observatory, LIGO, has two facilities located in Hanford, Washington and Livingston, Louisiana. The LIGO detectors are dual recycled Michelson interferometers with Fabry-Perot cavities (Arrain and Mueller, 2008). The facilities were upgraded to improve sensitivity with the aim of achieving a factor of 10 improvement at the most sensitive part of the detection band. On September 14, 2015 a gravitational wave signal was simultaneously detected at both sites (Abbot, 2016) with a second detection following on December 26, 2015.

With the advancing of technology, the main goal for aLIGO is to become more sensitive such that the detections of gravitational waves can extend to other sources such as neutron-star binary mergers. A few differences between iLIGO and aLIGO are that the input laser power changed from 10 W to 180 W, the mirror mass changed from 10 kg to 40 kg, the power-recycled Fabry-Perot arm cavity Michelson became a dual-recycled Fabry-Perot arm cavity Michelson, the seismic isolation performance improved from 50 Hz to 12 Hz, and the mirror suspensions increased from a single pendulum to a quadruple pendulum (Weinstein, 2012). The suspension design of initial LIGO would have limited the sensitivity due to seismic noise and thus needed to be upgraded. Seismic noises come from the natural occurrences such as earthquakes and tidal waves to man-made sources such as traffic. Thermal noise results from the thermal energy of atoms and molecules in the mirrors and their suspensions which are at a finite temperature. According to the fluctuation dissipation theorem, off-resonance noise can be reduced by using low dissipation materials in our suspension system which stores most of this thermal energy close to the resonant modes (Callen and Welton, 1951). There are a number of noise sources that affect the sensitivity of the interferometer gravitational wave detector. The main sources of noise are seismic, gravitational gradient, thermal, and quantum noise.

A goal of physicists is to increase the amount of advanced gravitational wave networks because a global network of gravitational wave detectors will improve the ability to locate sources in the sky while also increasing the detection sensitivity. By multiplying the amount of interferometers globally, it will allow the system to enhance the network sky coverage and maximum time coverage. With the increase of interferometers, the detection confidence will increase because there will be an increase of signal detections for a single source. In order to increase the sensitivity, the current research is being done to decrease the different noises.

2 Motivation for Decreasing Thermal Noise in Advanced LIGO Detectors

The Advanced LIGO mirror suspensions have now been installed (Aston et al., 2012) and research has begun to look at possible future upgrades that would allow thermal noise to be further reduced. The Advanced LIGO suspensions use fused silica for the test masses and fibers. Fused silica helps reduce the amount of off resonance Brownian motion from the atoms and molecules. Brownian motion describes the random motion of particles from the dissipation and fluctuations within the system (Einstein et al., 1926). In order to reduce the thermal noise, one potential process is using cryogenic techniques (Rowan et al., 2005). The focus of this project is to investigate fused silica using Finite Element Analysis (FEA) to analyze their properties as cryogenically-cooled suspension fibers. The main goal of this project is to work towards building a full model suspension system to allow for direct calculation of the mechanical admittance. By using FEA, the project will analyze built models for the gravitational wave detector mirror suspensions, specifically for third generation detectors using a fused-silica hybrid type suspension for the interferometry mirrors.

3 The Methods Used to Model the Suspension Systems

In this project FEA was performed using the computer program ANSYS. ANSYS allows a user to use FEA for multiple models using different analyses. The analyses that have been used to become familiar with the program are Modal analysis and Static Structural analysis. The goal is to continue to become familiar with the program in order to begin modeling the actual suspension system. By comparing analytically and computationally, there will be multiple checks on the ANSYS program to ensure the program accounts for the "real world" forces and natural phenomenon correctly. Not only is ANSYS being checked, but also the modeling techniques. This can be seen by the importance of ensuring the models have the correct meshing and mesh density and are built in such a way as to accurately reflect the real physical system. This is important because by accurately describing the mirror suspensions computationally, then experiments will be made with small increments without making a physical suspension system.

4 Calculating Violin Modes and Frequencies

A simple pendulum model was used to test the accuracy of our ANSYS model when analytically and computationally calculating the violin modes and frequencies. There were two individual pendulum models, one wire had a diameter of 400 microns and the other wire had a diameter of 800 microns. The suspended mass on each wire was 10 kg and the length of each wire was 0.6 m long, shown in Figure 1. These values were chosen because aLIGO fiber suspensions are 400 microns in diameter, suspending 10 kg per wire.

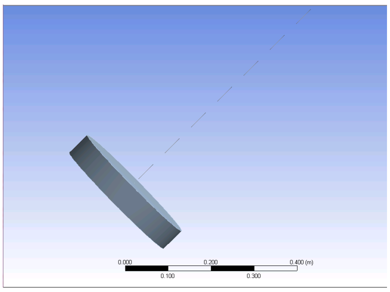
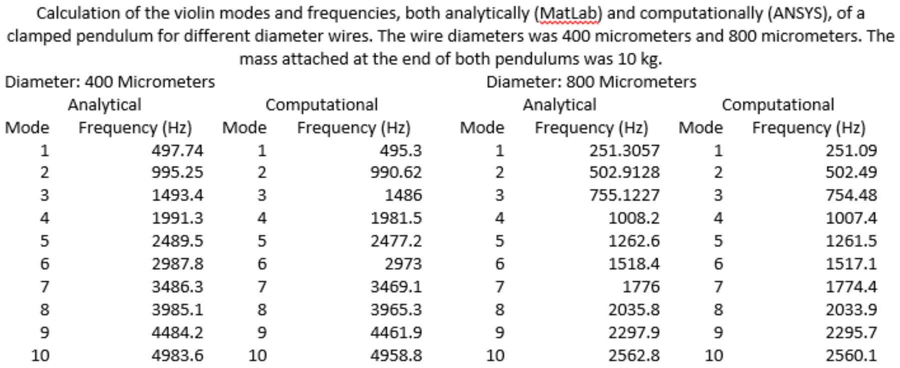


Figure 1: Simple pendulum model where the wire diameter is 400 microns with a length of 0.6 m and an attached mass of 10 kg.

The reason behind starting with a simple pendulum model was to check how ANSYS calculated the violin modes for a uniform wire. The complexity of the full suspension system increases processing power and in order to not waste time, smaller steps have been made to check that ANSYS is taking account of all factors. The actual fibers for the suspension system are tapered at the ends with varying diameters, which causes difficulty in analytically solving the violin modes by hand. By using a uniform fiber and analytically calculating the violin modes, the results can be compared reasonably to ANSYS in order to ensure that ANSYS in accurate. This will increase confidence in the program when using Finite Element Analysis (FEA) for the larger suspension system. The analytical equation for solving the violin frequencies was derived by Willems et al. (2002) which describes: $f_n = \frac{n}{(2l)}\sqrt{\frac{T}{\rho_l}}[1 + \frac{2}{l}\sqrt{\frac{EI}{T}} + \frac{EI}{2T}(\frac{n\pi}{l})^2]$, where n is the nth violin mode, T is the tension in the wire, l is the length of the wire, I is the moment of cross section, E is Young's Modulus of Elasticity, and ρ_l is the linear density.

For each different diameter of the wire, MATLAB was used to analytically solve for each violin frequency, shown in Figure 2.



Calculation of the violin modes and frequencies, both analytically (MatLab) and computationally (ANSYS), of a clamped pendulum for different diameter wires. The wire diameters was 400 micrometers and 800 micrometers. The mass attached at the end of both pendulums was 10 kg.

Diameter: 400 Micrometers				Diameter: 800 Micrometers			
	Analytical		Computational		Analytical		Computational
Mode	Frequency (Hz)	Mode	Frequency (Hz)	Mode	Frequency (Hz)	Mode	Frequency (Hz)
1	497.74	1	495.3	1	251.3057	1	251.09
2	995.25	2	990.62	2	502.9128	2	502.49
3	1493.4	3	1486	3	755.1227	3	754.48
4	1991.3	4	1981.5	4	1008.2	4	1007.4
5	2489.5	5	2477.2	5	1262.6	5	1261.5
6	2987.8	6	2973	6	1518.4	6	1517.1
7	3486.3	7	3469.1	7	1776	7	1774.4
8	3985.1	8	3965.3	8	2035.8	8	2033.9
9	4484.2	9	4461.9	9	2297.9	9	2295.7
10	4983.6	10	4958.8	10	2562.8	10	2560.1

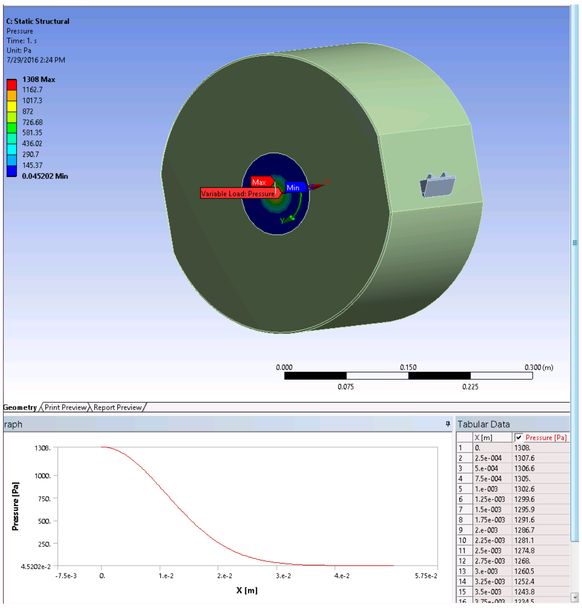
Figure 2: The analytical and computational comparison between the violin modes and frequencies of a beam in tension.

The results from the analytical solution were within 0.5% or below compared to the computational solution.

With this confirmation of the violin modes, then further study will go towards increasing the complexity of the model from a simple pendulum to the bottom stage of the mirror suspension system. The tapered thickness of the actual fibers can be analyzed using ANSYS through FEA. This will then be compared to other experimental ANSYS results in different studies in order to get a sense of using ANSYS towards modeling the full suspension system.

5 Calculation of Thermal Noise

Described by Levin, internal thermal noise can be calculated directly, by applying a pressure that mimics the light beam intensity and then calculating the energy dissipated in the mirror and suspension (Levin, 1998). A Gaussian pressure was applied to a face of a test mass with only the ears and a test mass with both the ears and fibers. The strain data was then exported from the test mass, ears, and fibers. The Gaussian pressure as a function of the radius can be shown from Coyne as $f(r) = \frac{1}{\pi(r_o)^2}e^{\frac{-r^2}{(r_o)^2}}$ where r_o is 1.56 cm. In Figure 3 the Gaussian pressure is shown in the static structrual analysis of ANSYS on the face of a test mass without wires.



	X [m]	☑ Pressure [Pa]
1	0.	1308.
2	2.5e-004	1307.6
3	5.e-004	1306.6
4	7.5e-004	1305.
5	1.e-003	1302.6
6	1.25e-003	1299.6
7	1.5e-003	1295.9
8	1.75e-003	1291.6
9	2.e-003	1286.7
10	2.25e-003	1281.1
11	2.5e-003	1274.8
12	2.75e-003	1268.
13	3.e-003	1260.5
14	3.25e-003	1252.4
15	3.5e-003	1243.8

Figure 3: ANSYS model of a Gaussian force applied to the face of an aLIGO optic.

The thermal noise can be calculated using Levin's formulation:

$$S_x(f) = \frac{2k_BT}{\pi^2 f^2}\frac{W_{diss}}{F_0^2}.$$

The strain energies were extracted from ANSYS in order to find the total loss angle of the system. While Levin's formulation of thermal noise allows the direct calculation of it, the method to follow Cumming et al., was used instead. This is where ANSYS is used to find the total loss in the system and then use a modal summation technique to calculate the resulting thermal noise. An analytical calculation using MatLab was used to calculate the power spectrum of displacement noise for a single pendulum with four wires. The noise calculation was made by using a modal summation of a pendulum and vertical modes for a 40kg mass made out of fused silica with 60cm length wires. In Table 1 the strain energies are shown for separate sections of the suspension. Figure 4 shows the resulting suspension thermal noise for the pendulum mode of the suspended optic.

Situation	Strain Energy (J)
Whole Part	1.212
Test Mass Only	1.121E-05
Both Ears Only	1.106E-04
Wires Only	1.212

Table 1: The computational results of the strain energies of the lower stage of the mirror suspension systems using FEA in ANSYS.

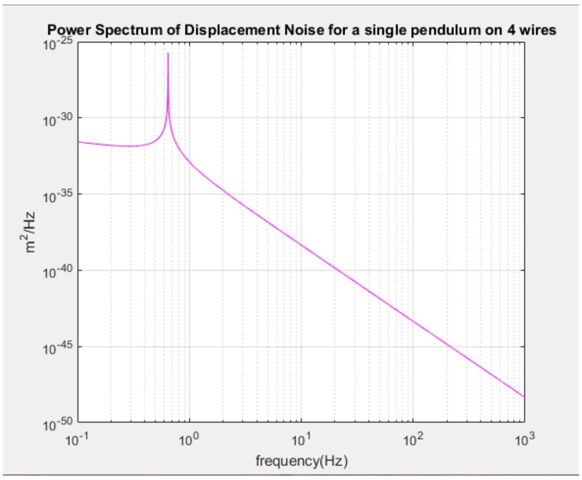


Figure 4: The Gaussian Force once applied to the face of the surface.

These calculations indicate the pendulum mode of the resonance frequency. Fused silica was chosen to be the fibers, test mass, and ears because one of the characteristics of fused silica is a high quality factor (Q). This means that the mechanical loss of fused silica is less. However, when the temperature decreases, the mechanical loss increases. The ongoing investigation into different materials such as silicon and sapphire is due to the decreased Q of fused silica as the temperature decreases. The strain energies can be used to directly calculate the thermal noise of the fibers when calculating $W_{diss} = E_{wire}\rho_{wire}$. The Levin approach allows the direct calculation of the fluctuation dissipation by using the strain energies of the system and the Gaussian force applied to the test mass face. This project was able to calculate the thermal noise and make the first few steps towards computationally calculating the noise using the Levin approach.

6 Conclusion

In conclusion, the continued effort to understand how to increase the sensitivity in the aLIGO mirror suspension systems will help the overall performance of aLIGO in detecting gravitational waves. The project shows the preliminary results through basic modal summation in order to give approximate measurements of the thermal noise in room temperature fused silica suspension. The final stage of the suspension model was implemented into ANSYS in order to compare the frequencies to the analytical modal summation analysis. Using ANSYS static structural model, strain energies were extracted from the wires. Overall, this project set the ground work for enhancing multiple techniques in using FEA for continual analysis of the suspension system.

7 Future Goals

The sensitivity curve has yet to reach the full potential of aLIGO and further improvements will be made in increasing the sensitivity. One goal is to calculate the thermal noise in the suspension systems at varying temperatures in order to compare and contrast different material for the test masses. Another goal is to directly apply the Levin technique using FEA in a way that allows us to sweep the frequency of the applied Gaussian force and to directly extract the thermal noise without using a modal summation technique. The basis of developing the suspension models will ultimately help analyze the performance of cryogenically cooled mirrors. The main goal is to find ways to decrease the thermal noise in the test mass, fibers, and ears. This work will pave the way for future projects in developing and building the full fledged mirror suspension system.

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9 References

Abbott, B. P., et al. (2016). Observation of Gravitational Waves from a Binary Black Hole Merger. Physical Review Letters.

Arain, M.A., and Mueller, G. (2008). Design of the Advanced LIGO recycling cavities. Opt. Express Optics Express, 16(14), 10018. doi:10.1364/oe.16.010018

Aston, S. M., Barton, M. A., Bell, A. S., Beveridge, N., Bland, B., Brummitt, A. J., . . . Wilmut, I. (2012). Update on quadruple suspension design for Advanced LIGO. Class. Quantum Grav. Classical and Quantum Gravity, 29(23), 235004. doi:10.1088/0264-9381/29/23/235004

Callen, H. B., & Welton, T. A. (1951). Irreversibility and Generalized Noise. Phys. Rev. Physical Review, 83(1), 34-40. doi:10.1103/physrev.83.34

Coyne, D. (2002, July 11). Thermal Noise Calculation with Inhomogeneous Loss using the Finite Element Method: Application to Test Mass Optics with Coating Loss, Attachments and Composite Assemblies [LIGO Laboratory/LIGO Scientific Collaboration]. LIGO.

Cumming, A., Heptonstall, A., Kumar, R., Cunningham, W., Torrie, C., Barton, M., . . . Rowan, S. (2009). Finite element modelling of the mechanical loss of silica suspension fibres for advanced gravitational wave detectors. Class. Quantum Grav. Classical and Quantum Gravity, 26(21), 215012. doi:10.1088/0264-9381/26/21/215012

Einstein, A., Furth, R., & Cowper, A. D. (1926). Investigations on the theory of the Brownian movement. London: Methuen &.

Einstein, A., & Rosen, N. (1937). On gravitational waves. Journal of the Franklin Institute, 304-316.

Levin, Y. (1998). Internal thermal noise in the LIGO test masses: A direct approach. Physical Review D Phys. Rev. D, 57(2), 659-663. doi:10.1103/physrevd.57.659

Rowan, S., Hough, J., & Crooks, D. (2005). Thermal noise and material issues for gravitational wave detectors. Physics Letters A, 347(1-3), 25-32. doi:10.1016/j.physleta.2005.06.055

Saulson, P. R. (1990). Thermal noise in mechanical experiments. Physical Review D Phys. Rev. D, 42(8), 2437-2445. doi:10.1103/physrevd.42.2437

Weinstein, A. (2012, June 21). Status of the Search for Gravitational Waves [The description of the Laser Interferometer Gravitational-Wave Observatory (LIGO) detectors around the world and the status of detecting Gravitational waves through technical advancements.].

Willems, P., Sannibale, V., Weel, J., & Mitrofanov, V. (2002). Investigations of the dynamics and mechanical dissipation of a fused silica suspension. Physics Letters A, 297(1-2), 37-48. doi:10.1016/s0375-9601(02)00380-8