引力波

LIGO-G1601928-v1

Control system in Gravitational Wave Detectors

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Introduction ~ Control?

- Gravitational wave detection
  Laser displacement sensor
  Requires linear displacement detection

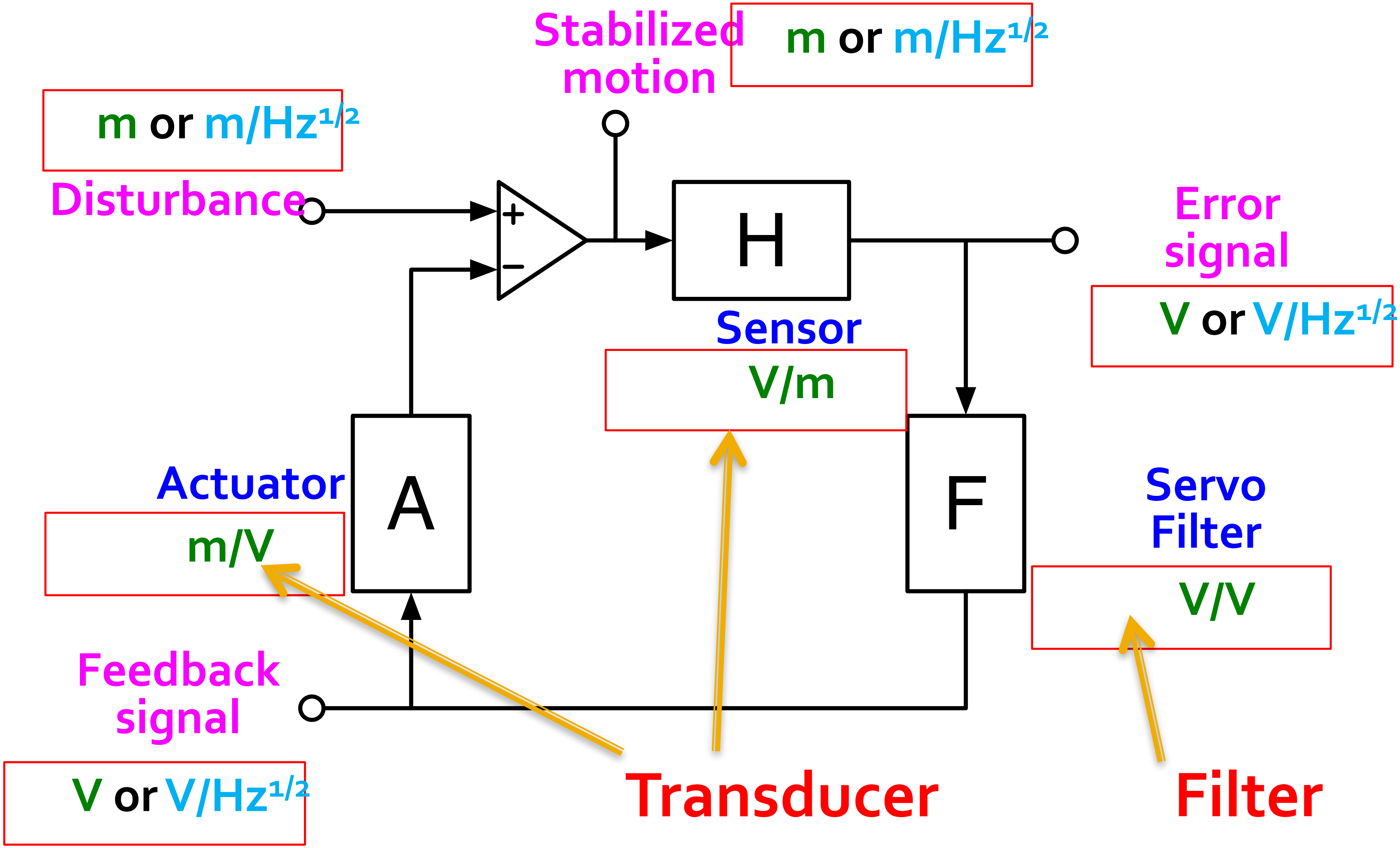
- Control for measurement
  Laser interferometer = nonlinear device
  Feedback control => linearization

Introduction ~ Control?

- What is the feedback control?
  - A scheme to monitor and modify output(s) of a system by changing the input(s) depending on the output(s)
- Examples
  - Shower temperature
  - Car driving
  - Tight rope walking
  - Air conditioning
  - Bike riding
  - Inverted bar on a hand
- Imagine what happens
  - If the response is too slow?
  - If the response is too fast?

Introduction ~ Control?

- Block diagram: Elements of a feedback loop



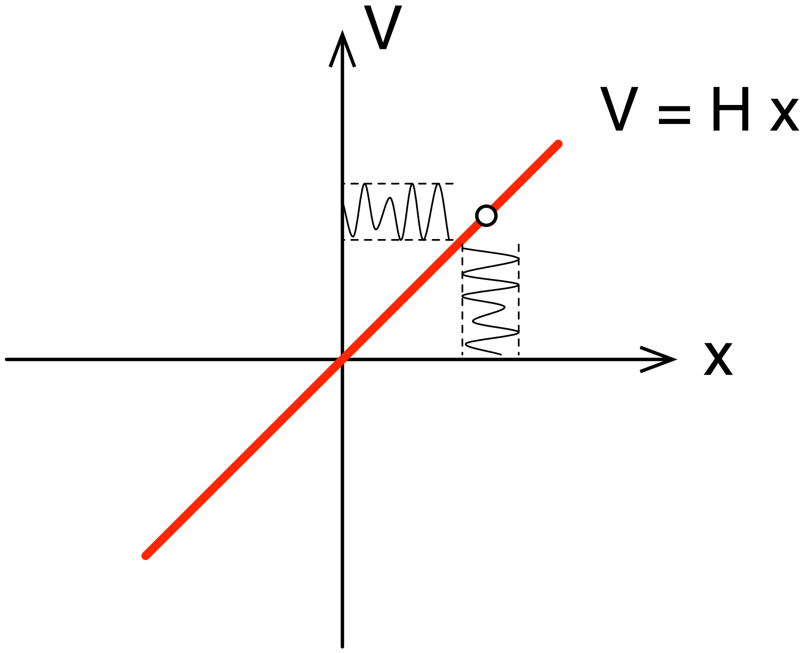
Introduction ~ Control?

- **Sensor:**

  Transducer for displacement-to-voltage conversion

- **<u>If the sensor is completely linear</u>**

  (and has or no frequency dependence)

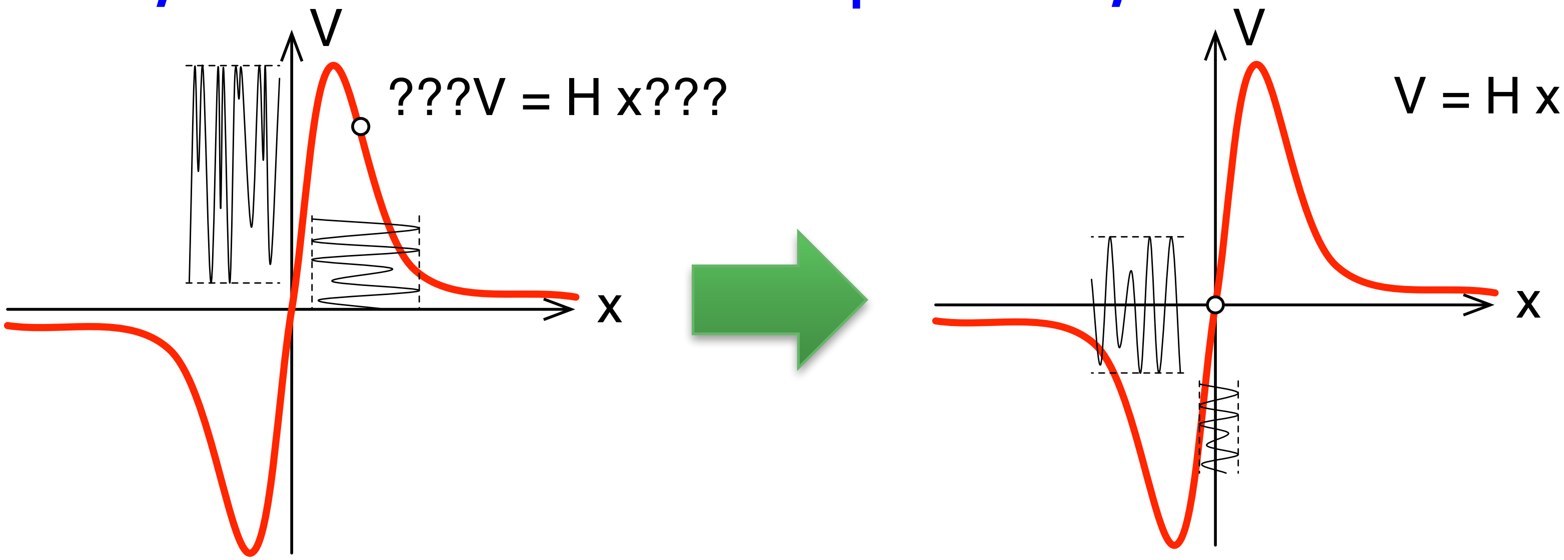


<u>We don't need feedback control!</u>

Introduction ~ Control?

- **In reality:**

Sensors, laser interferometers in particular, are nonlinear!



- <u>Enclose the operating point in the linear region</u>

  => The system recovers linearity

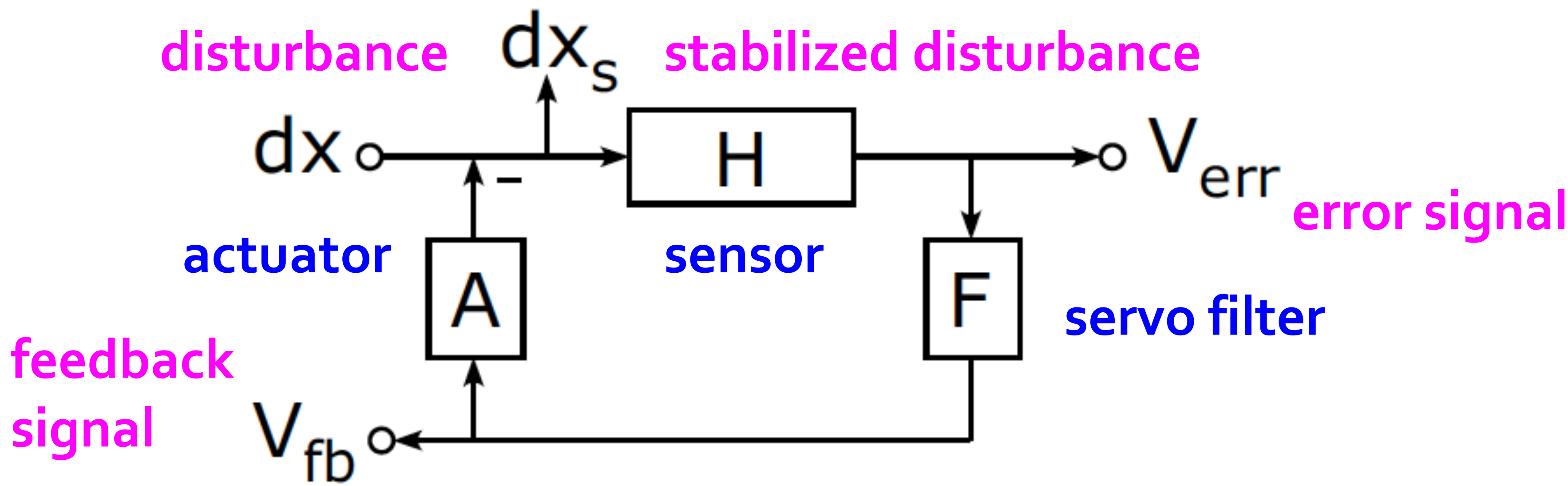
- **Was the displacement modified by the feedback?**

  => Precise knowledge of the control system

  for signal reconstruction

Introduction ~ Control?

- Elements of a feedback loop



$$dx_s = dx - G\,dx_s$$

$$\Rightarrow dx_s = dx\,/\,(1+G)$$

$$\Rightarrow dx = V_{err}\;(1+G)\,/\,H$$

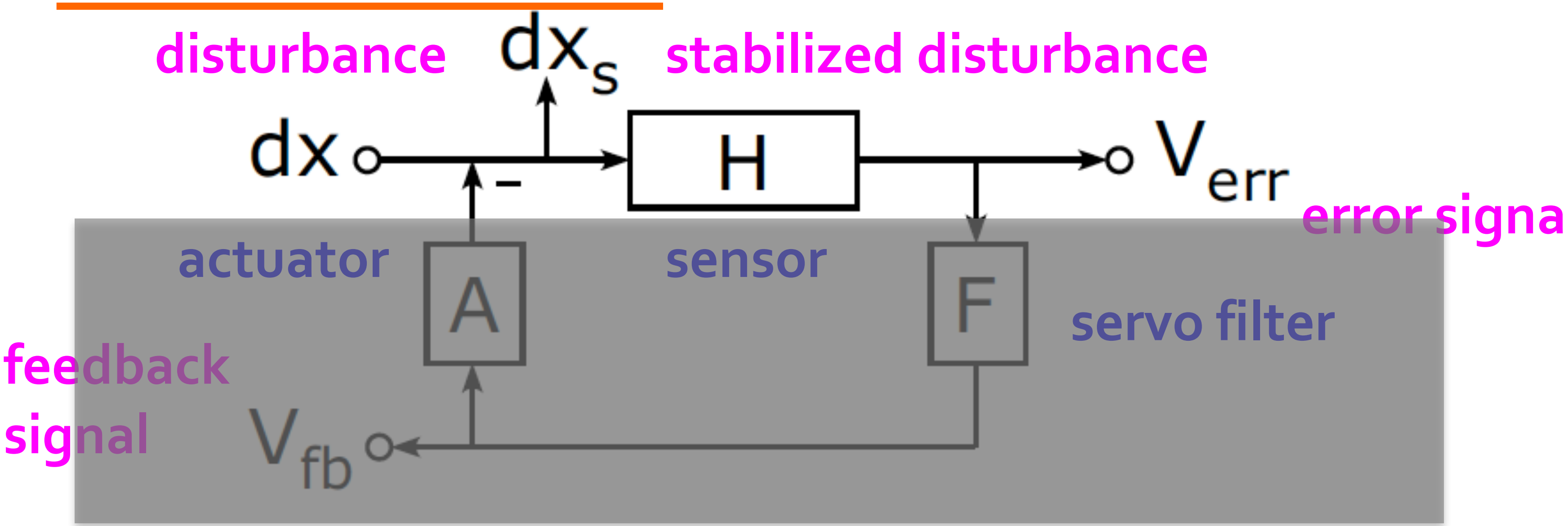
$$dx = V_{fb}\,A\,(1+G)\,/\,G$$

Open loop transfer function

$$G \stackrel{def}{=} H\,F\,A$$

Introduction ~ Control?

- **When G is small:**



Open loop transfer function

$$G \overset{\text{def}}{=} H\,F\,A$$

$$dx_s = dx - G\,dx_s$$

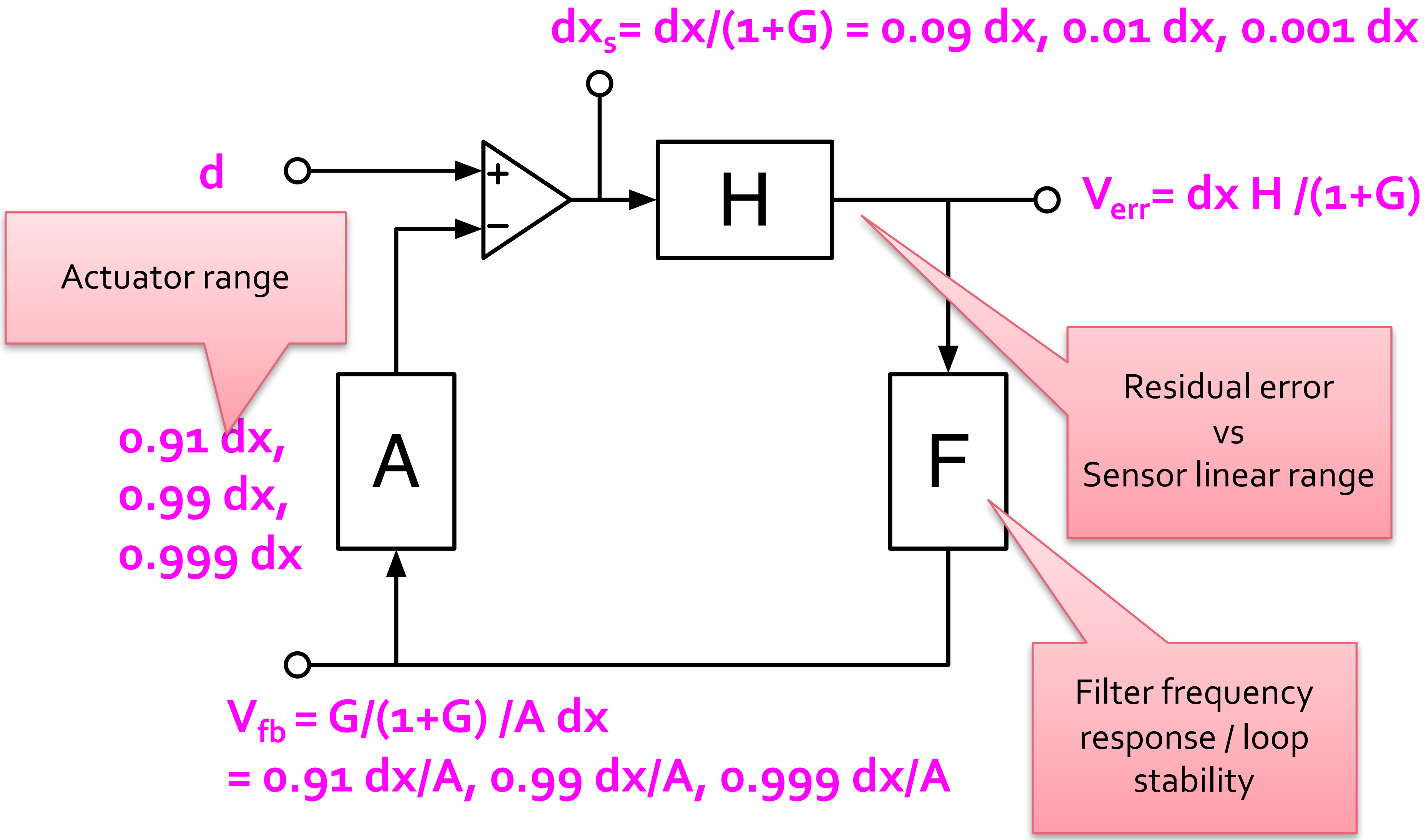
$$\Rightarrow dx_s = dx \,/\, (1+G)$$

$$\Rightarrow dx = V_{err}\,(1+G) \,/\, H$$

$$dx = V_{fb}\,A\,(1+G) \,/\, G$$

Introduction ~ Control?

- When G is big: e.g. G = 10, 100, or 1000



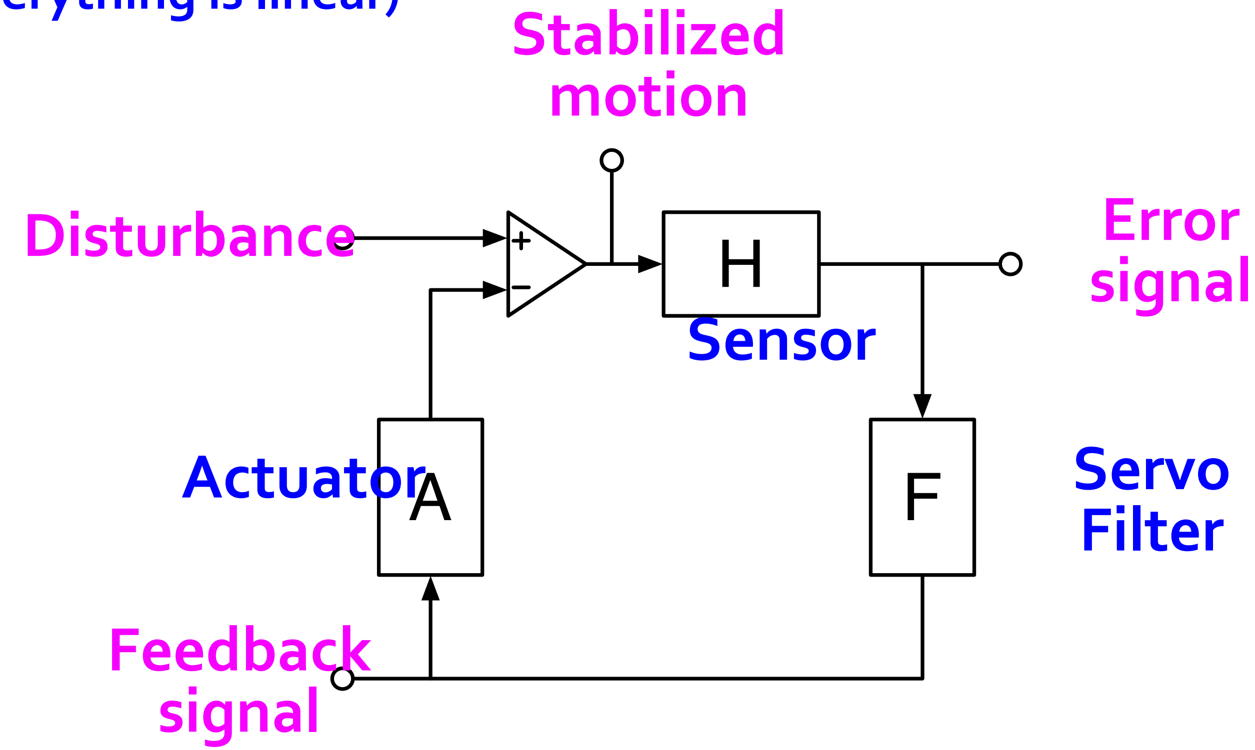
Introduction ~ Control?

- When the openloop gain G is >>1, the error signal gets suppressed

- "Wow! our sensor signal became smaller!"
  - Is our system more sensitive now? => No
  - Then, can we still measure gravitational waves even if the error signal is almost zero? => Yes

Introduction ~ Control?

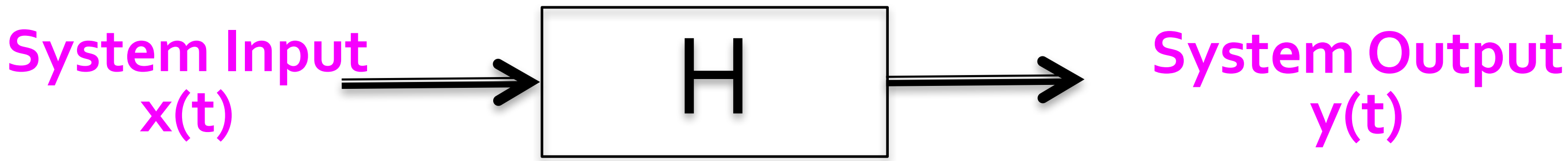
- Important difference between
  - "Feedback control for stabilization" and "Feedback control for measurement"
  - Feedback control changes the stabilized motion but reconstructed Disturbance is not modified by the loop* (*if everything is linear)



Linear systems and their stability

Linear systems and their stability

- A deterministic and time-invariant system: H



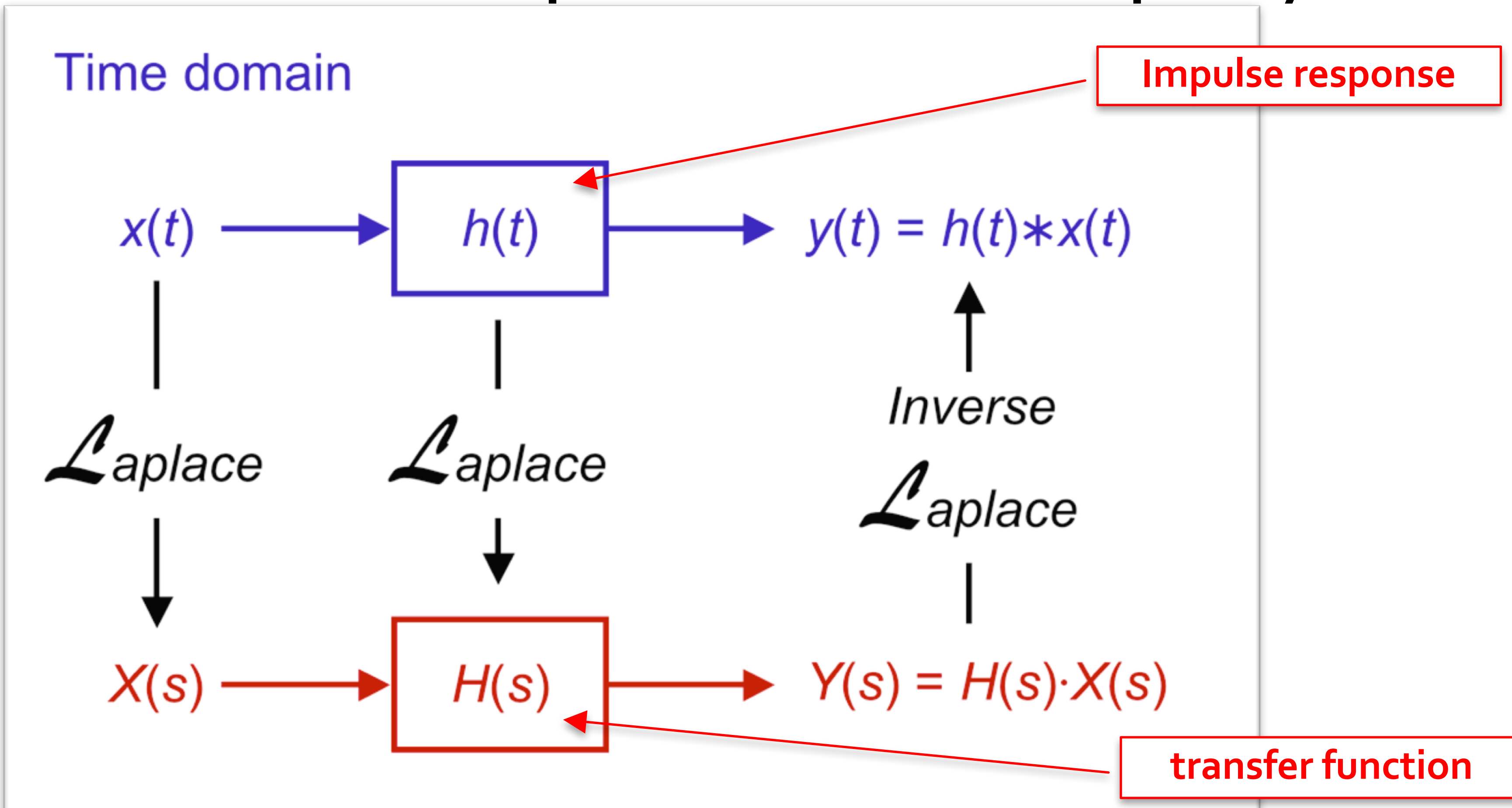
- The system H is LTI (linear & time-invariant) when

$$y_1(t) = H\left\{x_1(t)\right\}$$
$$y_2(t) = H\left\{x_2(t)\right\}$$
$$\Longrightarrow \alpha y_1(t) + \beta y_2(t) = H\left\{\alpha x_1(t) + \beta x_2(t)\right\}$$

- We can deal with such a system using Laplace transform (or almost equivalently Fourier Transform)

Linear systems and their stability

- **Time domain vs Laplace (or Fourier) frequency domain**



http://en.wikipedia.org/wiki/Linear_system
http://en.wikipedia.org/wiki/LTI_system_theory

Linear systems and their stability

- **It is easy to convert from an ordinary differential equation to a transfer function**

$$\frac{d}{dt} \Longrightarrow s$$

Laplace Transform

$$\Longrightarrow i\omega = i2\pi f$$

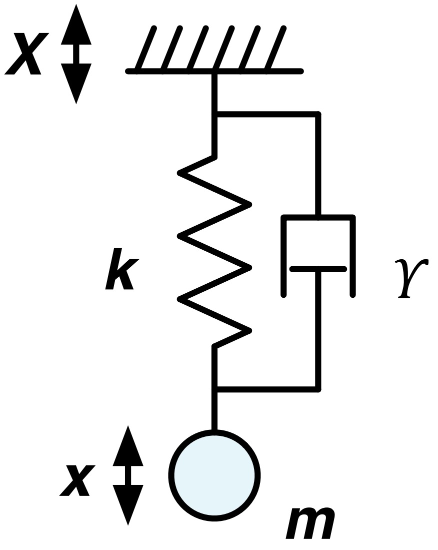
Fourier Transform

- **e.g. Damped oscillator**

$$m\ddot{x}(t) = -kx(t) - \gamma\dot{x}(t) + F(t)$$

$$ms^2X(s) = -kX(s) - \gamma sX(s) + F(s)$$

$$H(s) \equiv \frac{X(s)}{F(s)} = \frac{1}{ms^2 + \gamma s + k}$$



Linear systems and their stability

- **e.g. Damped oscillator**

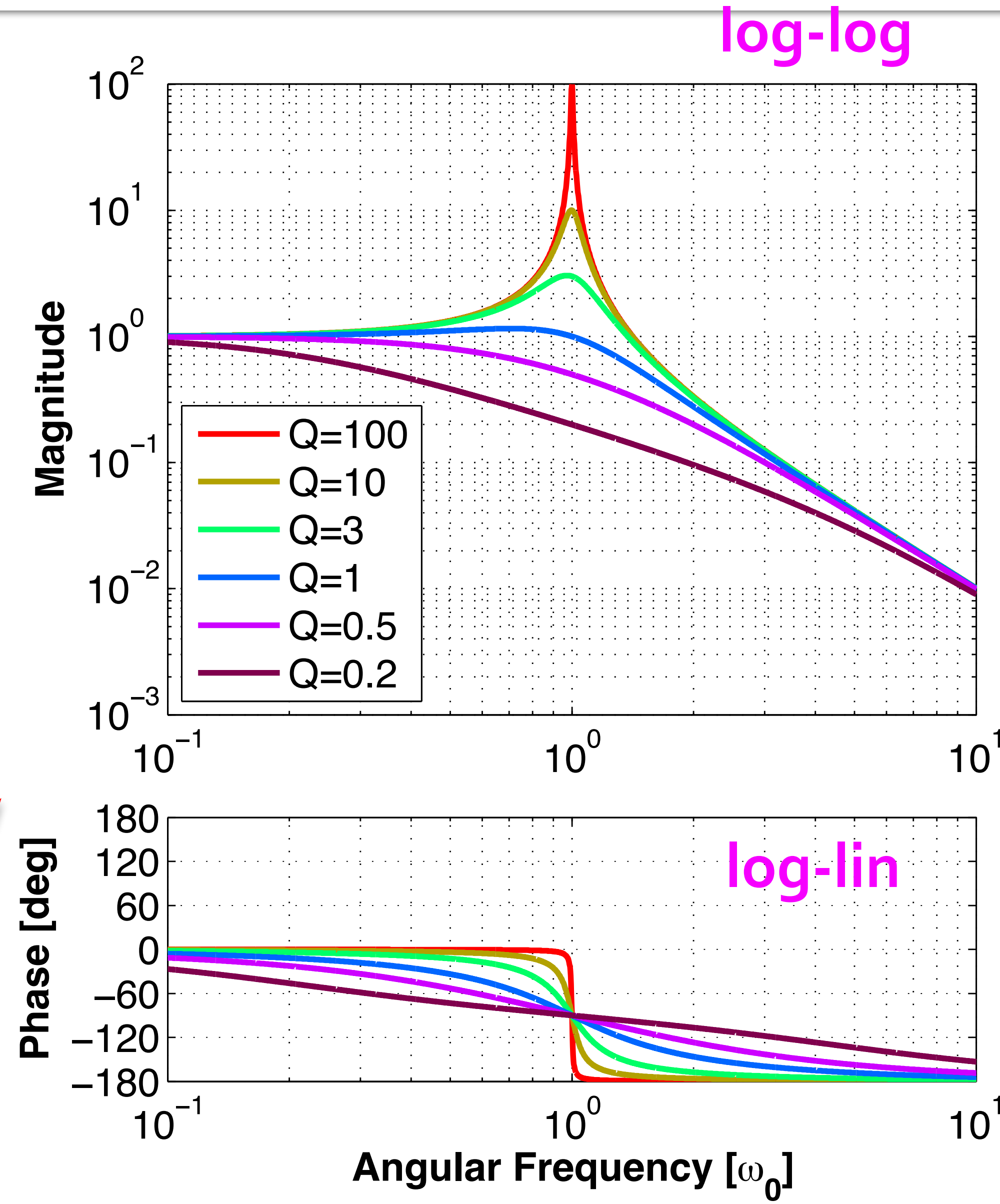
$$H(s) = \frac{1}{ms^2 + \gamma s + k}$$

$$H(s) = \frac{1}{m}\frac{1}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$

$$H(\omega) = \frac{1}{m}\frac{1}{-\omega^2 + i\frac{\omega_0}{Q}\omega + \omega_0^2}$$

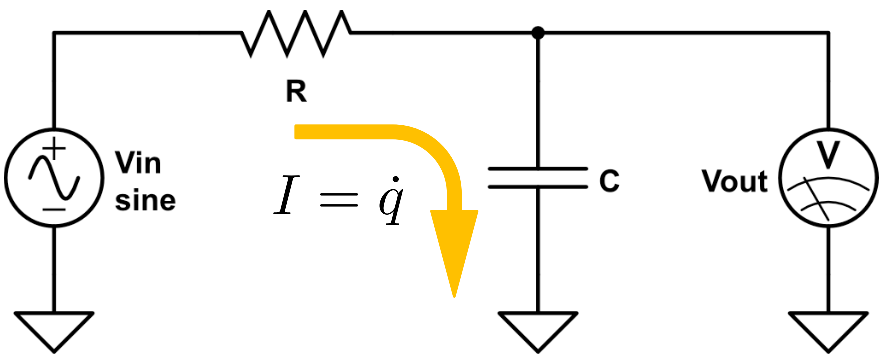
$$\omega_0 = \sqrt{k/m}, \quad \gamma = m\omega_0/Q$$

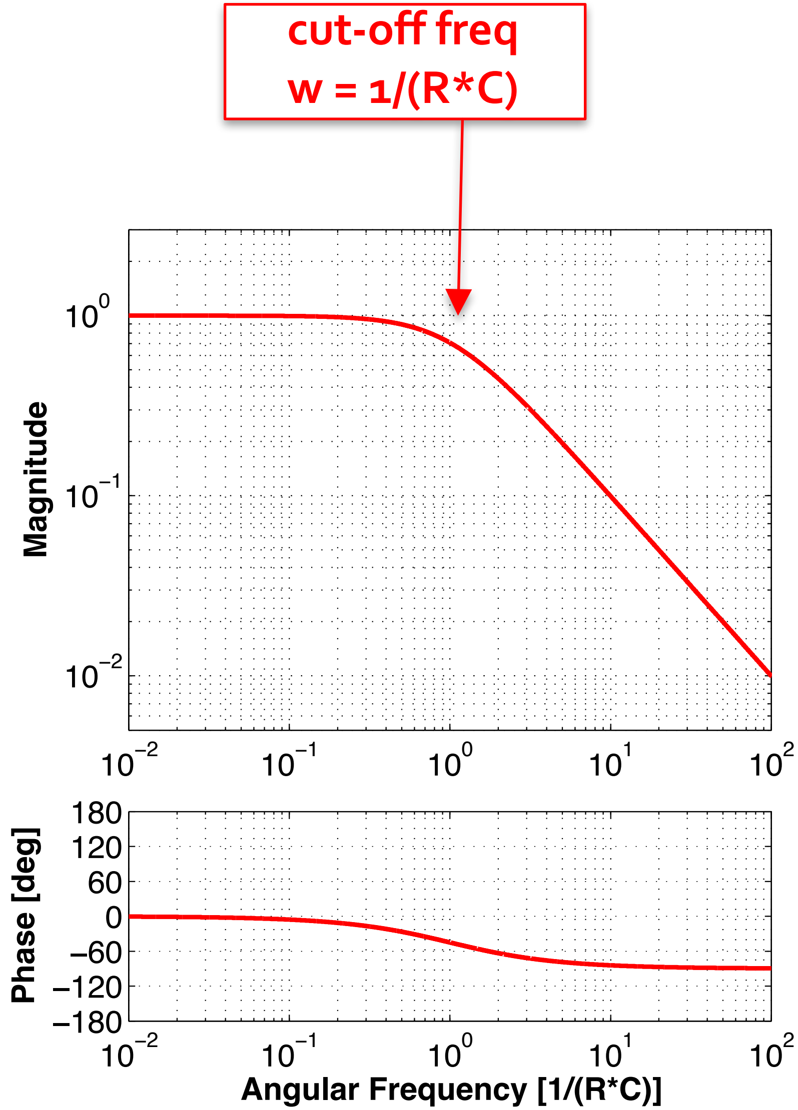
Bode diagram



Linear systems and their stability

- **e.g. RC filter**





$$V_{\text{out}} = q/C$$

$$\dot{q} = (V_{\text{in}} - V_{\text{out}})/R$$

$$\Longrightarrow i\omega C V_{\text{out}}(\omega) = (V_{\text{in}}(\omega) - V_{\text{out}}(\omega))/R$$

$$\Longrightarrow \frac{V_{\text{out}}(\omega)}{V_{\text{in}}} = \frac{1}{1 + i\omega RC}$$

Linear systems and their stability

- **In most cases, a system TF can be expressed as:**

$$H(s) = \frac{b_0 + b_1 s + b_2 s^2 + \dots + b_m s^m}{a_0 + a_1 s + a_2 s^2 + \dots + a_n s^n}$$

- **The roots of the numerator are called as "zeros" and the roots of the denominator are called as "poles"**

$$H(s) = \frac{b_m \prod_{i=1}^{m} (s - s_{zi})}{a_n \prod_{j=1}^{n} (s - s_{pj})}$$

- **Zeros (s_{zi}) and poles (s_{pi}) are**

  real numbers (single zeros/poles)

  or

  pairs of complex conjugates (complex zeros/poles)

Linear systems and their stability

- **Poles rule the stability of the system!**

  H(s) can be rewritten as

$$H(s) = \sum_{j=1}^{n} \frac{K_j}{(s - s_{pj})}$$

- **Each term imposes exponential time impulse response**

$$\text{T.F.: } H_j(s) = \frac{1}{s - s_{pj}} \Longleftrightarrow \text{I.R.: } h_j(t) = e^{s_{pj} t}$$

- **Therefore, if there is ANY pole with $\mathrm{Re}(s_{pj}) > 0$ the response of the system diverges**

Linear systems and their stability

- **Poles rule the stability of the system!**

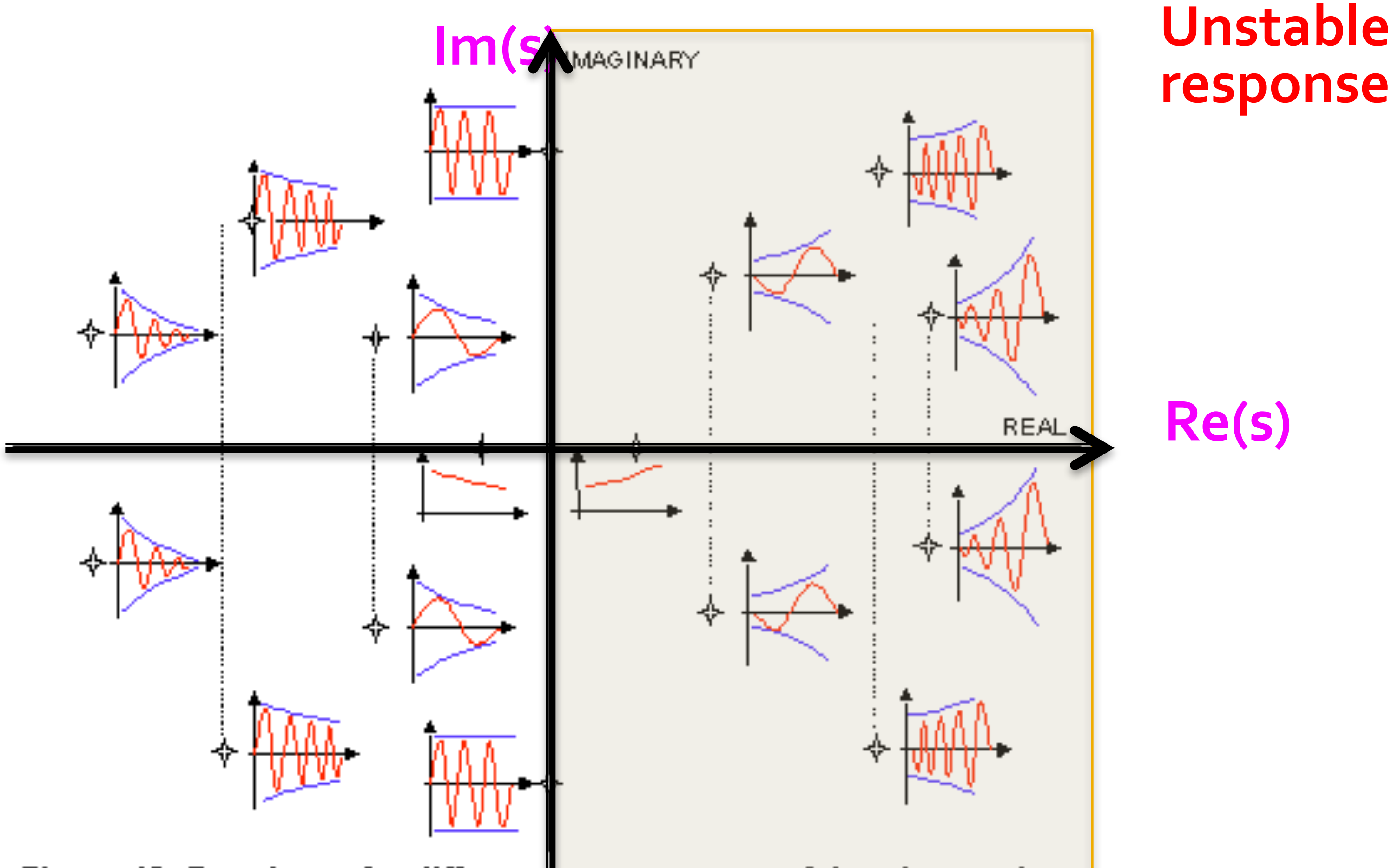
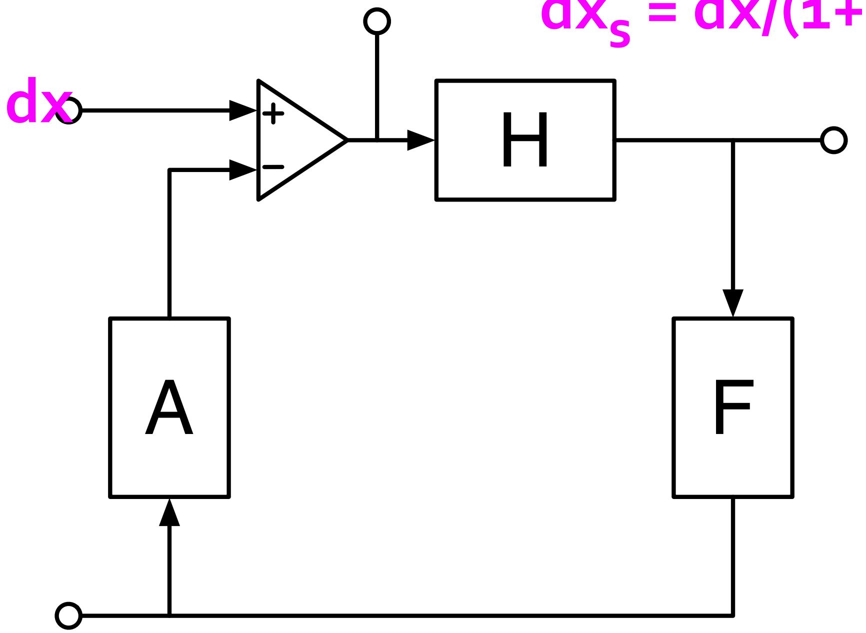


Figure 12: Root locus for different arrangements of the eigen values

http://nupet.daelt.ct.utfpr.edu.br/_ontomos/paginas/AMESim4.2.0/demo/Misc/la/SecondOrder/SecondOrder.htm

Linear systems and their stability

- Now we eventually came back to this diagram



$dx_s = dx/(1+G)$

Open loop TF:
$G = H\,F\,A$

Closed loop TF:
$G_{CL} = 1/(1+G)$

(!)

Requirement:

All the roots for 1+G should be in the left hand side of Laplace plane

Linear systems and their stability

- **Remarks**

Requirement:

All the roots for 1+G should be in the left hand side of Laplace plane

  - **This does not mean all H, F, A needs to be stable.**
    e.g. Unstable mechanical system A can be stabilized by a control loop. (cf. An inverted Rod)

  - **We usually play with F to tune the result.**
    It is awkward to evaluate the stability of 1/(1+G) every time.
    There is a way to tell the stability only from G

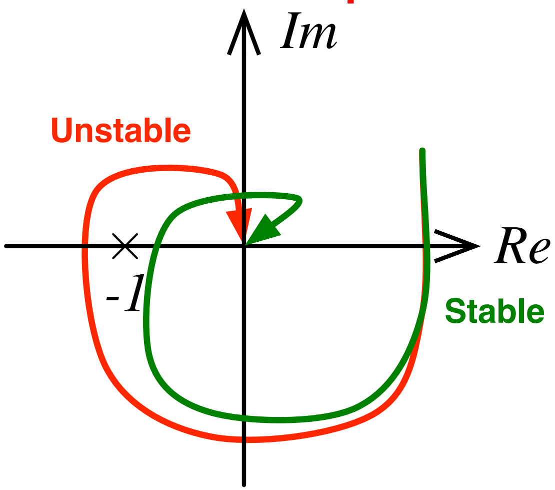
Open loop TF:
$G = H F A$

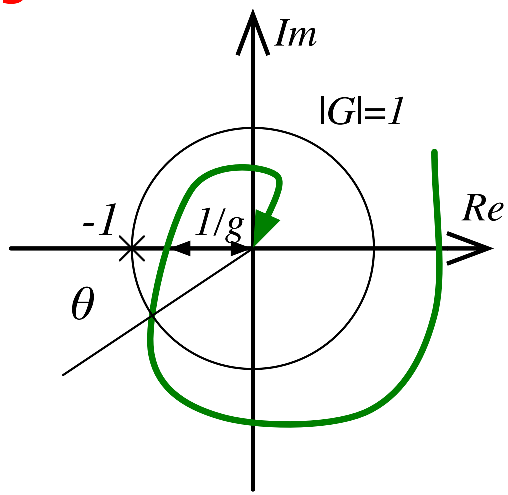
Closed loop TF
$G_{CL} = 1/(1+G)$

Nyquist's stability criterion

Linear systems and their stability

- **Nyquist stability criterion**
  - Plot openloop gain G in a complex plane (i.e. Nyquist diagram)
  - If the locus of $G(f)$ from f=0 to ∞, goes to 0 looking at the point (-1 + 0 i) at the left side => Stable
  - If the locus sees the point (-1+0 i) at the right side => Unstable

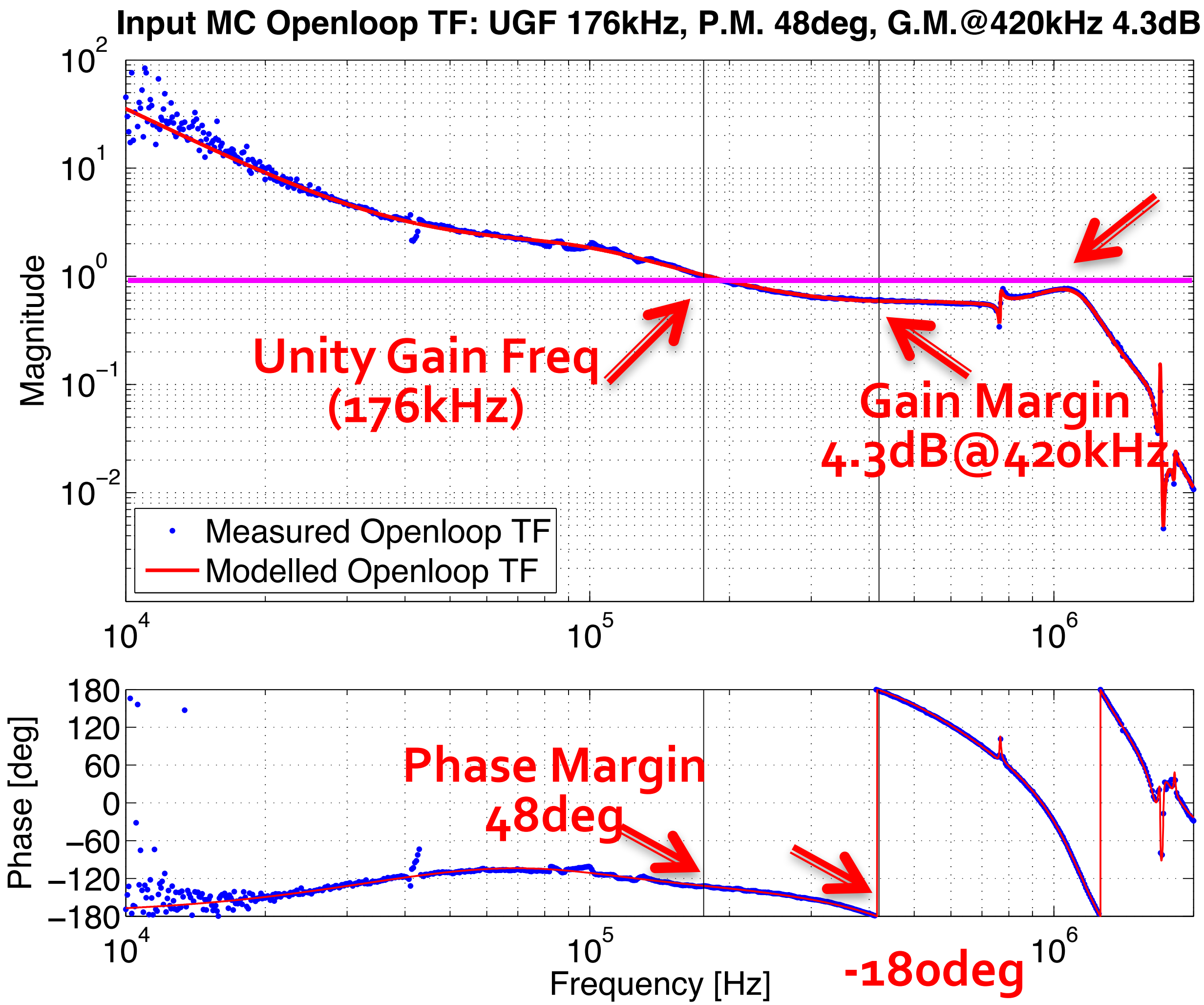




- Unity gain frequency f_{UGF} : for $|G(f_{UGF})| = 1$
- Phase margin ϑ : $\vartheta = \mathrm{Arg}(G(f_{UGF}))$
- Gain margin g: $g = 1/|G(f_o)|$ where $\mathrm{Arg}(G(f_o)) = -\pi$

Linear systems and their stability

- **Phase Margin / Gain Margin in Bode diagram**
  - Most of the case, a bode diagram of G is enough to see the stability



A rough standard of a stable servo loop:

Phase Margin > 40deg

Gain Margin > 10dB

Linear systems and their stability

- **Building blocks ("zpk" representation)**
  - **Single pole**

$$H(s) = \frac{s_p}{s + s_p} \qquad (s_p \in \mathbb{R}, s_p > 0)$$

  - **Single zero**

$$H(s) = \frac{s + s_z}{s_z} \qquad (s_z \in \mathbb{R}, s_z > 0)$$

  - **A pair of complex poles**

$$H(s) = \frac{s_p s_p^*}{(s + s_p)(s + s_p^*)} \qquad (s_p \in \mathbb{C},\ \Re(s_p) > 0)$$

  - **A pair of complex zeros**

$$H(s) = \frac{(s + s_z)(s + s_z^*)}{s_z s_z^*} \qquad (s_z \in \mathbb{C},\ \Re(s_z) > 0)$$

  - **Gain**

$$H(s) = K \qquad (K \in \mathbb{R})$$

Linear systems and their stability

- **Relationship between pole/zero locations and wo&Q**

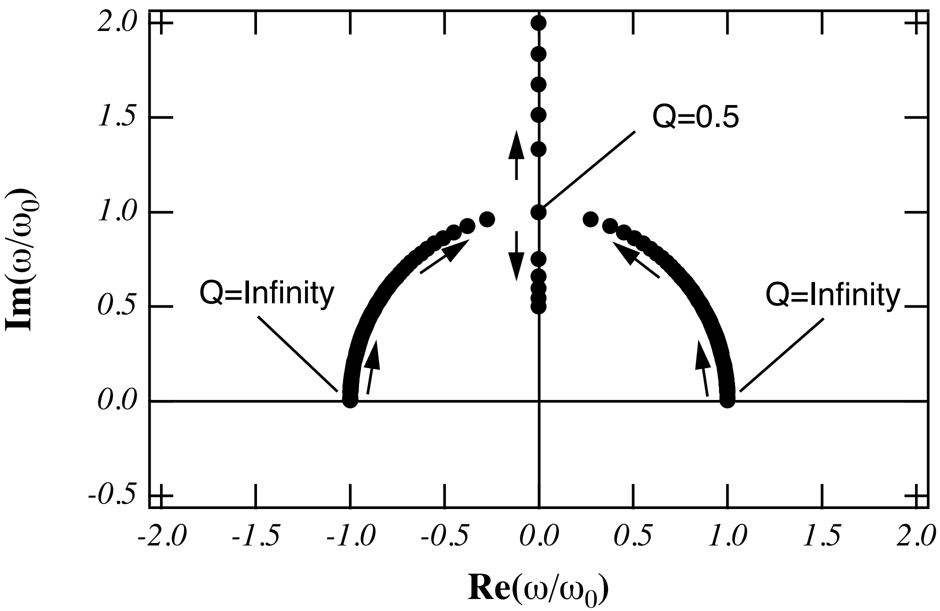
$$H(s) = \frac{s_p s_p^*}{(s+s_p)(s+s_p^*)}$$
$$= \frac{|s_p|^2}{s^2 + 2\Re(s_p)s + |s_p|^2}$$

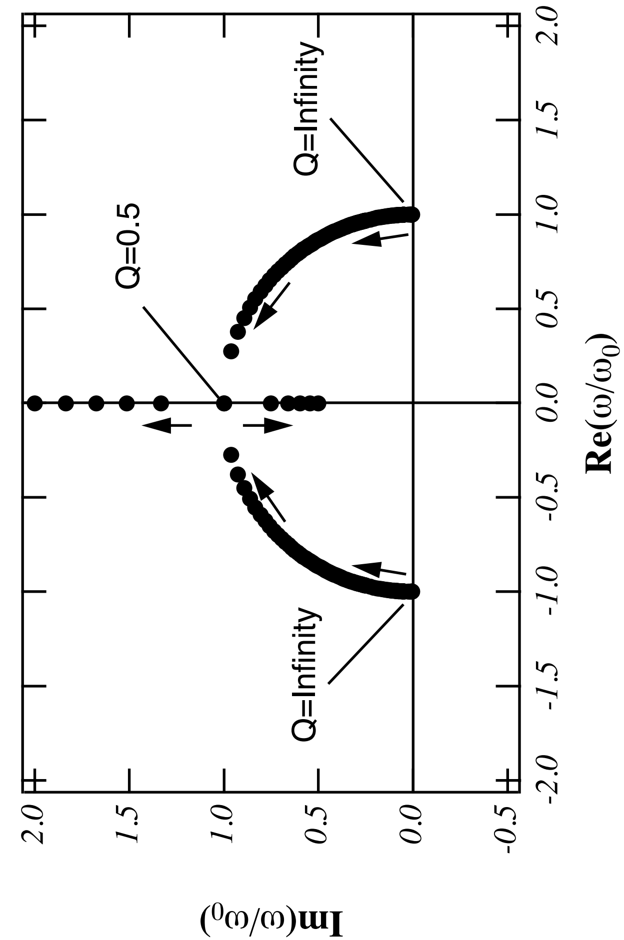
- **To be compared with**

$$H(\omega) = \frac{\omega_0^2}{-\omega^2 + i\omega_0\omega/Q + \omega_0^2}$$

$$\Longrightarrow \quad \omega_0 = |s_p|,\ Q = \frac{|s_p|}{2\Re(s_p)}$$

Linear systems and their stability





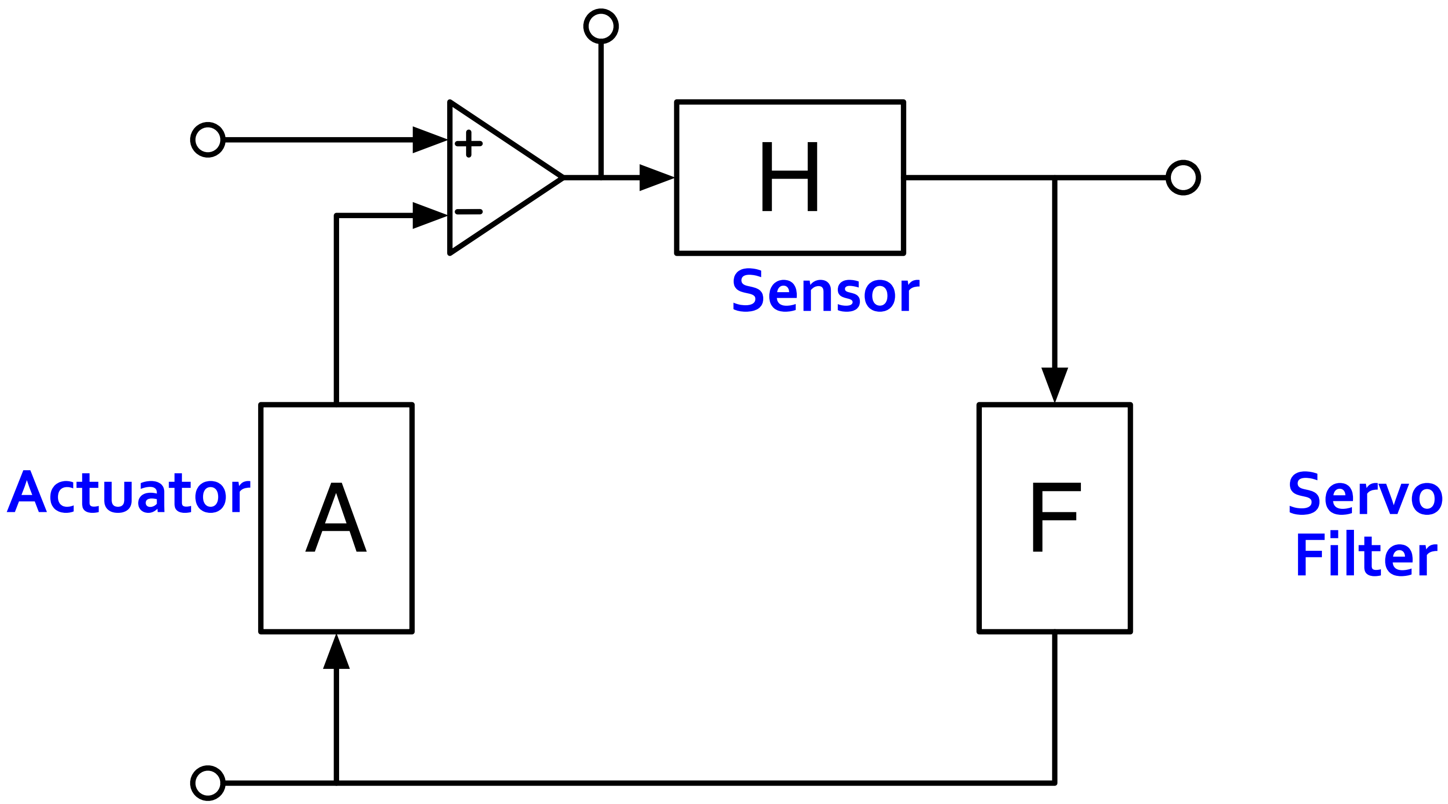
Linear systems and their stability

- Summary
  - Classical control theory
  - Design locations of poles and zeros
  - Stability: tuning of open loop transfer function is important

Control system components in GW detectors

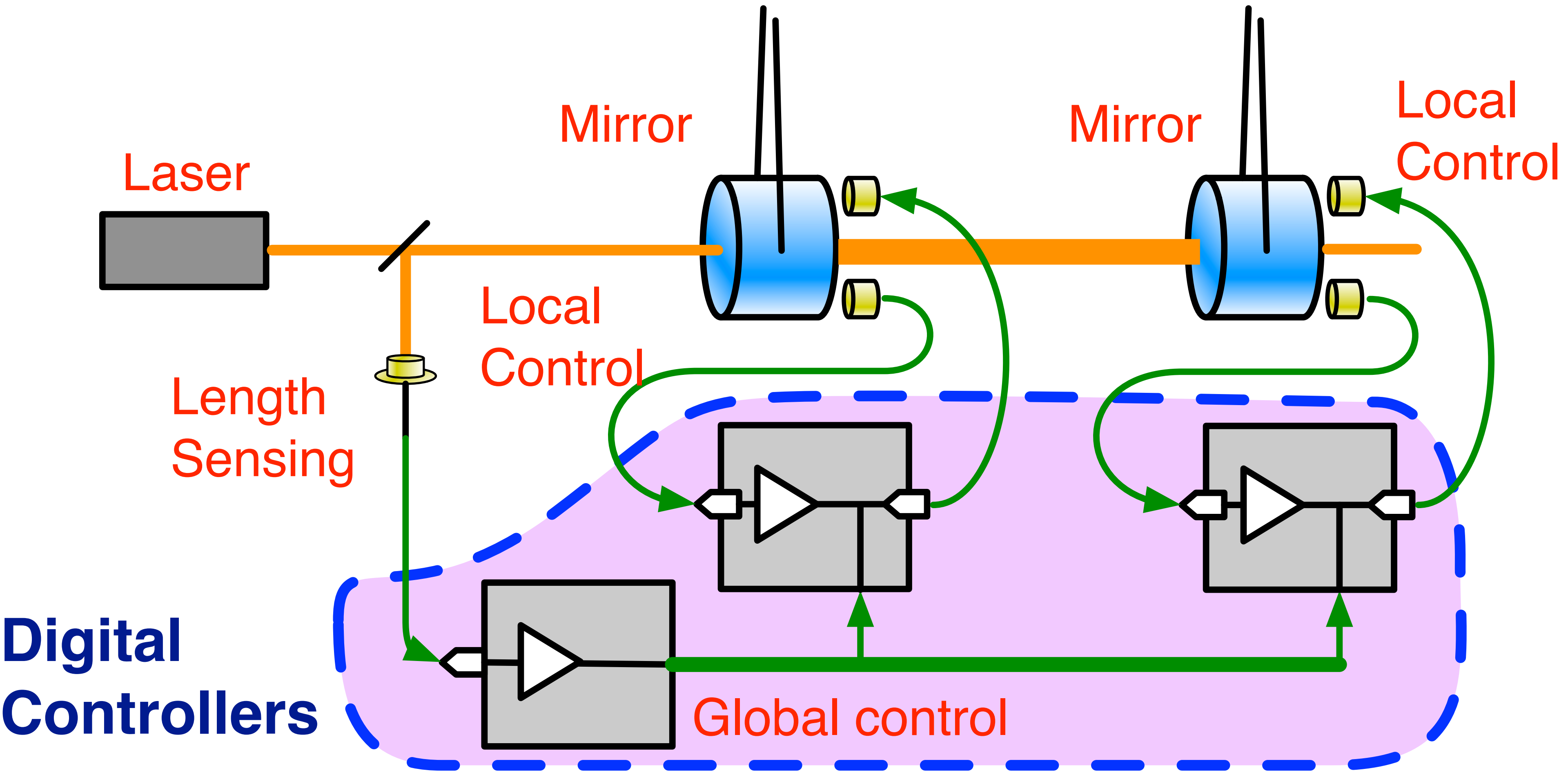
Control systems

- **Elements of a feedback loop (again)**



Interferometer control system

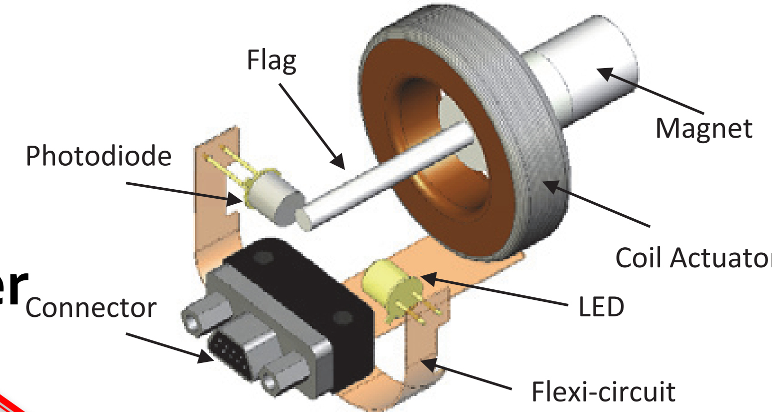
- Local control vs global control



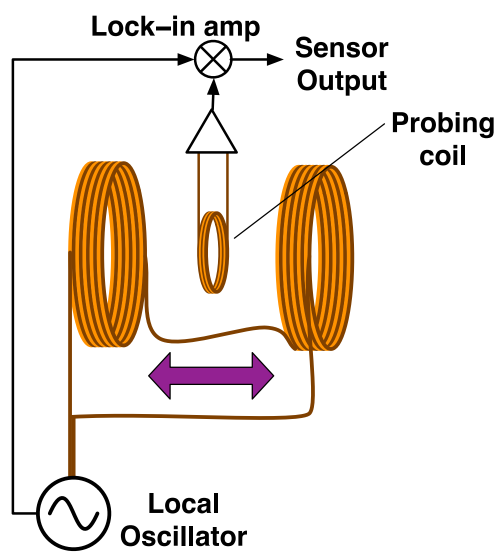
Local Sensors

- Shadow sensor (relative displacement sensor)
  - aLIGO: Birmingham Optical Sensor and Electro-Magnetic actuator (BOSEM)

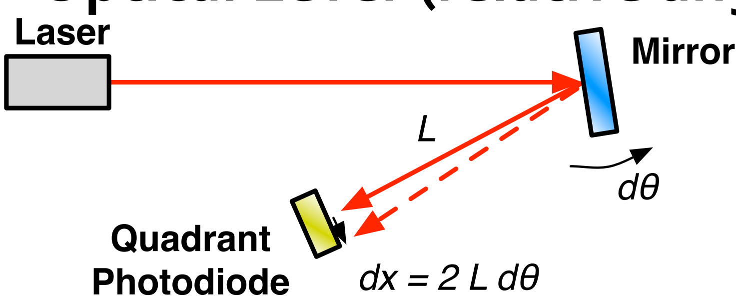
    L Carbone, Class. Quantum Grav. 29 (2012) 115005

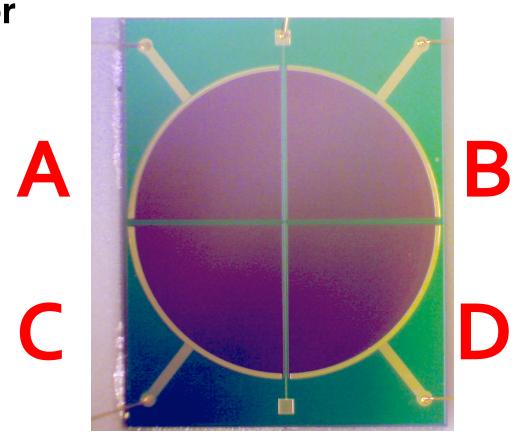


- Linear Variable Differential Transducer (relative disp. sensor)



- Optical Lever (relative angular sensor)

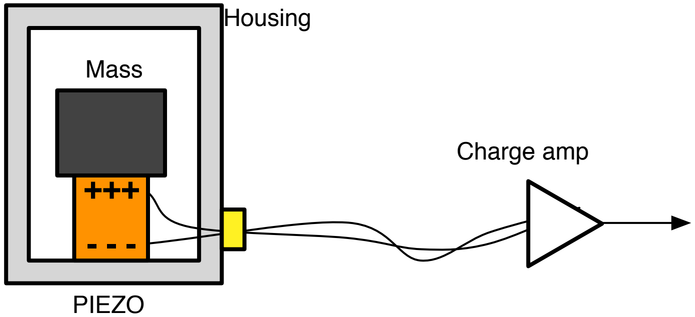




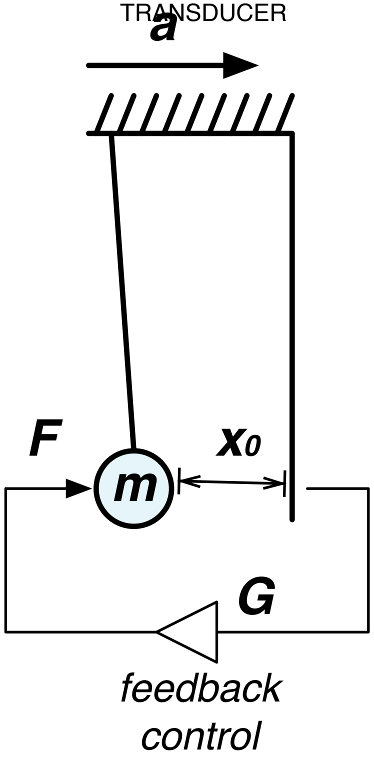
SUM = A+B+C+D
X = A−B+C−D
Y = A+B−C−D

Local Sensors

- Accelerometers (<u>Inertial sensor</u>)
  - Piezo Accelerometer



  - Servo Accelerometer



Local Sensors

- Servo Accelerometer (Inertial sensor)
  - Above the resonant freq: Limited by the sensor noise
  - Below the resonant freq:
    Steep rise of the noise as the mass does not move
    in relative to the ground
    => Low resonant freq is beneficial

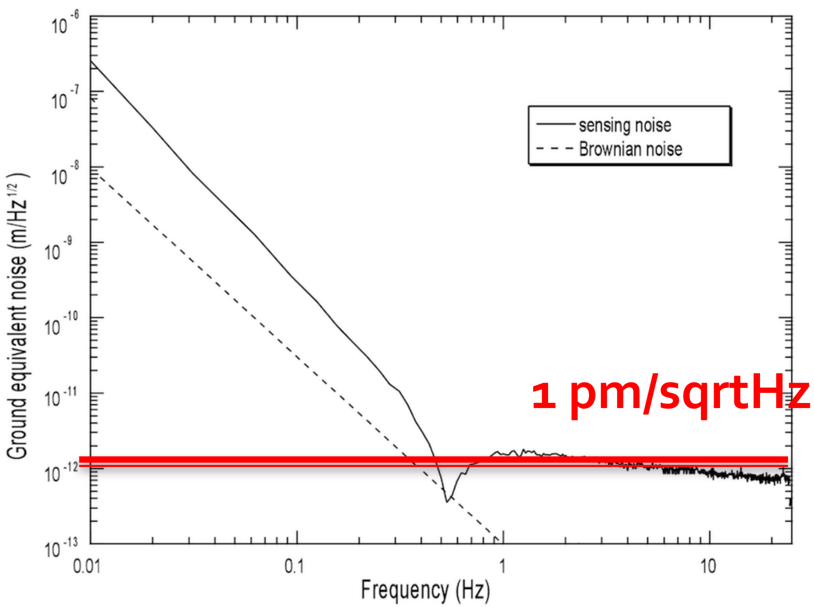
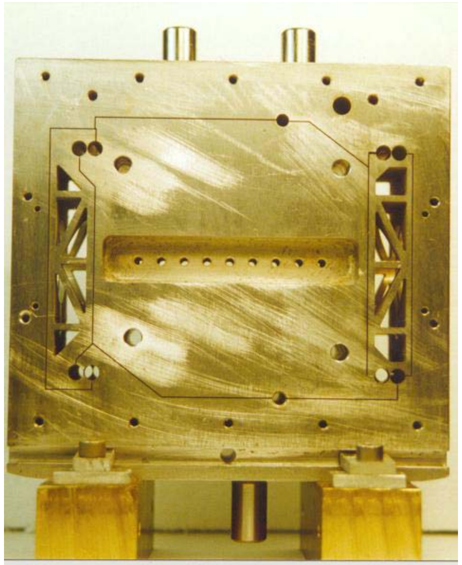
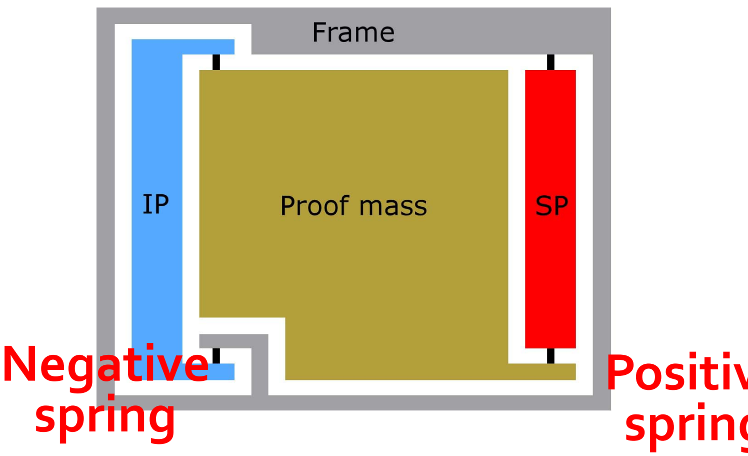


Fig.7. Equivalent frame displacement noise.

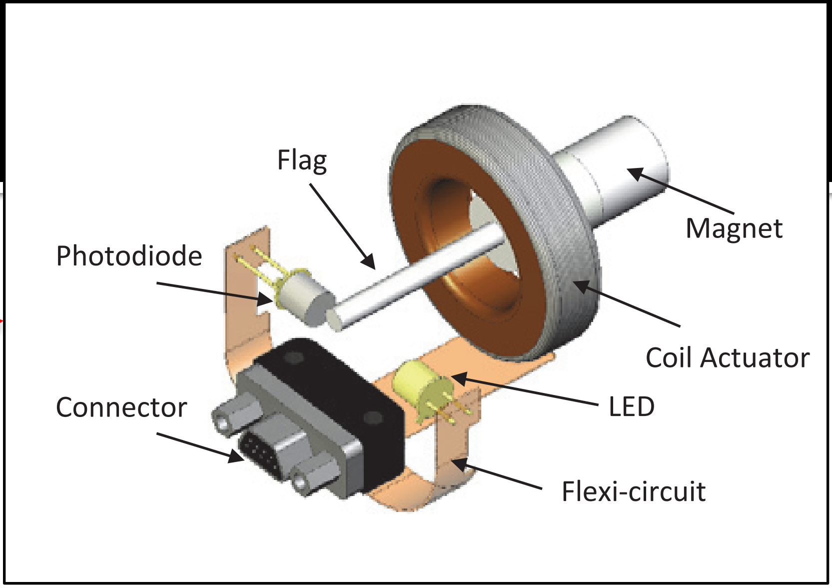
A. Bertolini et al, Nuclear Instruments and Methods in Physics Research A 564 (2006) 579–586

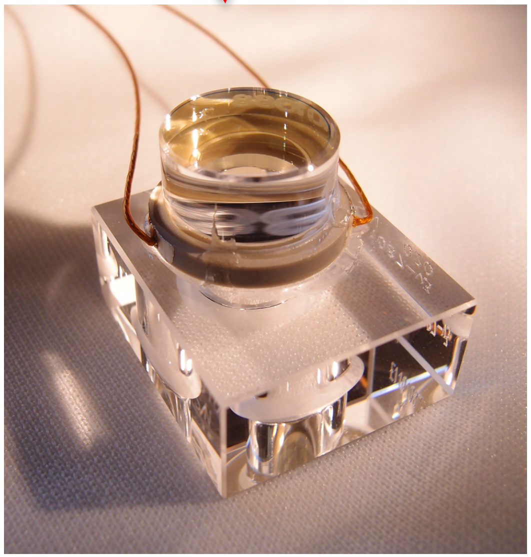


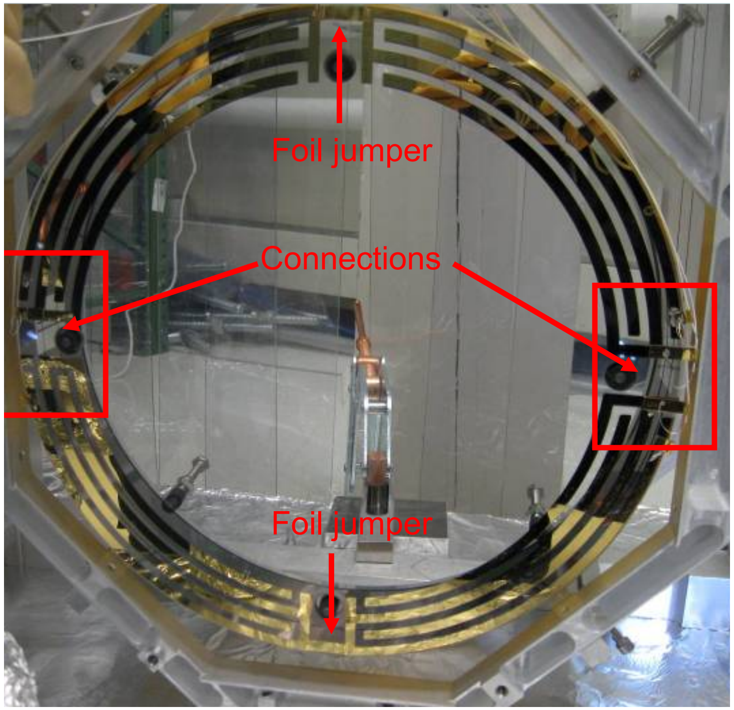


Acutuators

- Mechanical actuators
  - Coil Magnet actuator
  - Electro Static Driver (ESD)
  - Piezo (PZT) actuator

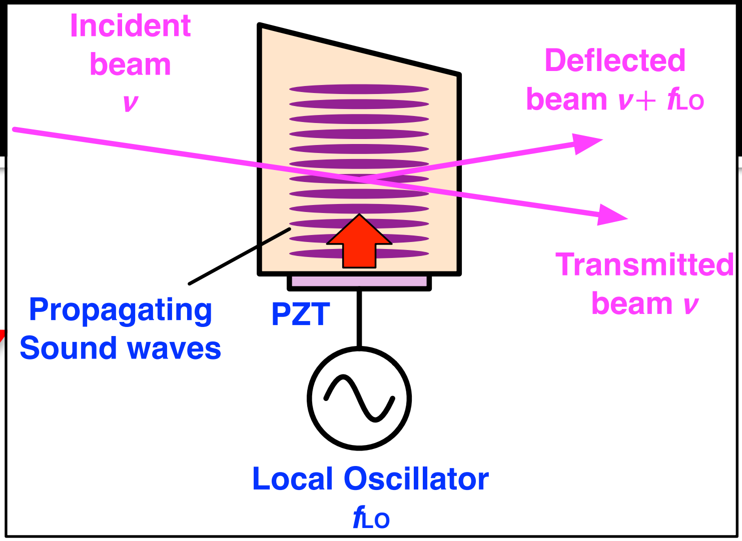




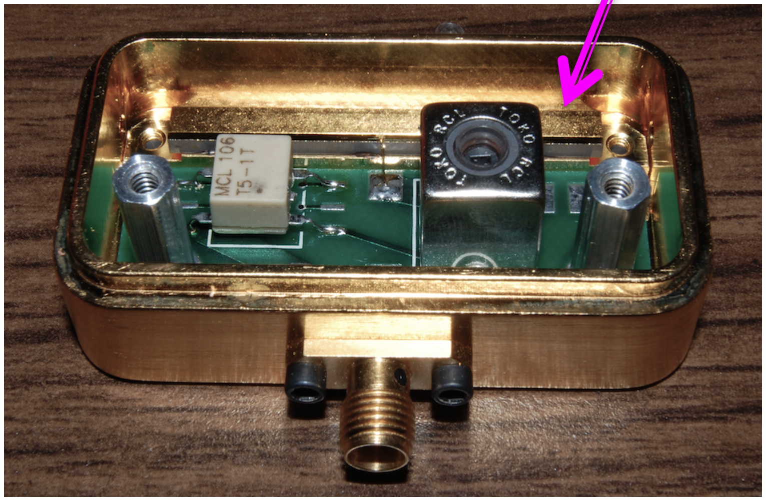


Acutuators

- Optical actuators
  - Acousto-Optic Modulator
  - Electro-Optic Modulator
  - Laser Frequency actuator
    1. Thermal actuator
    2. Fast piezo actuator
    3. External EOM

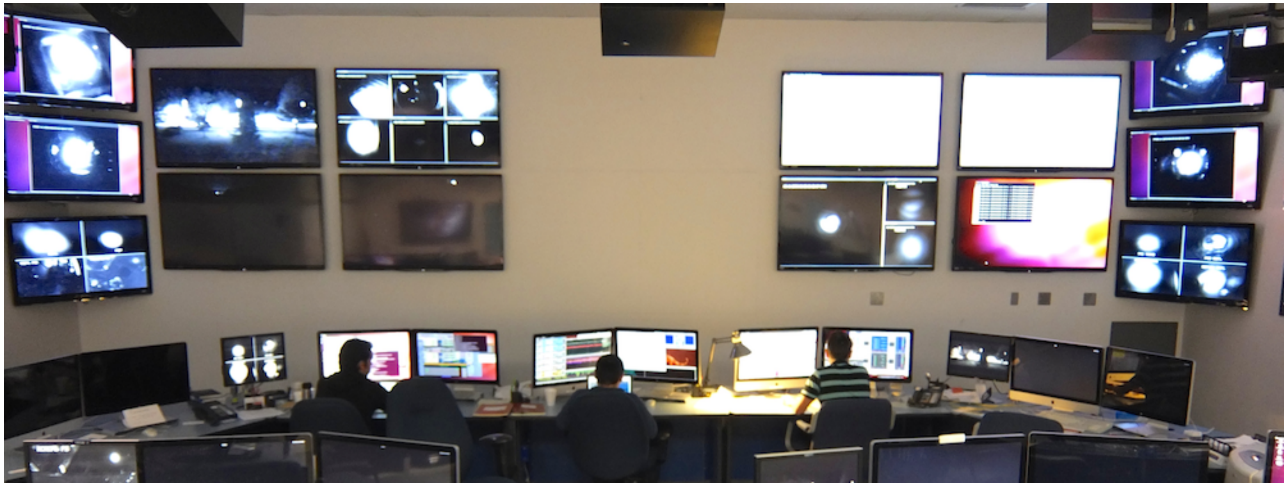


LiNbO3 crystal



Analog servo vs digital servo

- Comparison of the control room in the analog and digital eras



aLIGO (2014)

TAMA300 (2001)