引力波

LIGO-G1601927-v1

Noises in Gravitational Wave Detectors

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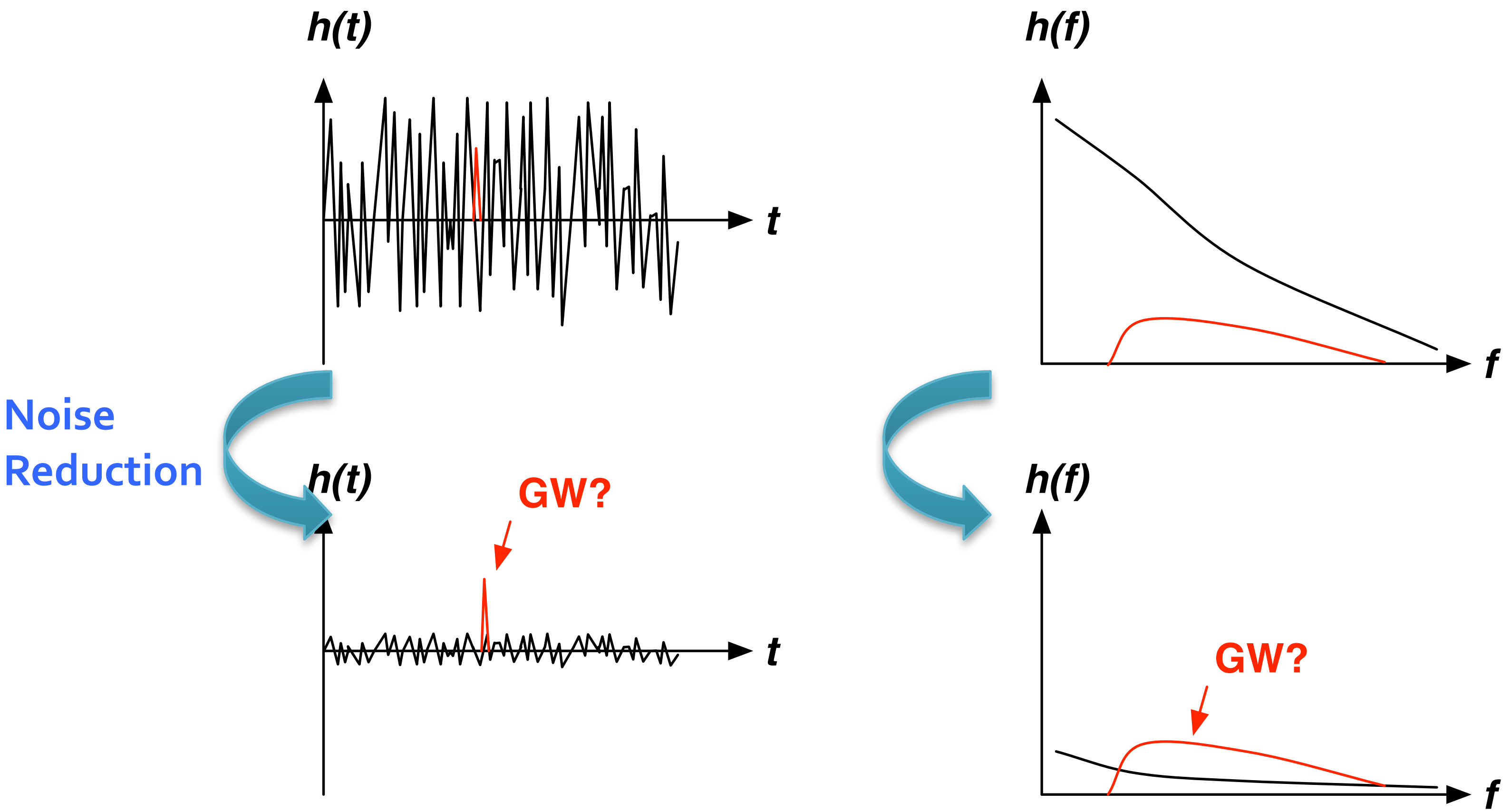
GW mini-school: Beijing Normal University 2016/9/15~18

Introduction ~ Noise?

- **GW detection**
  Data stream of differential arm strain
- **Once recorded:**
  Signals and noises are indistinguishable
  What we can do is to catch “likely” features
- **Reduce any kind of noises!**

Introduction ~ Noise?

- Time domain vs frequency domain



- Time domain: transient noises
  Frequency domain: stationary noises

Introduction ~ Noise?

- **Power Spectral Density (PSD)**
  Double sided PSD (-Infinity < f < Infinity)

$$S_{\mathrm{DS}}(f) = \lim_{T\to\infty} \frac{1}{T} \left| \int_{-T/2}^{T/2} x(t) e^{-2\pi i f t} \mathrm{d}t \right|^2$$

- **Single sided PSD (0 <= f < Infinity)**

$$S_x(f) = 2S_{\mathrm{DS}}(f) \qquad [x_{unit}^2 / \mathrm{Hz}]$$

- **Linearized PSD or Amplitude Spectral Density (ASD):**

$$G_x(f) = \sqrt{S_x(f)} \qquad [x_{unit} / \mathrm{sqrtHz}]$$

Introduction ~ Noise?

- **Parseval's Theorem** for signal RMS and PSD

$$\overline{x^2(t)} = \int_0^\infty S_x(f)df$$
$$\equiv x^2_{\mathrm{RMS}}$$

Root Mean of *x*(*t*):

average signal power density (per sec)
(cf. variance, std deviation)

PSD *Sx*(*f*):

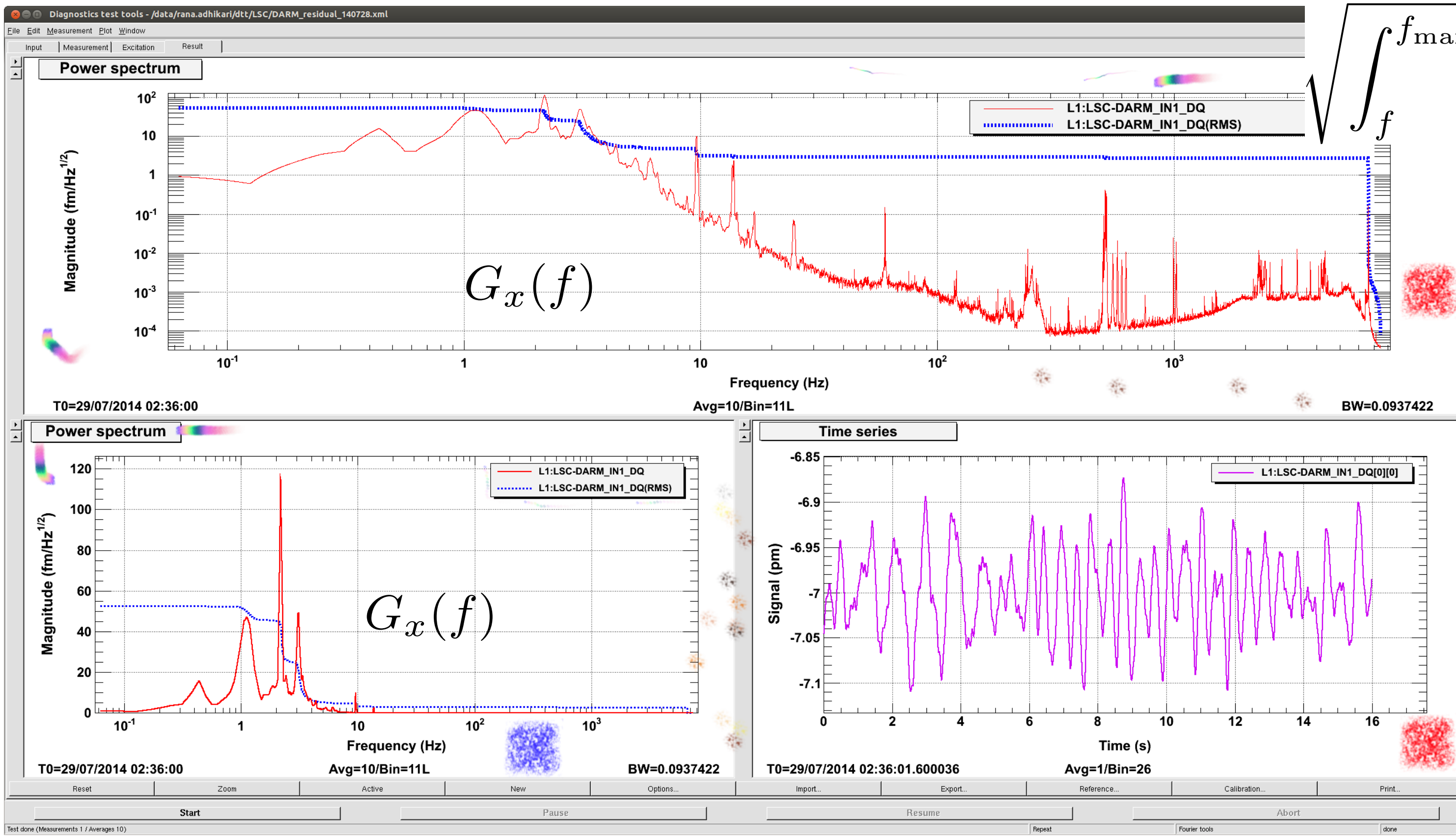
power density per frequency (per sec)

Introduction ~ Noise?

Example

PSD [fm/sqrtHz] in log-log scale, RMS [fm] ~ 50fm = 0.05pm

$$\sqrt{\int_f^{f_{\max}} G_x(f')^2 df'}$$



$x(t)$

PSD [fm/sqrtHz] in log-lin scale

Time series [pm]

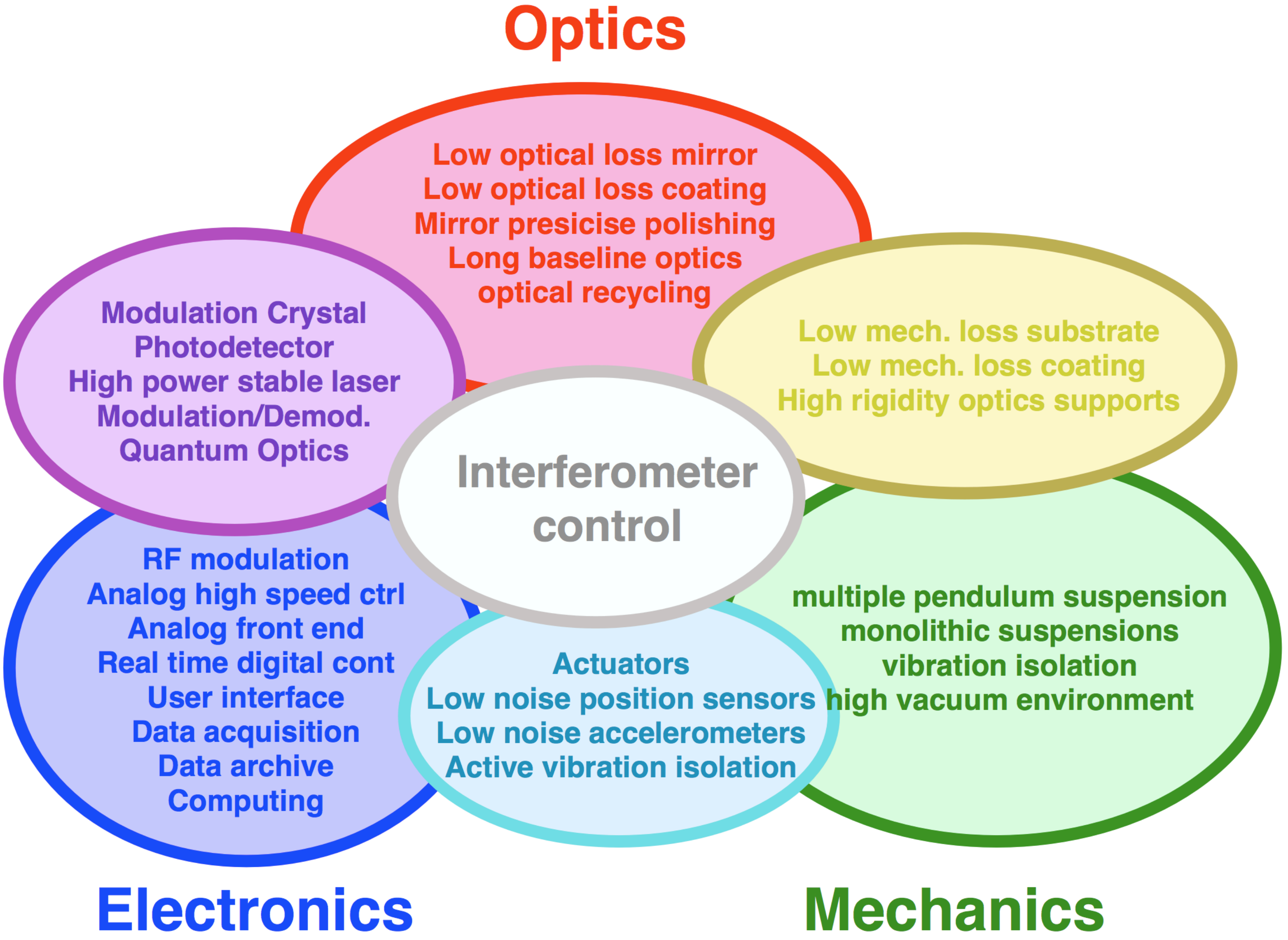
RMS [fm]

Components of the interferometer

- 3 fundamental elements of GW detectors
- Mechanics
- Optics
- Electronics

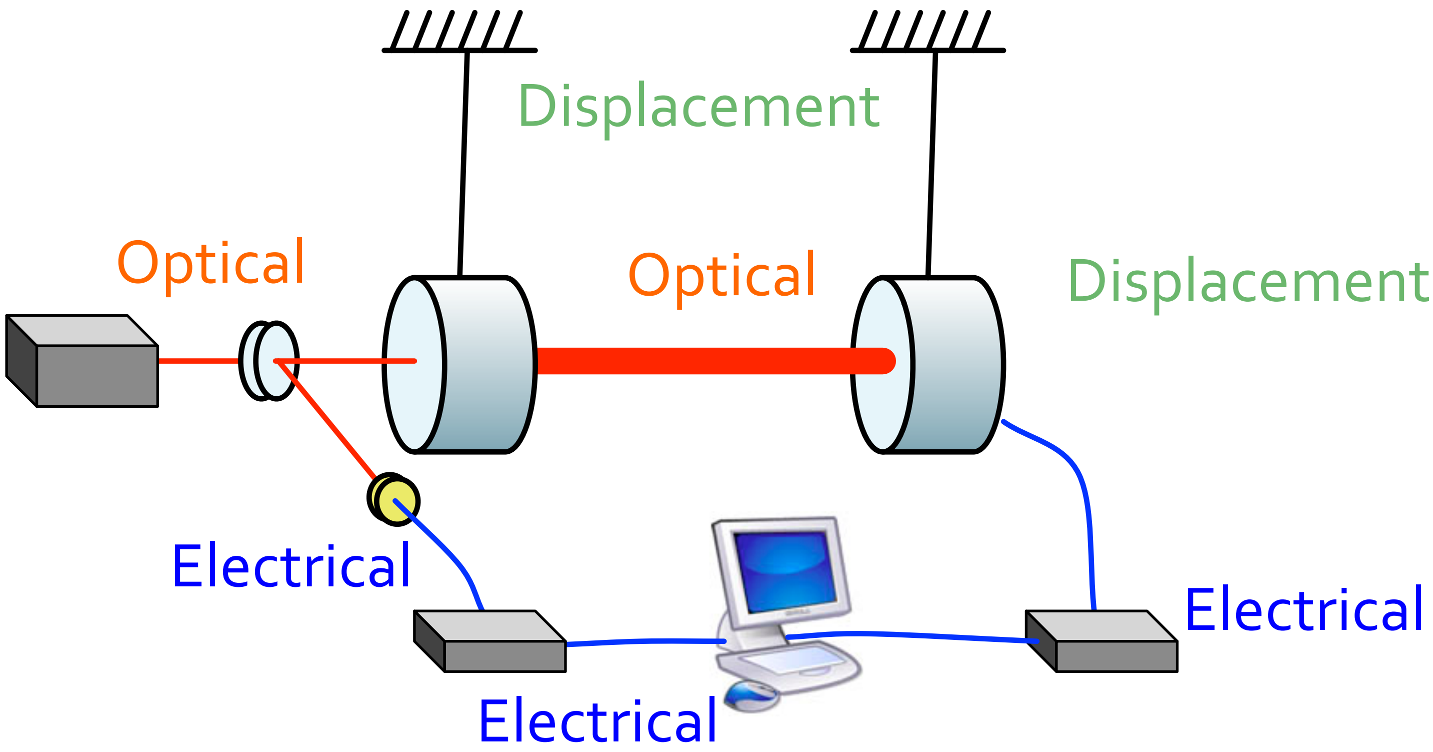
Components of the interferometer

- 3
- M
- O
- El



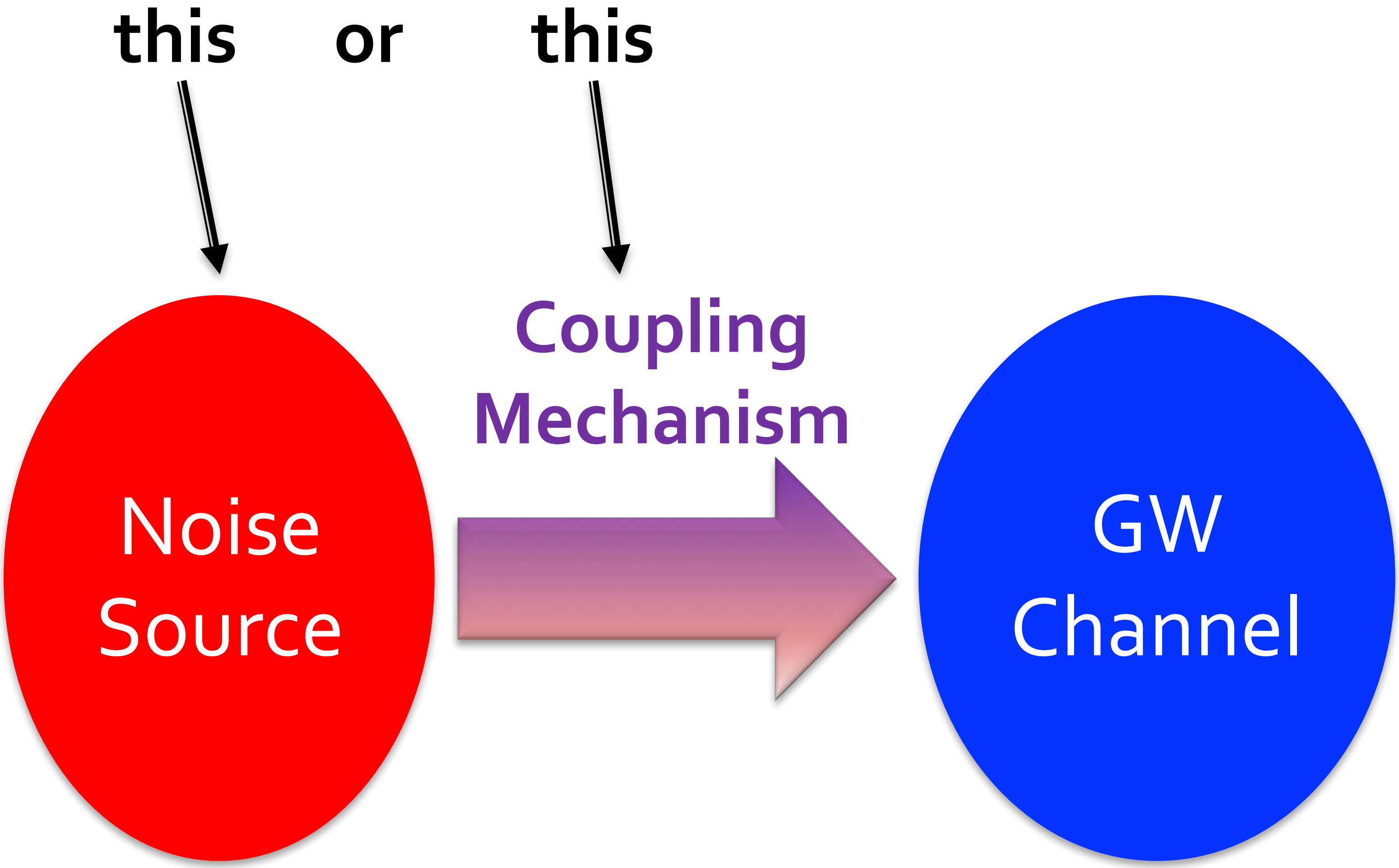
Noise categories

- 3 fundamental elements of GW detectors
- Mechanics -> Displacement noises
- Optics -> Optical noises
- Electronics -> Electrical noises



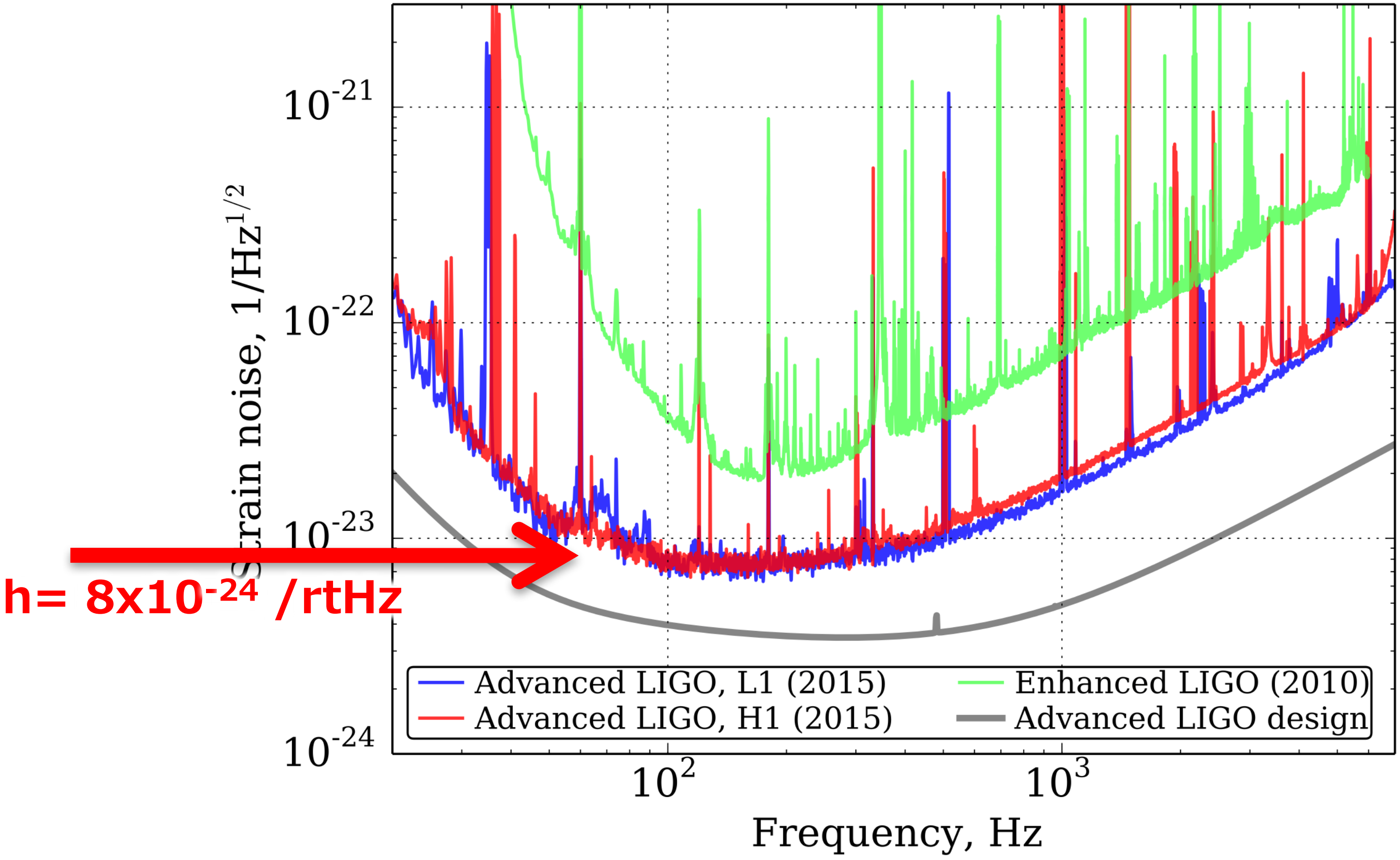
Noise coupling

- Noise source / Noise coupling
- Reduce



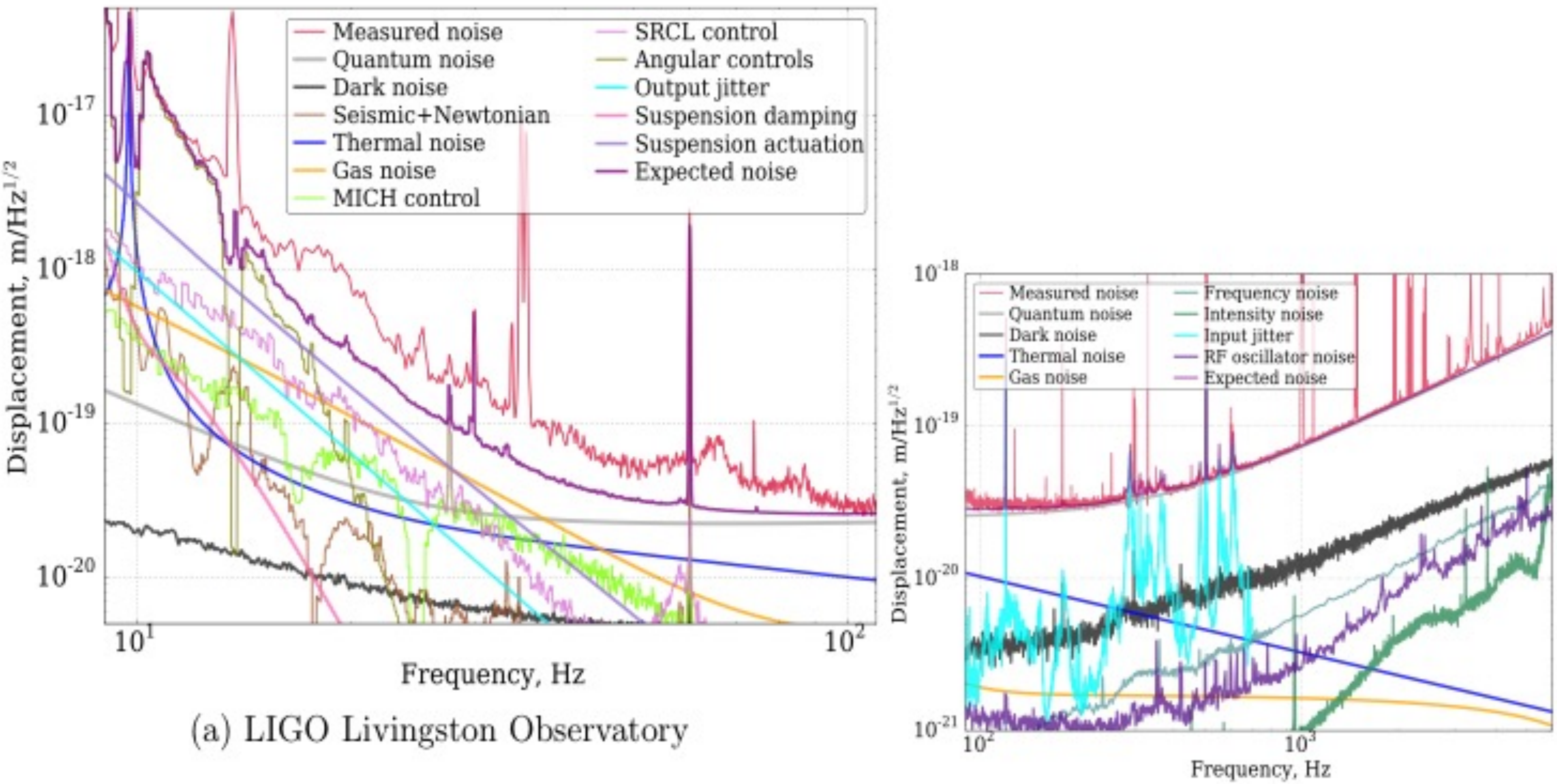
Sensitivity and noise

- Sensitivity (=noise level) of Advanced LIGO
- Current sensitivity



Sensitivity and noise

- Sensitivity (=noise level) of Advanced LIGO
- Noise budget



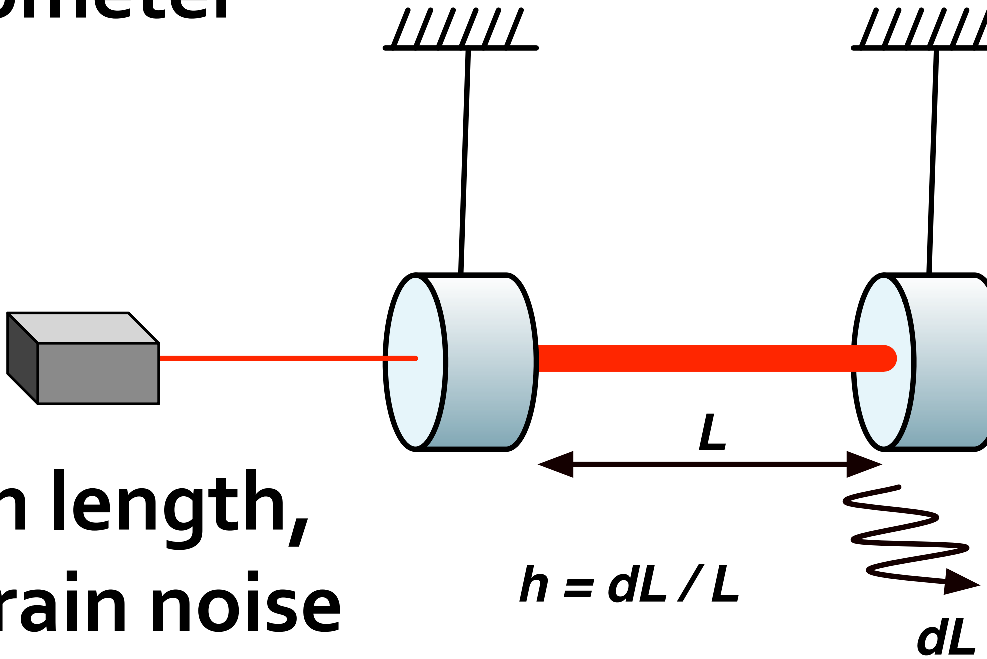
(a) LIGO Livingston Observatory

(b) LIGO Hanford Observatory

Displacement noises

Displacement noise

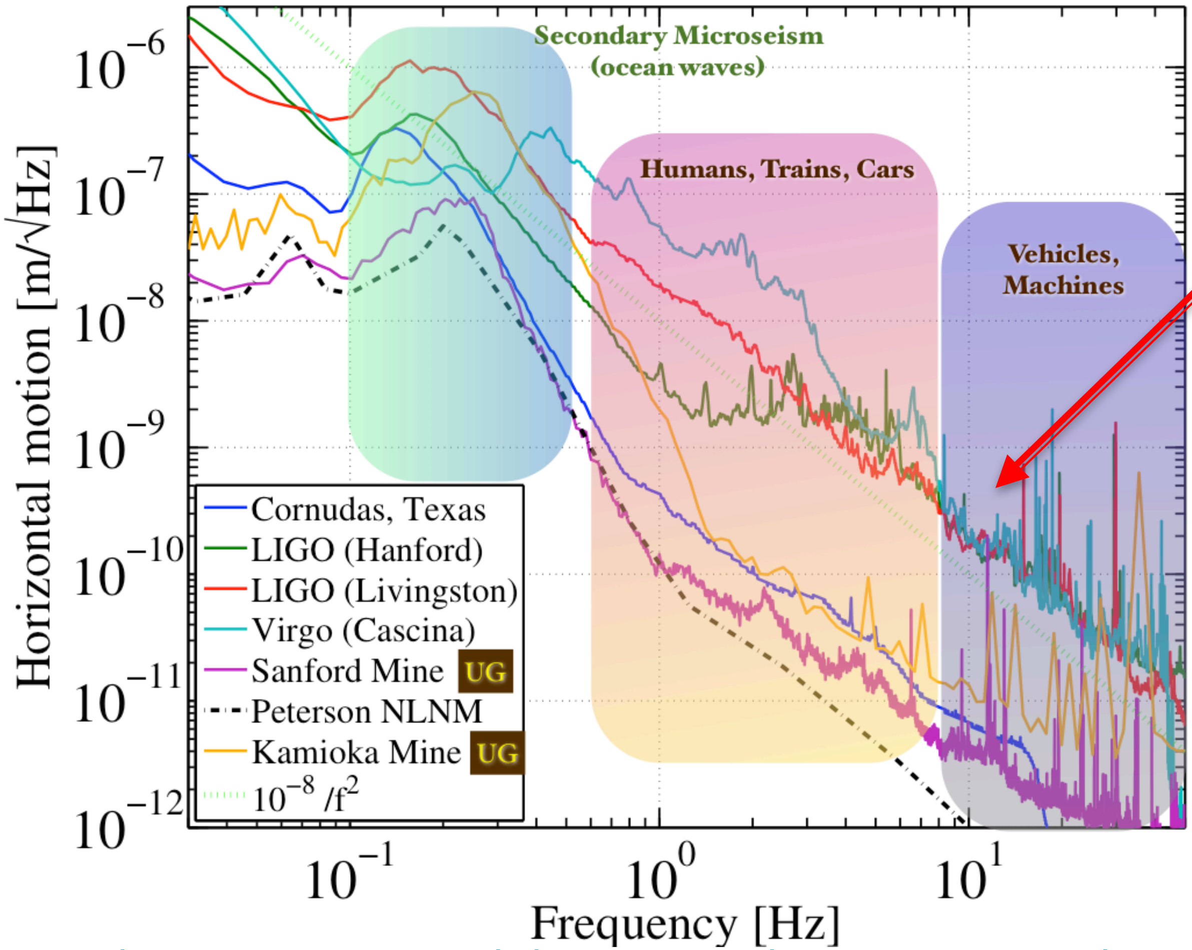
- Mechanical displacement sensed by a laser interferometer



- The longer the arm length, the smaller the strain noise
  - Seismic noise
  - Thermal noise
  - Newtonian Gravity noise

Displacement noise

- Seismic noise
  - Even when there is no noticeable earth quake…



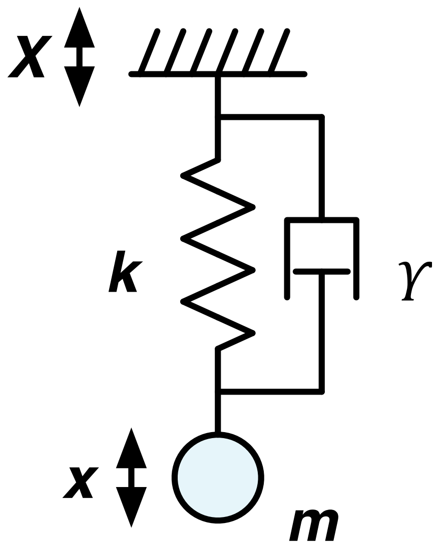
Ground motion @10Hz 10^{-10}m/rtHz

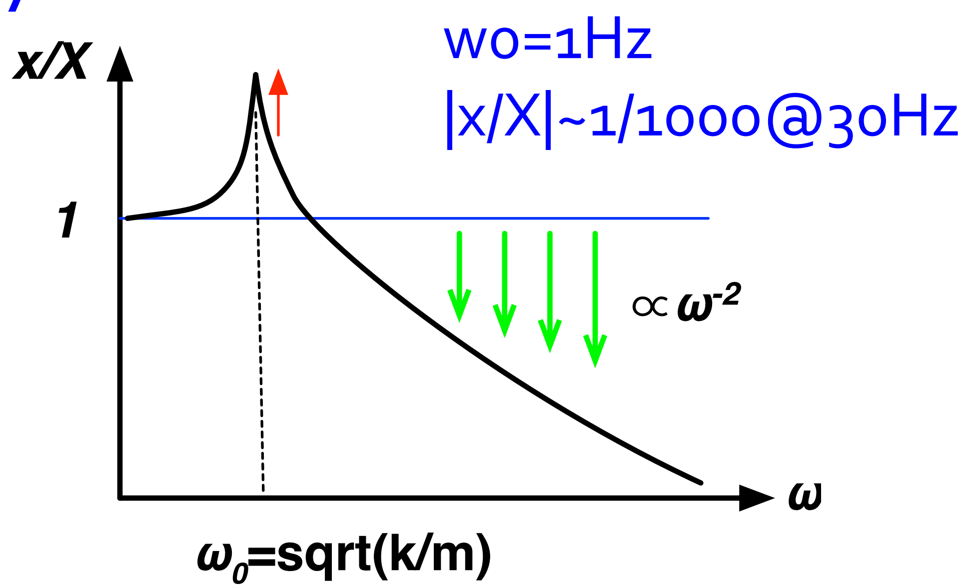
Target disp. noise 10^{-20}m/rtHz

http://link.aps.org/doi/10.1103/RevModPhys.86.121 (http://arxiv.org/abs/1305.5188)

Displacement noise

- Vibration isolation ~ utilize a harmonic oscillator
  - A harmonic oscillator provides vibration isolation above its resonant frequency





$$m\ddot{x} = -k(x - X) - \gamma(\dot{x} - \dot{X})$$

$$\left(\omega_0^2 + i\frac{\gamma}{m}\omega - \omega^2\right)\tilde{x} = \left(\omega_0^2 + i\frac{\gamma}{m}\omega\right)\tilde{X}$$

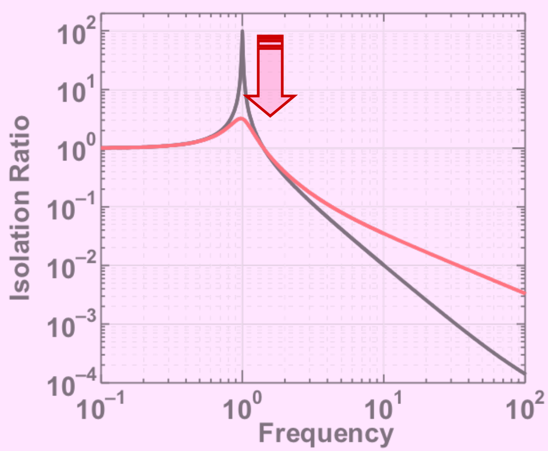
$$\frac{\tilde{x}}{\tilde{X}} = \frac{\omega_0^2 + i\frac{\gamma}{m}\omega}{\omega_0^2 + i\frac{\gamma}{m}\omega - \omega^2}$$

Displacement noise

- How to get more isolation?

Damping

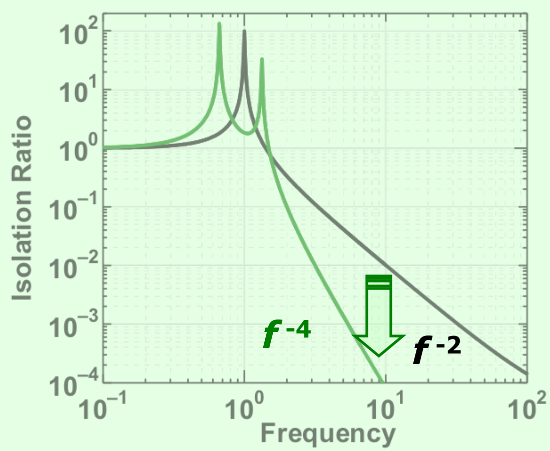
Lower the peak height



Worse isolation

Multi stage

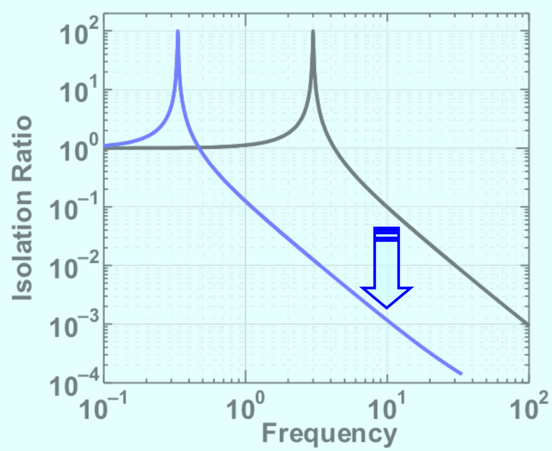
Steeper isolation curve



More peaks

Lower resonant freq

Better isolation

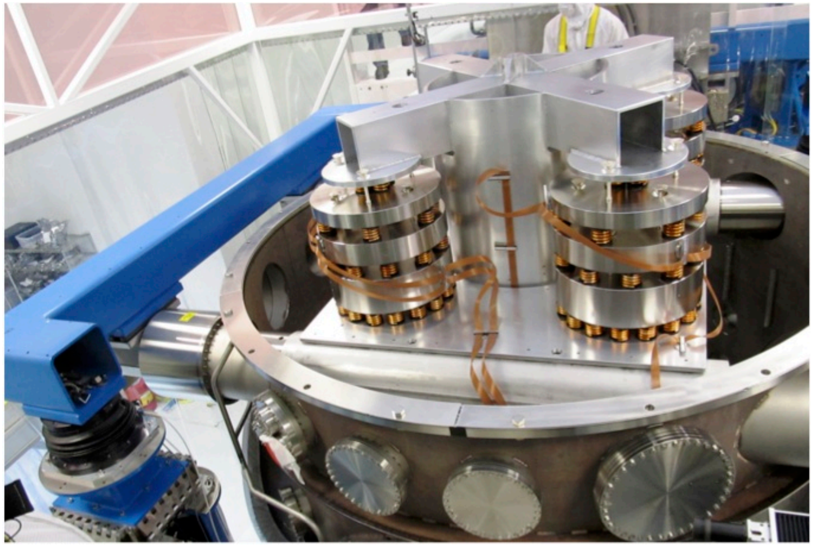


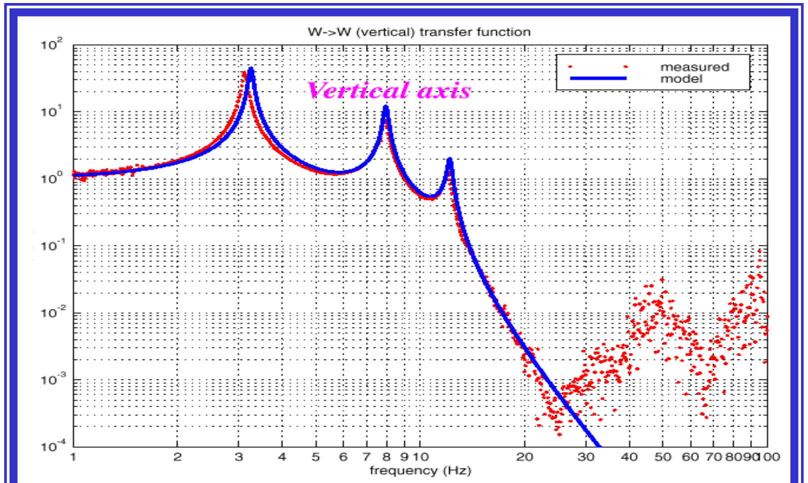
Complex to realize

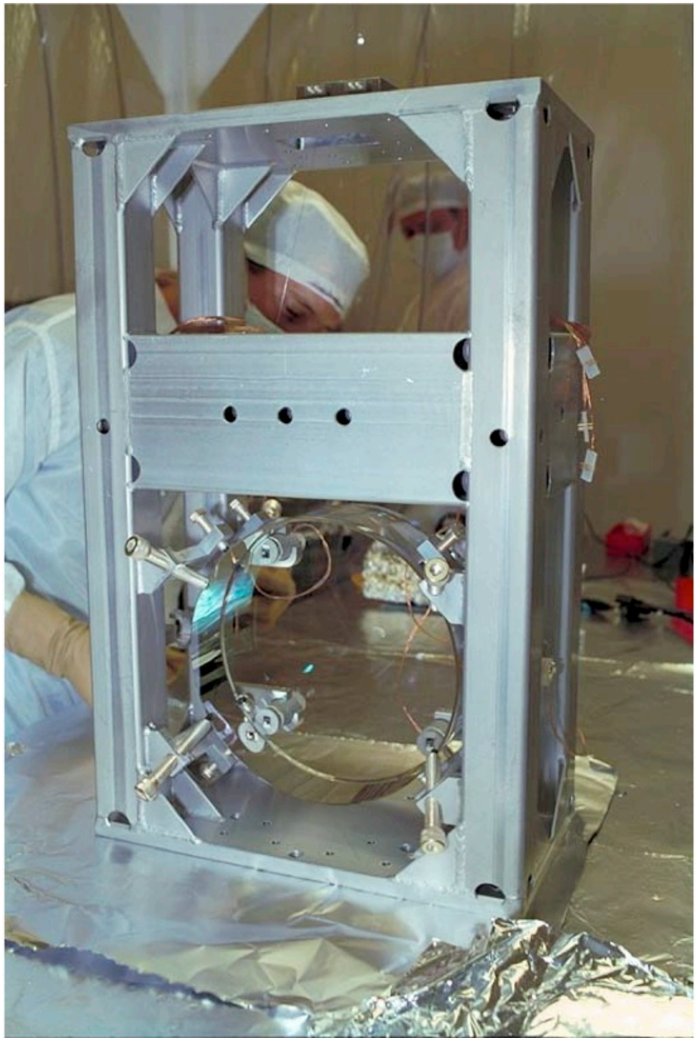
- In practice: employ combination of these measures

Displacement noise

- iLIGO vibration isolation
- Hydraulic active isolation / Isolation stack / Single Pendulum

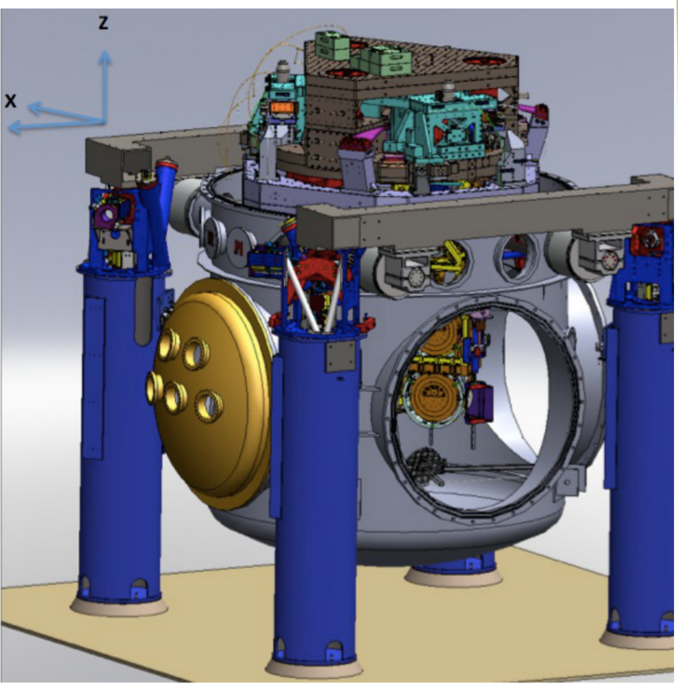


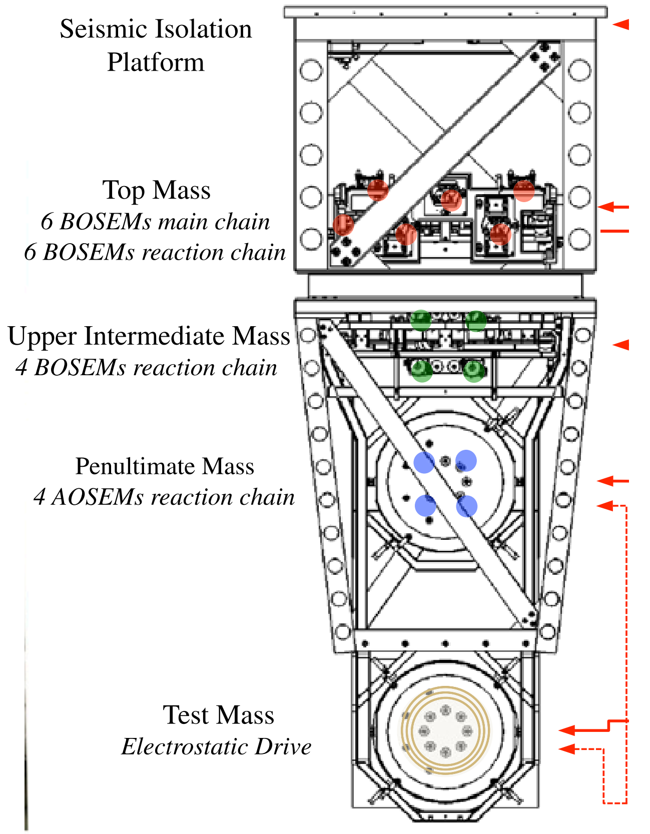




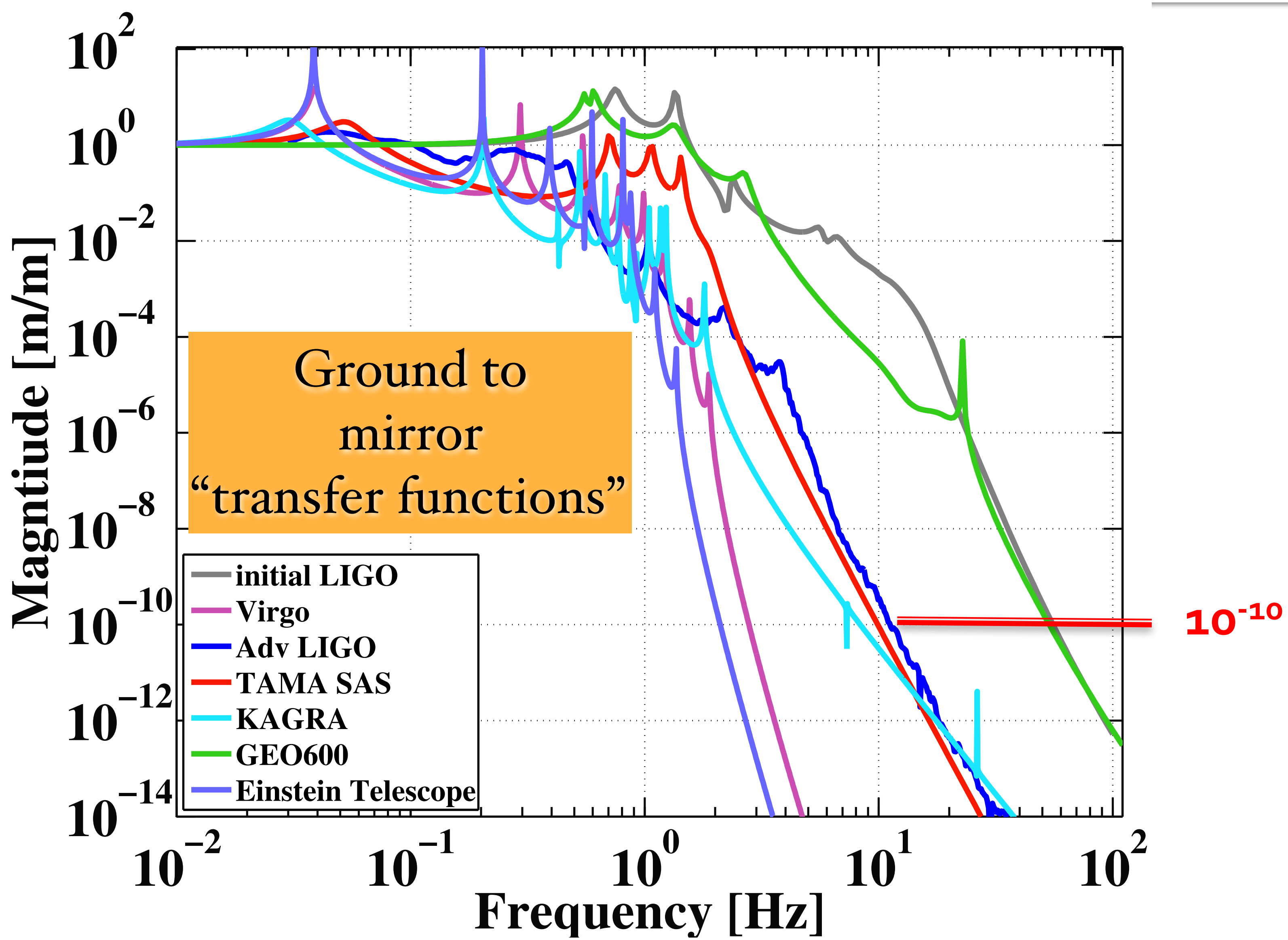
Displacement noise

- aLIGO vibration isolation
- Hydraulic active isolation / Invacuum Active Isolation Platforms / Multiple Pendulum





Displacement noise

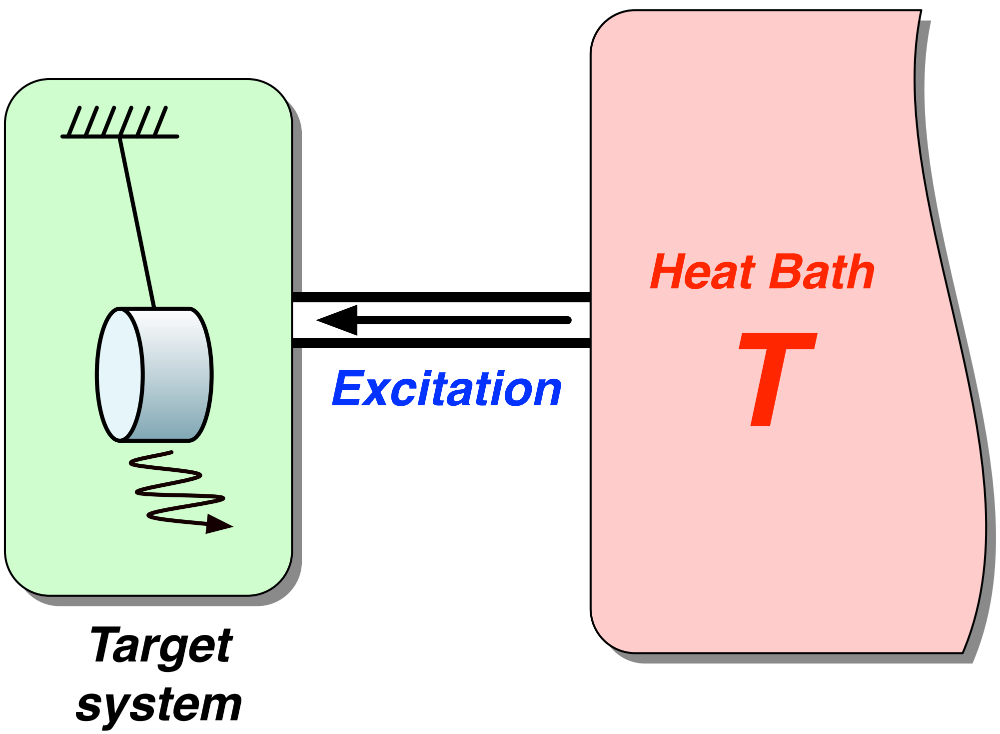


http://link.aps.org/doi/10.1103/RevModPhys.86.121 (http://arxiv.org/abs/1305.5188)

Displacement noise

- **Thermal noise:**
- **Thermally excitation of the motion of a system**
  - Each d.o.f. has <E> = ½ kb T   (Equipartition theorem)

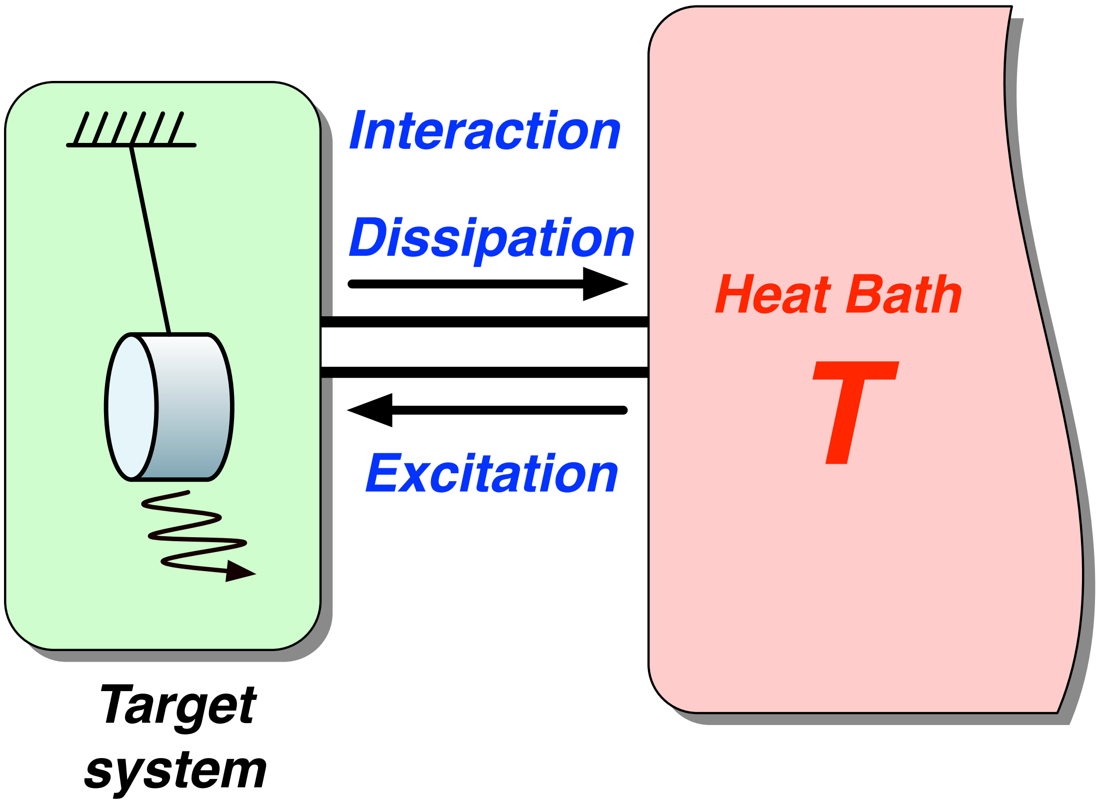
能量均分定理



$$\langle E_x \rangle = \frac{1}{2} k_{\mathrm{B}} T$$

Displacement noise

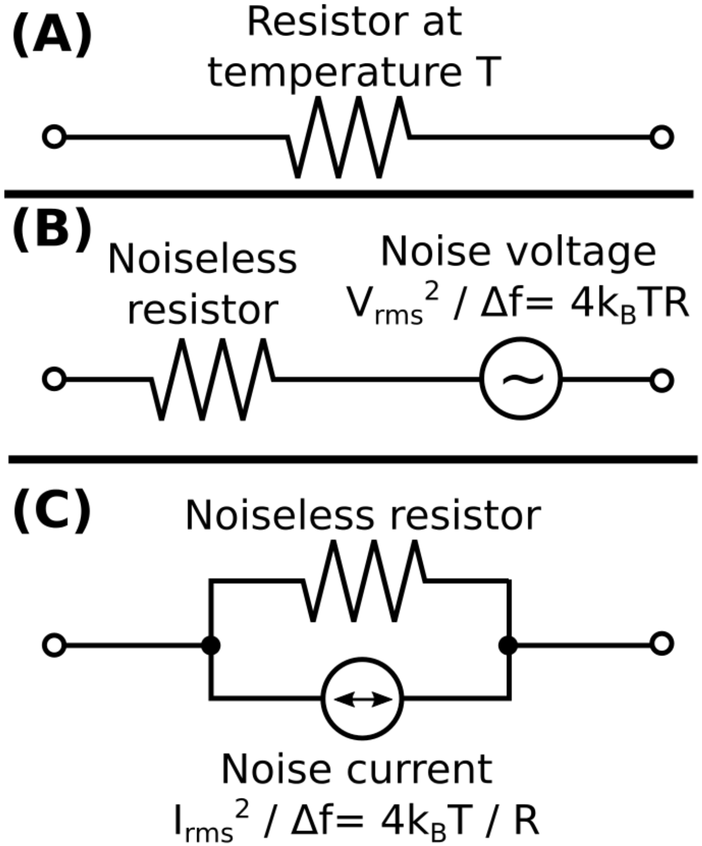
- **Thermal noise:**
- **Fluctuation dissipation theorem (FDT)** 涨落耗散定理



Displacement noise

- cf Johnson noise:
- Resistor R: Thermal voltage noise

$$G_{\mathrm{V}}(f) = \sqrt{4k_{\mathrm{B}}TR}\ \ [\mathrm{V}/\sqrt{\mathrm{Hz}}]$$
$$= 0.129\sqrt{R}\ \ [\mathrm{nV}/\sqrt{\mathrm{Hz}}]$$



- Measurement of R:

  R = V / I

Wikipedia: Johnson noise

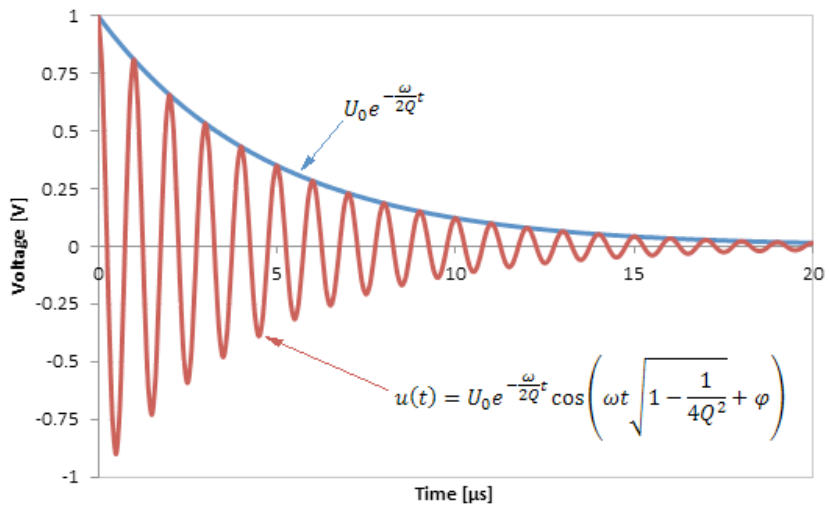
Displacement noise

- **Suspension thermal noise**

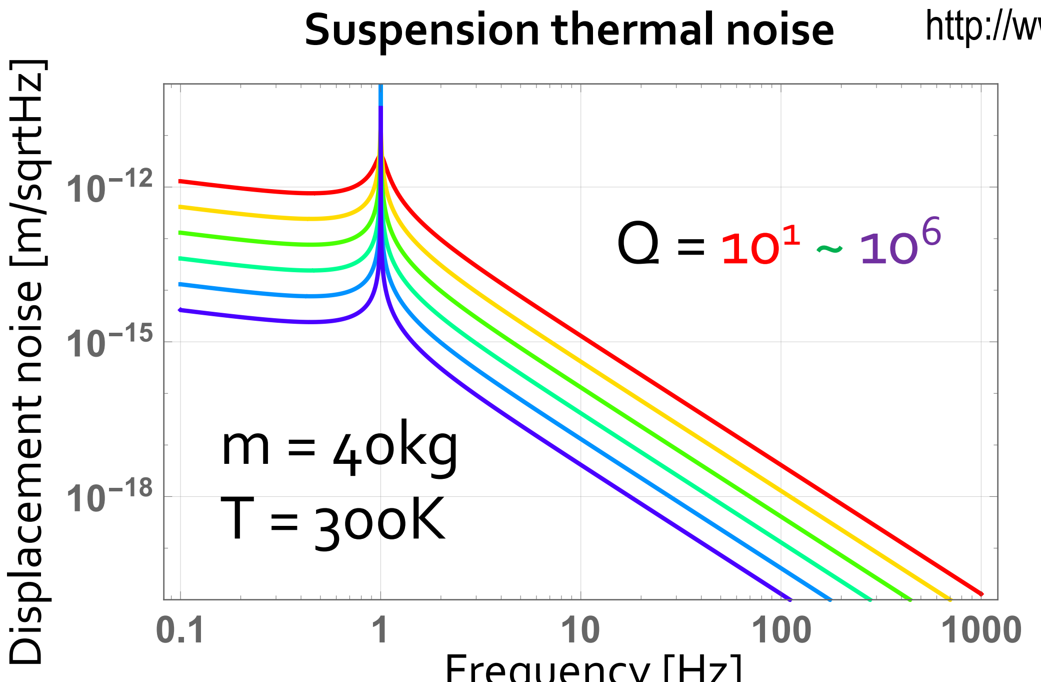
  Measurement of loss:

  -> Q factor

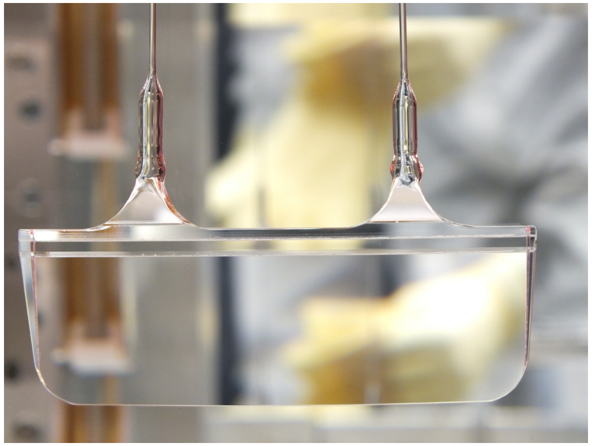
$$\sqrt{\langle \tilde{x}^2_{\mathrm{thermal(sd)}} \rangle} = \sqrt{\frac{4k_{\mathrm{B}}T\omega_0^2}{m\omega Q}\frac{1}{|\omega_0^2-\omega^2+i\omega_0^2/Q|^2}}$$



http://www.giangrandi.ch/electronics/ringdownq/ringdownq.shtml

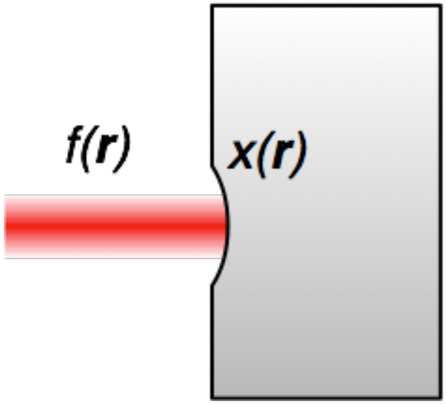


Monolithic suspension for high pendulum Q



Displacement noise

- **Mirror thermal noise** Y. Levin PRD **57**, 659-663 (1998)
- Sensing of the mirror surface deformation with a laser beam (with intensity profile of *f*(r))



- Apply periodic pressure with profile of *f*(r)

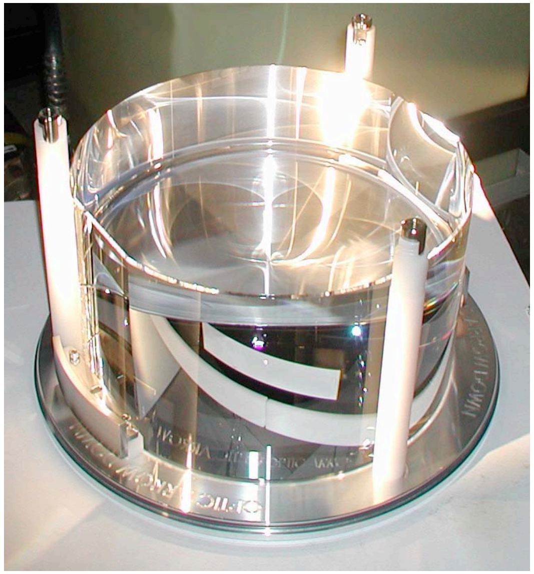
$$P(\mathbf{r}) = F_0 e^{\mathrm{i}\omega t} f(\mathbf{r})$$

- Calculate the rate of dissipation Wdiss

  analytically, using FEA, or etc

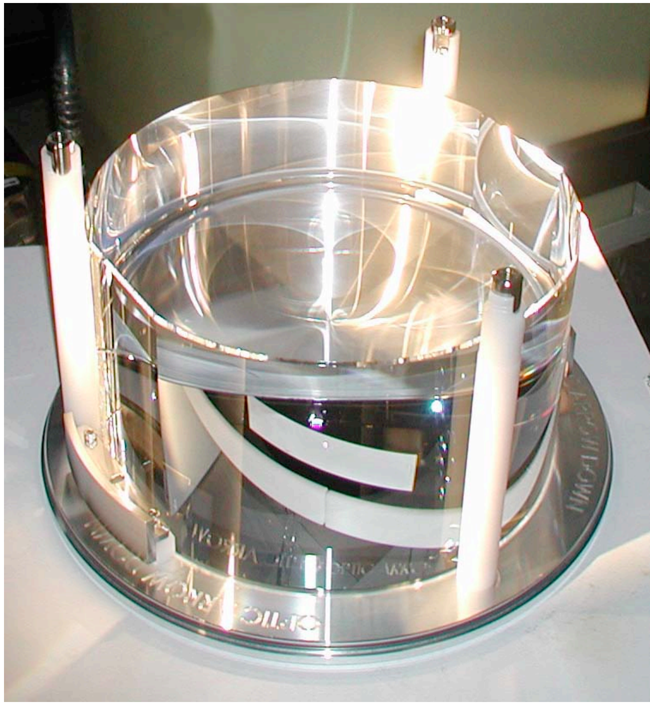
- Put this into the formula

$$S_x(\omega) = \frac{8 k_\mathrm{B} T W_\mathrm{diss}}{F_0^2 \omega^2}$$



Displacement noise

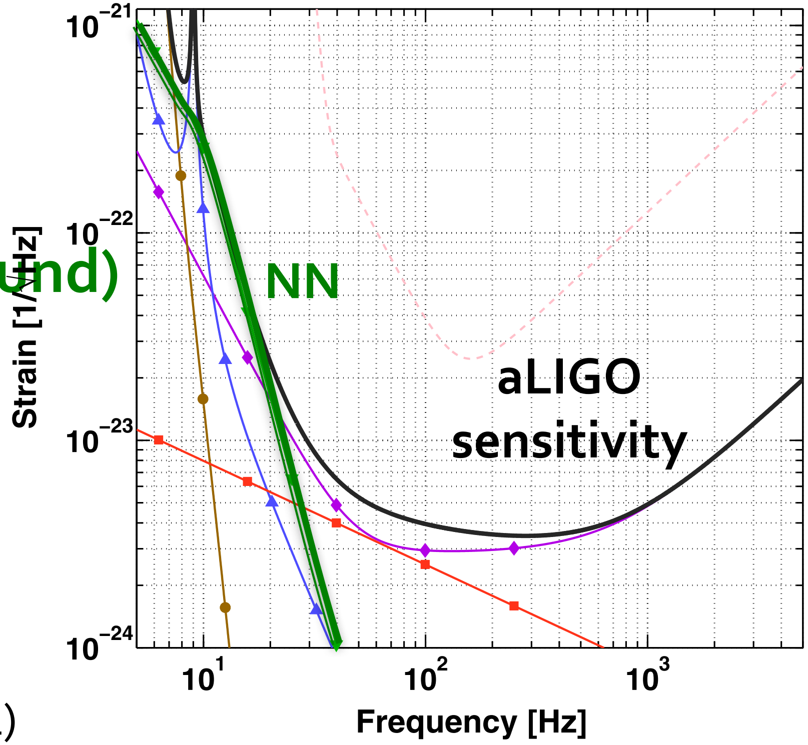
- Mirror substrate thermal noise



  - Brownian motion
    Mechanical loss associated
    with the internal friction
    ⇔Thermally excited body modes
    Optical coating (high mechanical loss)
    will be limiting noise source in aLIGO
  - Thermo elastic noise
    Elastic strain & thermal expansion coefficient
    => cause heat distribution & flow in the substrate
    ⇔Temperature fluctuation causes mirror displacement
  - Thermo-refractive noise
    ⇔Temp. fluctuation causes fluctuation of refractive index

Displacement noise

- **Newtonian Gravity noise**
  - Mass density fluctuations around the test masses
    => test mass motion via gravitational coupling
  - Dominant source of Newtonian noise
    = Seismic surface wave
  - Mitigation
    1) Going to quiet place (underground)
    2) Feedforward subtraction
    3) Passive reduction by shaping
       local topography

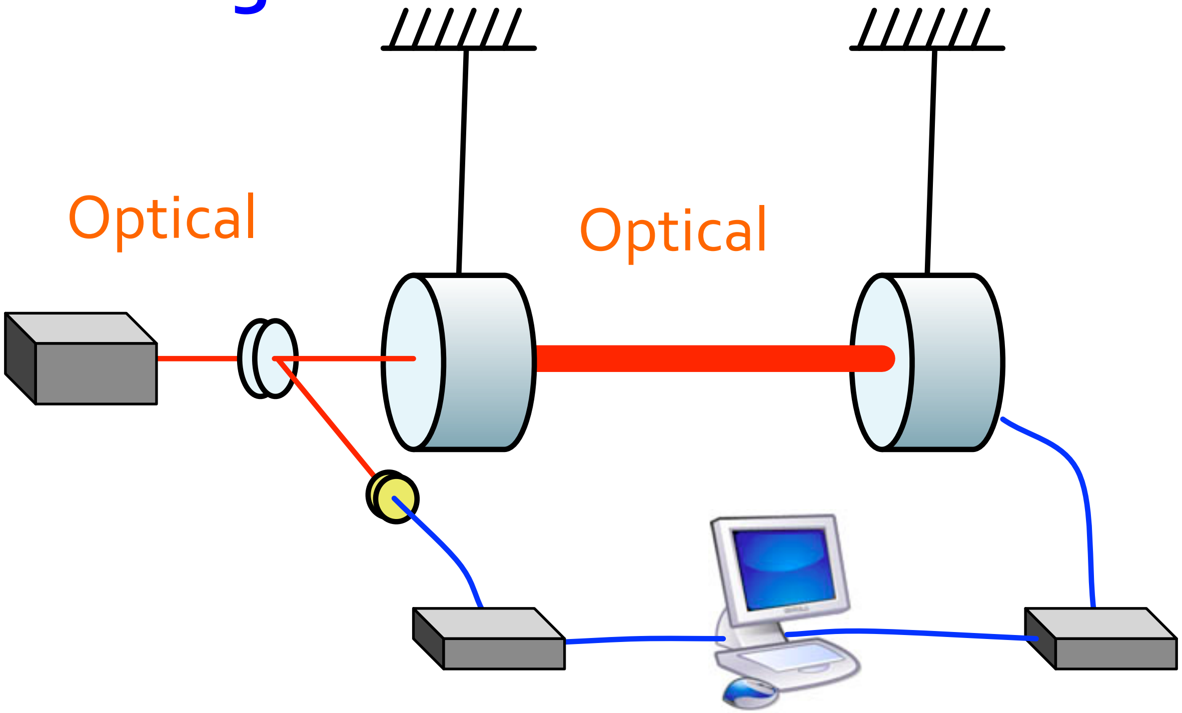


J Driggers, et al, PRD 86, 102001 (2012)
J Harms, et al, Class. Quantum Grav. 31 185011 (2014)

Optical noises

Optical noises

- Noises that contaminate the readout signal
  - Quantum noises (shot noise, radiation pressure noise)
  - Laser technical noises (frequency/intensity noise)
  - Modulation noises
  - Scattered light noise



Optical noises

- **Quantum noises: Shot noise** $\propto 1/\sqrt{P_{\rm in}}$
  - Photon shot noise associated with photodetection

$$i_{\rm shot} = \sqrt{2 e i_{\rm DC}} \quad [{\rm A}/\sqrt{\rm Hz}]$$

  - Michelson interferometer

$$i_{\rm DC} = \frac{e \eta P_{\rm in}}{h\nu} \frac{1 - \cos\delta\phi}{2} \quad [{\rm A}]$$

$i_{\rm DC}$: DC Photocurrent
η : PD Quantum Efficiency
ν : Optical Frequency

$$i_{\rm shot} / \frac{d i_{\rm DC}}{d\phi} = \sqrt{\frac{2 h\nu}{\eta P_{\rm in}}} \quad [{\rm rad}/\sqrt{\rm Hz}]$$

at the limit of dϕ->0

Shot-noise limit of the Michelson phase sensitivity

  - Michelson response (@DC)

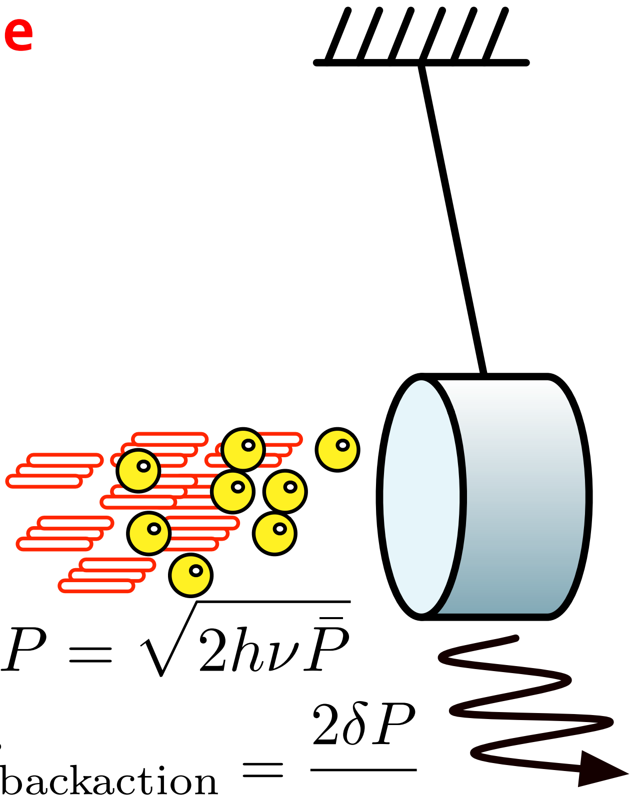
$$\frac{\delta\phi}{h_{\rm GW}} = \frac{4\pi L \nu}{c} \quad [{\rm rad/strain}]$$

Michelson Strain Sensitivity

1.3x10^{-20} 1/sqrtHz @1W, 1064nm, 4km

Optical noises

- Quantum noises ~ Radiation pressure noise $\propto \sqrt{P_{\mathrm{in}}}$
  - Photon number fluctuation in the arm cavity
    => Fluctuation of the back action force
  - Vacuum fluctuation injected
    from the dark port
    => Differential power fluctuation
    => Cause the noise in the GW signal



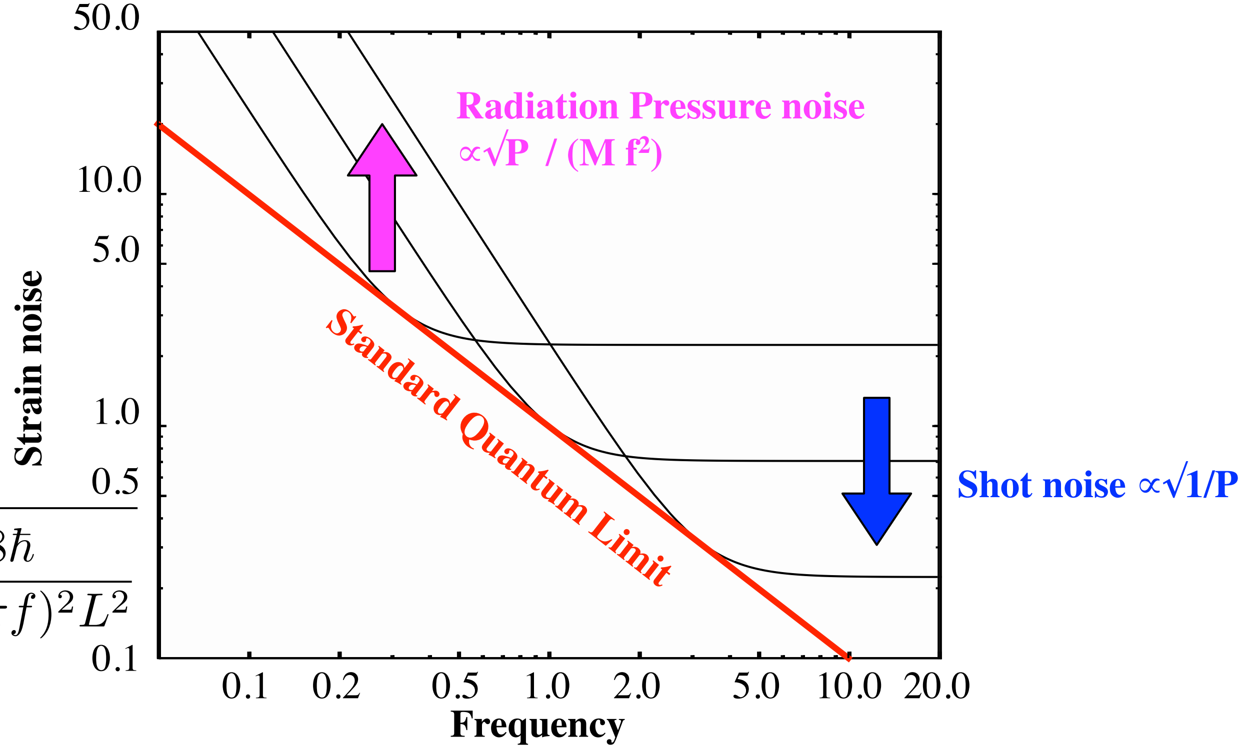
$$\delta P = \sqrt{2h\nu\bar{P}}$$

$$f_{\mathrm{backaction}} = \frac{2\delta P}{c}$$

$$\tilde{x} = \frac{f_{\mathrm{backaction}}}{M\omega^2}$$

Optical noises

- **Quantum noises**
  - **Standard Quantum Limit (SQL)**



$$\sqrt{S_h^{\mathrm{SQL}}} = \sqrt{\frac{8\hbar}{M(2\pi f)^2 L^2}}$$

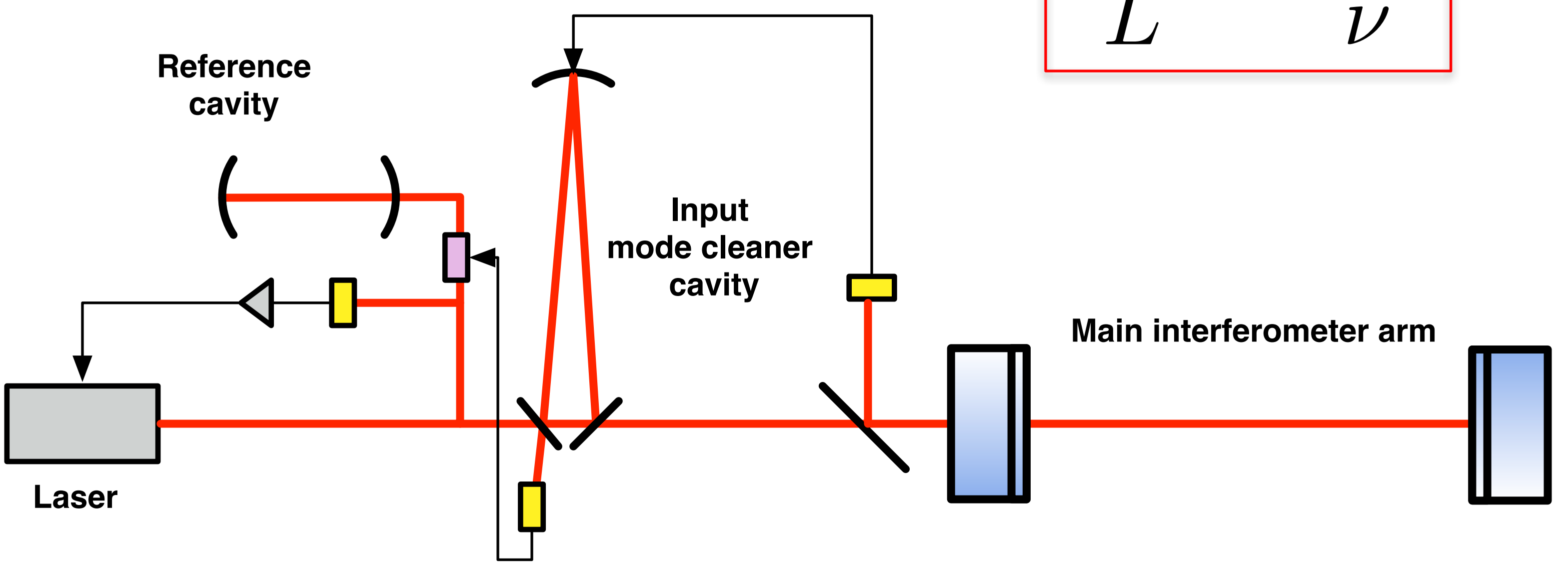
- Trade-off Between Shot Noise and Radiation-Pressure Noise
- Uncertainty of the test mass position due to observation

=> Quantum Optical techniques to overcome SQL

Optical noises

- Laser frequency noise
  - Laser wavelength ($\lambda = c / \nu$)
    = reference for the displacement measurement
  - Optical phase ϕ = 2 pi ν L / c
    dϕ = 2 pi / c (L dν + ν dL) <= indistinguishable
  - Laser frequency stabilization

$$\frac{dL}{L} = \frac{d\nu}{\nu}$$

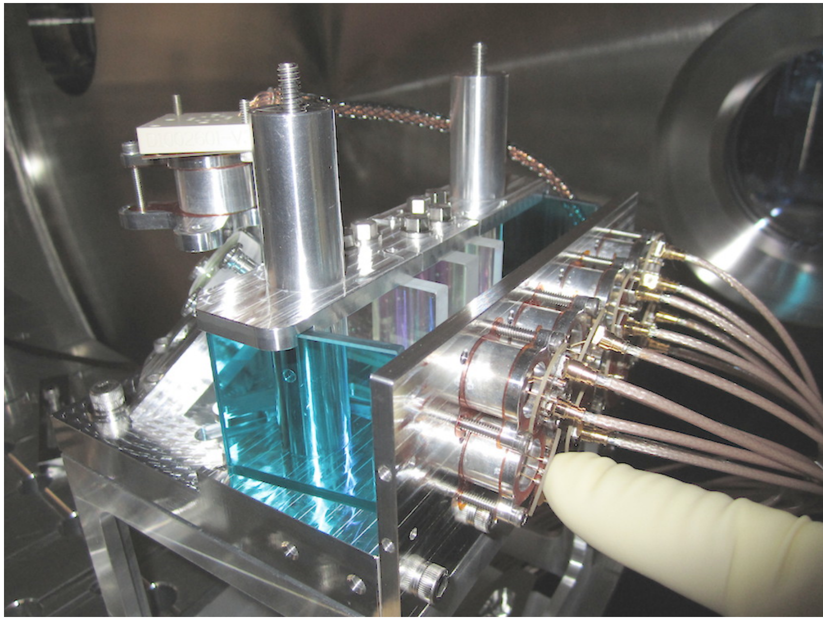


Optical noises

- Laser intensity noise
  - Relative Intensity Noise (RIN): dP/P
  - Sensor output V = P x
    => dV = P dx + x dP <= indistinguishable

$$\frac{dx}{x_{\text{offset}}} = \frac{dP}{P}$$

In-vacuum 8-branch
Photodiode array



  - Laser intensity stabilization

    P. Kwee et al,
    Optics Express **20** 10617-10634 (2012)

Electrical noises

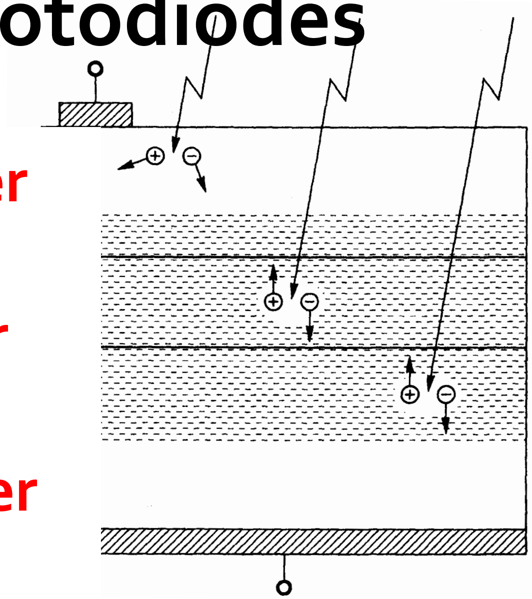
Noise in photodetectors

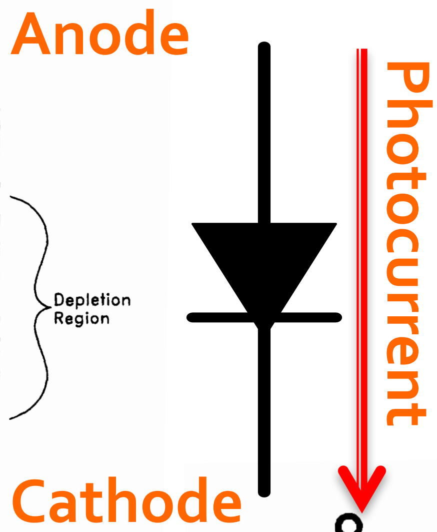
- Photodiodes

P-layer

I-layer

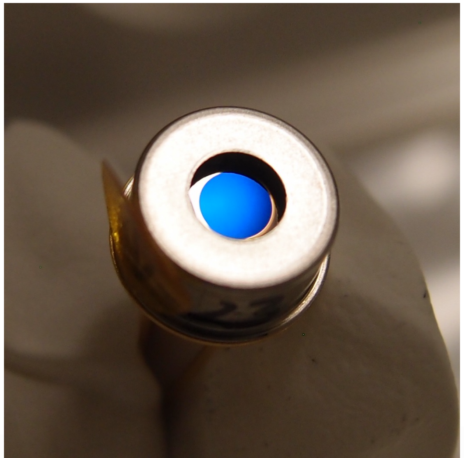
N-layer





"Photodiode Amplifiers",
J. Graeme (McGrawHill 1995)

aLIGO InGaAs PD (ϕ3mm)



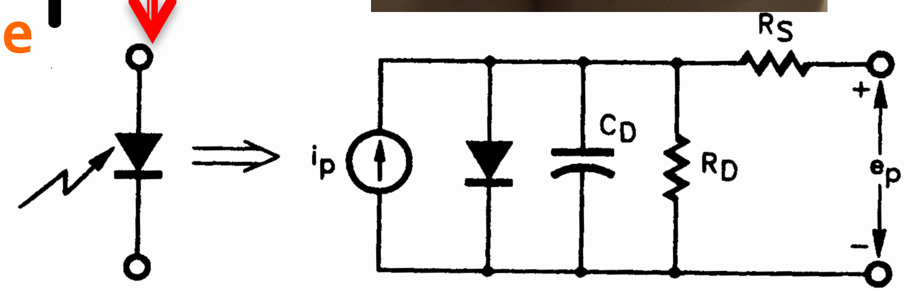


Figure 1.3 The circuit model of a photodiode consists of a signal current, an ideal diode, a junction capacitance, and parasitic series and shunt resistances.

- Photodiode equivalent circuit
  - Shunt Capacitance R_D
  - Junction Capacitance C_D
  - <u>Series Resistance</u> R_S

input referred noise current

$$i_{Rs} \sim \omega C_d \sqrt{4 k_{\mathrm{B}} T R_s}$$

Digitization (Quantization) noise

- Analog signals (~+/-10V) -> Digital signal
  - Digitized to a discrete N bit integer number

quantization

quantization error

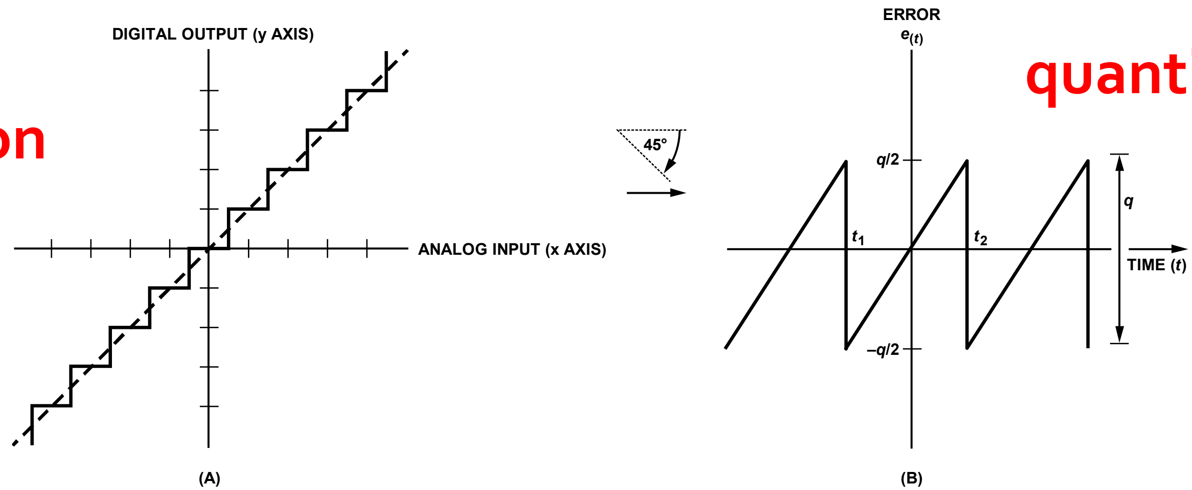


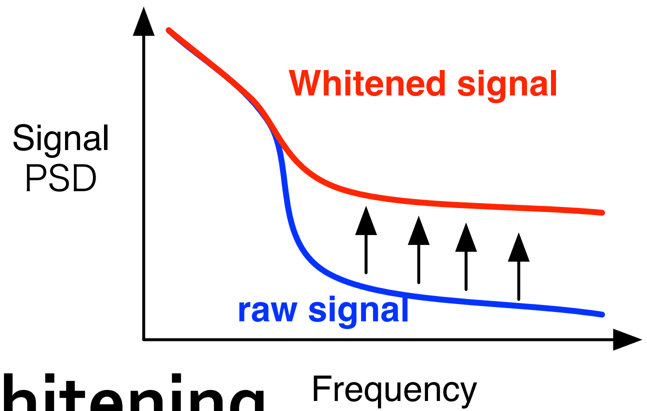
Figure 1. Ideal ADC Transfer Function (A) and Ideal N-Bit ADC Quantized Noise (B)

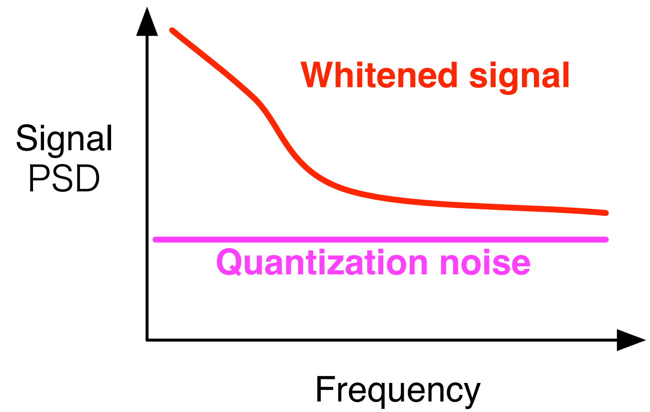
http://www.analog.com/static/imported-files/tutorials/MT-229.pdf

  - Quantization causes a white noise $V_n = \frac{\Delta}{\sqrt{12}}\ [\mathrm{V}/\sqrt{\mathrm{Hz}}]$
    e.g. +/-10V 16bit => Δ = 0.3mV => Vn ~ 100 μV/sqrtHz
    cf. Input noise of a typical analog circuit 10nV/sqrtHz

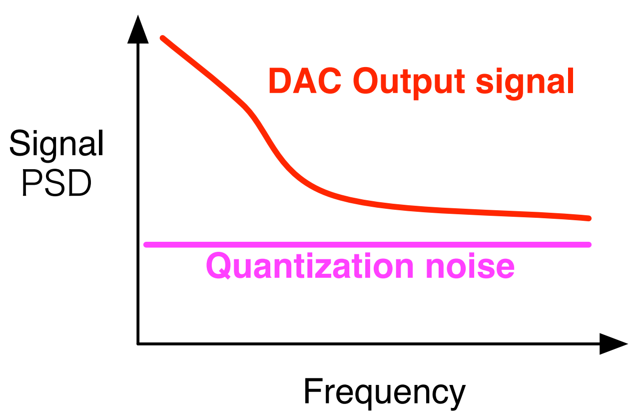
Digitization (Quantization) noise

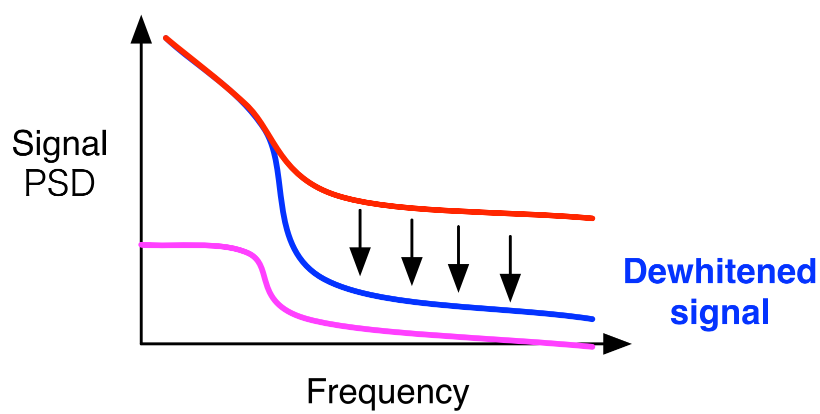
- **Whitening**
  - Amplify a signal in the freq band where the signal is weak





- **Dewhitening**
  - Amplify a signal in the freq band where the signal is weak





Actuator noise

- Actuator noise appears in the GW signal as an external disturbance

  - Mitigation
    1) Make the noise itself smaller
    2) Make the actuator response smaller

  - We need to keep sufficient actuator strength for lock acquisition
    => Transition to a low-noise mode after achieving lock

Summary

Summary

- Summary
  - There are such large number of noises
  - They are quite omni-disciplinary
  - Even only one noise can ruin our GW detection

  - GW detection will be achieved by
    - Careful design / knowledge / experience
    - Logical, but inspirational trouble shooting
  - Noise "hunting"