引力波

LIGO-G1601926-v1

Interferometer configurations for Gravitational Wave Detectors

Koji Arai (新井宏二) – LIGO Laboratory / Caltech

GW mini-school: Beijing Normal University 2016/9/15~18

Optical configurations

Laser interferometry

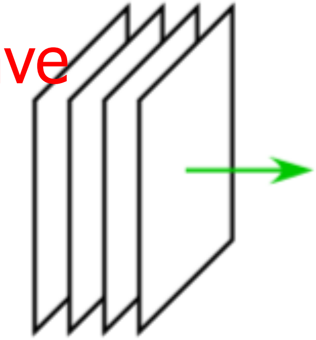
- GW detection
  = High sensitive detection of displacement

- Laser interferometry is an indispensable technology (lambda ~ 1um)

- **We use optical cavities in many places**
  What is an optical cavity?

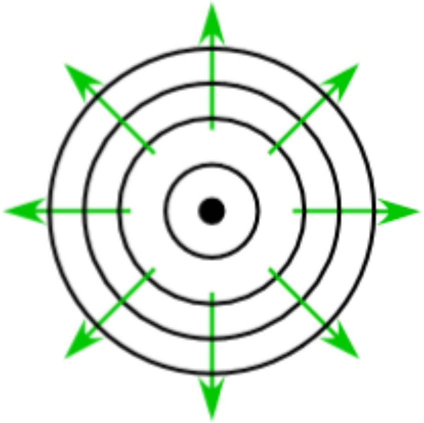
Gaussian beam

- Propagation of EM waves
  - Plane wave

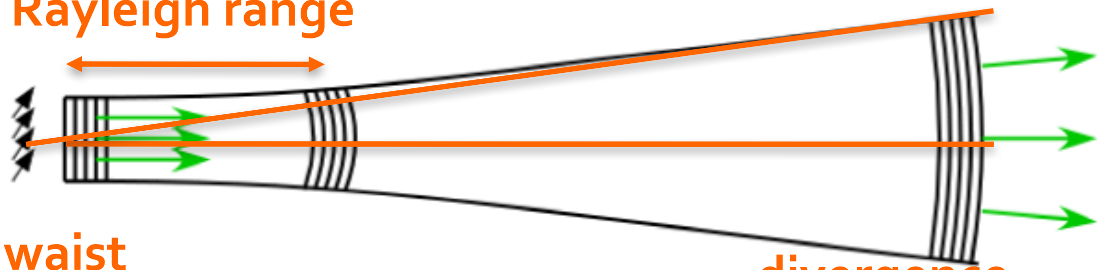


Equiphase Planes

Spherical wave



  - Gaussian beam:
    A solution to paraxial approximated wave equation
    Amplitude distribution of the beam is Gaussian



Rayleigh range

Coherent Light source

Spherical-wave like

waist

divergence angle

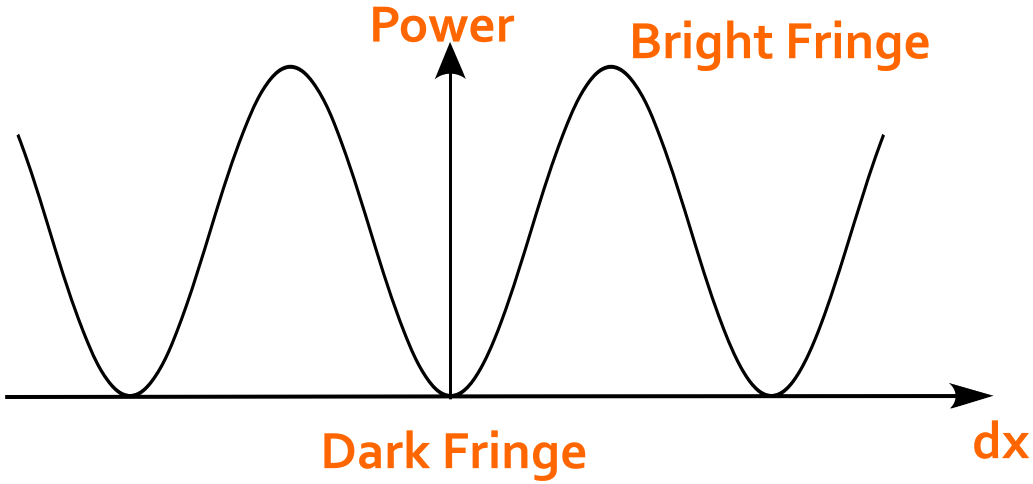
Plane-wave like

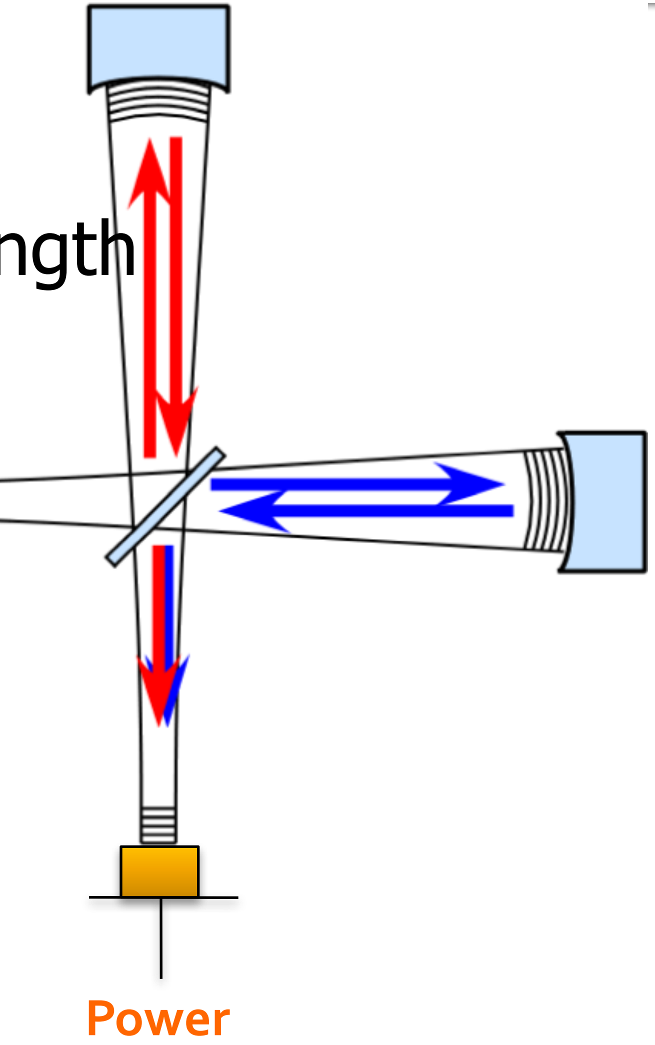
Fringes

- Two beams overlap at the BS
- Differential change of the arm length => fringe condition changed

$$P = E\cdot E^{*} = P_0(e^{if1} - e^{if2})(e^{-if1} - e^{-if2})$$

$$= 2P_0(1 - \cos df)$$





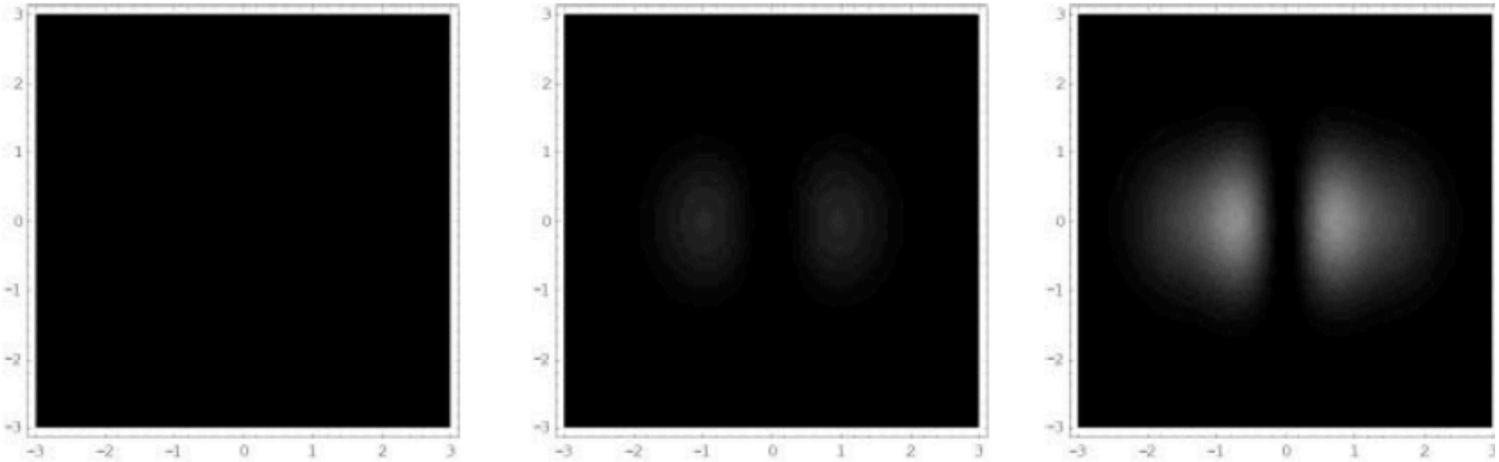
Fringes

- Contrast

$$C = \frac{Imax - Imin}{Imax + Imin}$$

Contrast Defect with mirror tilting

Dark Fringe



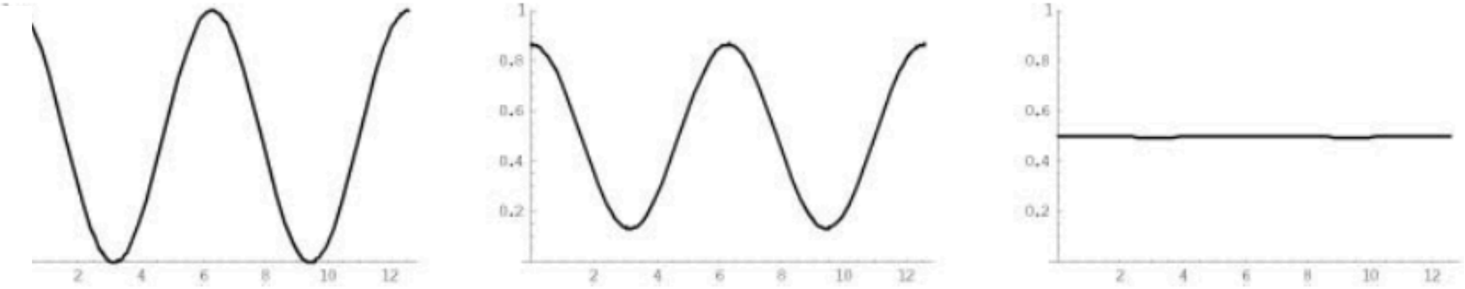
Contrast

C = 1

C ~0.7

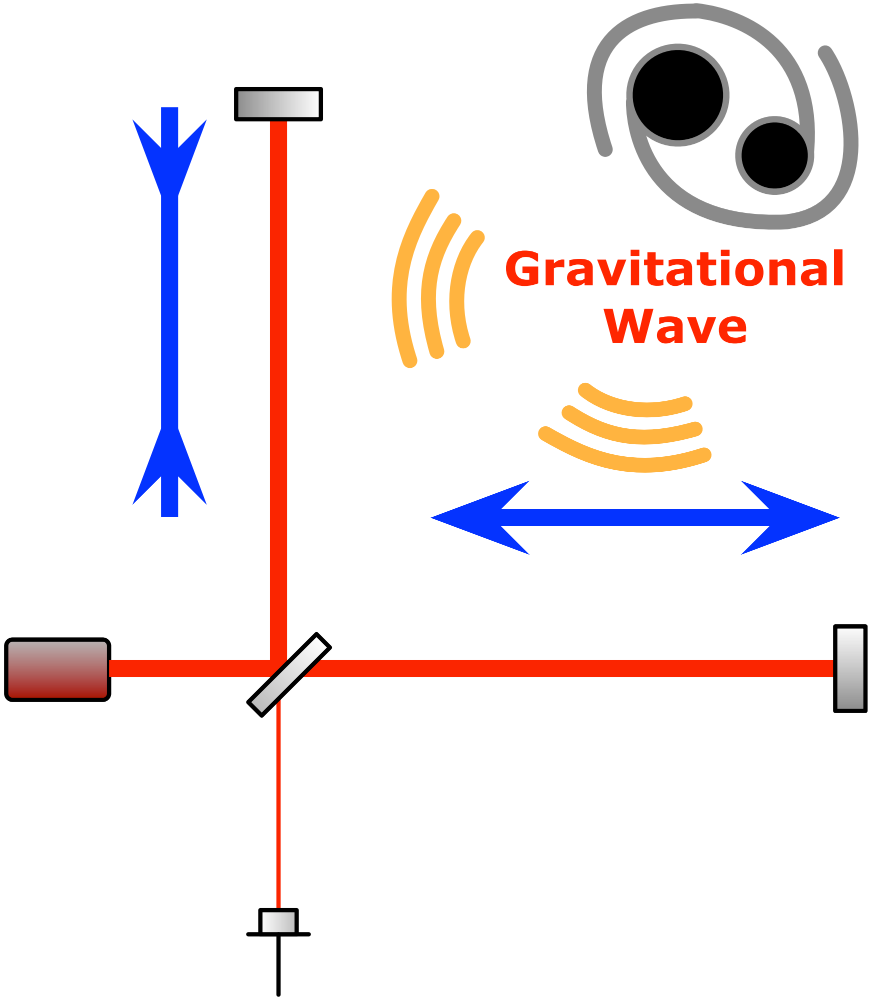
C ~ 0

Light intensity at the output with a mirror swept



Simplified interferometer GW detector

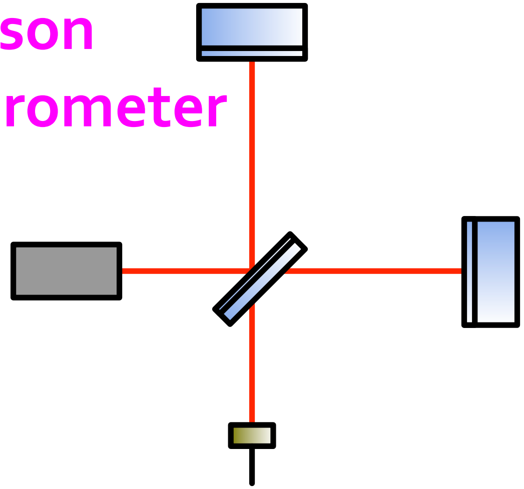
- Differential motion => Michelson interferometer



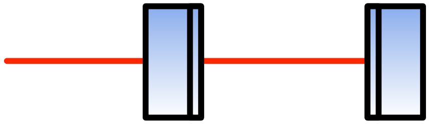
Introduction ~ Interferometer?

- Interferometry
  - Utilize characteristic of a Michelson interferometer as a length comparator

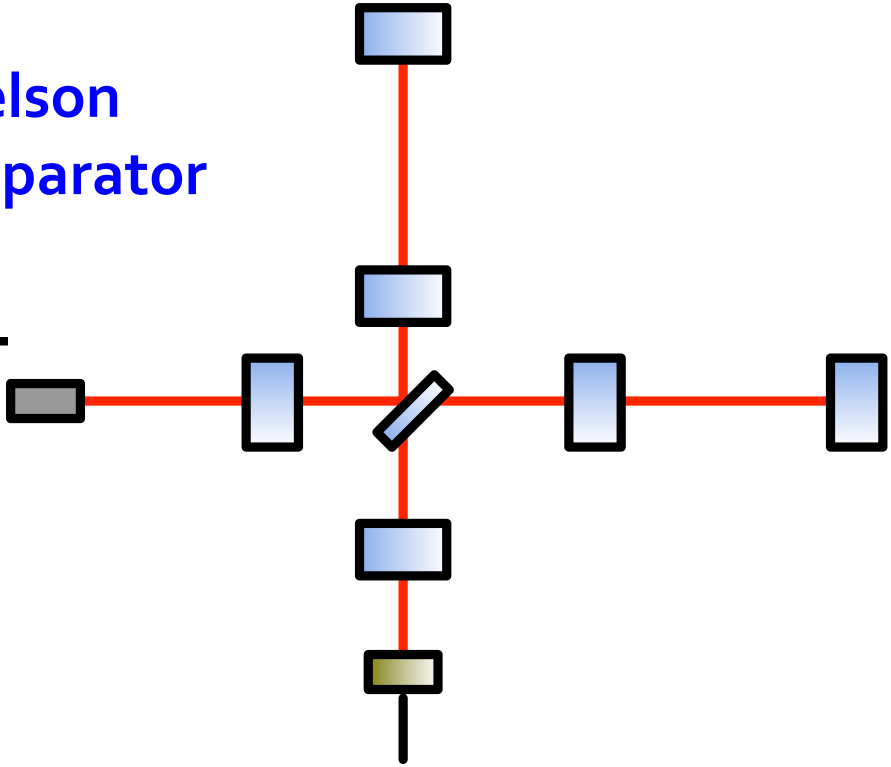
Michelson Interferometer



Fabry-Perot cavity



In reality…



Advanced LIGO

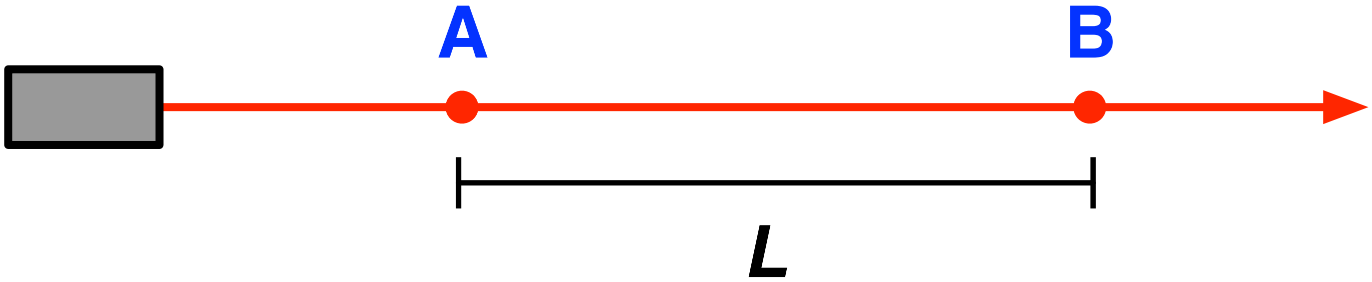
Dual-Recycled Fabry-Perot Michelson Interferometer

No worries: It's just a combination of MI and FPs

Plain wave (single mode) model

Frequency $\nu = c/\lambda$

Angular frequency $\Omega = 2\pi\nu$



$$E_{\mathrm{A}} = E_0 e^{i\Omega t}$$

$$E_{\mathrm{B}} = E_0 e^{i\Omega(t-L/c)}$$

$$\boxed{E_{\mathrm{B}} = E_{\mathrm{A}} e^{-i\phi} \qquad (\phi = \Omega L/c = L/\lambda)}$$

Michelson interferometer

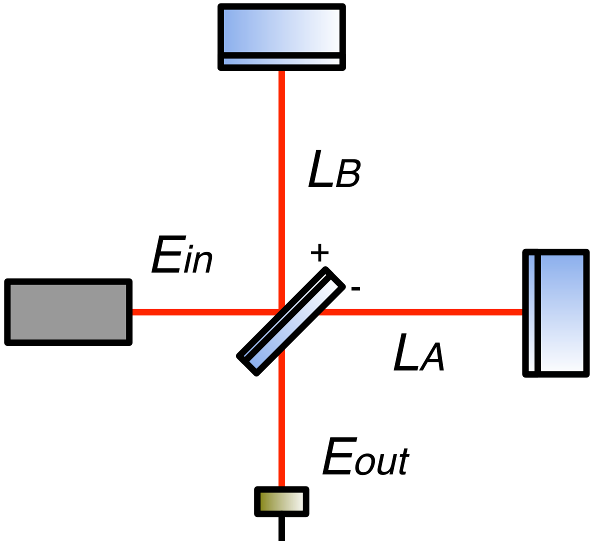
- Light intensity at the output port
  - Difference of the electric fields from the arms

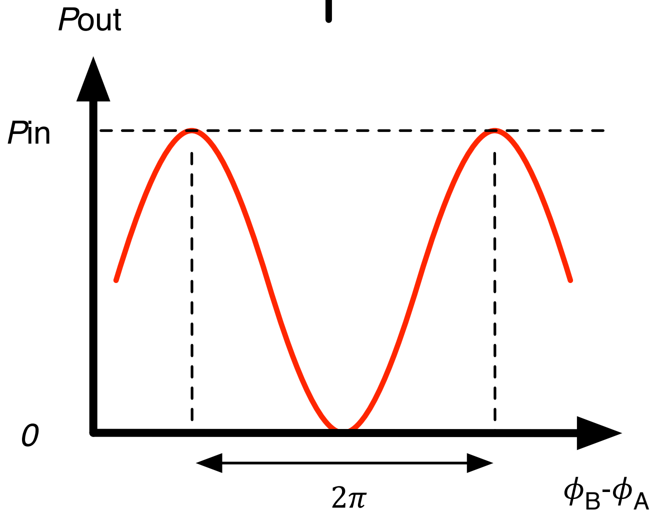
$$E_{\rm out} = \frac{1}{2}\left(e^{-i\phi_{\rm B}} - e^{-i\phi_{\rm A}}\right) E_{\rm in}$$

$$E_{\rm out} = \left[ie^{-i(\phi_A+\phi_B)/2} \sin\frac{\phi_A - \phi_B}{2}\right] E_{\rm in}$$

$$P_{\rm out} = E_{\rm out}E^*_{\rm out} = \left(\sin^2\frac{\phi_A - \phi_B}{2}\right) E_{\rm in}$$

$$= [1 - \cos(\phi_A - \phi_B)]\frac{P_{\rm in}}{2}$$





Output intensity is sensitive to the differential phase

Michelson interferometer

- **Frequency response of the Michelson to GWs**

$$\phi_A - \phi_B = \int_{t-2L/c}^{t} \Omega \left[1 + \frac{1}{2}h(t)\right] dt - \int_{t-2L/c}^{t} \Omega \left[1 - \frac{1}{2}h(t)\right] dt$$

$$= \int_{t-2L/c}^{t} \Omega h(t) dt$$

Frequency response
of the Michelson interferometer

$$h(t) = h_0 e^{i\omega t}$$

$$\phi_A - \phi_B = \boxed{\frac{2L\Omega}{c} e^{-iL\omega/c} \frac{\sin(L\omega/c)}{L\omega/c}} \cdot h_0 e^{i\omega t}$$

$$= \frac{4\pi L}{\lambda_{\mathrm{opt}}} e^{-i2\pi L/\lambda_{\mathrm{GW}}} \frac{\sin(2\pi L/\lambda_{\mathrm{GW}})}{2\pi L/\lambda_{\mathrm{GW}}} \cdot h_0 e^{i\omega t}$$

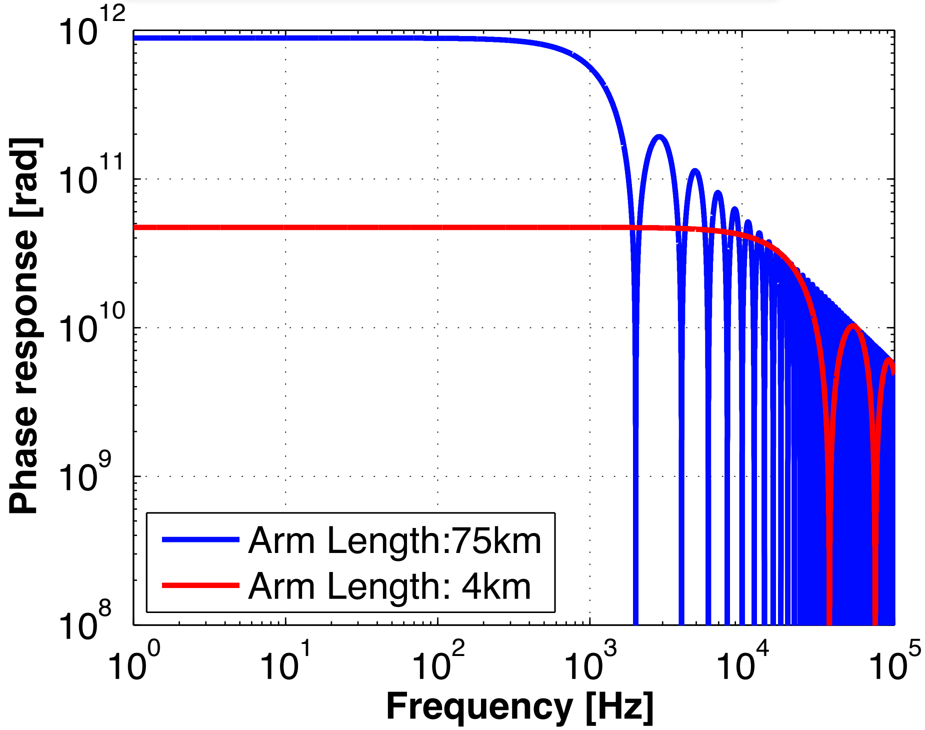
Ω: optical angular frequency, λ_{OPT} laser wavelength
ω: angular frequency of GW, λ_{GW} wavelength of GW

Jean-Yves Vinet, et al
Phys. Rev. D 38, 433 (1988)

Michelson interferometer

- **Frequency response of the Michelson to GWs**

$$\Delta\phi = \boxed{\frac{2L\Omega}{c} e^{-iL\omega/c} \frac{\sin(L\omega/c)}{L\omega/c}} \cdot h_0 e^{i\omega t}$$



DC Response
longer ->
larger

Cut off freq
longer -> lower

Notch freq
f = n c / (2 L)

Michelson arm length optimized for 1kHz GW -> 75km, too long!

Michelson shot noise limit

- **Quantum noises: Shot noise**
  - **Photon shot noise associated with photodetection**

$$i_{\rm shot} = \sqrt{2 e i_{\rm DC}} \quad [{\rm A}/\sqrt{\rm Hz}]$$

  - **Michelson interferometer**

$$i_{\rm DC} = \frac{e \eta P_{\rm in}}{h\nu} \frac{1-\cos\delta\phi}{2} \quad [{\rm A}]$$

$i_{\rm DC}$: DC Photocurrent

η : PD Quantum Efficiency

ν : Optical Frequency

$$\boxed{i_{\rm shot} / \frac{d i_{\rm DC}}{d\phi} = \sqrt{\frac{2 h\nu}{\eta P_{\rm in}}} \quad [{\rm rad}/\sqrt{\rm Hz}]}$$

at the limit of $d\phi$->0

Shot-noise limit of the Michelson phase sensitivity

  - **Michelson response (@DC)**

$$\frac{\delta\phi}{h_{\rm GW}} = \frac{4\pi L \nu}{c} \quad [{\rm rad/strain}]$$

Michelson
Strain Sensitivity

1.3x10^{-20} 1/sqrtHz
@1W, 1064nm, 4km

(5x10^{-17} m/sqrtHz)

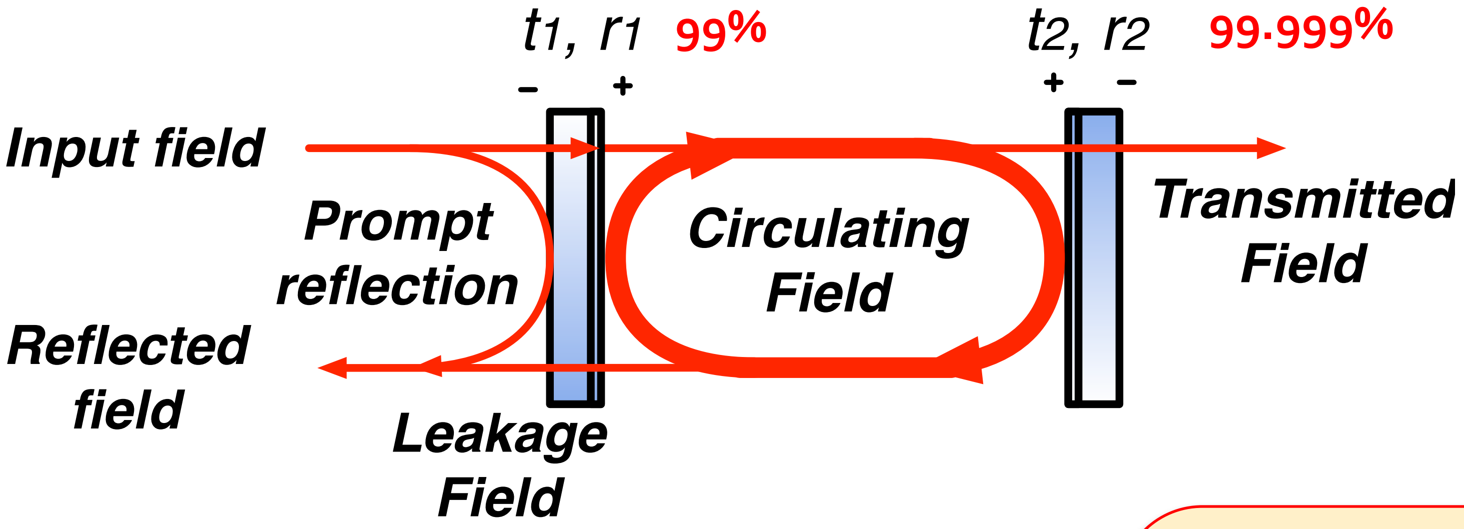
Supplemental slide

Shot noise derivation

- Take an average of Current $I(t)$ for a period of T, and sample it every T.
- Number of photons in this period T is $N = \bar{I}T/e$.
- Fluctuation of photon number in T is $\sigma_N = \sqrt{N}$. cf Poisson statistics
- Thus, the standard deviation (RMS) of $\bar{I}$ is $\sigma_I = e\sqrt{N}/T = \sqrt{e\bar{I}/T}$
- Think about the transfer function of this box car average filter. It is $H(f) = \mathrm{sinc}(\pi f T)$
- Parsevals theorem: $\sigma_I = \int_0^\infty H(f)^2 i_s^2 df$, where i_s is the linear power spectrum density of the current (white spectrum).
- According to the above integration, $i_s = \sigma_I\sqrt{2T}$.
- Therefore we obtain $i_s = \sqrt{2e\bar{I}}$.

Fabry-Perot optical resonator

- Storing light in an optical cavity



  - Input vs circulating: constructive
    Prompt reflection vs leakage: destructive
    => The circulating field grows up

  - N times more power
    Equivalent to have N times longer arm

Finesse

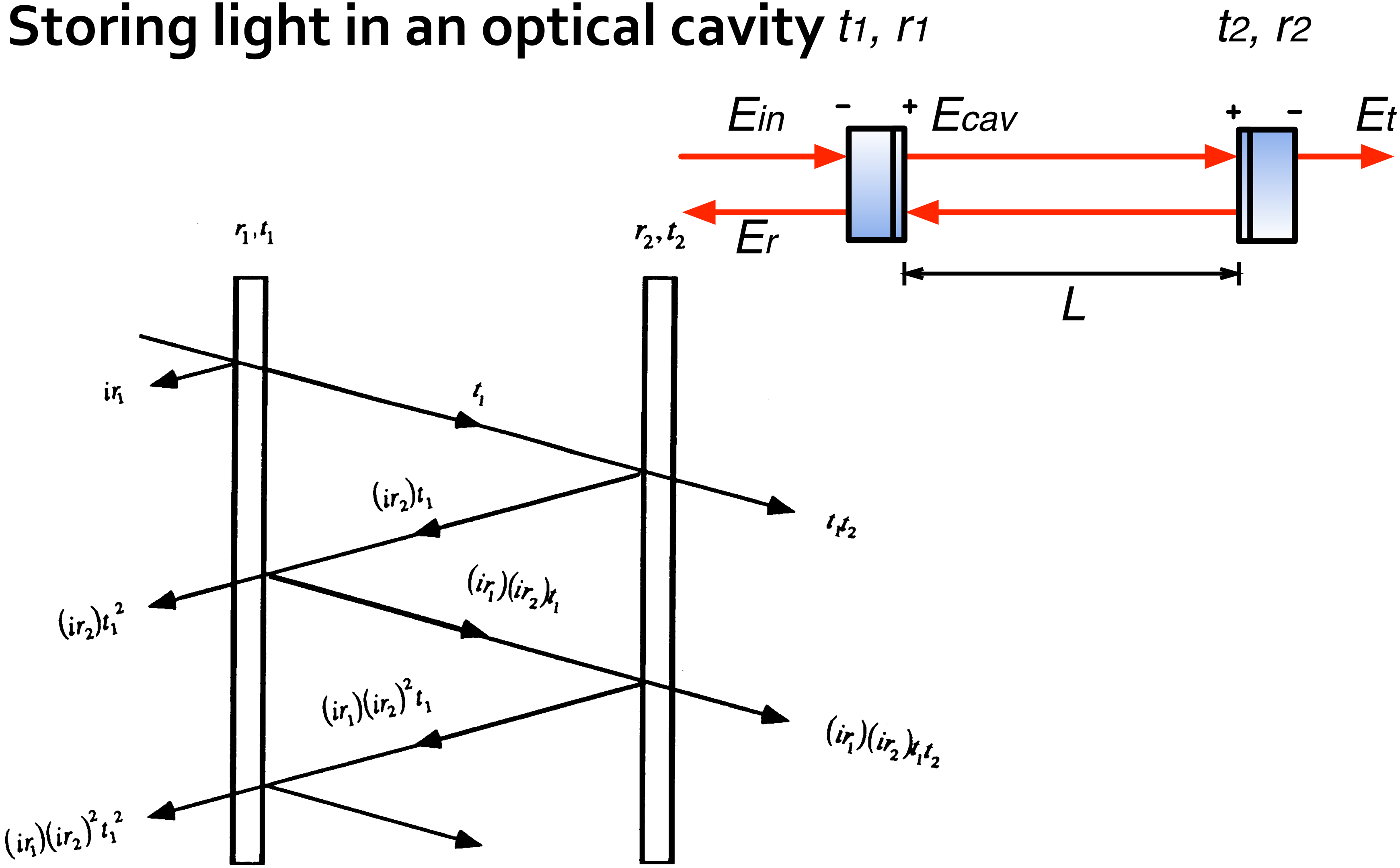
$$\mathcal{F} = \frac{\pi\sqrt{r_1 r_2}}{1 - r_1 r_2}$$

Folding Number

$$N = 2\mathcal{F}/\pi$$

Fabry-Perot optical resonator

- **Storing light in an optical cavity**



Fabry-Perot optical resonator

- **Storing light in an optical cavity**
  - **Field equations**

$$\boxed{E_{\mathrm{cav}} = t_1 E_{\mathrm{in}} + r_2 e^{-i\phi} E_{\mathrm{cav}}}$$

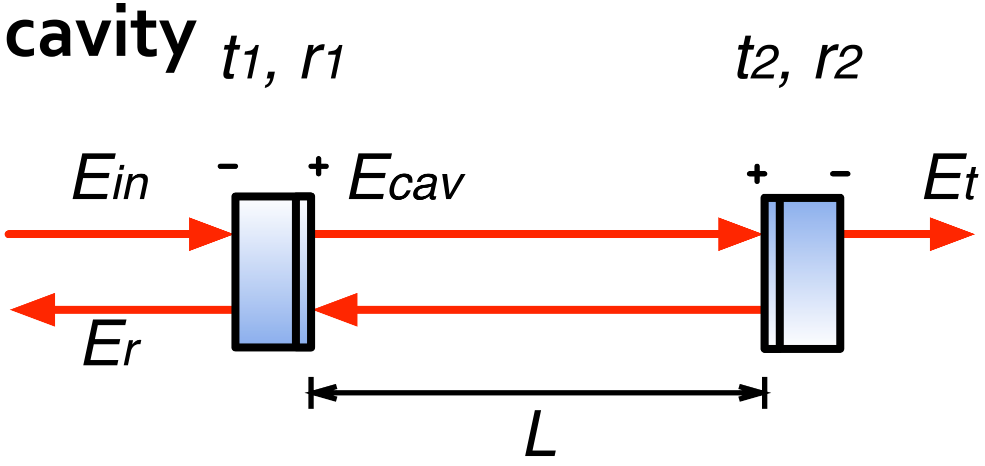
$$E_{\mathrm{t}} = t_2 e^{-i\phi/2} E_{\mathrm{cav}}$$

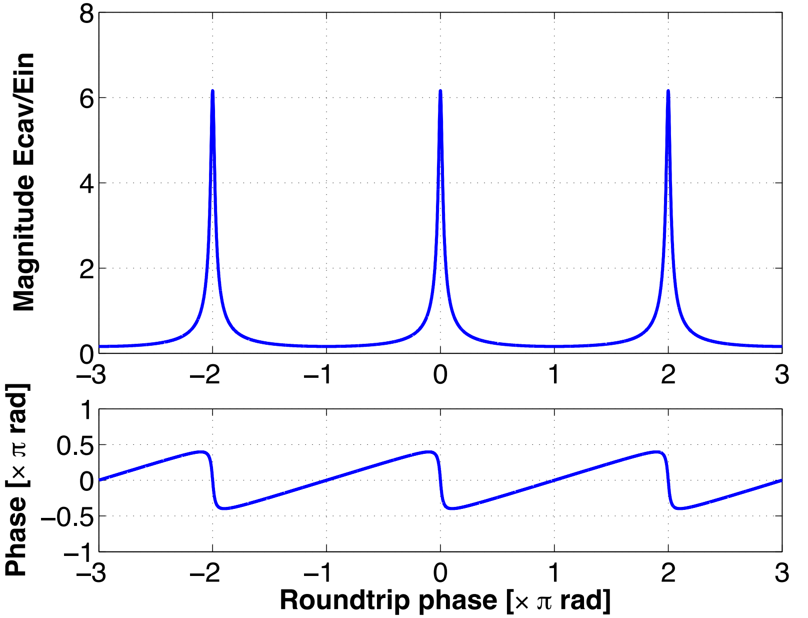
$$E_{\mathrm{r}} = -r_1 + t_1 r_2 e^{-i\phi} E_{\mathrm{cav}}$$

$$\boxed{\frac{E_{\mathrm{cav}}}{E_{\mathrm{in}}} = \frac{t_1}{1 - r_1 r_2 e^{-i\phi}}}$$

$$\frac{E_{\mathrm{r}}}{E_{\mathrm{in}}} = -r_1 + \frac{t_1^2 r_2 e^{-i\phi}}{1 - r_1 r_2 e^{-i\phi}}$$

$$\frac{E_{\mathrm{t}}}{E_{\mathrm{in}}} = \frac{t_1 t_2 e^{-i\phi/2}}{1 - r_1 r_2 e^{-i\phi}}$$

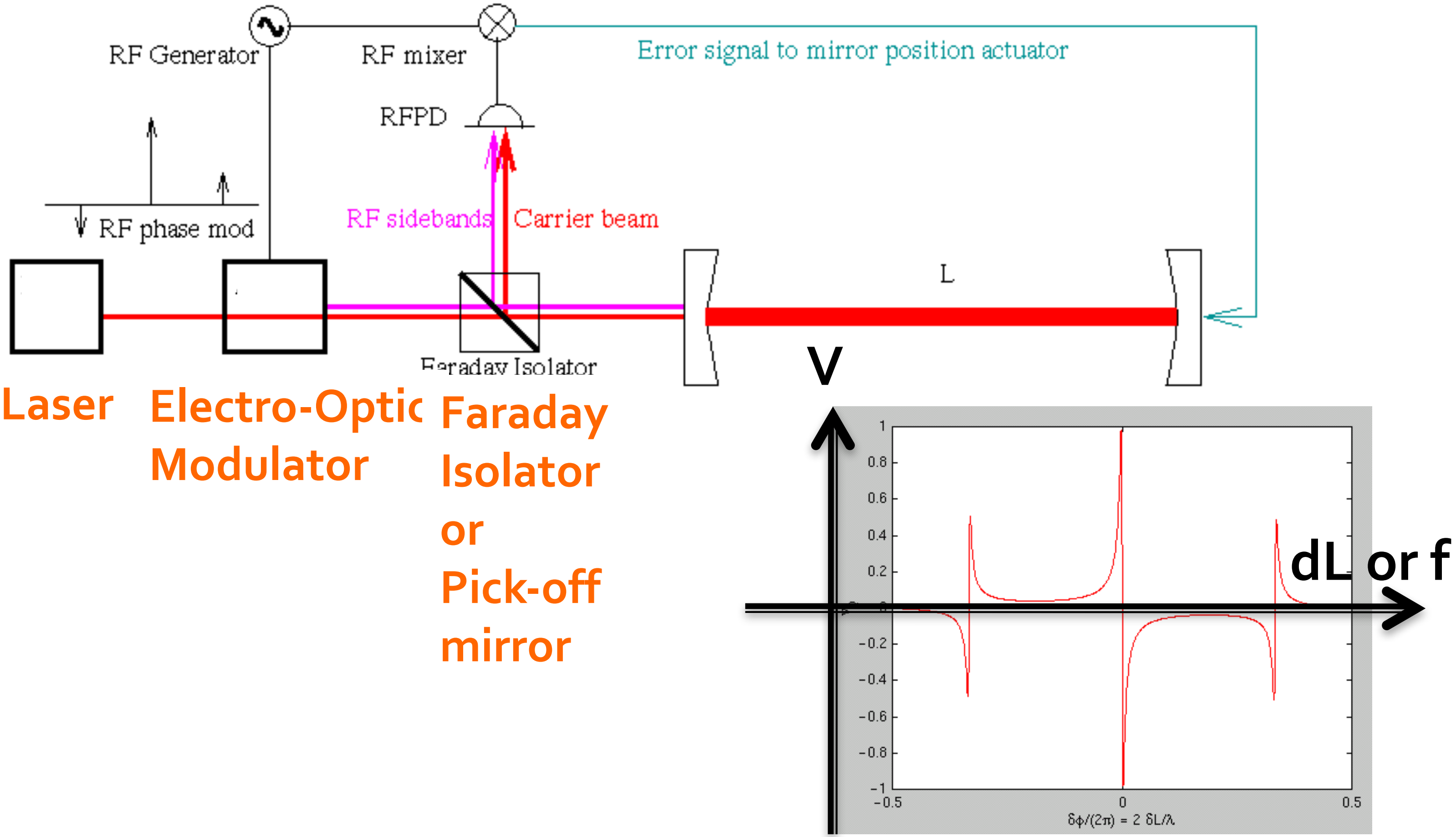




Very fast phase response

Pound-Drever-Hall technique (PDH)

- **Signal extraction scheme for the cavities**
  - Phase modulation -> RF optical sidebands
  - Reflected beam -> detected / demodulated



Laser

Electro-Optic Modulator

Faraday Isolator or Pick-off mirror

Fabry-Perot optical resonator

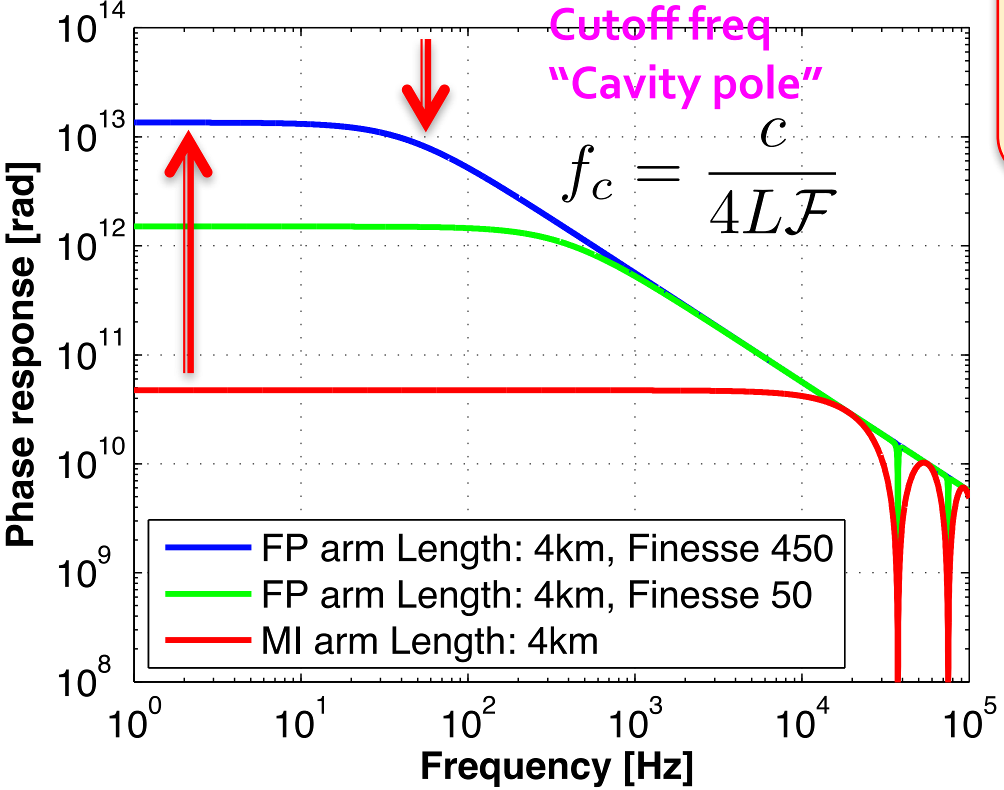
- **Storing light in an optical cavity**

Finesse

$$\mathcal{F} = \frac{\pi\sqrt{r_1 r_2}}{1 - r_1 r_2}$$

DC Response amplification

$$N = 2\mathcal{F}/\pi$$

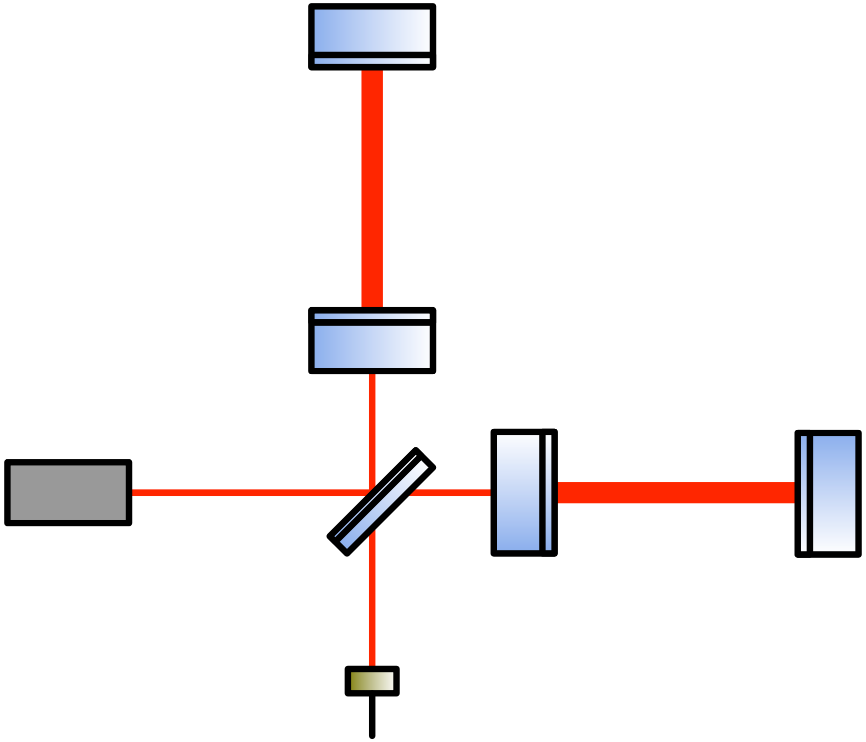


1. FP increases stored power in the arm
2. FP increases accumulation time of the signal

=> Above the roll-off, increasing F does not improve the response

Fabry-Perot Michelson Interferometer

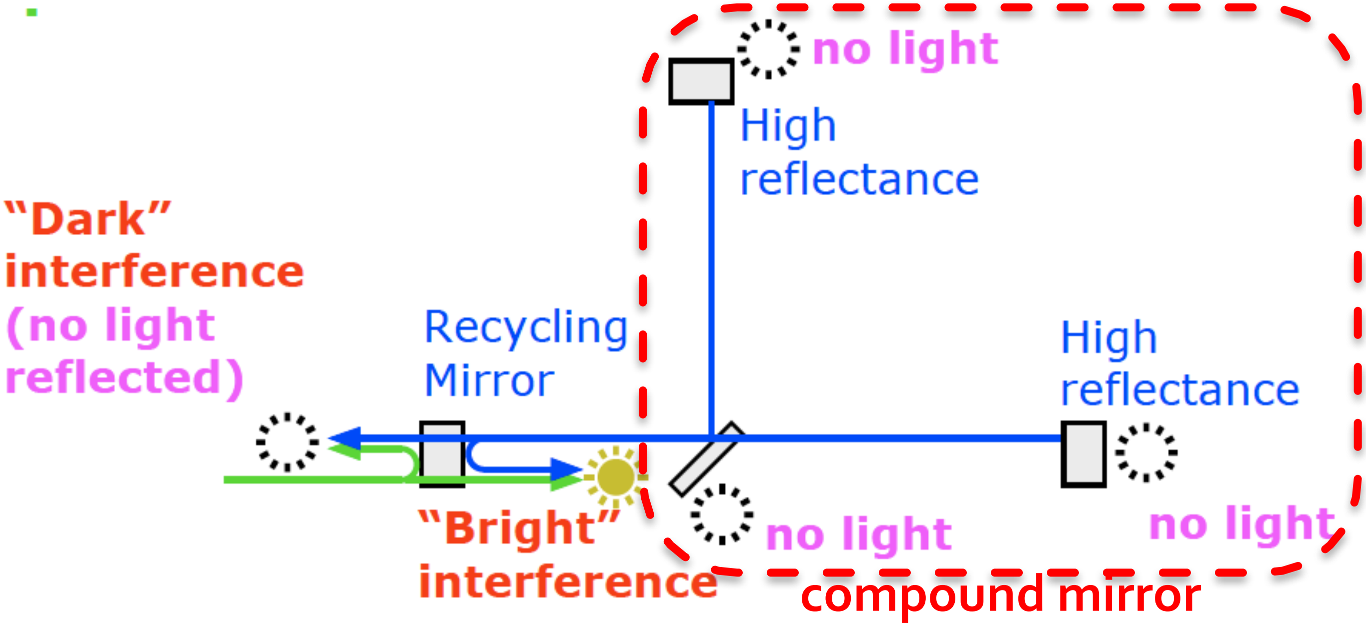
- Differential nature of the Michelson
  + Longer photon storage time of Fabry-Perot cavities
  = Fabry-Perot Michelson Interferometer



Basic form of the modern interferometer GW detector

Optical Recycling Technique

- Power recycling
  - Place a mirror in front of the interferometer to form a cavity with the Michelson (compound mirror) "Power Recycling Mirror"



no light

High reflectance

"Dark" interference (no light reflected)

Recycling Mirror

High reflectance

"Bright" interference

no light

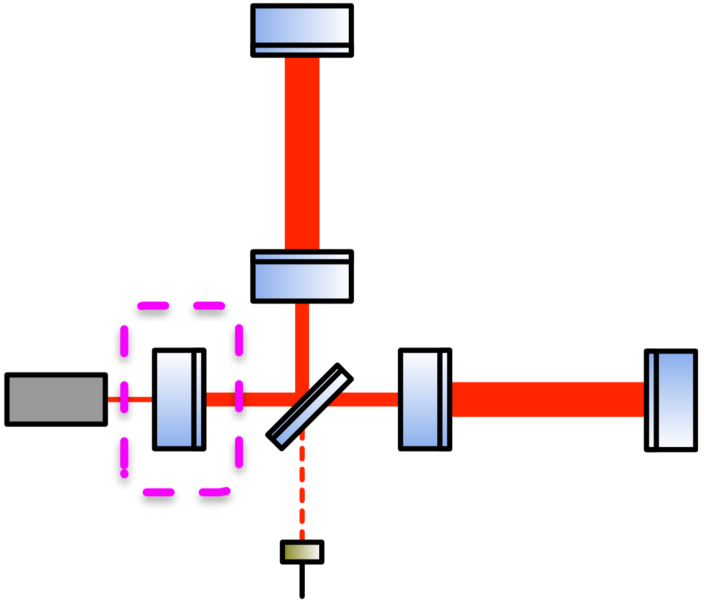
no light

compound mirror

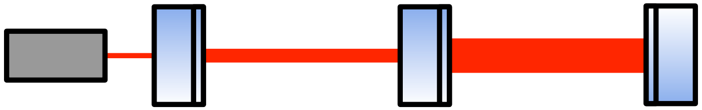
  - The internal light power is increased = equivalent to the increase of the input laser power

Optical Recycling Technique

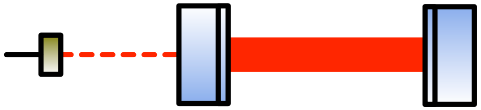
- Power-recycled Fabry-Perot Michelson Interferometer
  - Internal light power in the arms is increased



From the laser side /
common arm length change
It looks like a three mirror caivty
= high finesse cavity

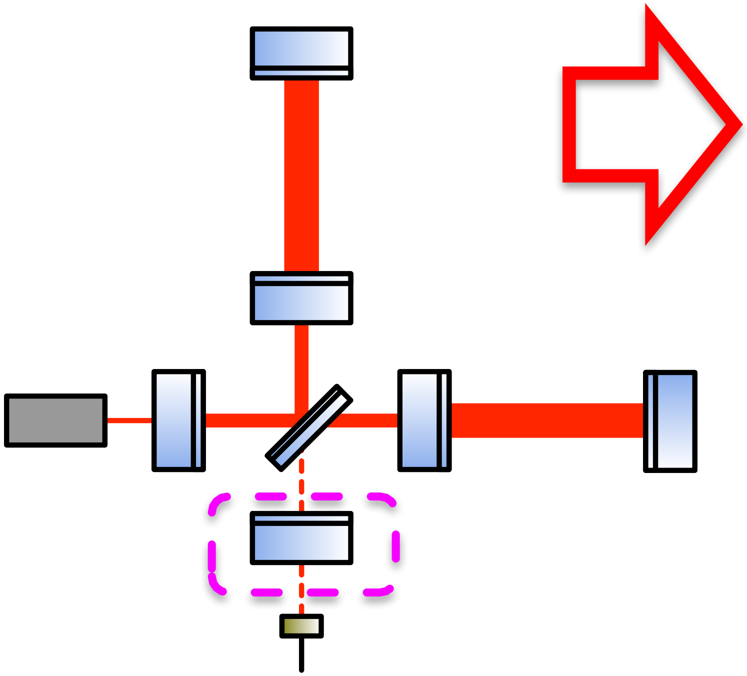


For the differential motion (=GW)
It looks like just an arm cavity



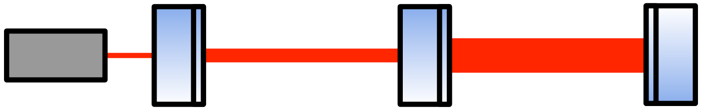
Optical Recycling Technique

- Dual-recycled Fabry-Perot Michelson Interferometer
  - Another mirror is added at the dark port
    "Signal Recycling Mirror"
  - Dual recycling allows us to set different storage times for common and differential modes



Common mode

= high finesse three mirror cavity

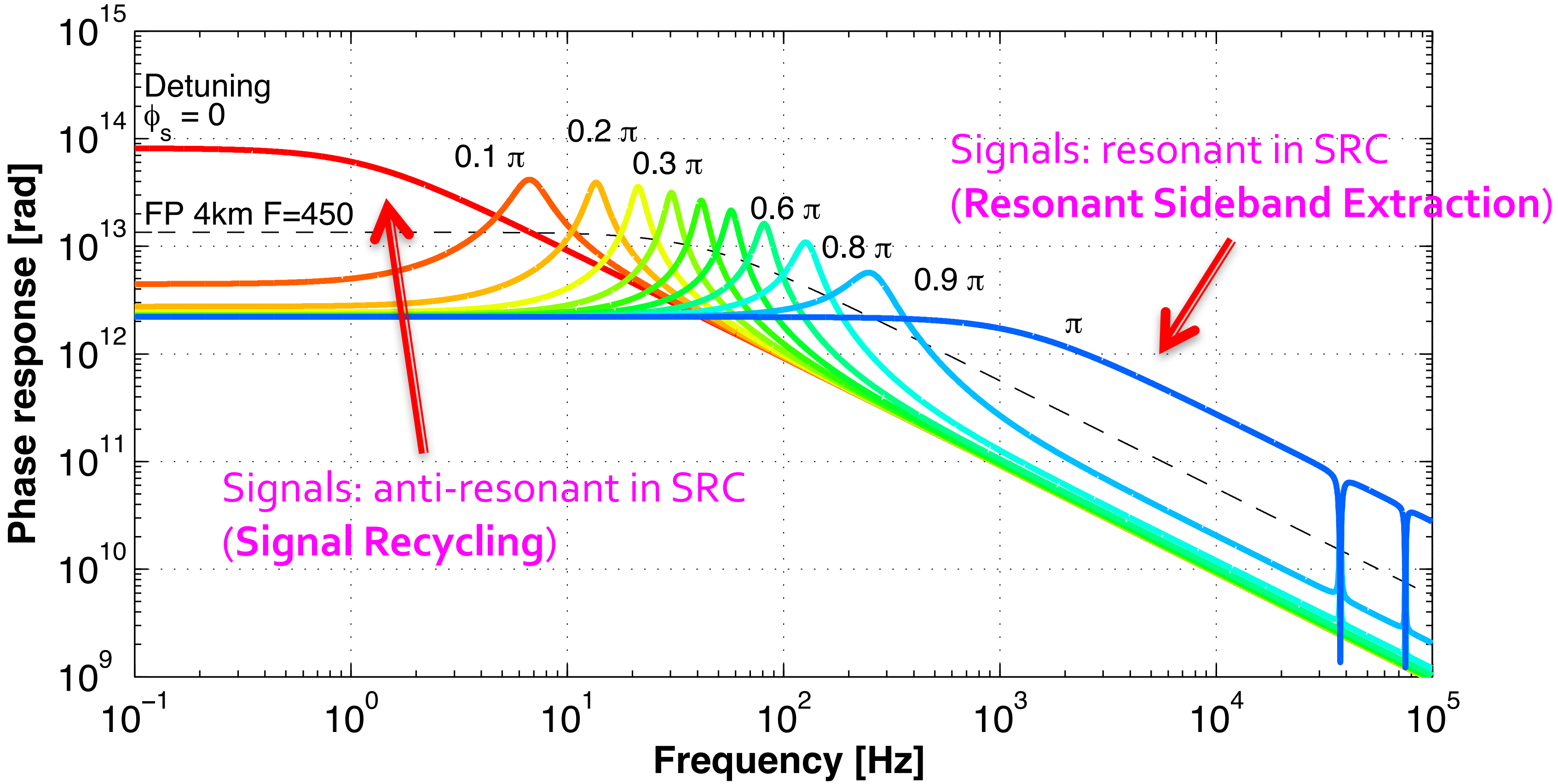


Differential mode (=GW)

= low (or high) finesse three mirror cavity

To tuned or not to tune

- Bandwidth of the detector can be changed
  - by changing the resonant phase of the signal recycling cavity (SRC)



Signals: resonant in SRC
(Resonant Sideband Extraction)

Signals: anti-resonant in SRC
(Signal Recycling)

- Optimize the curve depending on the noise shape
- Dynamic signal tracking

Summary

- Optical phase measurement => Interferometry

- Michelson interferometer: requires too long arm
- Fabry-Perot arm: longer light storage time

- Optical recycling technique:
  allows us to set different storage time for the incident light and the GW signals
  - Power recycling: maximize the stored light power
  - Resonant sideband extraction: optimize the signal bandwidth