Using Continuous GWs from Known Pulsars to Measure Gravitational Wave Speed: **First Project Update**

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1 Project Description

Gravitational waves (GWs) were predicted by Einstein in 1916 during his development of the Theory of General Relativity (GR) and finally detected directly in September 2015 by the Laser Interferometer Gravitational wave Observatory (LIGO)[1]. Now that direct measurements of the phenomena are possible, we can start to analyze their properties in more depth. We will look into the measurement and bounding of the speed of continuous gravitational waves by comparing variances observed in long-term signals using the motion of the detector relative to the source.

Unlike the black hole mergers from the three observations, certain sources of gravitational waves are constant. These objects gives signals with very consistent and long lifespans but small amplitudes, called continuous waves (CWs). This research will focus on pulsars which could emit CWs.

The unmodified strain signal takes the form of a sinusoid (thus gravitational *waves*). However, since the Earth is moving relative the source, there is a significant shift due to Doppler Effects. Research into the methods of doing such (including heterodyning data) have been well researched[4–6], but were generally unsuccessful in estimating the speed of GWs. However, since the first of those papers was published there have been major improvements to the sensitivity of the device.

2 Problem Description

Before we get the data, a lot of things happen to the signal to make it not a perfect sine wave.

The motion of the source leads to a frequency/phase evolution of the wave. For a simplified version, we know that the pulsar spins at ν (revolutions per second) but then has a slowdown of $\dot{\nu}$ (revolutions per second per second). Because of this slowdown factor, we would need to have a wave with changing frequency. In reality, we have electromagnetic observations that tell us the actual spinning rate at any time, which involves some unpredictable breaks from smoothness.

The rotation of the Earth leads to two more transformations.

There is a factor called the Rømer delay which deals with the time necessary to travel the extra distance when the observatory is farther than when it is closer. This has a period of a day (with some terms with half a day because the CWs can go through Earth).

There is a major factor in the change in the sensitivity of the antennas. Certain angles optimize the sensitivity of the detectors, dependent on the source position. This leads to an amplitude modulation across a half day. This is actually most of the signal we want to see.

Once the wave travels to the detector, the raw data goes through a long process in order to hope to see a signal. For continuous waves with a known source, the process follows the following path:

1. Raw data is collected
2. Filters (e.g. Butterworth) applied
3. Heterodyne based on source motion (assumes c)
4. Downsample

The first process is beyond the interests of this research, but it suffices to mention that there is plenty of noise, i.e. it's almost exclusively noise.

The second process is also not yet applicable, but it removes some of the aforementioned noise and much of the high frequency things.

The third process is where much of the research is happening. In this step, the waveform is multiplied by a complex sinusoid with phase based on the source. This step deals with the Rømer delay and should leave one fast sinusoid and one slower one (for the Crab nebula, the fast one would be at about 100 Hz and the slow one would be at about 0.00002 Hz). The faster sinusoid is a fast version of the signal while the slow sinusoid represents the amplitude modulation from the antenna patterns.

The fourth process reduces the number of points significantly in order to ignore the fast signal. This leads to just the amplitude modulation, with magnitude proportional to the strain on the observatory.

3 Progress

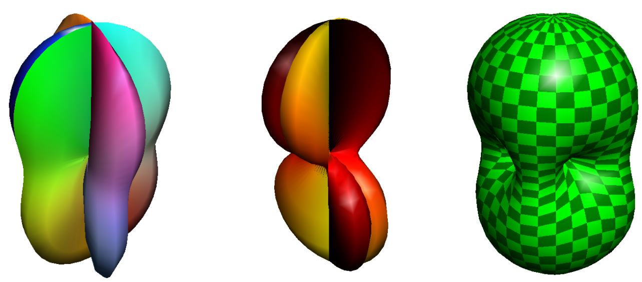
GO FROM h(t) to lambda(t) (That's both heterodyning and downsampling explained mathematically)

There were four major things that needed to be broken down for the project: antenna patterns, amplitude modulation, frequency modulation, and Rømer delay.

The antenna pattern represents the sensitivity to the detector in a certain direction to a certain polarization of GWs. This can be calculated for a single detector either analytically or numerically. The general formula for the sensitivity of a detector with two arms and zenith $\mathbf{d}_x$, $\mathbf{d}_y$, and $\mathbf{d}_z$ and a source with rotation direction and axes $\mathbf{w}_x$, $\mathbf{w}_y$, and $\mathbf{w}_z$ is

$$F_+ = \frac{1}{2}[(\mathbf{w}_x \cdot \mathbf{d}_x)^2 - (\mathbf{w}_x \cdot \mathbf{d}_y)^2 - (\mathbf{w}_y \cdot \mathbf{d}_x)^2 + (\mathbf{w}_y \cdot \mathbf{d}_y)^2]$$
$$F_\times = (\mathbf{w}_x \cdot \mathbf{d}_x)(\mathbf{w}_y \cdot \mathbf{d}_x) - (\mathbf{w}_x \cdot \mathbf{d}_y)(\mathbf{w}_y \cdot \mathbf{d}_y)$$

This is derived from the tensors for both polarizations. Plotting these for different relative orientations yields the following plots[9]:



(a) Plus (F_+) (b) Cross ($F_\times$) (c) $\sqrt{F_+^2 + F_\times^2}$

Figure 1: Antenna patterns for the two GR polarizations

The problem becomes interesting when one considers the fact that the Earth is revolving and therefore all the detector's vectors are completely dependent on time. One can prove with some private time with the above equations that the sensitivity of the detector should have periodicity of both a day and half of a day.

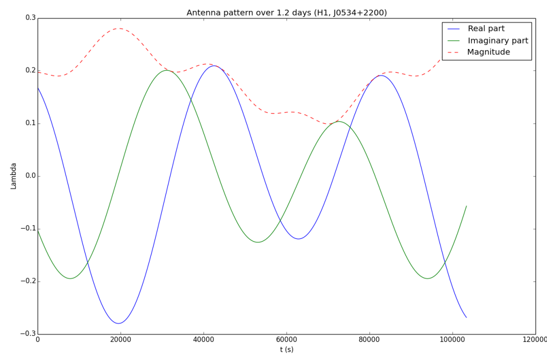
We know the ideal signal $h(t)$ consists of both polarizations in a predicable way. We know that since the signal is real and has a sinusoidal element, there exists a function $\Lambda(t)$ and a function $\theta(t)$ such that

$$h(t) = \Lambda(t)e^{i\theta(t)} + \Lambda^*(t)e^{-i\theta(t)}$$

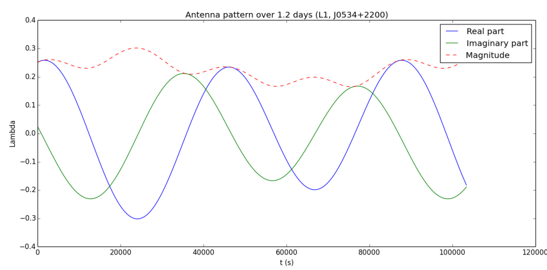
Based on previous work[CITATION PLEASE,9], we know how to calculate this using the antenna patterns and the magnitude of the strain, h_0:

$$\Lambda(t) = h_0 \left(\frac{1 + \cos^2 \iota}{4} F_+(t) + \frac{i \cos \iota}{2} F_\times(t) \right) \quad (1)$$

We implement these equations in order to actually discuss what would happen if a constant magnitude signal came by (as all CWs pretty much are). For simplicity's sake, the following graphs use data from the Crab pulsar (well understood):



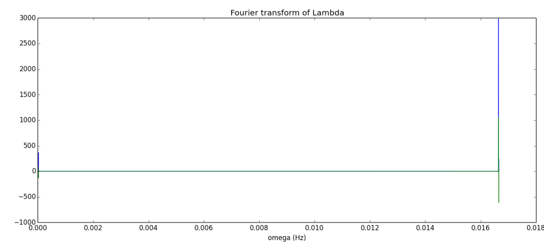
(a)



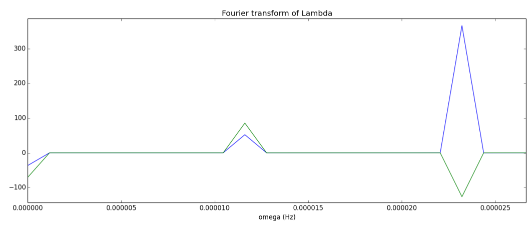
(b)

Figure 2: Overall sensitivity to Crab pulsar over a day at Hanford and Livingston Observatories

In order to check that this has the two frequencies we expect, we Fourier transform it. If we look closely, we can see the two frequencies we anticipate, as shown below.



(a)



(b)

Figure 3: (a) Fourier transform for the whole range (to double the Nyquist frequency) then (b) zooming in on the barely noticeable bumps near 0; blue is real, green is imaginary. The three visible bumps represent an inherent term at 0, the diurnal cycle at about 1/86400 Hz, and the twice diurnal cycle at about 1/43200 Hz.

From this variable sensitivity, we know that the signal will have an amplitude modulation and therefore we need to extract it from raw data. As mentioned, the original signal starts as

$$h(t) = \Lambda(t)e^{i\theta(t)} + \Lambda^*(t)e^{-i\theta(t)}$$

For our purposes, $\theta(t)$ is well understood, so we are able to heterodyne the signal. This process involves just multiplying by a complex sinusoid in order to cancel out some of the properties of the original, i.e.

$$h_{het}(t) = e^{-i\theta(t)} \cdot h(t) = \Lambda(t) + \Lambda^*(t)e^{-2\theta(t)}$$

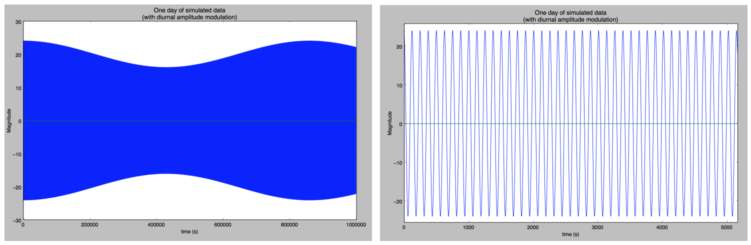
which involves a very slow wave and a very fast one. It may be important to note here that even though the signal was purely real, we now are dealing in the complex domain.

In order to get rid of the fast wave, we can downsample. If we break the entire set of data into minute sized chunks indexed by k, we can say that their average value can be our downsampled value, i.e.

$$B_k = \frac{1}{M}\sum_{i=1}^{M} h_{het}(t_i)$$

where M is the total number of samples in a minute.[PitkinThesis] For actual LIGO data, this takes the sampling rate from 16384 Hz to $\frac{1}{60}$ Hz.

To show this in practice, we generated a day of approximate signal with an amplitude modulation with a much lower frequency. This data is shown below:



(a) (b)

Figure 4: (a) The simulated signal for a day; the blue coloring is actually due to a very fast sinusoid present which is visible when zoomed in as in (b). Note the amplitude modulation obvious in (a).

The first step is to heterodyne. The phase evolution of this signal is known, so $\theta(t)$ leads us to

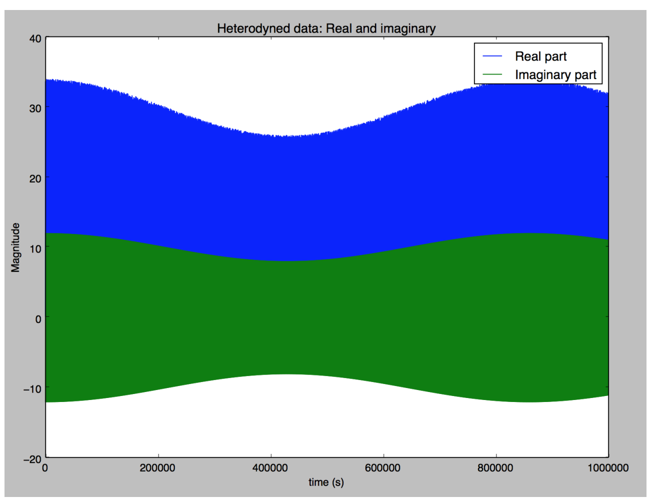


Figure 5: Heterodyne of the above data with known frequency evolution. Again, the regions look filled due to the very fast signal.

Next, we take that data and downsample significantly (here, downsampling was by a factor of a few thousand), taking an average over every segment. This eliminates the high frequency data. This is not how actual LIGO data does it; since

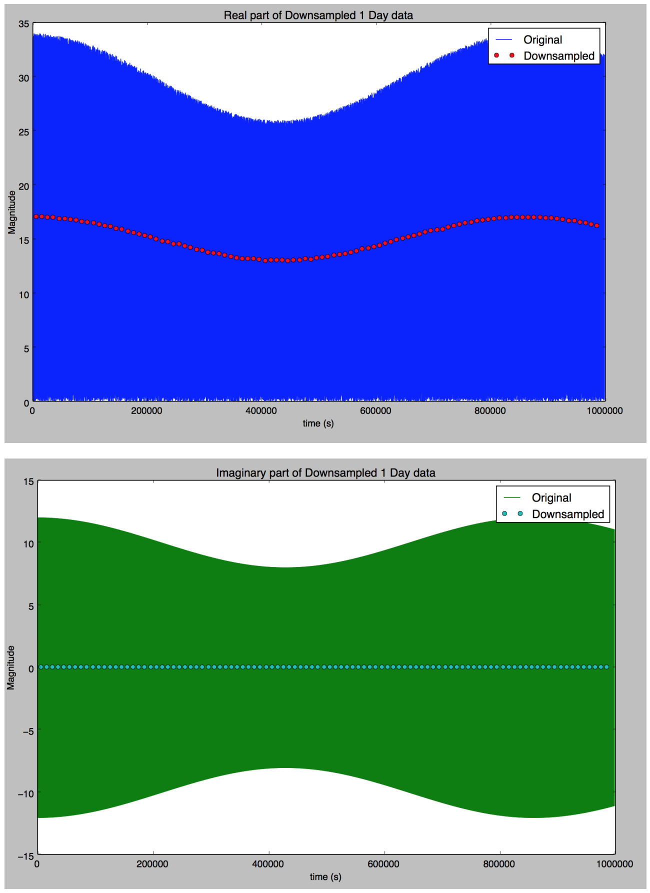


Figure 6: Real and imaginary parts of the downsampling from the heterodyned data. Note how the real part represents half of the amplitude of the final wave and the imaginary part goes to zero; this reflects the original data's real part.

In the above example there were some simplifications. The sinusoids were all constant in their frequency (i.e. $d^2\theta/dt^2 = 0$). In reality, this is a close approximation, but not exact. We know that the source has a rotational speed of $\nu(t)$ which varies significantly. With older pulsars, this is a pretty smooth rate, but for younger pulsars, unexpected errors happen at a significant rate. These glitches are seen in the EM spectrum, so if $c_g \approx c$, these should align pretty well in time and allow us to use astronomical observations during the time of observation in order to model the frequency of the expected wave. If $c_g << c$, this phase evolution may hurt the search, but that case is not expected[CITATION].

There is a small modulation of the time it takes GWs to get to the Earth. Over a day, this delay (known as the Rømer delay) would be significant in the transformation of the wave. This allows us to measure the Doppler shift of the wave.

To explain why that is, we need to understand the formula for the delay. We have

$$\Delta_R = \frac{\vec{r} \cdot \hat{n}}{c_g}$$

where $\vec{r}$ points from the barycenter of the solar system (SSB) to the detector and $\hat{n}$ is the unit vector pointing from the source to the SSB. We will assume $\hat{n}$ is constant with value $\hat{n}_0$, as the source is effectively at rest relative the SSB. We know $\vec{r}$ has two components: $\vec{r} = \vec{R} + \vec{s}$ where $\vec{R}$ points from the SSB to the center of Earth and $\vec{s}$ points from Earth to the detector. $\vec{R}$ oscillates over a year while $\vec{s}$ has periodicity of a day. For most purposes, we will mostly ignore the influence of $d\vec{R}/dt$. We can calculate the Rømer delay across a day some day in the year:

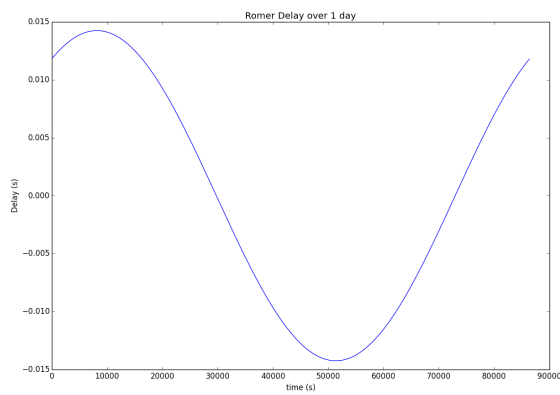


Figure 7: The Rømer delay over a day makes a sinusoid

Now that we understand the basics, we can start to analyze the possibility of this process falsely assuming $c_g = c$. Combining all of the above, we can find the following equation:

$$h(t) = \Lambda(t)e^{i\theta(t)} + \Lambda^*(t)e^{-i\theta(t)} \qquad \text{where } \theta(t) = 2\phi(t) \approx 2(\phi_0 + \nu(t+\Delta_R) + \frac{\dot{\nu}}{2} \cdot (t+\Delta_R)^2 + \frac{\ddot{\nu}}{6}(t+\Delta_R)^3$$

where $\phi(t)$ is the motion of the source. If we let $\theta(t; v)$ be the angle assuming a speed v of gravitational waves. The heterodyne assumes $c_g = c$, so

$$h_{het}(t) = \Lambda(t)e^{i(\theta(t;c_g)-\theta(t;c))} + \Lambda^*(t)e^{-i(\theta(t;c_g)+\theta(t;c))}$$

Downsampling/ low pass filtering removes the fast signal. Then dividing by the anticipated antenna pattern yields

$$\frac{h_{ds,het}(t)}{\Lambda(t)} = \exp\left(i(\theta(t;c_g) - \theta(t;c)\right)$$

Seeing this modulation amid the noise is now equivalent to finding the speed of gravitational waves.

4 Goals

I am hoping to be able to search for a signal in data soon using χ^2 distributions. I will then create different waveforms and compare the templates for different speeds of GWs. After that, I will focus on moving away from the toy models that I am currently creating, implementing with actual LIGO data. Once we've analyzed data from O1, then we will hope to see the modulation in any way, at least giving a bound on the difference between the two waves.

5 Challenges

So far, the largest challenges have been in understanding the math and implementing it. I have had days lost to geometry arguments and that yields a lot of frustration. However, it has been incredibly rewarding to work through these problems, even sometimes spurred by a significant push in the right direction by my mentors.

6 Resources

I will eventually start to use a cluster to analyze data if my laptop stops being sufficiently fast, but otherwise, my mentors have been helpful enough for me to make it through.

7 Citations

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