Use of the Bayes Factor to Improve the Detection of Binary Black Hole Systems

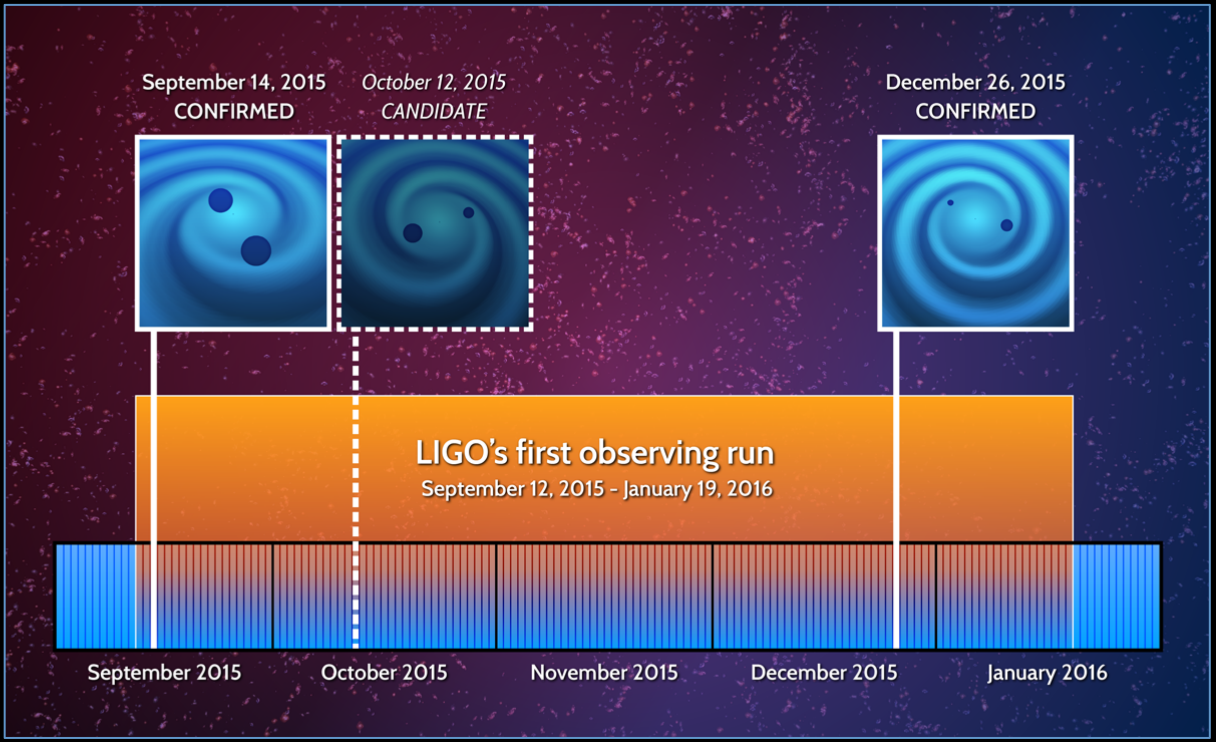
Avi Vajpeyi
Rory Smith, Jonah Kanner

LIGO SURF 16

Summary

- Introduction
  - Detection Statistic
  - Bayesian Statistics
- Selecting Background Events
- Bayes Factor
  - Results
  - Drawbacks
- Bayes Coherence Ratio
  - Results
  - Comparison with SNR

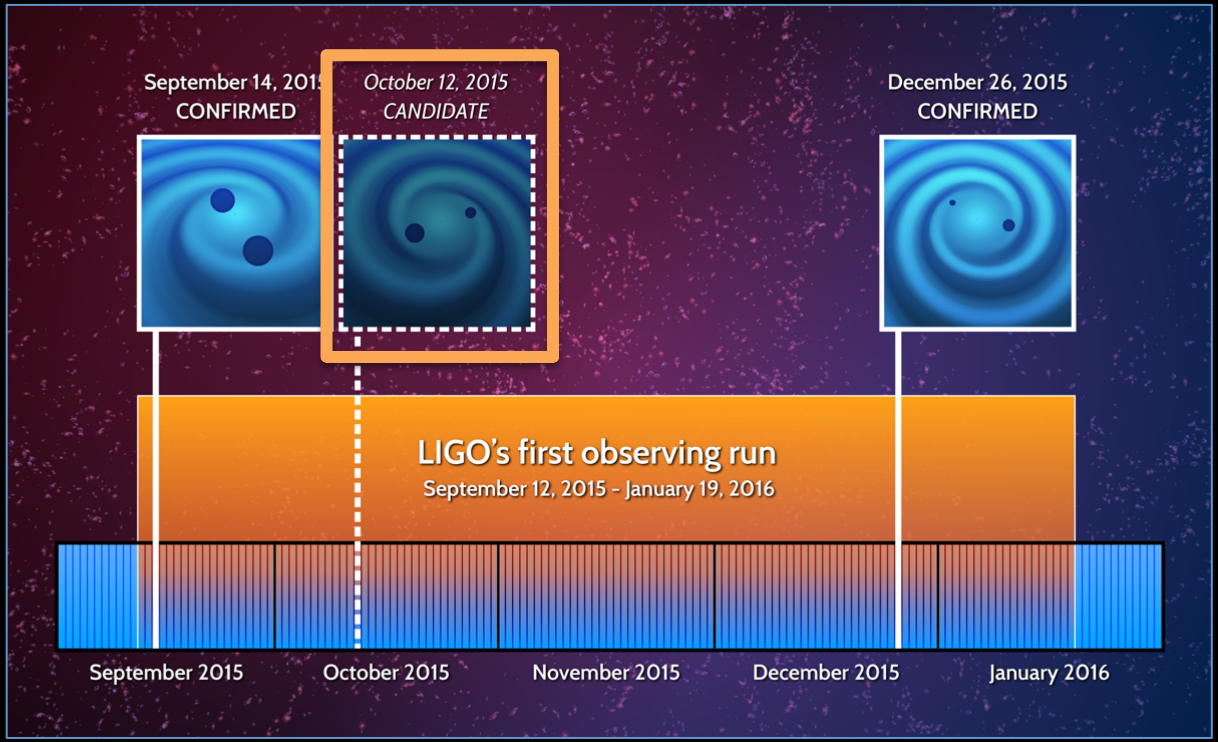
Overview



Some candidate events like LVT151012 have low Signal-to-Noise ratios which fall within the background distribution

LIGO

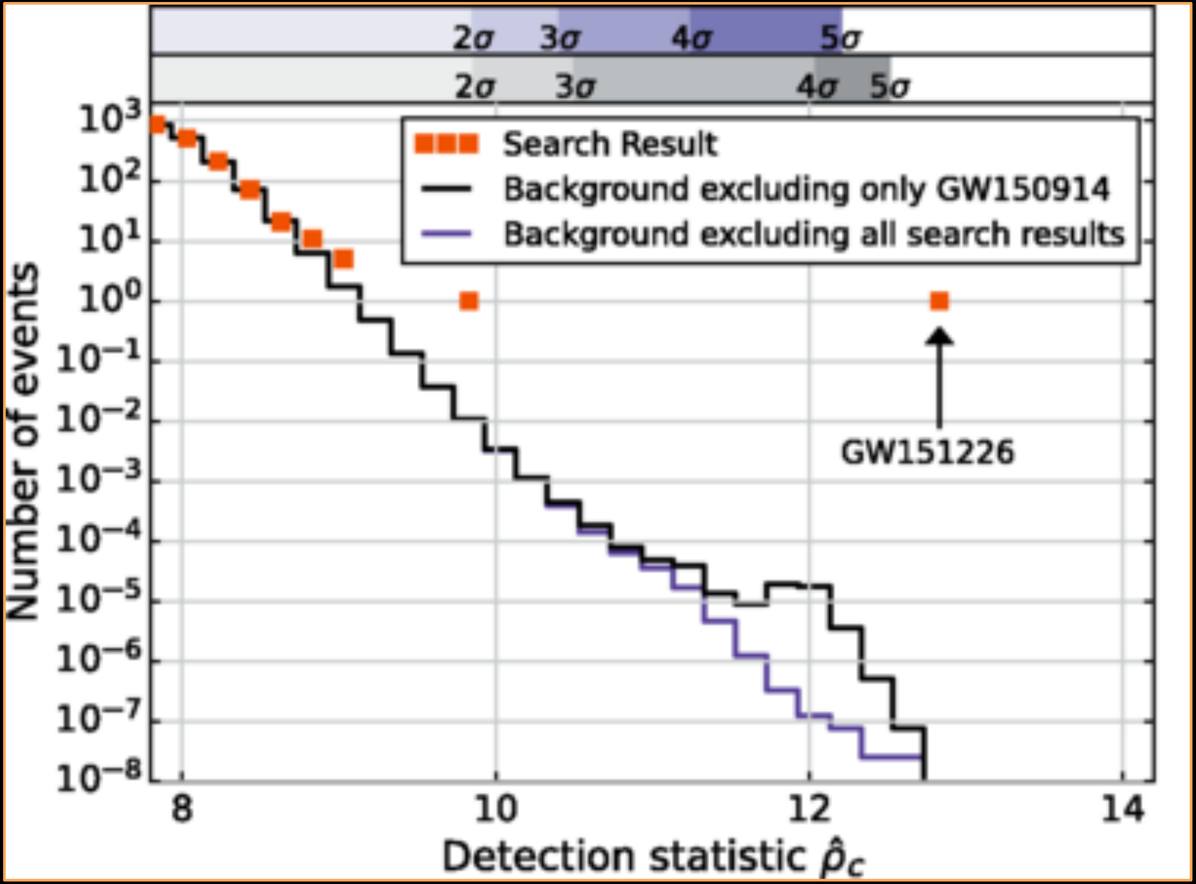
Overview



Can the Bayes factor help increase the detection confidence for binary black hole systems?

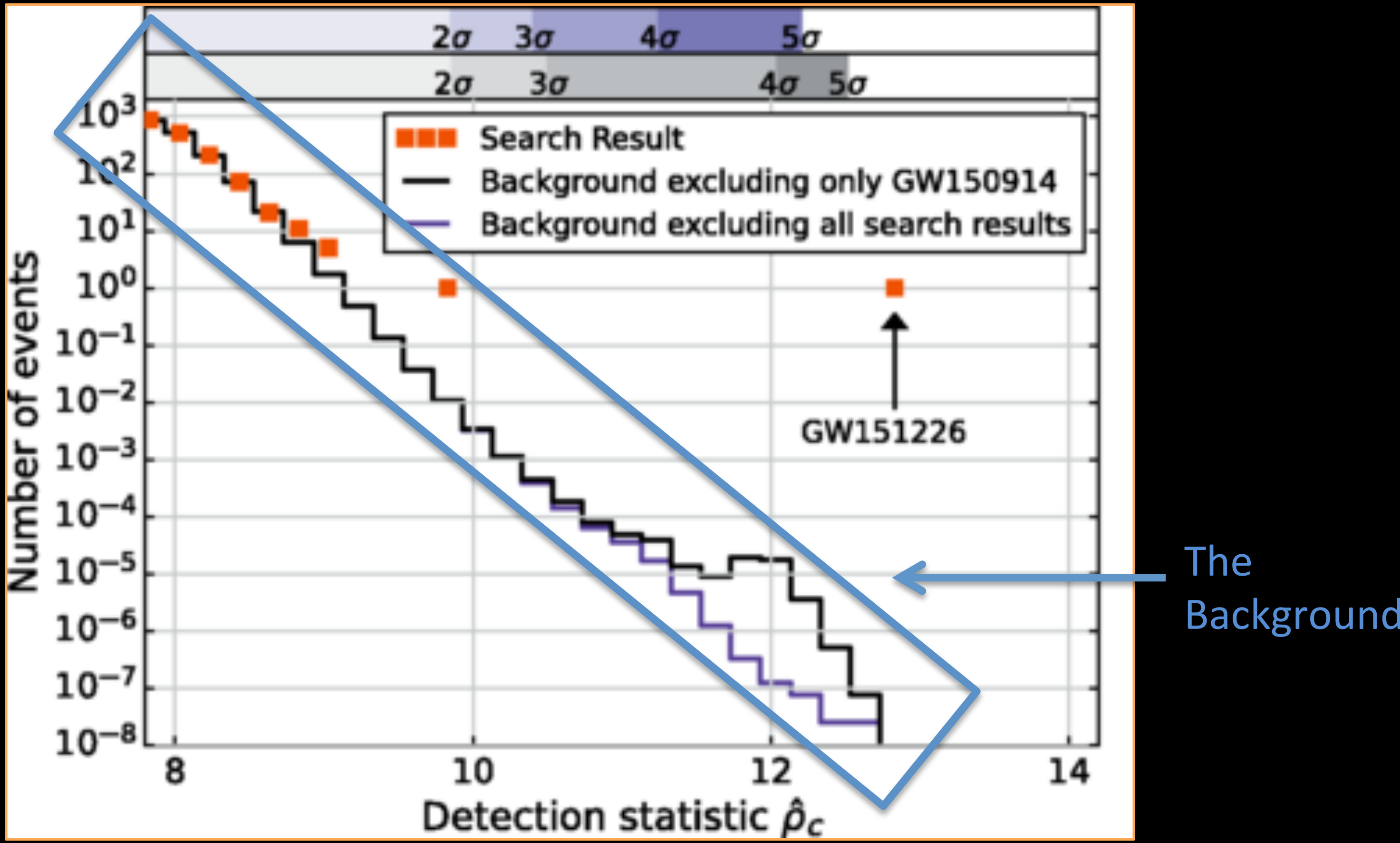
LIGO

Detection Statistic



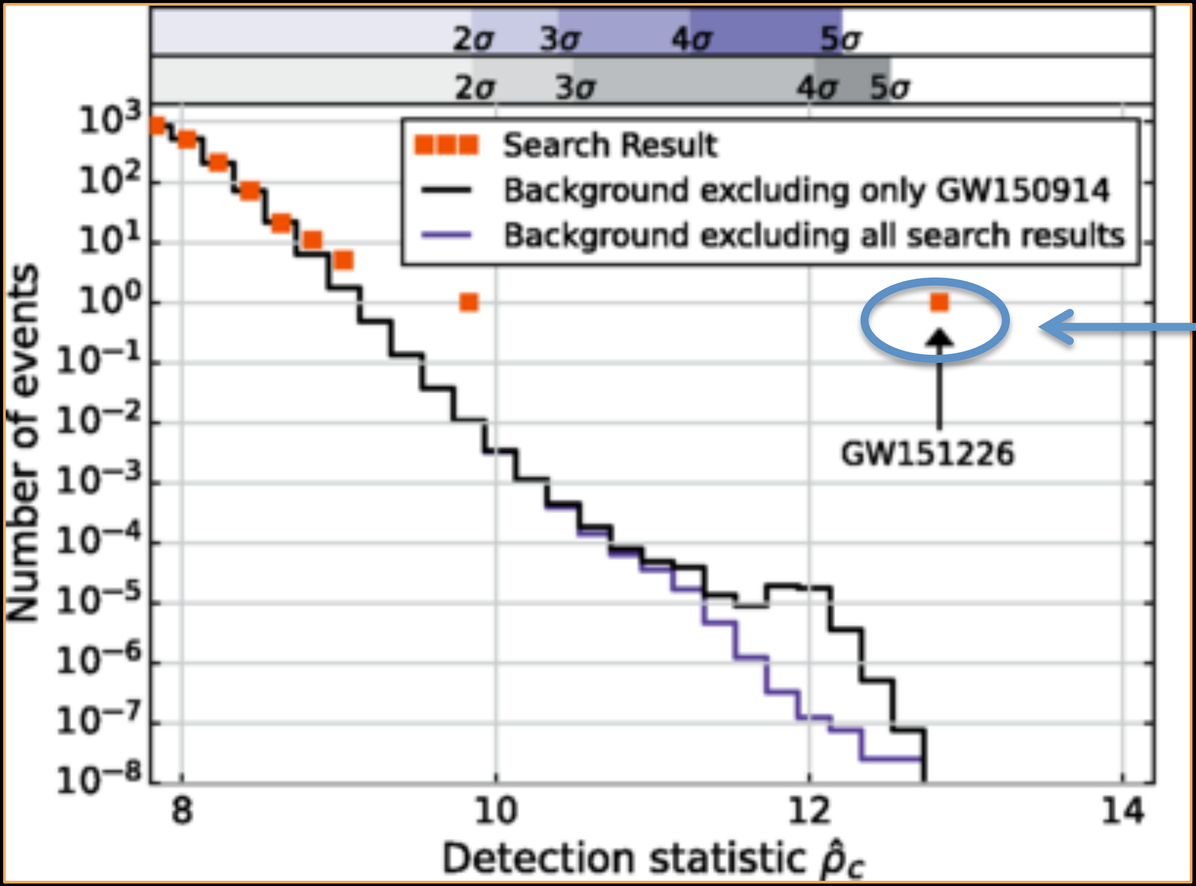
Search results from the two binary coalescence searches using a combined matched filtering signal-to-noise ratio

Detection Statistic



Search results from the two binary coalescence searches using a combined matched filtering signal-to-noise ratio

Detection Statistic

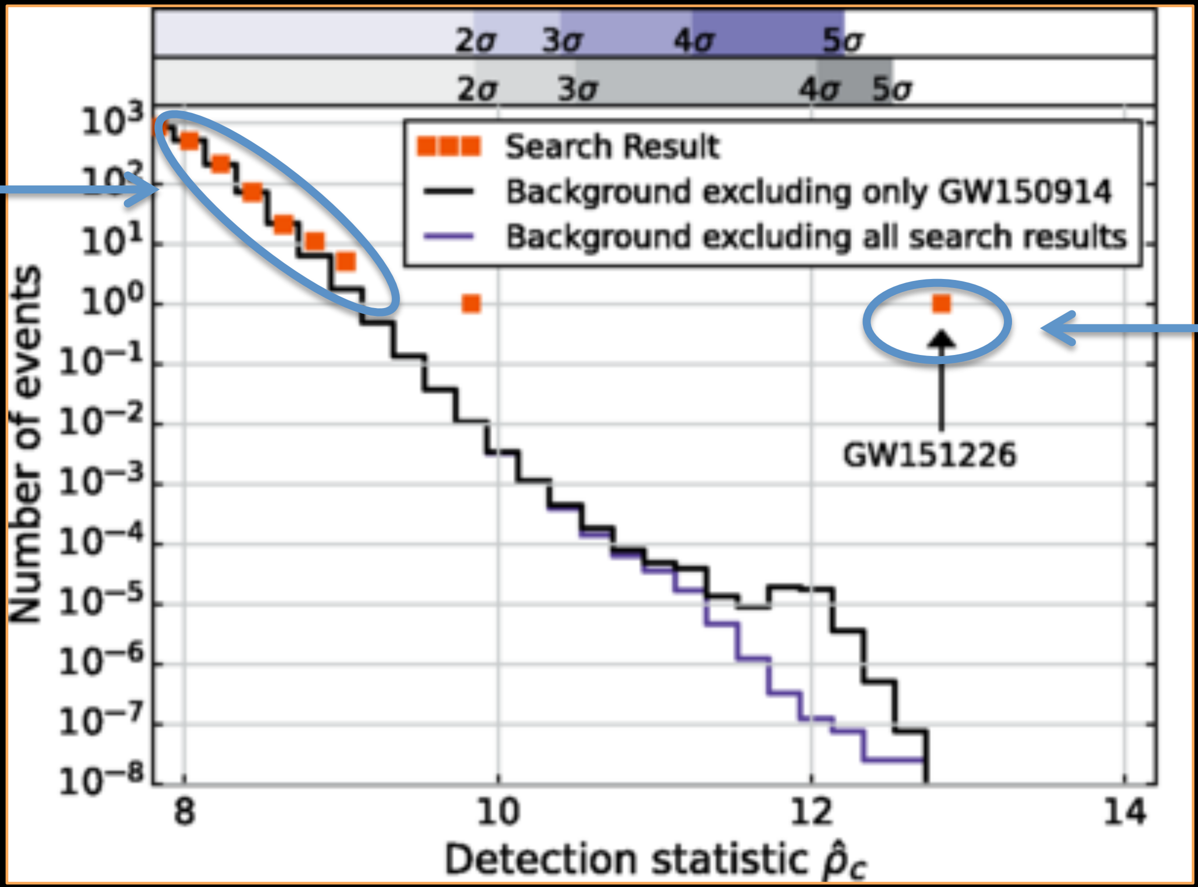


Some events stand out from background

Search results from the two binary coalescence searches using a combined matched filtering signal-to-noise ratio

Detection Statistic

Some events fall along the background



Some events stand out from background

Search results from the two binary coalescence searches using a combined matched filtering signal-to-noise ratio

LIGO

A Gentle Introduction
- Bayesian Statistics

Bayesian Statistics

Probability of a hypothesis, H conditional some data D
'Posterior Density'

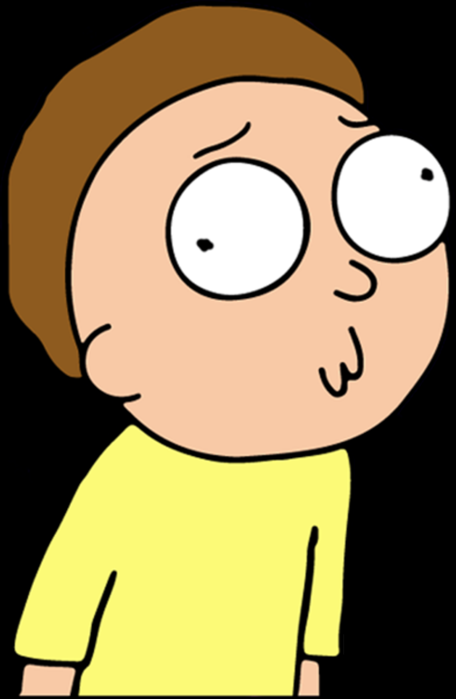
Probability of Data given Hypothesis
'Likelihood'

Probability of the Hypothesis
'Prior'

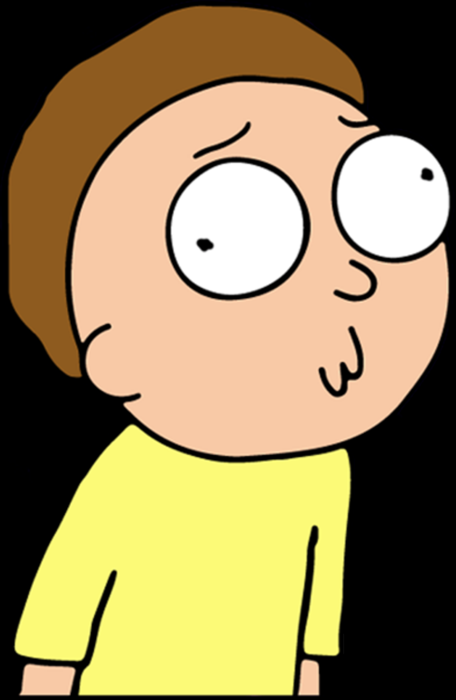
$$P(H \mid D) = \frac{P(D \mid H) \quad P(H)}{P(D)}$$

Probability of the data
'Evidence'

Bayesian Statistics



Bayesian Statistics

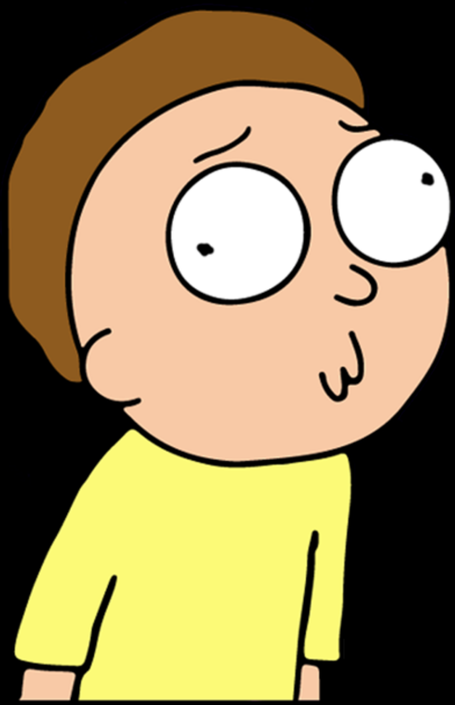


LIGO

Bayesian Statistics

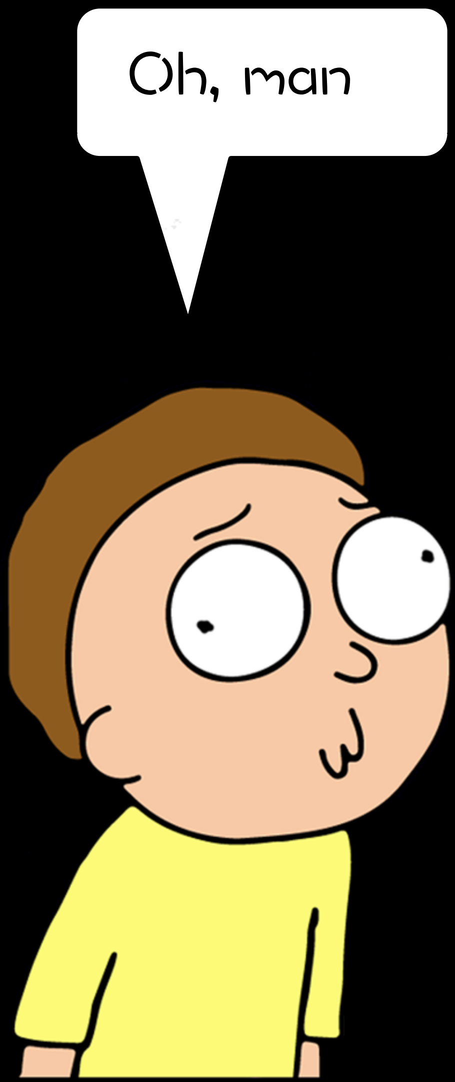
WebMD
HYPOTHESITIS

P(H) = YOU HAVE HYPOTHESITIS



LIGO

Bayesian Statistics



WebMD HYPOTHESITIS

$P(H)$ = YOU HAVE HYPOTHESITIS

$P(S|H)$ = PROBABILITY OF SYMPTOMS GIVEN THE HYPOTHESIS $= 0.95$

LIGO

Bayesian Statistics

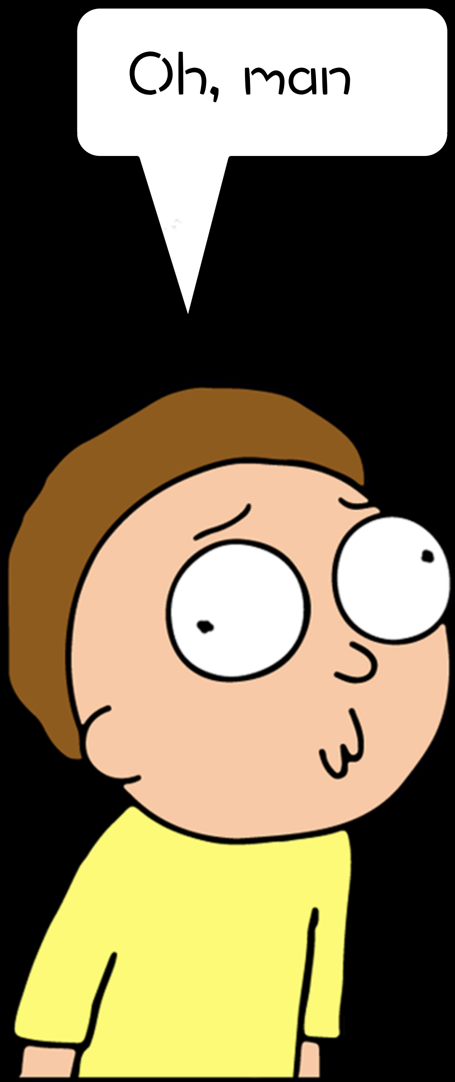
P(You Can Get Hypothesitis)

$$P(H \mid S) = \frac{P(S \mid H) \quad P(H)}{P(S)}$$

P (You Can Get the Symptoms)

LIGO

Bayesian Statistics



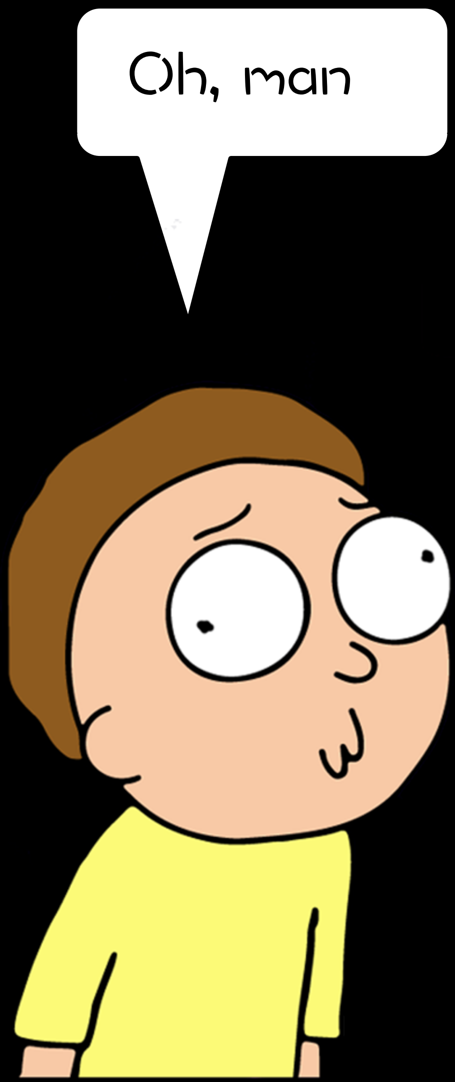
WebMD
HYPOTHESITIS

$P(H)$ = YOU HAVE HYPOTHESITIS = 0.00001

$P(S|H)$ = PROBABILITY OF SYMPTOMS GIVEN THE HYPOTHESIS = 0.95

LIGO

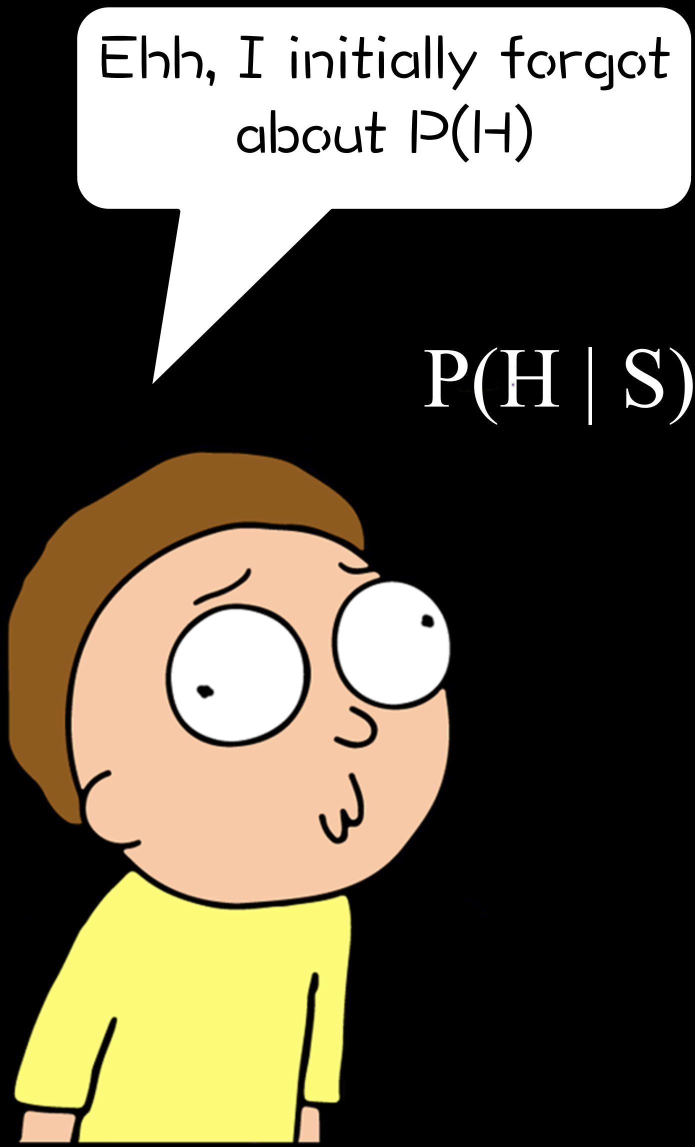
Bayesian Statistics



WebMD
HYPOTHESITIS

$P(H)$	=	YOU HAVE HYPOTHESITIS	=	0.00001
$P(S\|H)$	=	PROBABILITY OF SYMPTOMS GIVEN THE HYPOTHESIS	=	0.95
$P(S)$	=	THE EVIDENCE, OR PROBABILITY OF HAVING SYMPTOMS	=	0.01

Bayesian Statistics

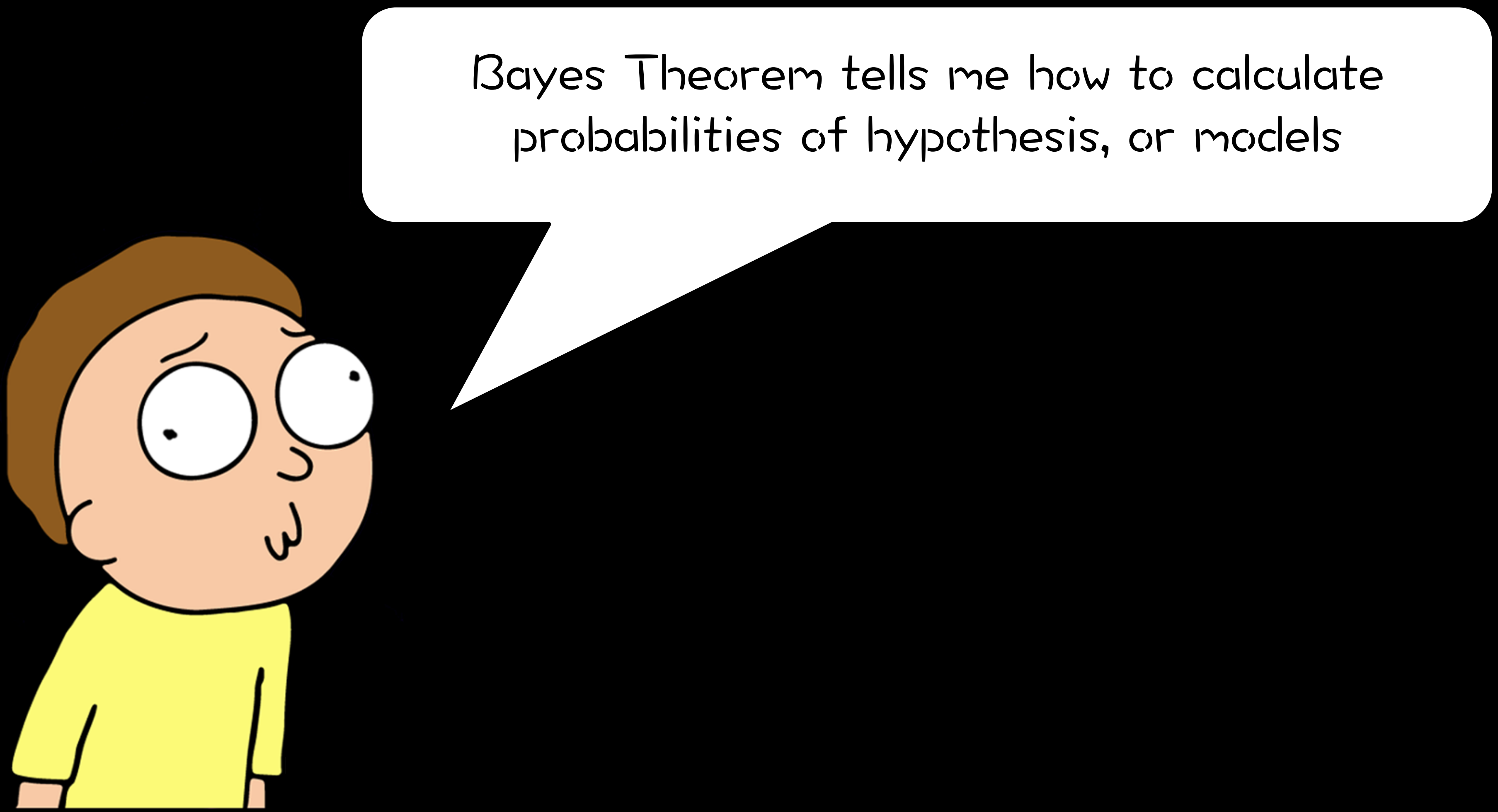


$$P(H \mid S) = \frac{(0.95)\,(0.00001)}{(0.01)}$$

$$= \quad 0.00095$$

LIGO

Bayesian Statistics



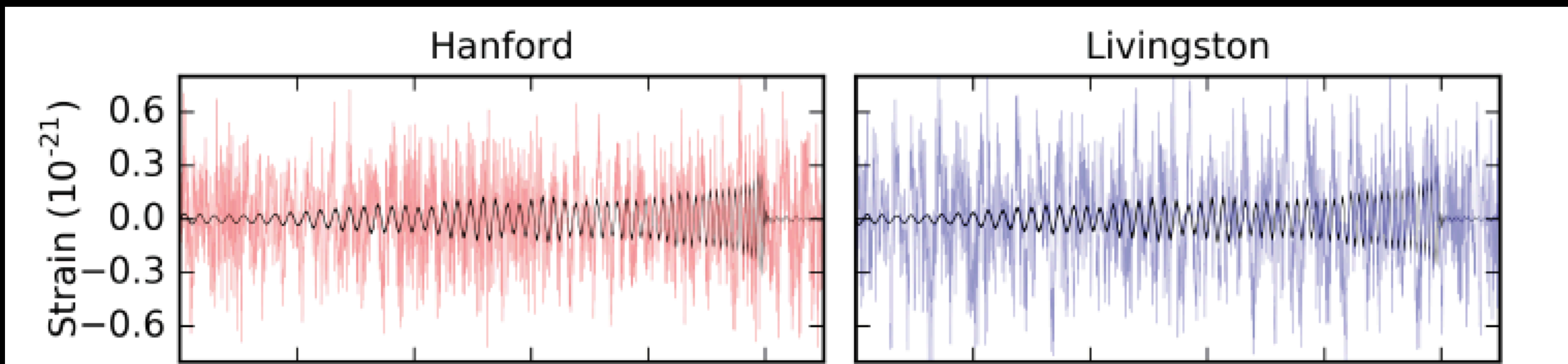
LIGO

Bayesian Statistics

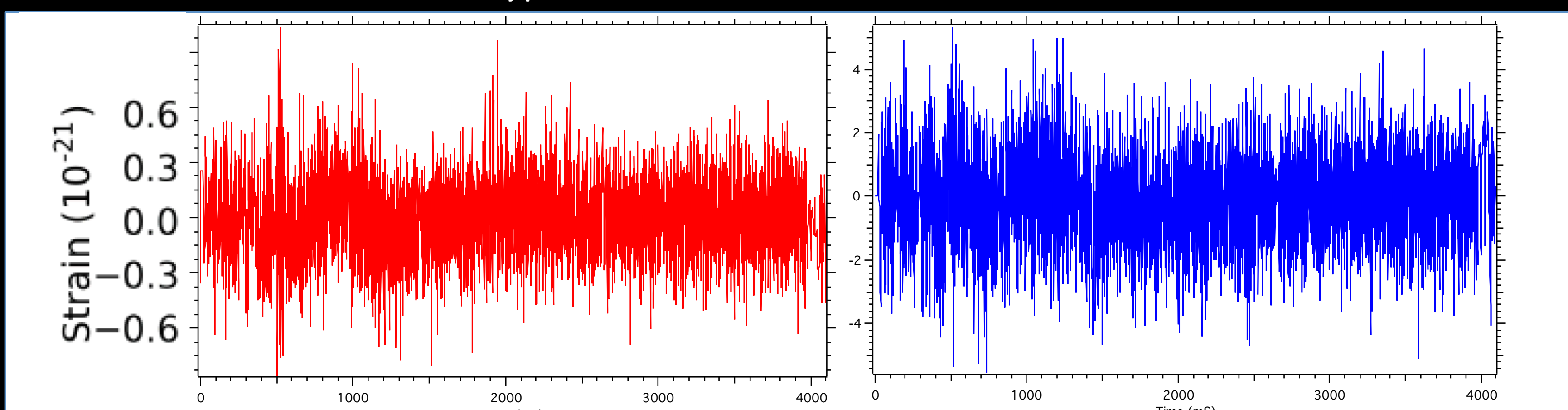
Helps compare different models!

Models in GW

Hypothesis 1 : data = Gaussian Noise + GW Strain



Hypothesis 2 : data = Gaussian Noise



Bayesian Statistics

$$\text{Bayes Factor} = \frac{\text{Evidence}(d = n + s)}{\text{Evidence}(d = n)}$$

Bayesian Statistics

$$\text{Bayes Factor} = \frac{\text{Evidence}(d = n + s)}{\text{Evidence}(d = n)}$$

$$Evidence = \int_{\Theta} (Prior)\ (Likelihood)\ d\vec{\theta}$$

↑

Product calculated for every set of parameters, Θ
(parameters like masses, spins etc of black holes)

Bayes Factor vs SNR

Bayes Factor

- Calculated using entire set of parameters (all possible templates)
- Takes into account spins orientations, and magnitudes

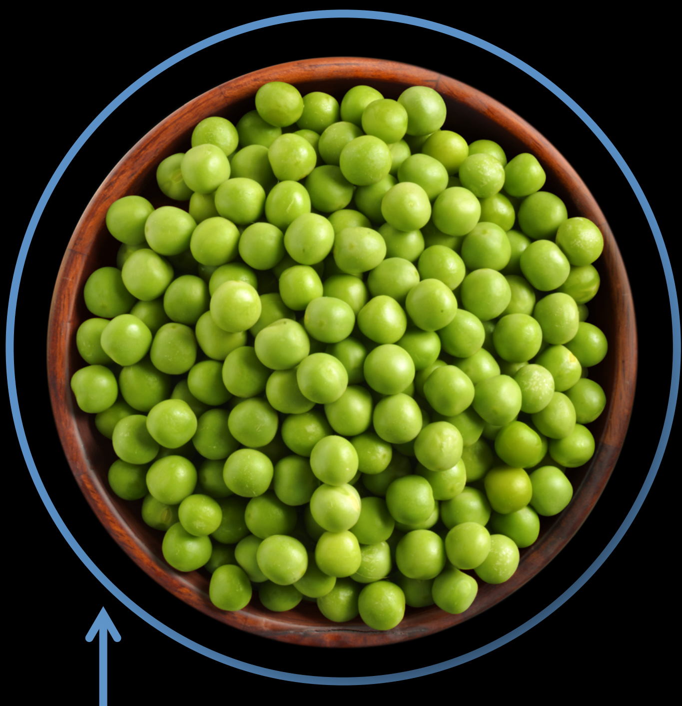
$$\text{Bayes Factor} = \frac{\text{Evidence}(d = n + s)}{\text{Evidence}(d = n)}$$

LIGO

Bayes Factor vs SNR

Bayes Factor

- Calculated using entire set of parameters (all possible templates)
- Takes into account spins orientations, and magnitudes



All Parameters Considered

$$\text{Bayes Factor} = \frac{\text{Evidence}(d = n + s)}{\text{Evidence}(d = n)}$$

Bayes Factor vs SNR

Signal to Noise Ratio

- Maximum Likelihood Estimator (uses one template)
- Does not consider spins orientations, and magnitudes

$$SNR = max_{\vec{\theta}} \frac{(d|h(\vec{\theta}))}{\sqrt{(h(\vec{\theta})|h(\vec{\theta}))}}$$

Bayes Factor vs SNR

Signal to Noise Ratio

- Maximum Likelihood Estimator (uses one template)
- Does not consider spins orientations, and magnitudes

One set of Parameters Considered

$$SNR = max_{\vec{\theta}} \frac{(d|h(\vec{\theta}))}{\sqrt{(h(\vec{\theta})|h(\vec{\theta}))}}$$

Project Motivations

Bayes Factor may prove to be more robust than the SNR

LIGO

Project Motivations

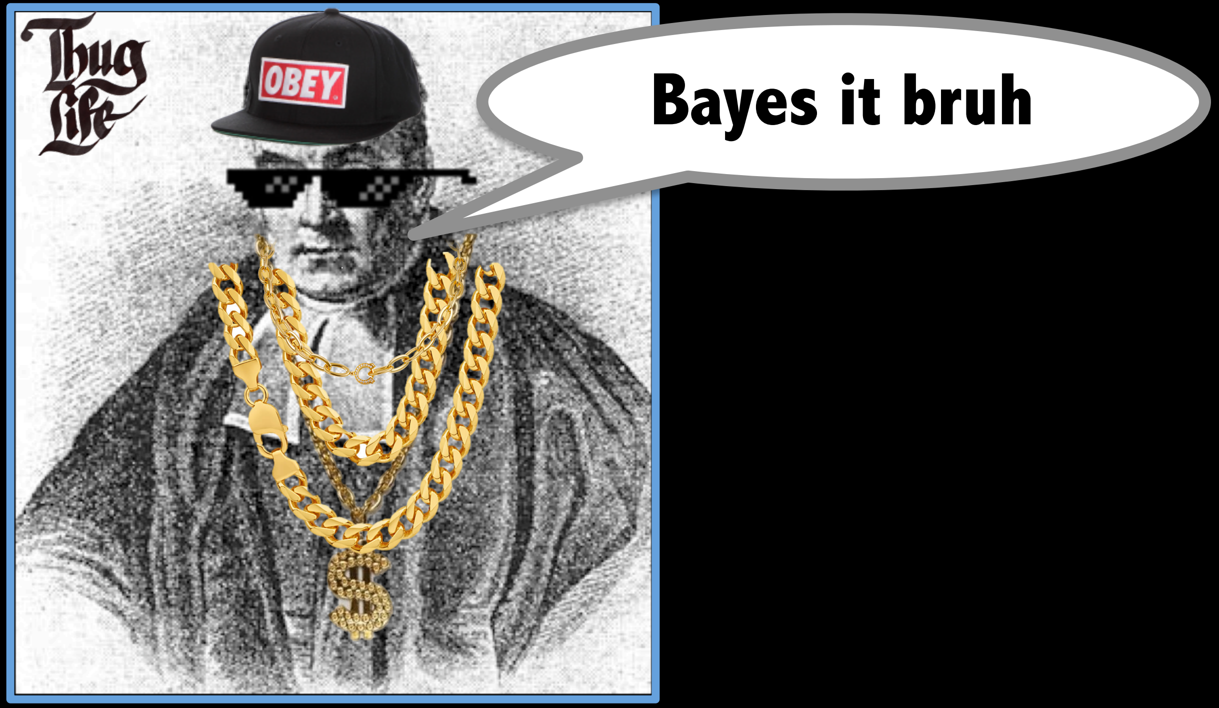
Bayes Factor may prove to be more robust than the SNR

Bayes it bruh

LIGO

Project Goals

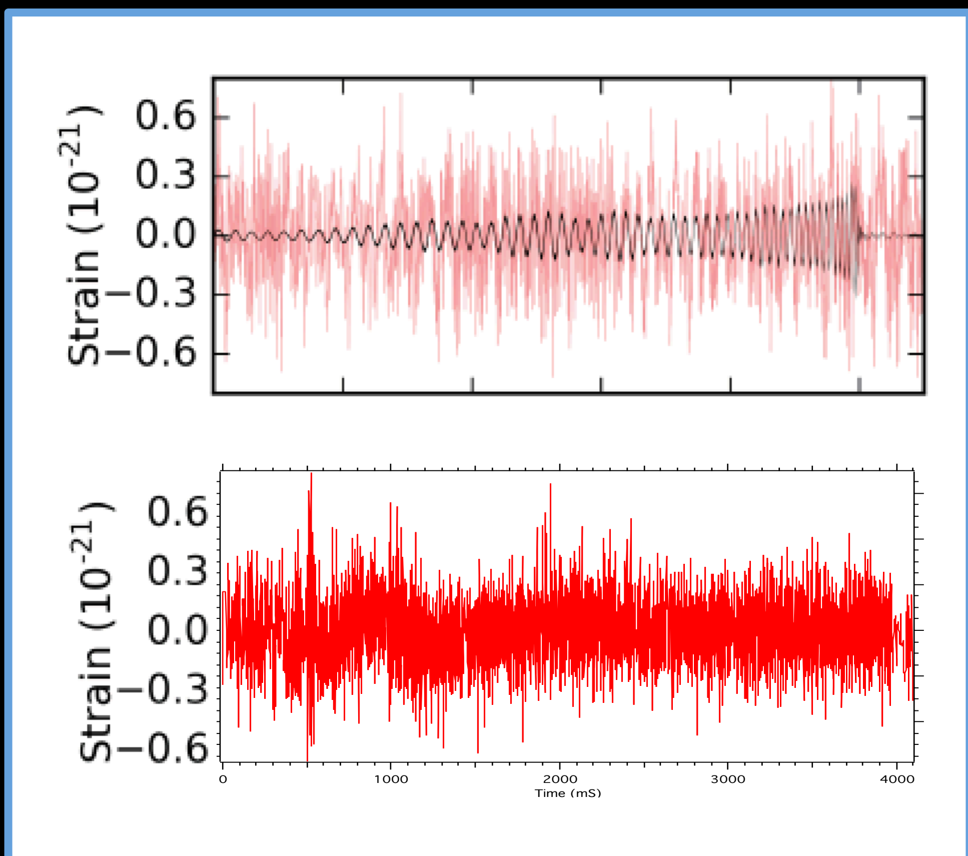
Can we use the Bayes factor as a detection statistic?



LIGO

Obtaining the Bayes Factor

Once we run Parameter Estimations for the events, we can calculate the Bayes Factor



Ln Bayes Factors - GW signals:

GW150914 – 289.8 ± 0.3

GW151226 – 60.2 ± 0.2

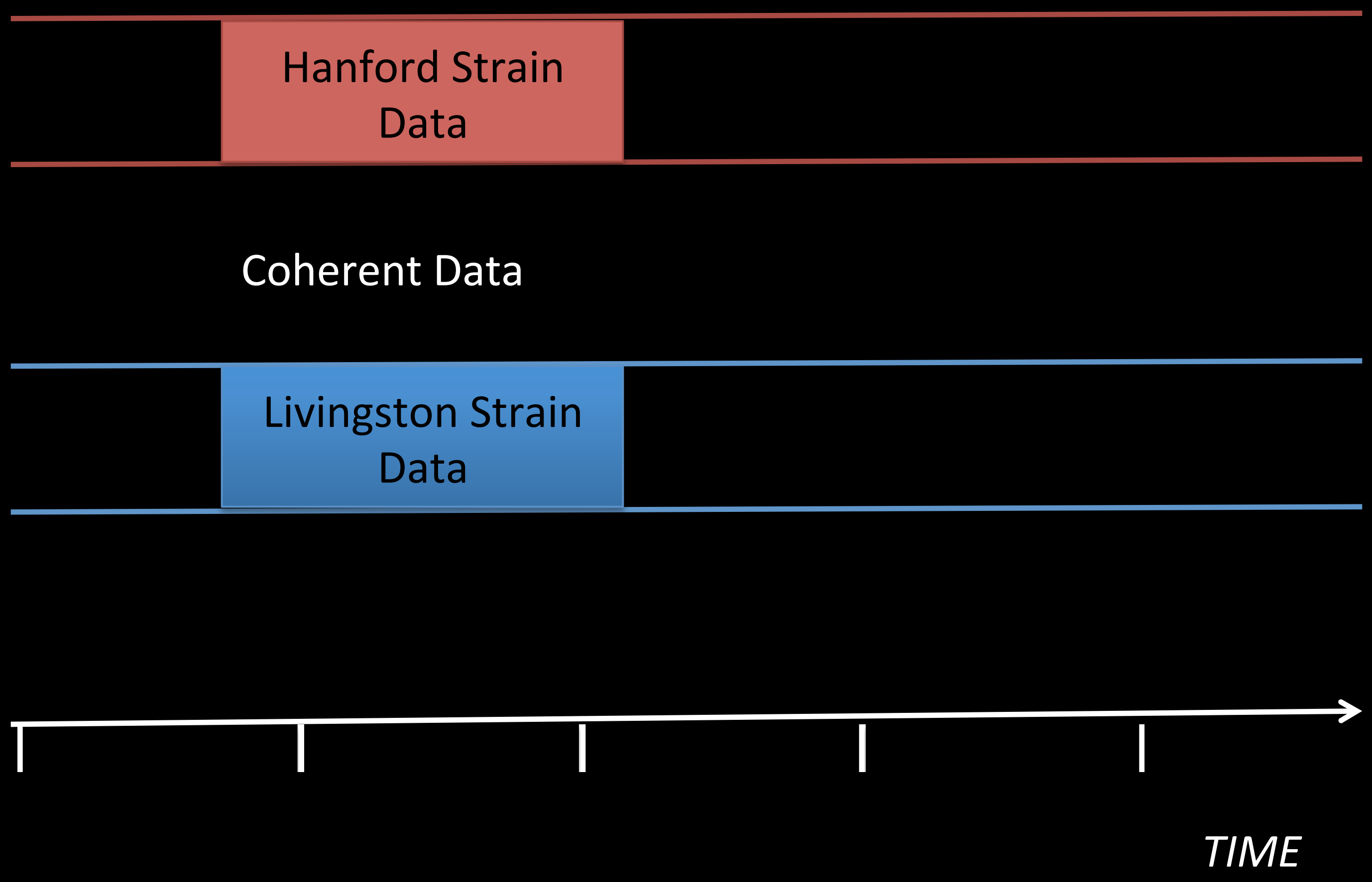
LVT151012 – 23.0 ± 0.1

Values in ~10's range

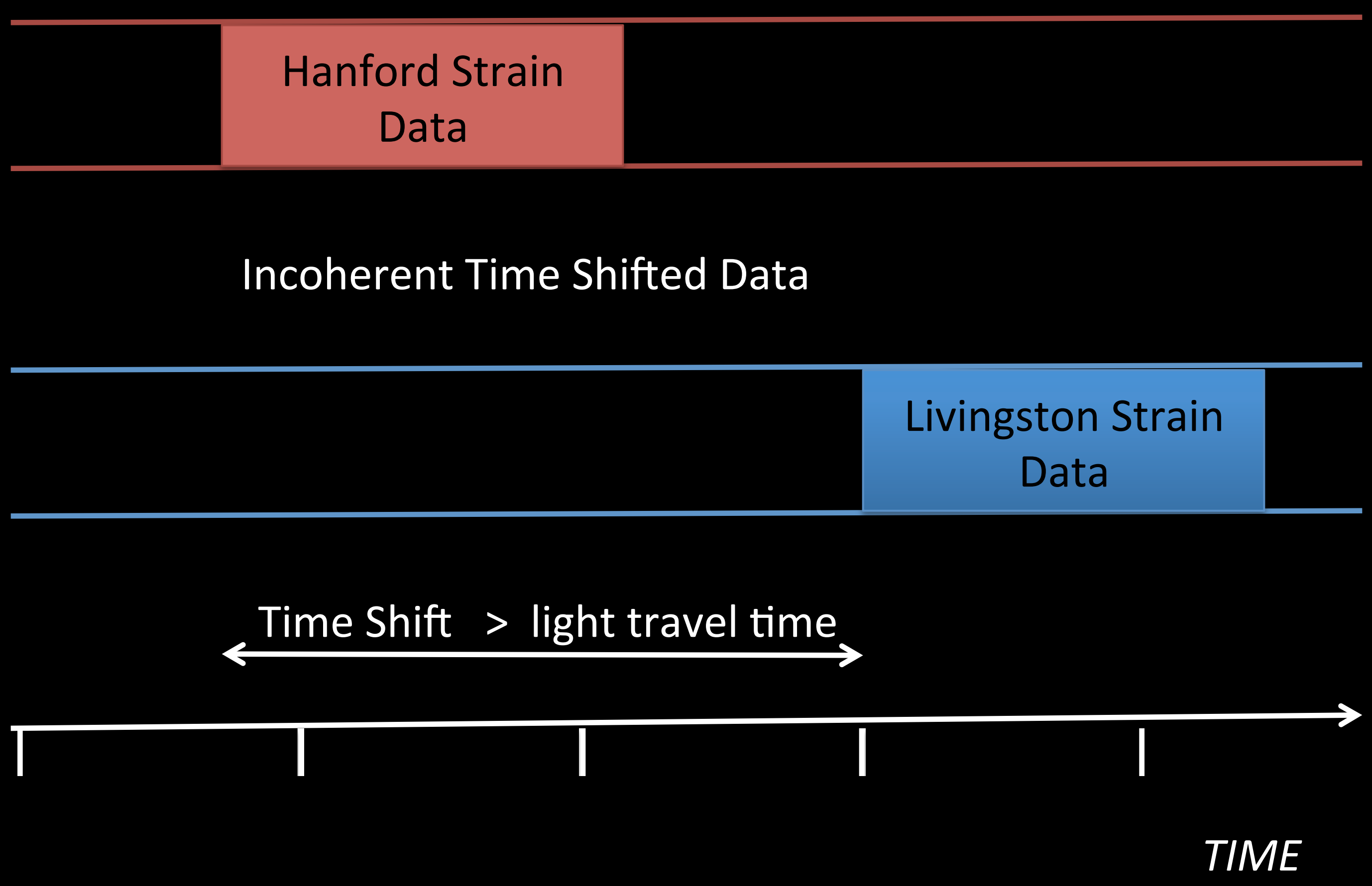
Ln Bayes Factors - Noise:

Values in ~1's range

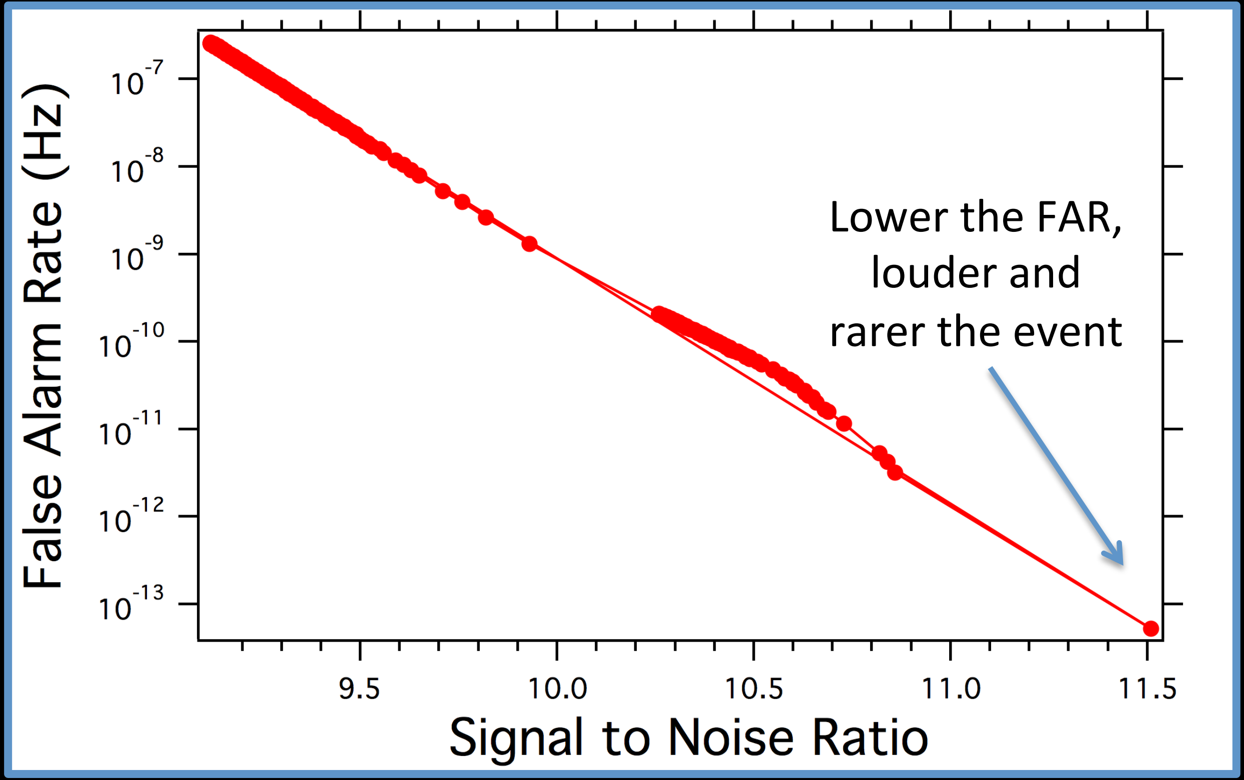
Generating Background Data



Generating Background Data

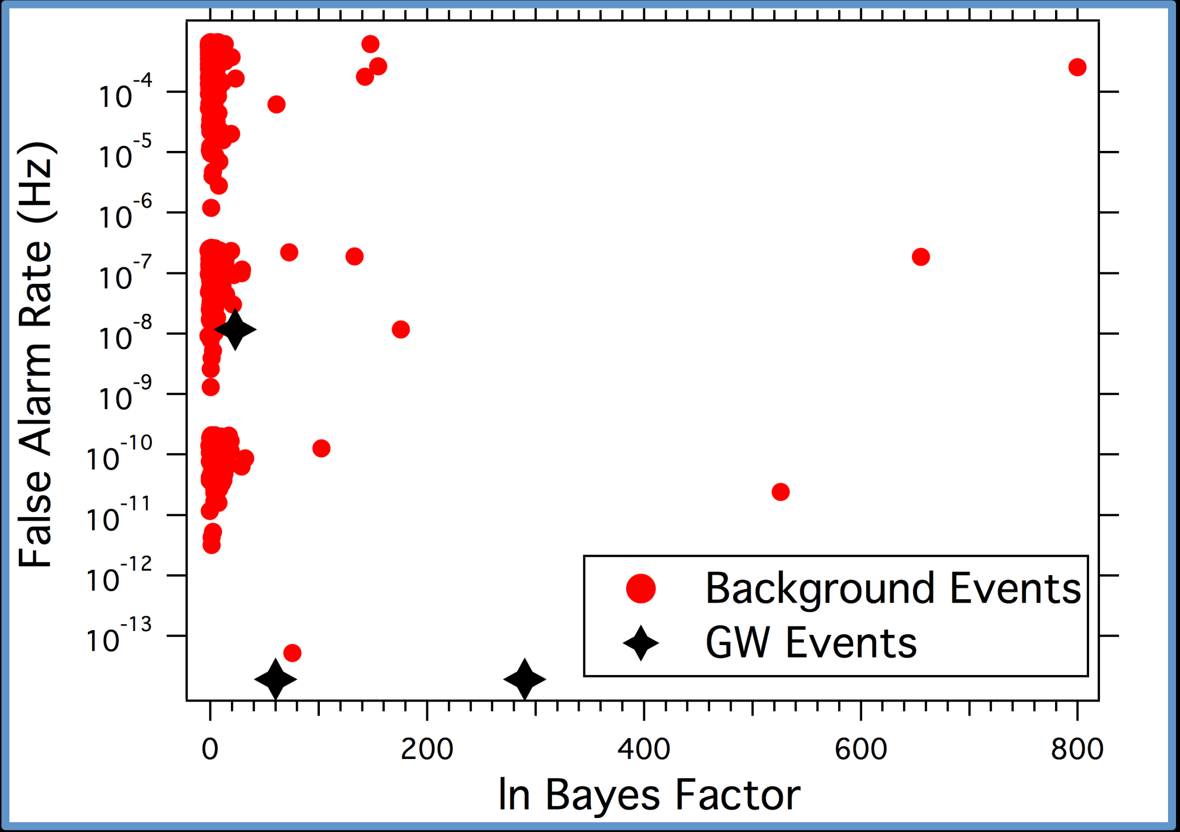


Generating Background Data



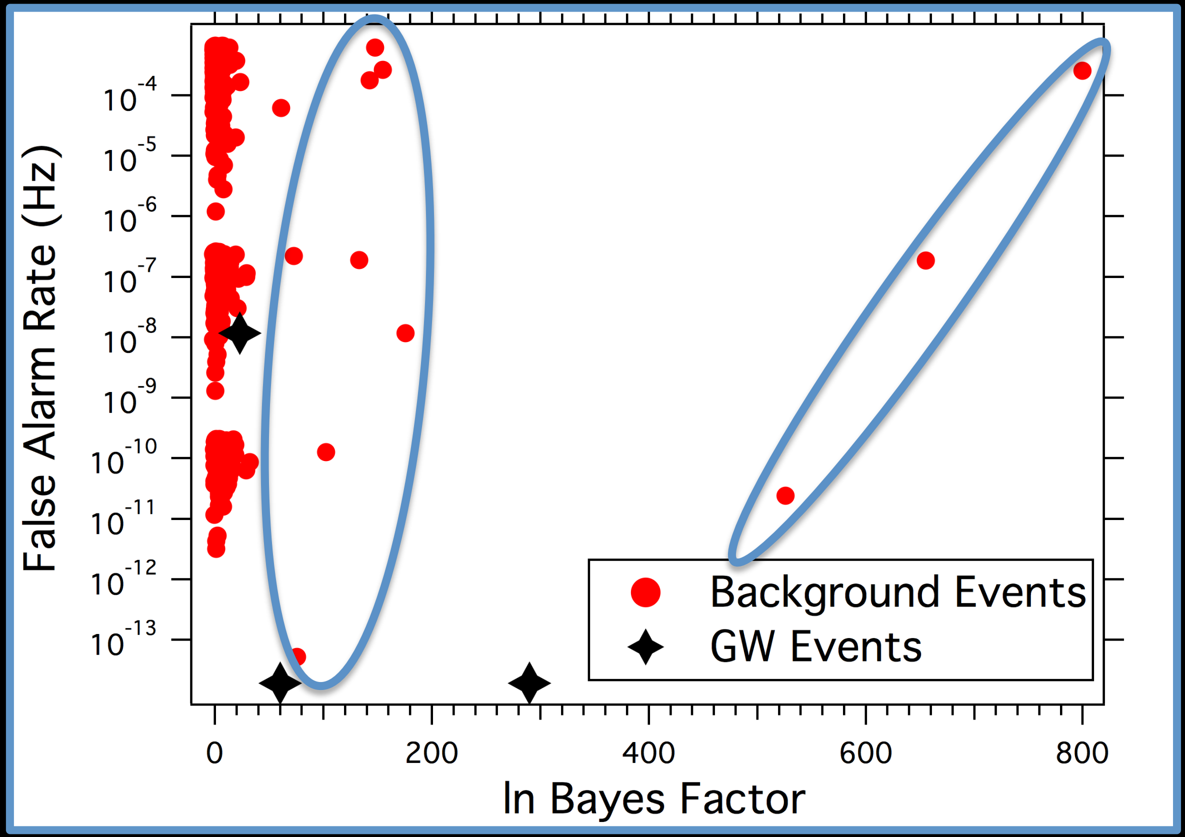
False Alarm Rate Plotted Against The SNR

Bayes Factor Results



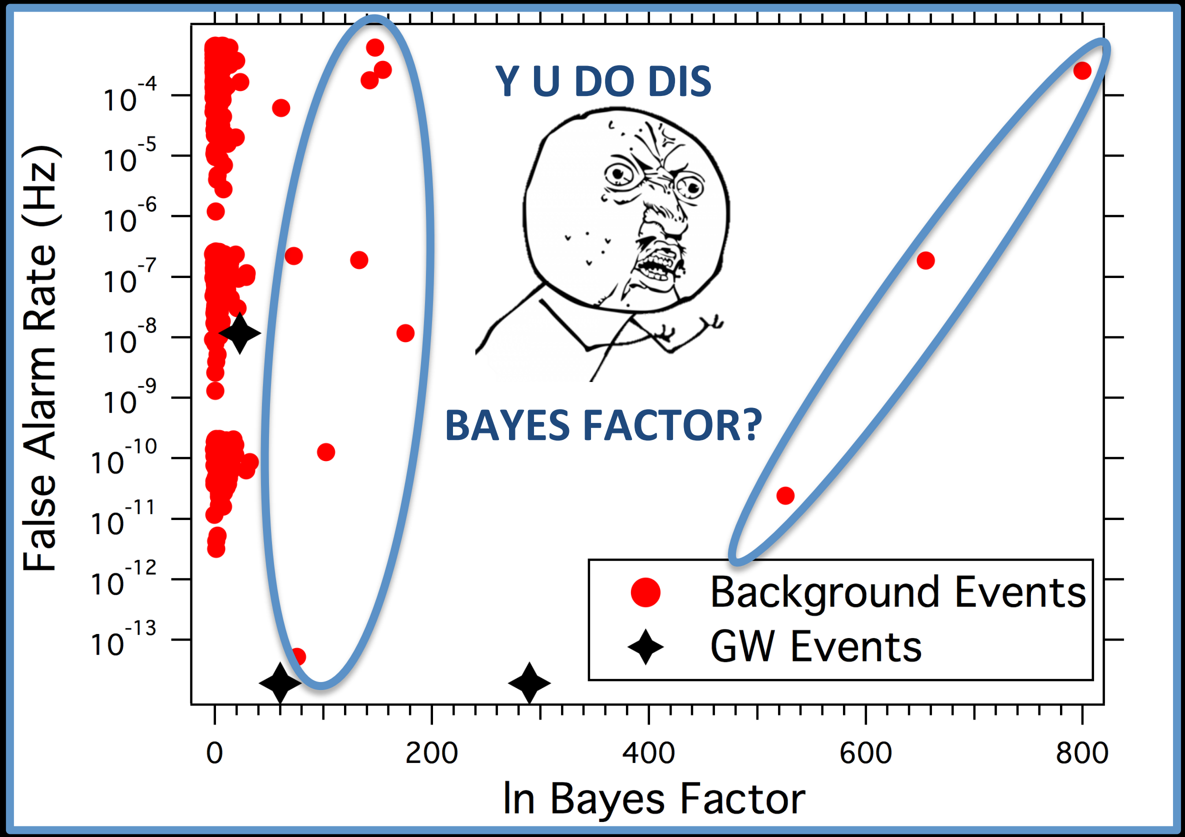
Bayes Factor as a Detection Statistic, using only Coalescing Binary Back Hole Templates

Bayes Factor Results



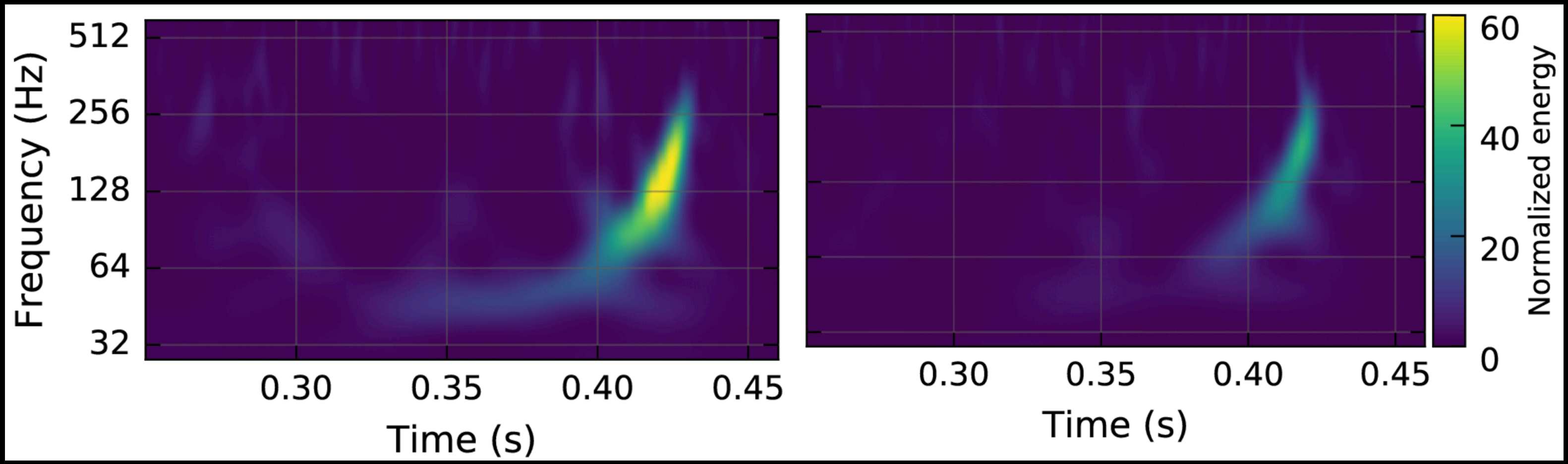
Bayes Factor as a Detection Statistic, using only Coalescing Binary Back Hole Templates

Bayes Factor Results



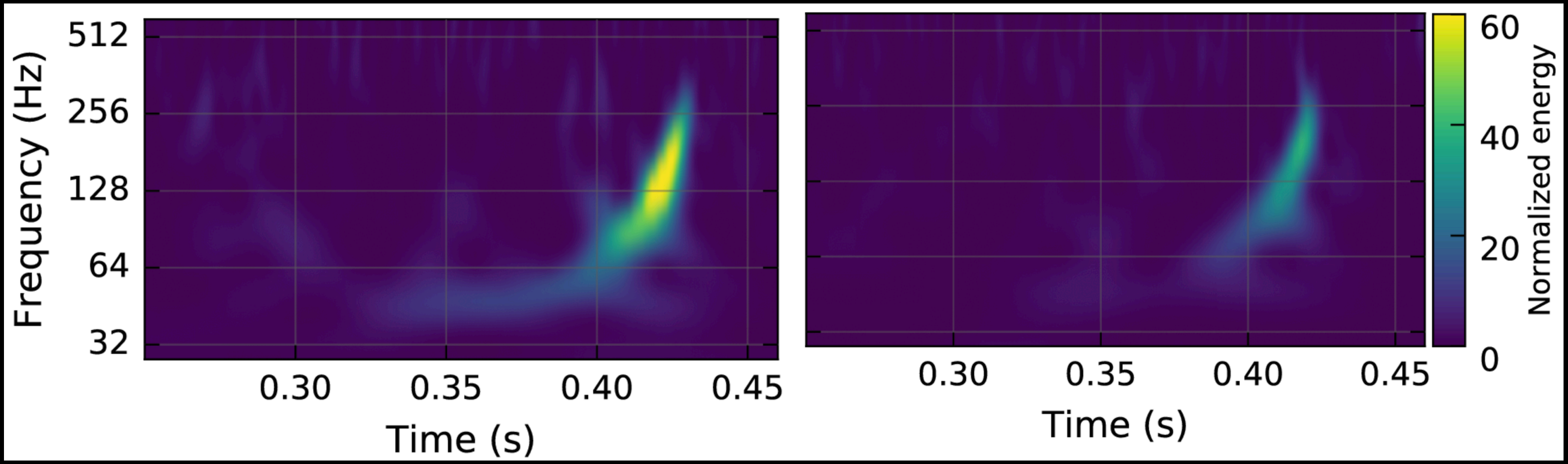
Bayes Factor as a Detection Statistic, using only Coalescing Binary Back Hole Templates

Issue with Bayes Factor

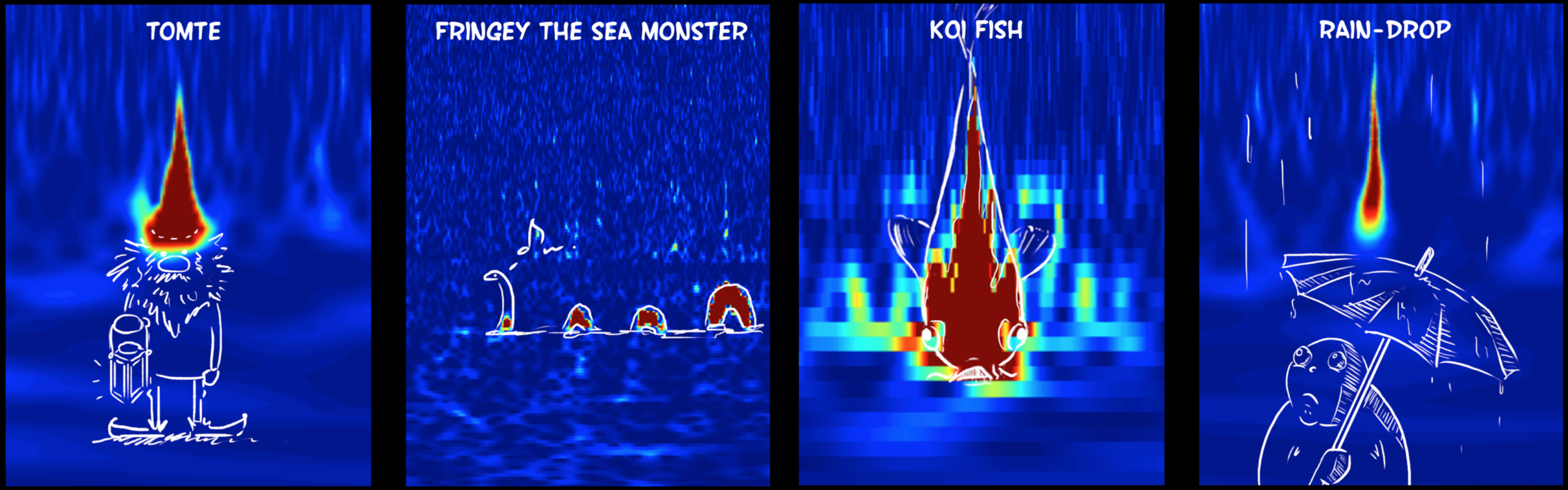


Real GW Spectogram

Issue with Bayes Factor



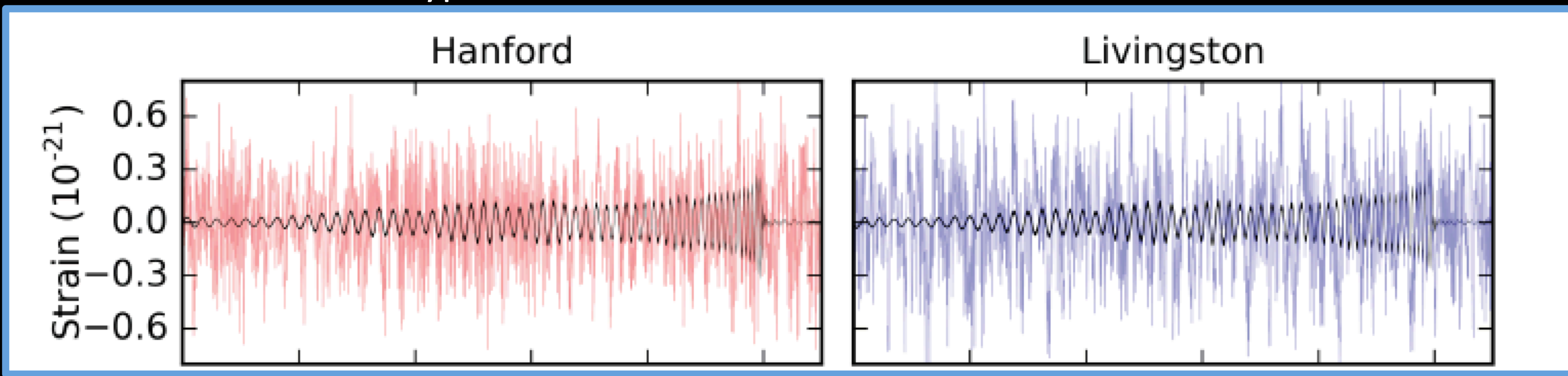
Real GW Spectogram



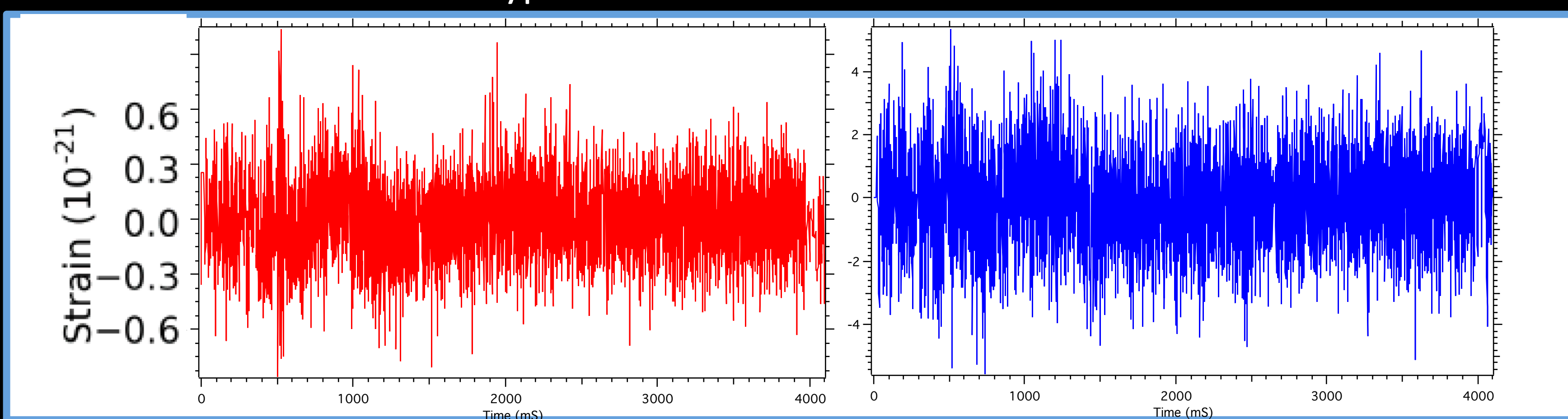
Glitch Spectograms

Issue with Bayes Factor

Hypothesis 1 : data = Gaussian Noise + GW Strain

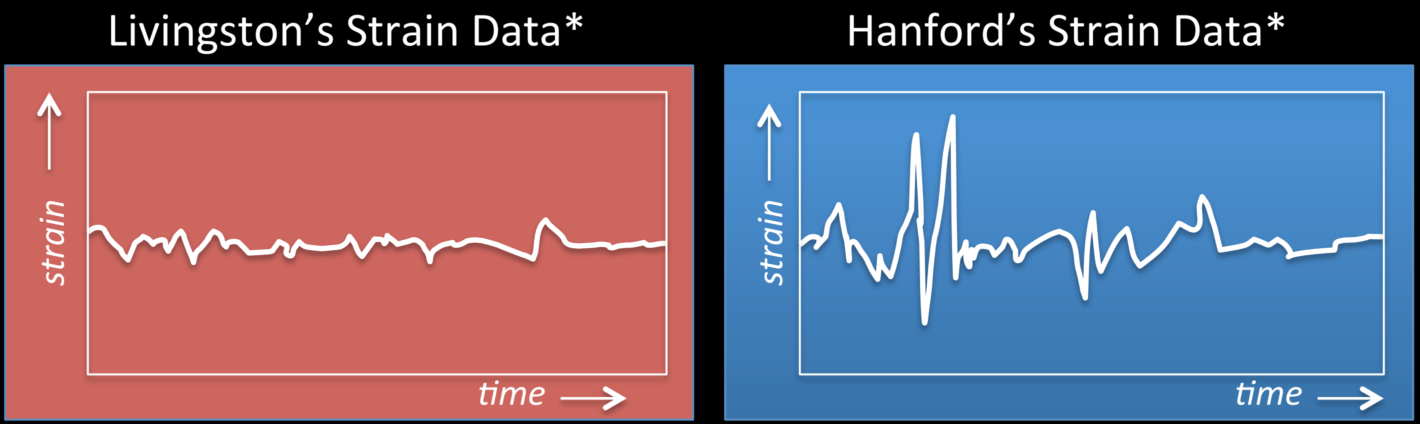


Hypothesis 2 : data = Gaussian Noise



Issue with Bayes Factor

A glitch in one detector's data inflates the Coherent Bayes Factor



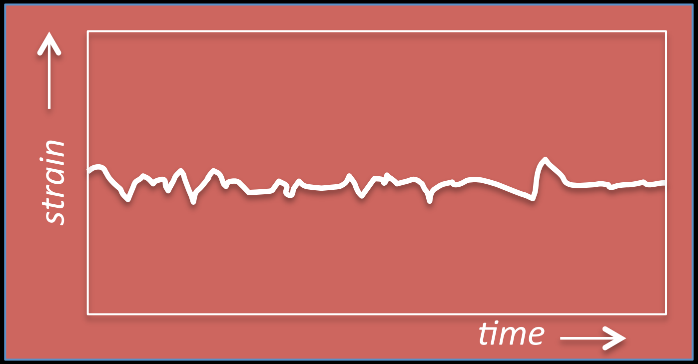
Coherent Bayes Factor = 142.82

* Figures are not real, Numbers are

Issue with Bayes Factor

A glitch in one detector's data inflates the Coherent Bayes Factor

Livingston's Strain Data*



Hanford's Strain Data*

Incoherent Bayes factor = 0.91

Incoherent Bayes factor = 152.58

Coherent Bayes Factor = 142.82

* Figures are not real, Numbers are

LIGO

Bayes Coherence Ratio

$$\mathcal{B}_R = \frac{\mathcal{B}_{SN}}{\mathcal{B}_{SN}^{I(H)} + \mathcal{B}_{SN}^{I(L)}}$$

The Bayes Coherence Ratio Reduces the error that appears in the Coherent Bayes Factor

Bayes Coherence Ratio

COHERENT BAYES FACTOR

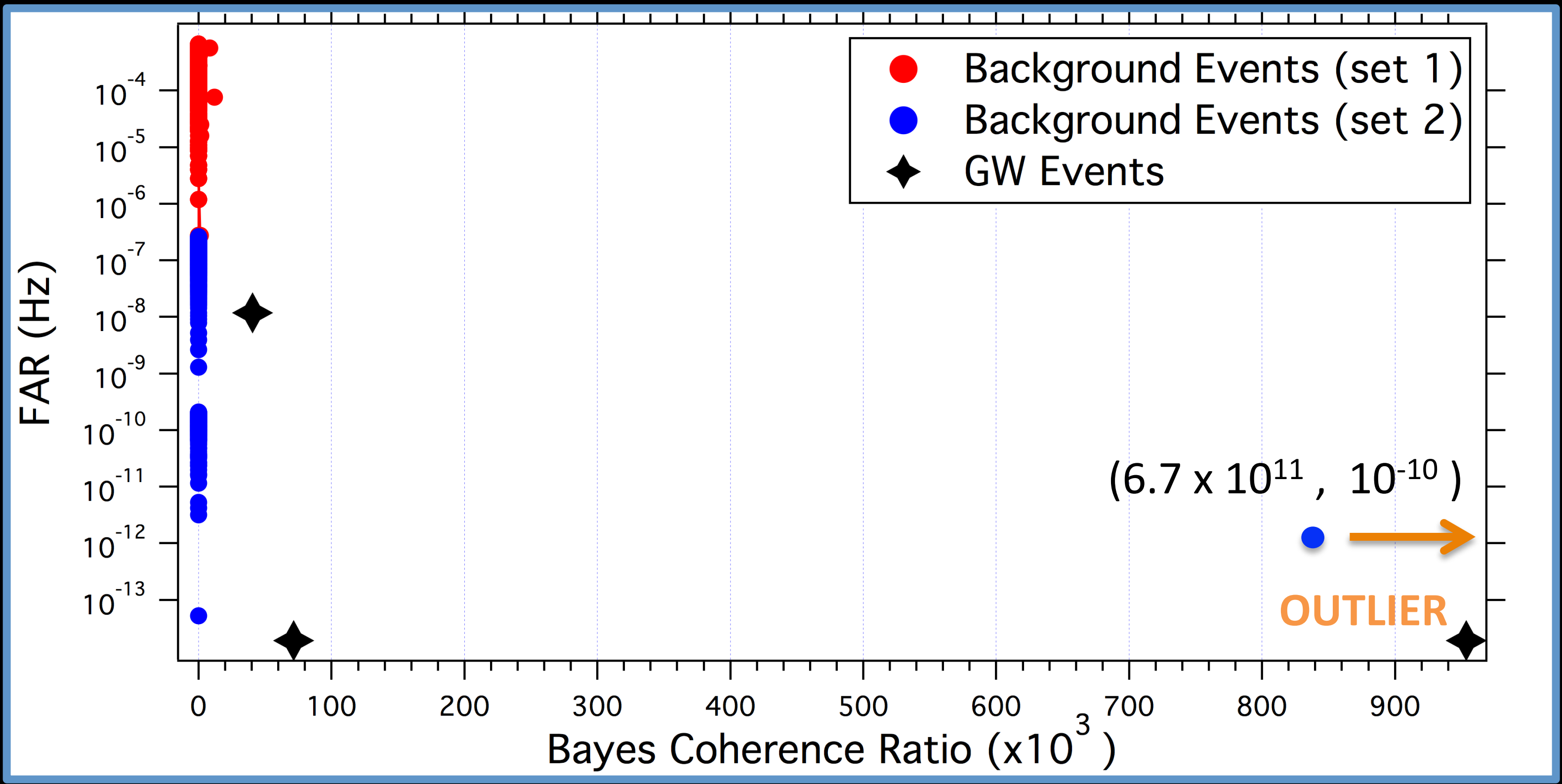
$$\mathcal{B}_R = \frac{142.82}{(0.91) + (152.58)}$$

$$= 0.93$$

SUM OF BOTH DETECTOR'S BAYES FACTORS

LIGO

Results for Bayes Coherent Ratio



Set 1 Chirp Mass Range: 6.6 – 45.9 $M_\odot$,

Set 2 Chirp Mass Range: 24.5 – 45.9 $M_\odot$

Bayes Coherence Ratio as a Detection Statistic

LIGO

Additional Information Available

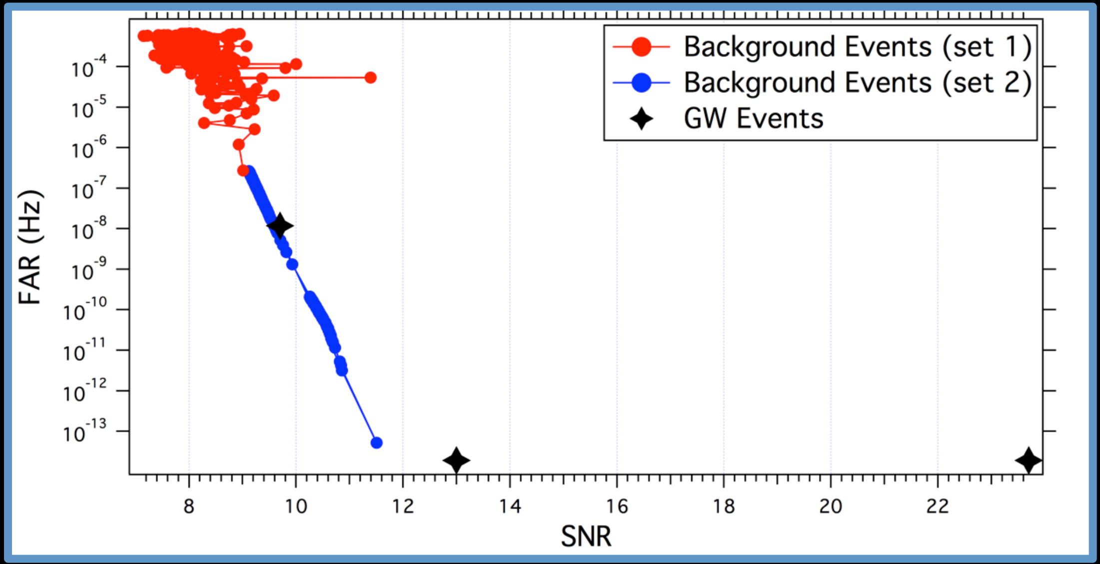
Top	Summary statistics		
	maP	maxL	stde
logw	-63.9420308376	-64.4376360327	16.05
redshift	0.00961537938354	0.00865935297158	0.000
tilt_spin2	2.48513498513	2.71409201537	0.414
tilt_spin1	1.28658153514	1.32002002608	0.056
f_ref	20.0	20.0	0.0
psdscaleflag	0.0	0.0	0.0
phi_jl	4.9058602691	4.96840312361	0.071
l1_optimal_snr	17.2586721755	17.5667235864	1.027
v1h1_delay	0.00751137733459	0.00826120376587	0.000
h1l1_delay	-0.00925827026367	-0.00932765007019	6.647
signalmodelflag	1.0	1.0	0.0
logl	-10309.6732874	-10309.0517577	2.927
deltalogll1	160.999763497	161.58278359	2.738
h1_cplx_snr_arg	0.411353471019	0.111878888247	0.425
deltaloglh1	2.29302471921	2.33153436158	1.084
sky_frame	1.0	1.0	0.0
polarisation	0.331237361641	0.248401261922	0.058
deltalogl	163.292788216	163.914317951	2.927
h1_cplx_snr_amp	2.39491070701	2.19945022169	0.125
l1v1_delay	0.00174689292908	0.00106644630432	0.000
mf_source	144.998641396	144.366620189	1.048
l1h1_delay	0.00925827026367	0.00932765007019	6.647
v1_end_time	1128443202.01	1128443202.01	0.000
eta	0.0990158458624	0.0990262339649	0.000
m1	132.375246642	131.678073268	0.981
rightascension	3.93346951378	3.96231717612	0.026
h1l1_relative_phase	0.386040056127	0.0600591965268	0.426
m2	16.6008881791	16.5156866008	0.137

We have a lot of additional information that we could potentially use to distinguish the outlier as a glitch

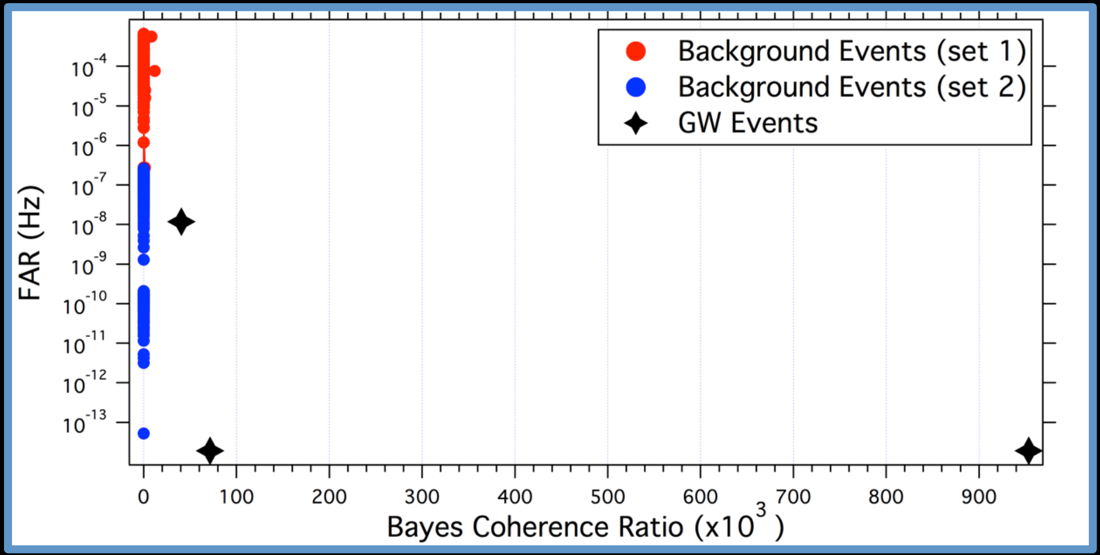
L1 optimal SNR : 17.2
H1 optimal SNR : 2.6

Comparing Detection Statistics

Signal-to-Noise Ratio as a Detection Statistic



Bayes Coherence Ratio as a Detection Statistic



Conclusions and Future Work

- Study the low FAR background events
- Determine if BCR can be used in addition with SNR as a detection statistic
- Expand the work for more mass bins
- Repeat the Study with Binary Neutron Star Signals

Acknowledgements

Thanks to NSF, Dr Rory Smith, Dr Jonah Kanner, Professor Alan Weinstein and the LIGO SURF pen.