# SURF Proposal:
# The effect of orbital eccentricity in binary black hole simulations on gravitational waveforms

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## 1 Background

### 1.1 Binary Black Holes and Gravitational Waves

The theory of General Relativity predicts gravitational wave (GW) emission from binary systems consisting of compact objects such as neutron stars and stellar mass black holes (BH). This prediction has been recently verified by the Laser Interferometer Gravitational-Wave Observatory (LIGO), which has detected GW emission from a binary black hole (BBH) merger [1]. Once detected, such GW signals can be used to infer source information, such as mass ratios and spins, through parameter estimation. However, the detection of GWs relies on high quality, accurate theoretical waveforms, which can be computed with either analytical approximations, such as post-Newtonian (PN) expansions, or by directly solving the full set of Einstein's equations using numerical relativity.

For large separations, PN waveforms are accurate enough to use for matched filtering detection methods and parameter estimation. Since the PN approximation is a perturbative solution which solves Einstein's equations in powers of $v/c$, when objects are very close and near merging, PN waveforms are no longer accurate. Although numerical relativity in principle can solve Einstein's equations to within numerical precision, it is highly computationally demanding, and as a result, only a small portion of the 7 dimensional parameter space (mass ratio, individual spins) for BBHs has been explored [2]. There are analytical models such as the effective-one-body and PhenomP models [3, 4] that, unlike PN, can compute binary black hole waveforms that include merger and ringdown, but these models must be calibrated to numerical relativity simulations.

For isolated systems, the orbit of a compact binary system will gradually become circular due to gravitational wave emission. When the GWs reach detection frequency, the eccentricity is expected to be negligible [5]. However, there are proposed mechanisms through which the orbital eccentricity becomes significant, such as the Kozai mechanism, where eccentricity oscillations occur in an inner

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binary system due to tidal forces between an outer binary system [6].

The GWs emitted from eccentric binaries are expected to be different from non eccentric (circular) binaries, as the emitted frequencies will span a spectrum instead of adhering to a single frequency. Also, the peak emitted power will be greater due to a greater orbital apastron, leading to more dynamical motion. Thus, characterizing the effect of eccentricity on emitted GWs is crucial in determining the ability to distinguish between binary sources with high eccentricity and sources with low or negligible eccentricity. Furthermore, computing a numerical relativity simulation with a desired eccentricity is not straightforward; the actual eccentricity of the orbit will differ from the desired eccentricity by some amount, even when simulating circular orbits. Because of this, it is important to determine how small the eccentricity of a numerical relativity simulation must be in order to justify treating it as zero.

### 1.2 SpEC

The Spectral Einstein Code (SpEC) is a multi-domain pseudo-spectral evolution code originally developed by Lawrence Kidder, Harald Pfeiffer, and Mark Scheel, that, given initial spins and a mass ratio, calculates inspiral orbits of compact binaries [7]. In order to solve Einstein's equations, solutions to the differential equations are found as a series approximation to the orthogonal basis functions (in space) and then are evolved in time [8]. While the first results for binary black hole orbits were computed using finite difference methods, in which fundamental variables are updated through time, pseudospectral methods have both increased the computational efficiency and accuracy of numerical relativity simulations. In this project we utilize SpEC to simulate BBHs and calculate the resulting waveforms.

## 2 Research Goals

The primary objectives of this SURF project are threefold:

- First, to determine the effect of eccentricity on the GW emission from BBHs and to compare the properties of the waveforms using some distinguishability criteria.
- Second, to investigate the experimental implications of using eccentric waveforms on detection and parameter estimation. We will aim to determine the smallest orbital eccentricity required in order to be experimentally indistinguishable from zero eccentricity.
- Third, to learn about the computational techniques used in numerical relativity. More specifically, to gain experience with the SpEC evolution code in application to BBH inspirals.

## 3 Methodology

### 3.1 Calculating Waveforms

Because numerical simulations are so computationally expensive, numerical relativity has only covered a small portion of the parameter space. Fortunately, there is a catalog of completed SpEC simulations. For this project, instead of running all of our own simulations, we will start by surveying the catalog for relevant simulations, which necessarily share the same initial mass and spins but differ in eccentricity.

For the simulations that remain, we will adapt the pre existing structure of the code to run simulations of varying eccentricity. The SpEC code currently runs an iterative eccentricity reducing process, which, for given mass and spins, adjusts the initial separation of the BHs, the initial time derivative of the separation, and the orbital frequency such that the orbit has successfully smaller eccentricity, until the eccentricity is less than $10^{-4}$ [8]. For the first iteration, the user can calculate approximate initial parameters using the PN formalism. Then, during each iteration, the inspiral is evolved for only $2-3$ orbits before the eccentricity is estimated. However, instead of evolving the iterative simulations for only $2-3$ orbits, we will allow these simulations to run to completion in order to extract the gravitational wave signal. This way, for a given BBH system, a range of eccentricities can be generated without implementing significant changes to the code.

### 3.2 Comparing Waveforms

After generating the waveforms, we will determine the effects of eccentricity by comparing waveforms emitted by BBH of the same mass ratio and spins, but varying eccentricities. One possible way to compare two GW signals, $h_1(t), h_2(t)$, is the least squared distance over a time interval:

$$\delta_{t_1,t_2}(h_1, h_2) = \int_{t_1}^{t_2} |h_1(t) - h_2(t)|^2 dt.$$

In the frequency domain, a similar notion of distance can be constructed during the inspiral phase during which frequency increases with time, so that the interval $[t_1, t_2]$ can be associated with the interval $[f_1, f_2]$:

$$\delta_{t_1,t_2}(h_1, h_2) = \int_{f_1}^{f_2} |\widetilde{h}_1(t) - \widetilde{h}_2(t)|^2 df.$$

Another way we can objectively compare the waveforms is through calculating the inner product, $(\delta h|\delta h)^{1/2}$, where $\delta h = h_1 - h_2$. The inner product should also take into account the detector properties, so we can define the inner product in terms of the one sided power spectral density $S_n(f)$:

$$(x|y) = 4\text{Re}\int_0^\infty \frac{\widetilde{x}(f)\widetilde{y}^*(f)}{S_n(f)} df.$$

This formula works for Gaussian noise and is calculated in the frequency domain, where data at different frequencies are independent. We could also normalize by the signal -to-noise ratio (SNR), as computed by either $h_1$ or $h_2$, since we expect that waveforms with large SNR will be easier to distinguish and thus distinguishability should be more stringent.

The above provides some examples of the quantitative measures that we plan to use to determine distinguishability between waveforms.

## 4 Timeline

Week 1-3: Spend time familiarizing with SpEC and learning how to extract waveforms from simulation output, while determining which pre-existing simulations can be used for the project. For certain simulations with the same mass ratio and spins, multiple orbits with different eccentricities may already exist. However, many simulations are expected to only contain the reduced eccentricity orbit. Also, as early as possible, we will start to run remaining simulations that are not already in the catalog. This will likely take longer than two weeks if many runs are needed. For more extreme cases of BBHs, simulations will take much longer to run.

Week 4-6: Continue to run simulations, and then summarize the complete database of waveforms and compare waveforms through the distinguishability criteria discussed. Any time spent waiting for simulations to complete can be used to learn how to extract waveforms and analyze them.

Week 7-8: Investigate additional additional questions related to eccentricity, including the lowest detectable/distinguishable eccentricity, degeneracies in the parameter space, and experimental implications of findings.

Week 9-10: Work on SURF paper and presentation and finalize results.

## REFERENCES


[1] B.P. Abbott et al., Phys. Rev. Lett. **116**, 061102 (2016).

[2] A.H. Mroué, M.A. Scheel, B. Szilágyi, et al., Phys. Rev. Lett. **111**, 241104 (2013).

[3] A. Taracchini et al., Phys. Rev. D **89**, 061502 (2014).

[4] M. Hannam et al., Phys. Rev. Lett. **113**, 151101 (2014).

[5] V. Tiwari et al., arXiv:1511.09240 (2015).

[6] M.C. Miller and D.P. Hamilton ApJ **576** 894, (2002).

[7] `http://www.black-holes.org/SpEC.html`.

[8] L.T. Buchman, H.P. Pfeiffer, M.A. Scheel, and B. Szilágyi, Phys. Rev. D **86**, 084033 (2012).

[9] L. Santamaría et al., Phys. Rev. D **82**, 064016 (2010).

[10] A. Buonanno et al., Phys. Rev. D **83**, 104034 (2011).

[11] A.H. Mroué, H.P. Pfeiffer, L.E. Kidder, and S.A. Teukolsky, Phys. Rev. D **82**, 124016 (2010).

[12] L. Lindblom, B.J. Owen, D.A. Brown, Phys. Rev. D **78**, 124020 (2008).

[13] L. Lindblom, J.G. Baker, and B.J. Owen, Phys. Rev. D **82**, 084020 (2010).