LIGO-G1601395-v1

Gravitational wave detection with laser interferometers

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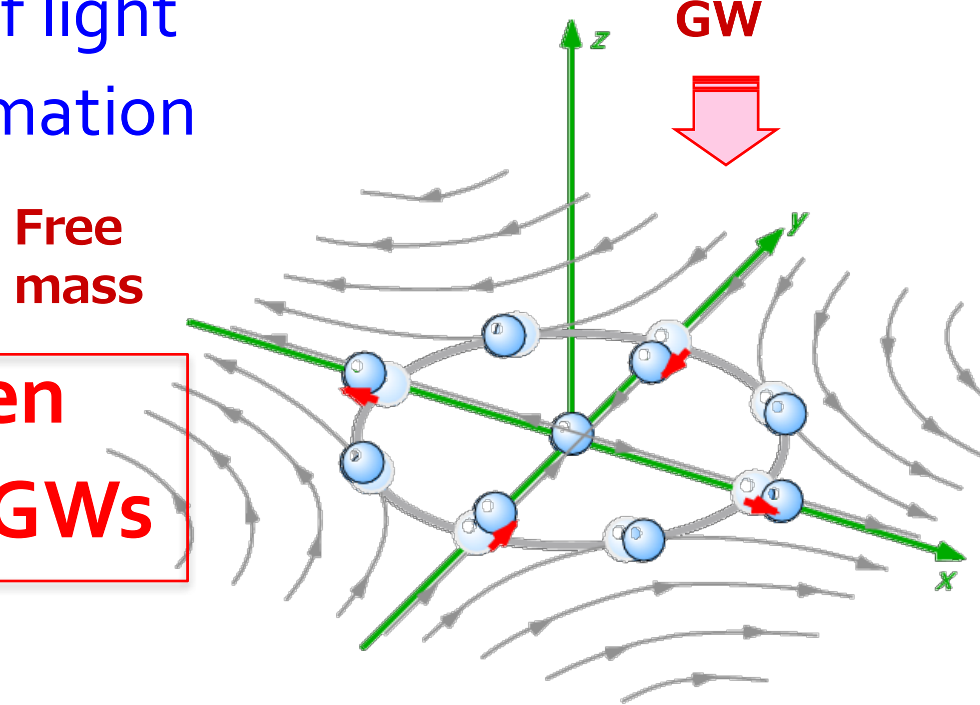
Introduction

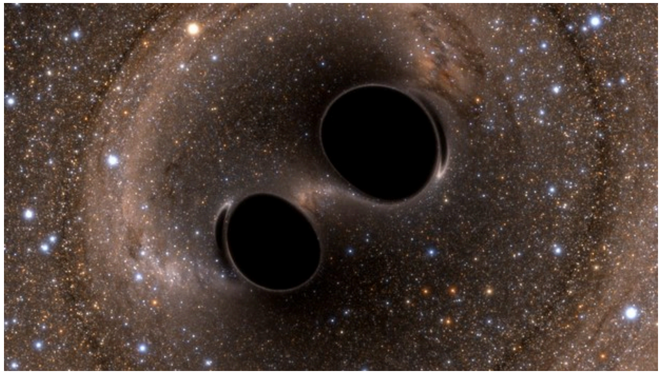
Gravitational wave

- **General Relativity**
  - Gravity = Spacetime curvature
  - Gravitational wave = Wave of spacetime curvature
- **Gravitational waves**
  - Generated by motion of massive objects
  - Propagates with speed of light
  - Cause quadrupole deformation of the spacetime



Measure strain between free masses to detect GWs

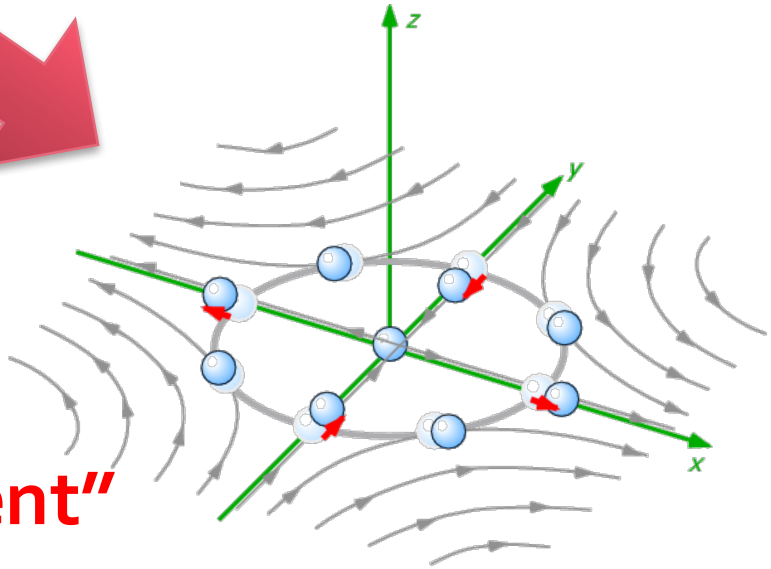
GW: Generation, Propagation, and Detection



Generation:
Change of quadrupole moment
Post-newtonian, NR

1/R

Propagation:
wave equation of
the spacetime metric



Detection:
Quadrupolar "displacement"
of the masses

GW Detection

- Measure strain between free masses

GW Detection

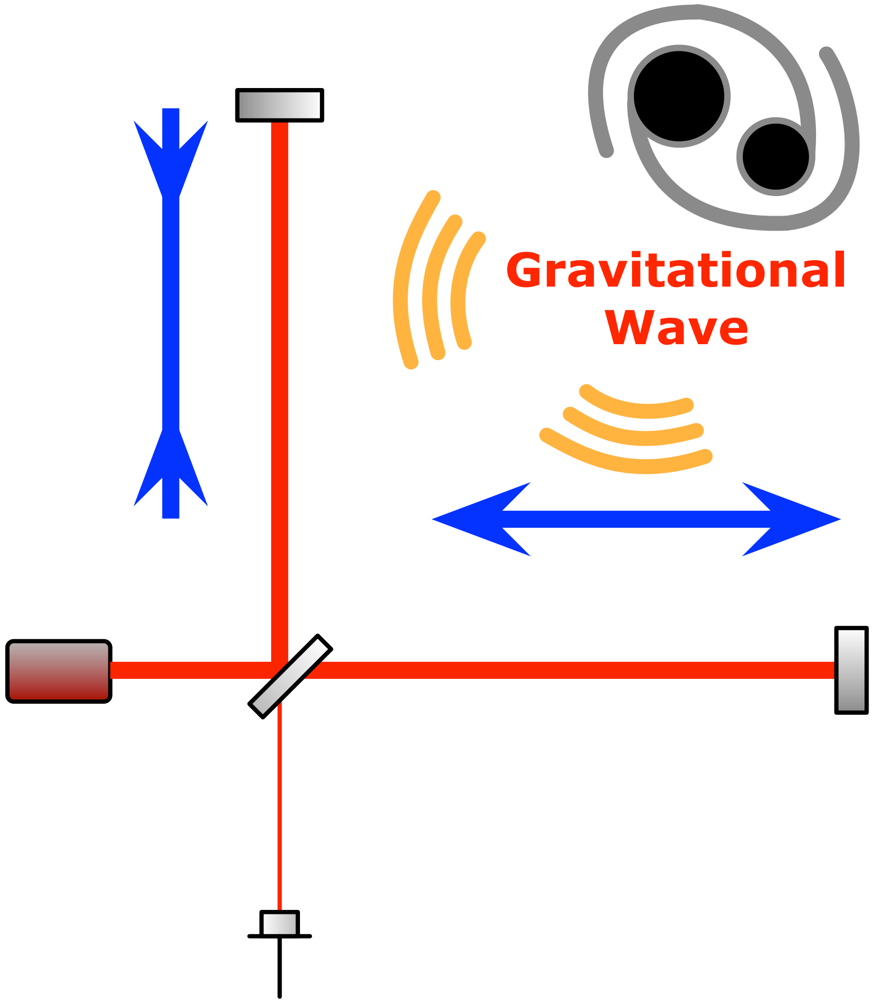
- Measure strain between free masses

  - GW does not appear in the local motion
  - Changes optical distance between the masses

- Longer the baseline, the bigeer change
  - (displacement dx) = (Strain h) x (baseline L)
- We need to measure phase of the laser light
  => use "laser interferometry"

Quadrupole nature of GWs

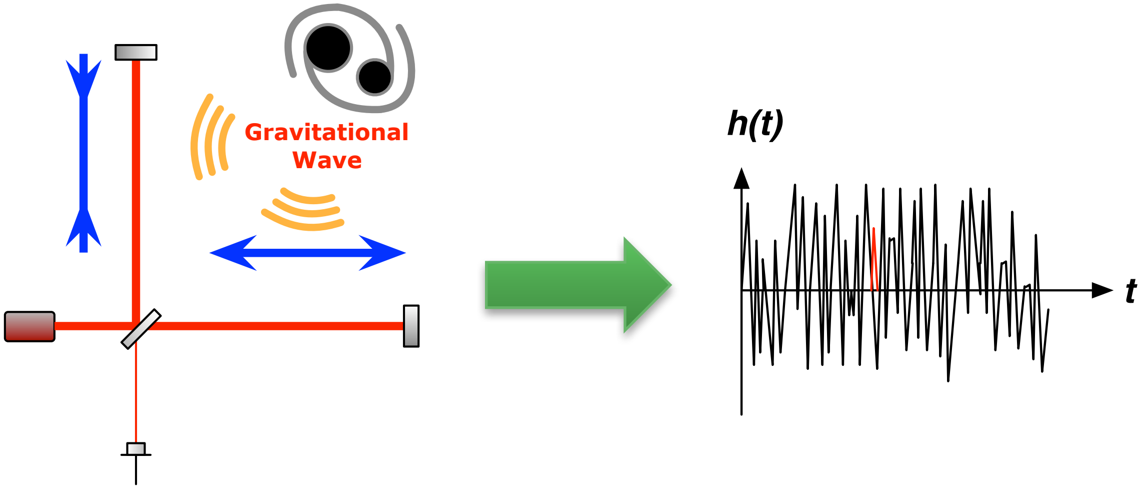
- Differential motion => Michelson interferometer



- Longer baseline
  -> bigger change
  - (displacement dx) = (Strain h) x (baseline L)
- Need to measure phase of the laser light
  => use "laser interferometry"

GW telescope?

- A continuous signal stream from an interferometer



- Fixed on the ground, can not be directed
- Poor directivity

=> More like an antenna

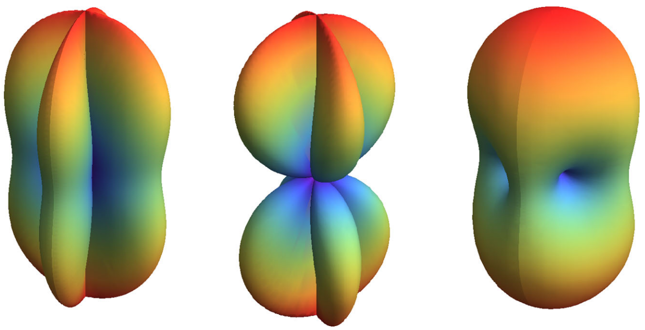
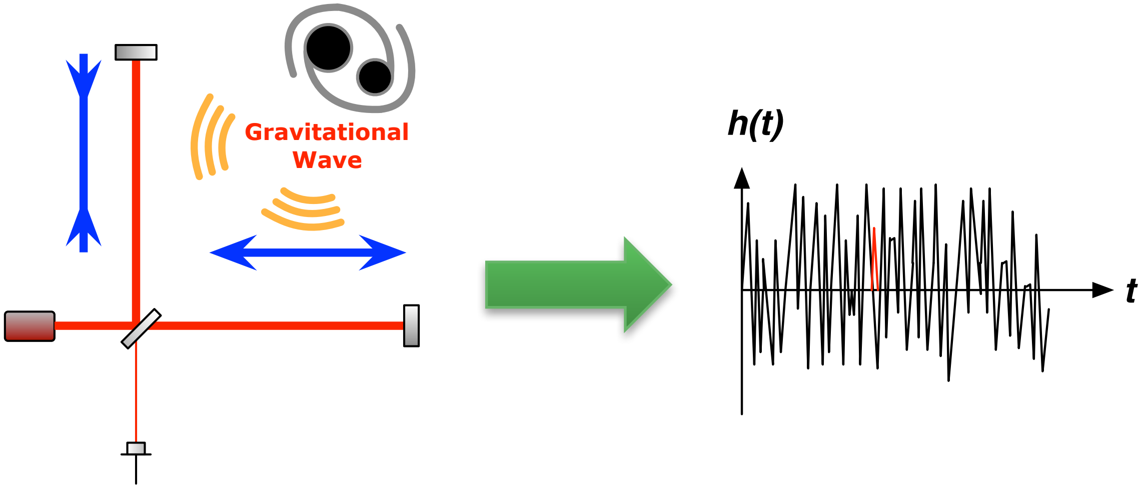


FIG. 2 (color online). Interferometer antenna response for (+) polarization (left), (×) polarization (middle), and unpolarized waves (right).

GW telescope?

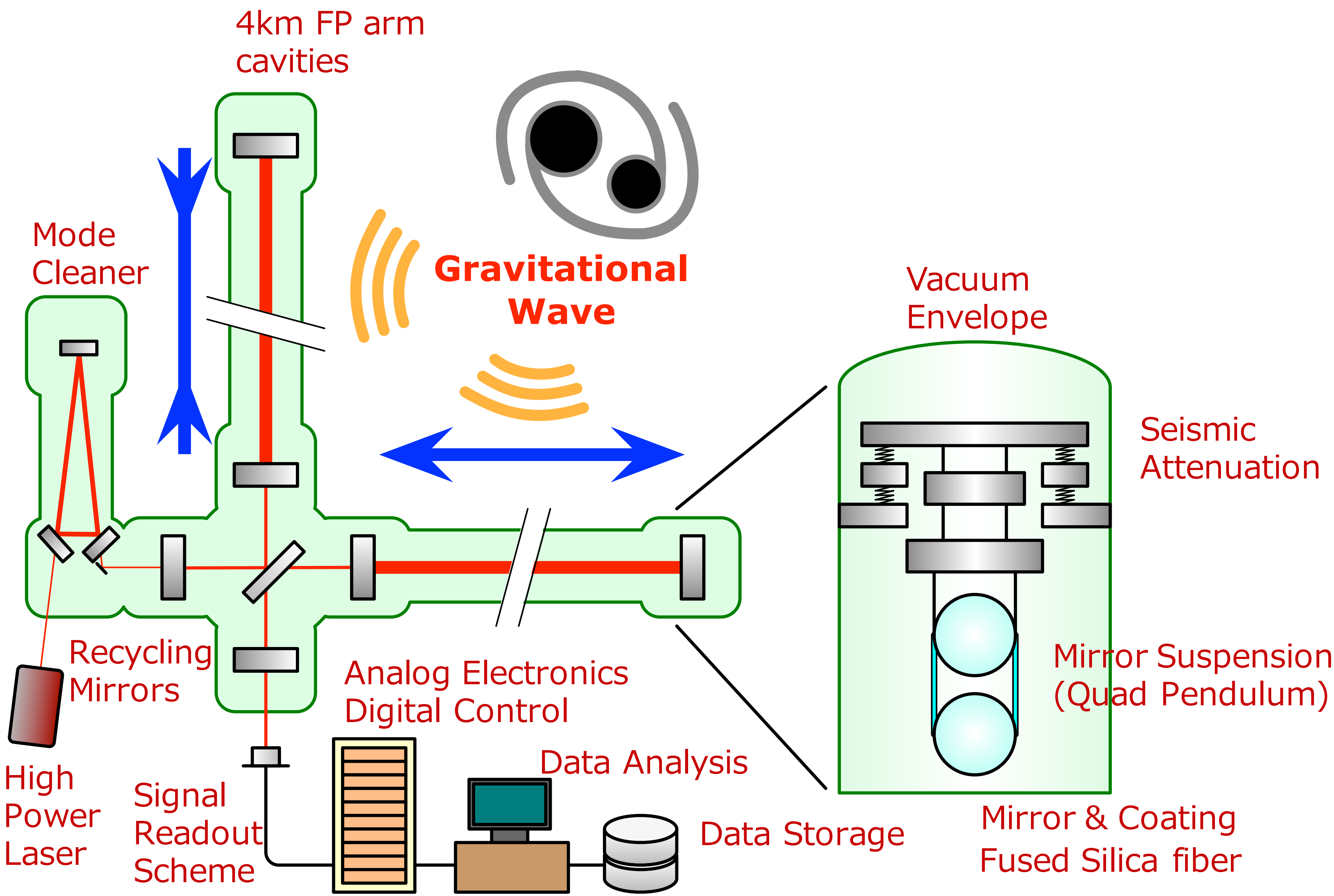
- A continuous signal stream from an interferometer



- GWs and noises: in principle, **indistinguishable**
  => Anything we detect is GW

- **Reduce noises!**
  - Obs. distance is inv-proportional to noise level
  - x10 better => x10 farther => x1000 more galaxies

Actual GW detector

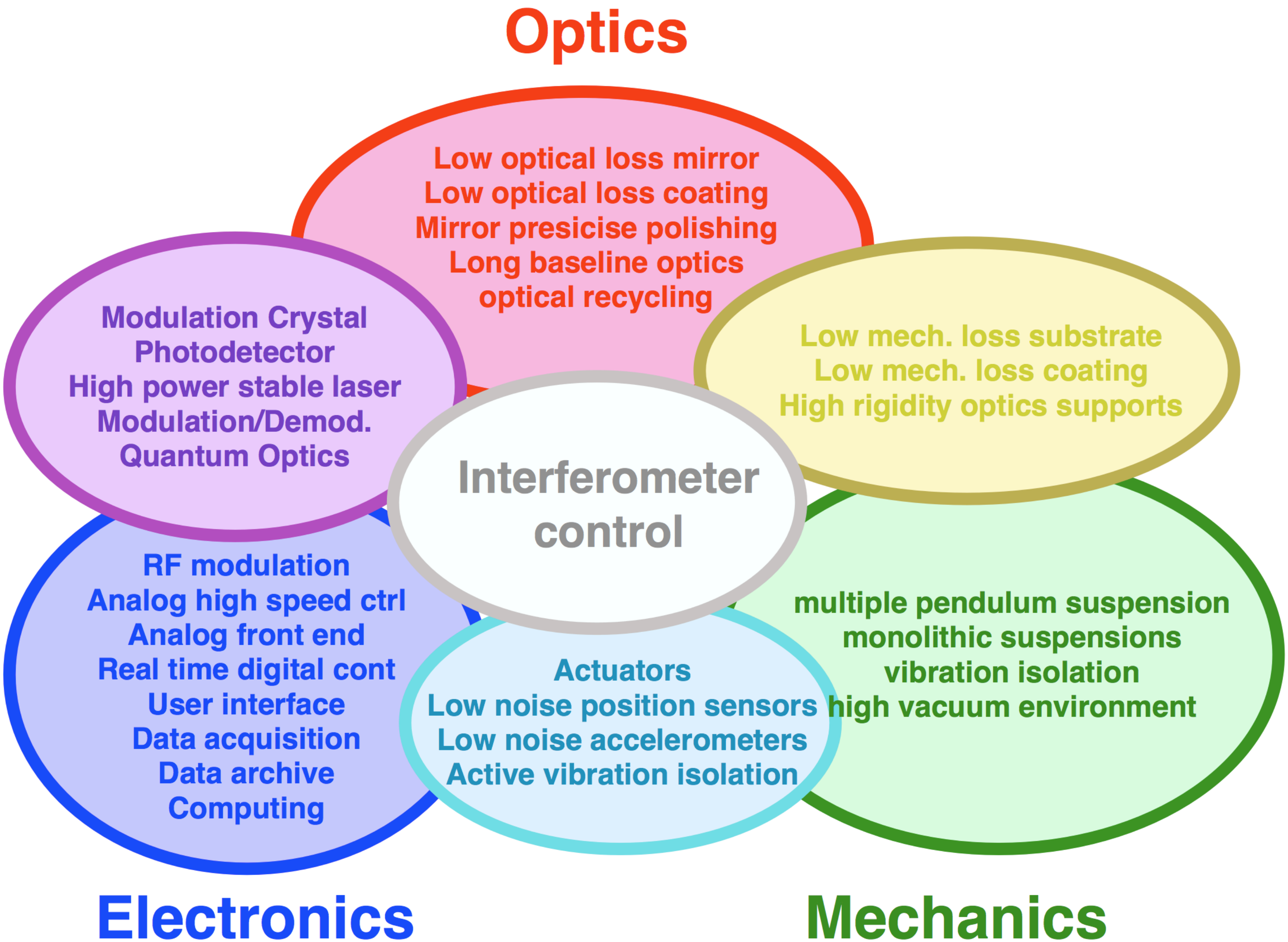


Components of the interferometer

- 3 elements of a GW detector
  - Mechanics
  - Optics
  - Electronics

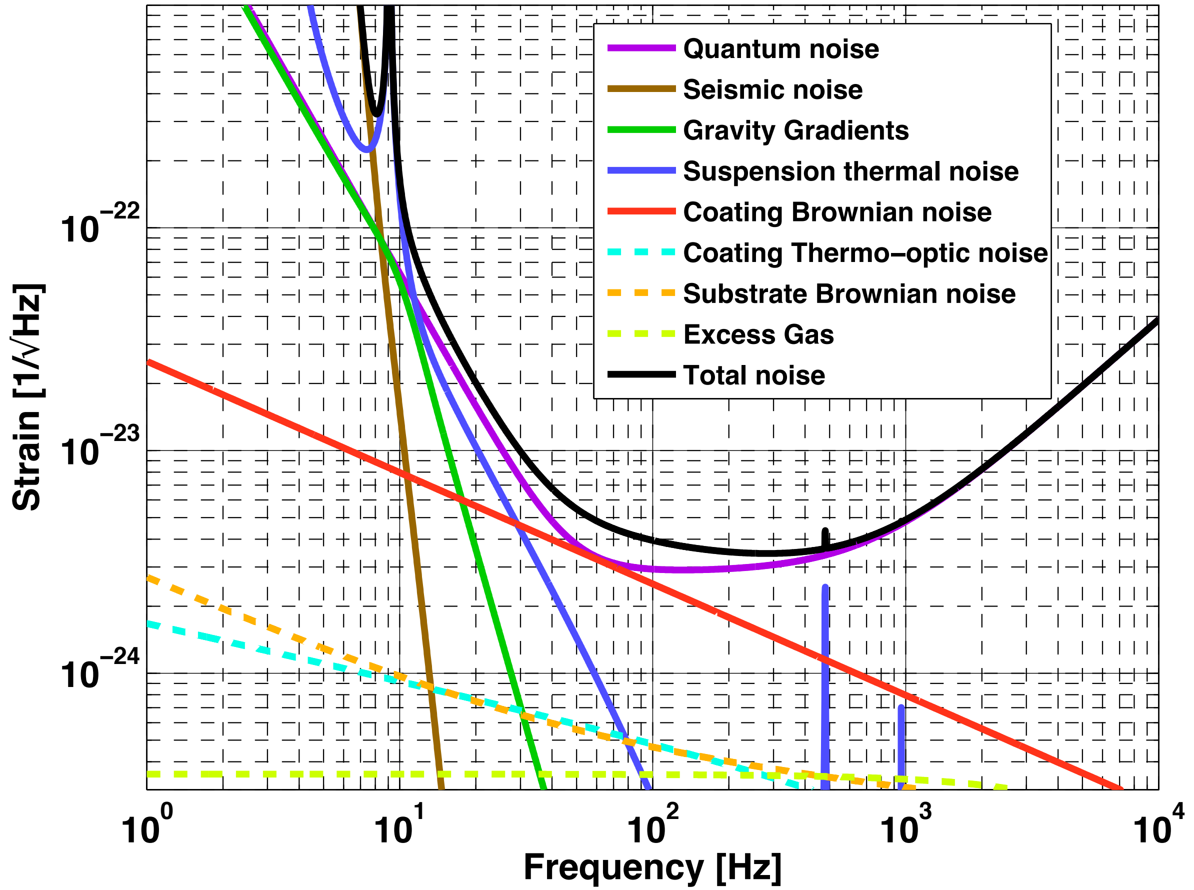
Components of the interferometer

- 3



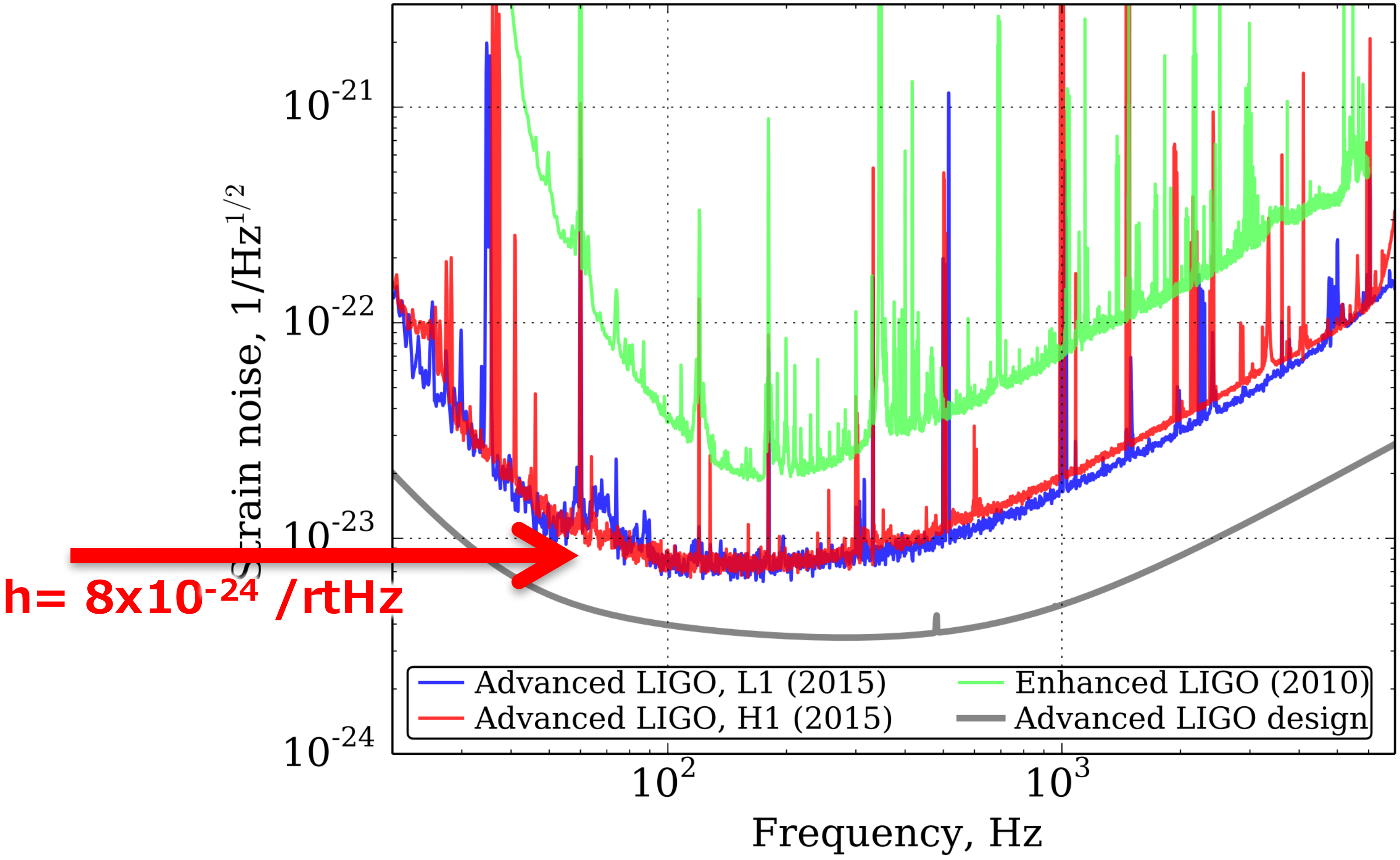
Sensitivity and noise

- Sensitivity (=noise level) of Advanced LIGO
- Design



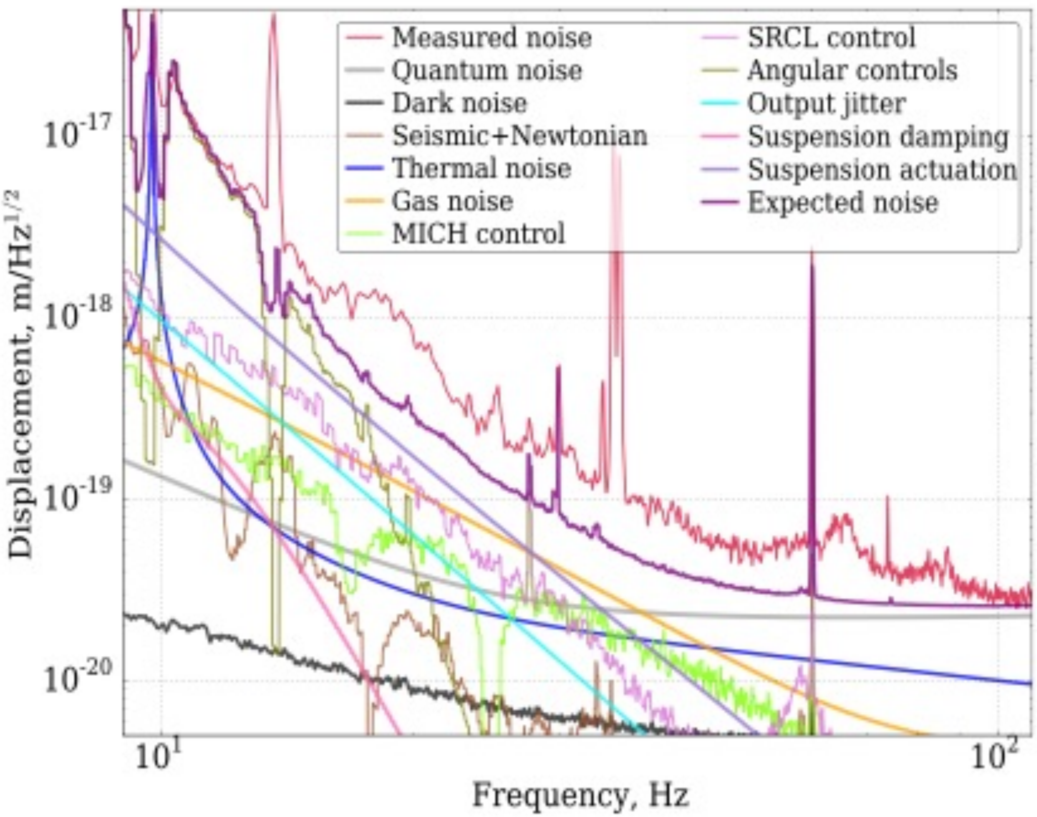
Sensitivity and noise

- Sensitivity (=noise level) of Advanced LIGO
- Current sensitivity

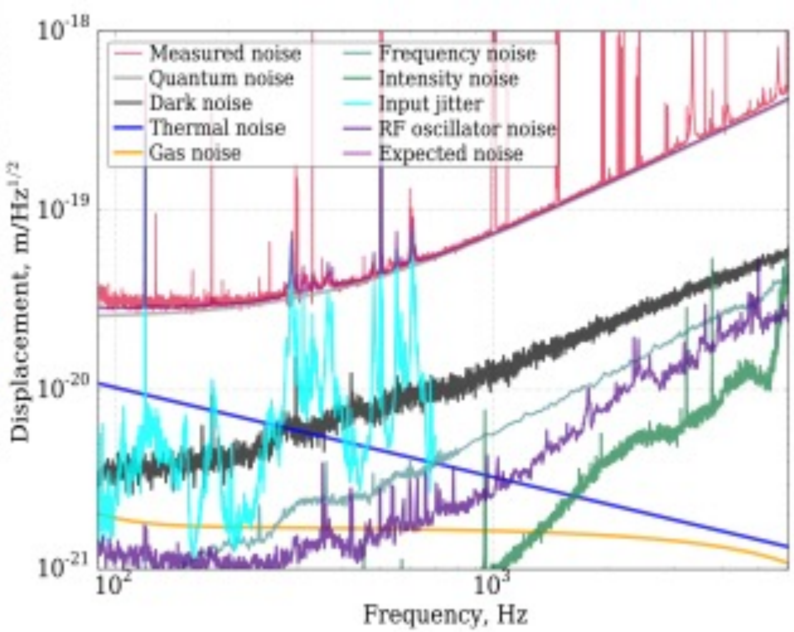


Sensitivity and noise

- Sensitivity (=noise level) of Advanced LIGO
- Noise budget



(a) LIGO Livingston Observatory



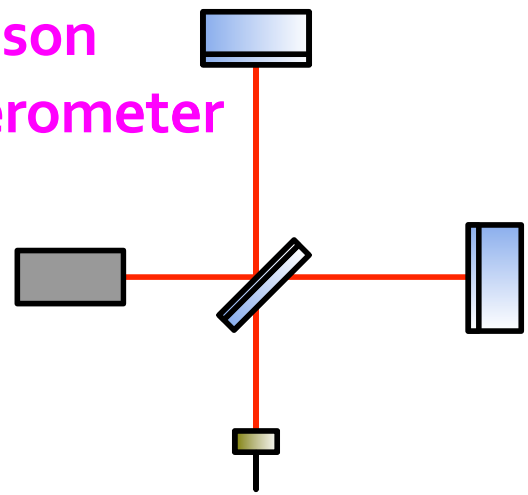
(b) LIGO Hanford Observatory

Optical configurations of laser interferometer GW detectors

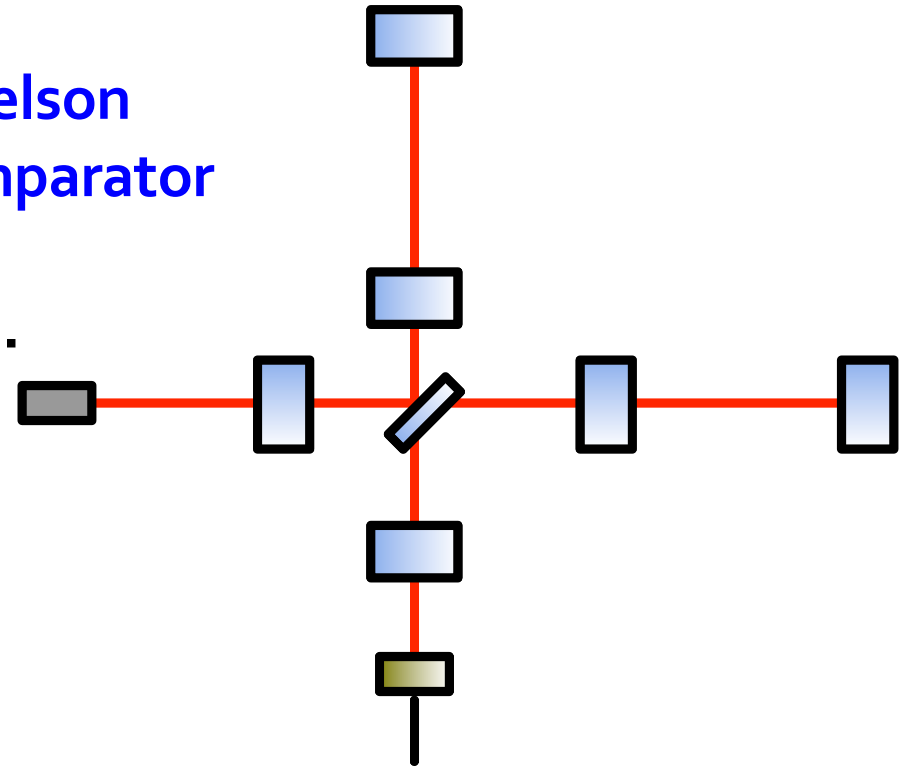
Introduction ~ Interferometer?

- Interferometry
  - Utilize characteristic of a Michelson interferometer as a length comparator

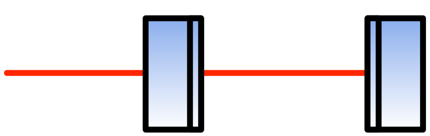
Michelson Interferometer



In reality...



Fabry-Perot cavity



Advanced LIGO
Dual-Recycled Fabry-Perot Michelson Interferometer

No worries: It's just a combination of MI and FPs

Michelson interferometer

- **Light intensity at the output port**
  - **Difference of the electric fields from the arms**

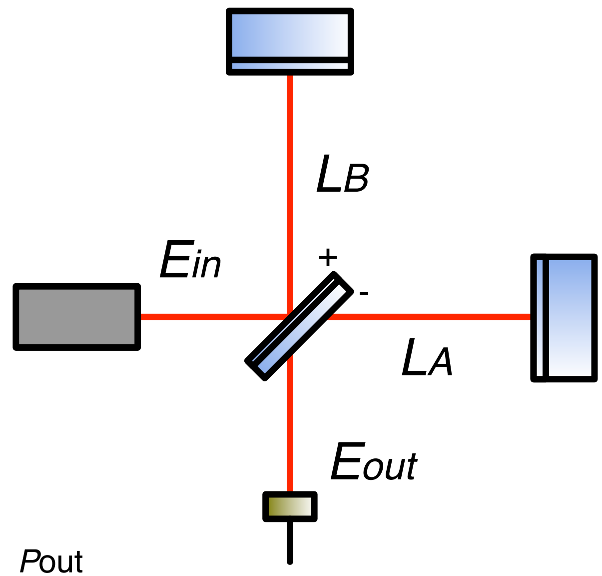
$$E_{\mathrm{out}} = \frac{1}{2}\left(e^{-i\phi_{\mathrm{B}}} - e^{-i\phi_{\mathrm{A}}}\right) E_{\mathrm{in}}$$

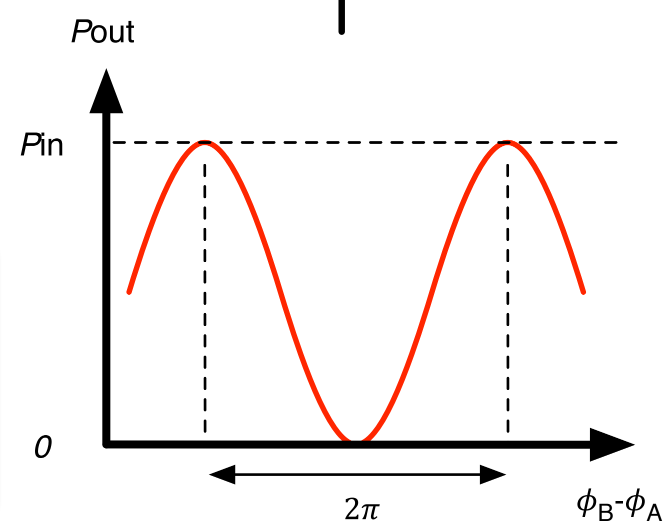
$$(\text{Roundtrip phase: } \phi_x = 4\pi\nu L_x/c)$$

$$E_{\mathrm{out}} = \left[i e^{-i(\phi_A+\phi_B)/2} \sin\frac{\phi_A-\phi_B}{2}\right] E_{\mathrm{in}}$$

$$P_{\mathrm{out}} = E_{\mathrm{out}} E^*_{\mathrm{out}} = \left(\sin^2\frac{\phi_A-\phi_B}{2}\right) E_{\mathrm{in}}$$

$$= [1-\cos(\phi_A-\phi_B)]\frac{P_{\mathrm{in}}}{2}$$





Output intensity is sensitive to the differential phase

Michelson interferometer

- **Frequency response of the Michelson to GWs**

$$\phi_A - \phi_B = \int_{t-2L/c}^{t} \Omega\left[1+\frac{1}{2}h(t)\right]dt - \int_{t-2L/c}^{t} \Omega\left[1-\frac{1}{2}h(t)\right]dt$$

$$= \int_{t-2L/c}^{t} \Omega h(t) dt$$

Frequency response of the Michelson interferometer

$$h(t) = h_0 e^{i\omega t}$$

$$\phi_A - \phi_B = \boxed{\frac{2L\Omega}{c} e^{-iL\omega/c} \frac{\sin(L\omega/c)}{L\omega/c}} \cdot h_0 e^{i\omega t}$$

$$= \frac{4\pi L}{\lambda_{\mathrm{opt}}} e^{-i2\pi L/\lambda_{\mathrm{GW}}} \frac{\sin(2\pi L/\lambda_{\mathrm{GW}})}{2\pi L/\lambda_{\mathrm{GW}}} \cdot h_0 e^{i\omega t}$$

Ω: optical angular frequency, λ_{OPT} laser wavelength
ω: angular frequency of GW, λ_{GW} wavelength of GW

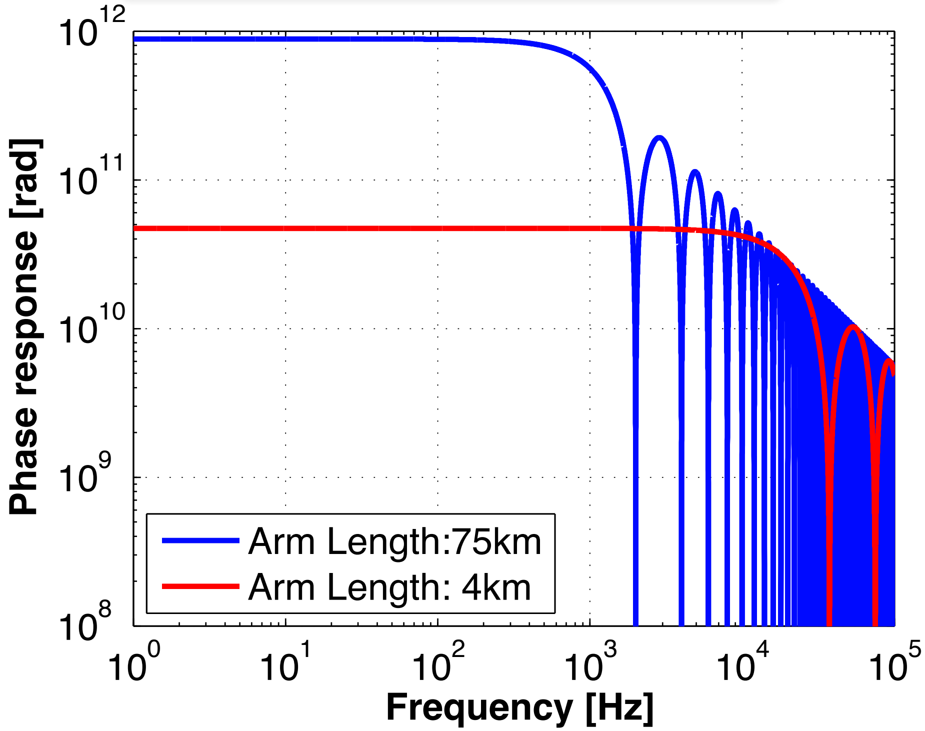
Jean-Yves Vinet, et al
Phys. Rev. D 38, 433 (1988)

Michelson interferometer

- **Frequency response of the Michelson to GWs**

$$\Delta\phi = \boxed{\frac{2L\Omega}{c} e^{-iL\omega/c} \frac{\sin(L\omega/c)}{L\omega/c}} \cdot h_0 e^{i\omega t}$$

DC Response longer -> larger



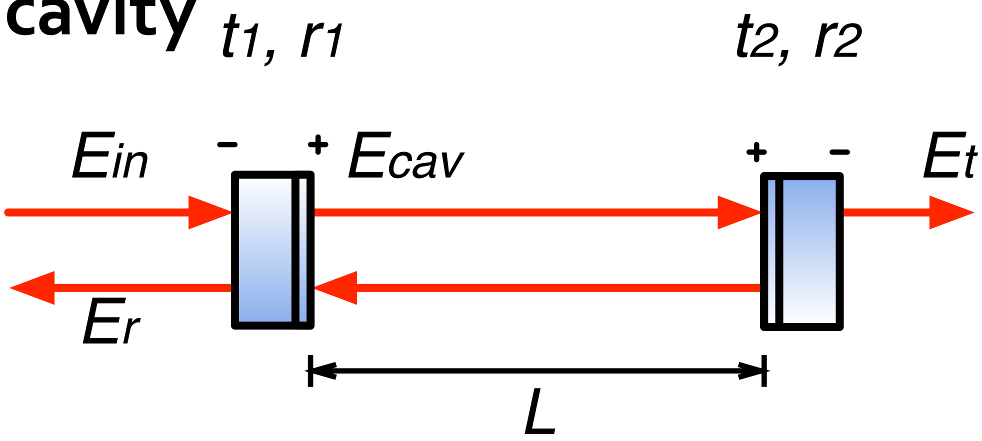
Cut off freq longer -> lower

Notch freq f = n c / (2 L)

Michelson arm length optimized for 1kHz GW -> 75km, too long!

Fabry-Perot optical resonator

- **Storing light in an optical cavity**
  - **Field equations**



$$E_{\text{cav}} = t_1 E_{\text{in}} + r_2 e^{-i\phi} E_{\text{cav}}$$

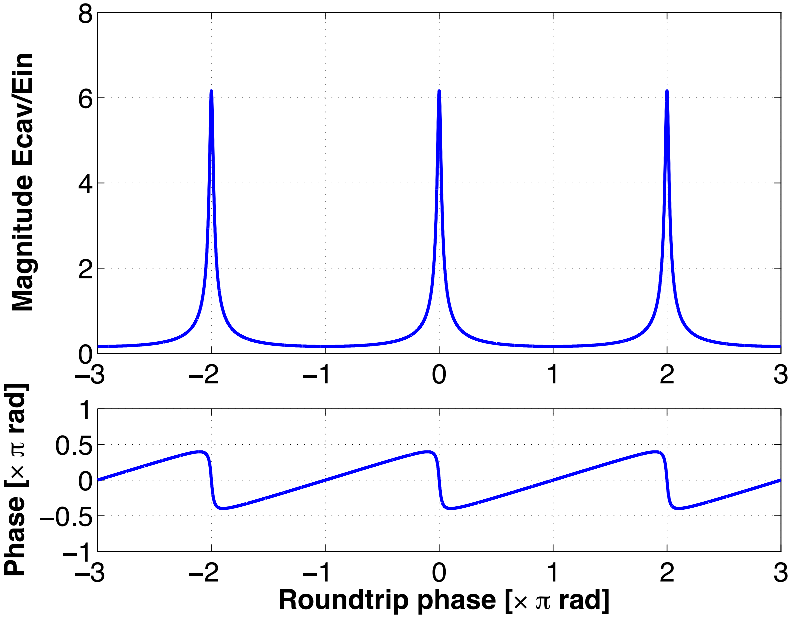
$$E_{\text{t}} = t_2 e^{-i\phi/2} E_{\text{cav}}$$

$$E_{\text{r}} = -r_1 + t_1 r_2 e^{-i\phi} E_{\text{cav}}$$

$$\frac{E_{\text{cav}}}{E_{\text{in}}} = \frac{t_1}{1 - r_1 r_2 e^{-i\phi}}$$

$$\frac{E_{\text{r}}}{E_{\text{in}}} = -r_1 + \frac{t_1^2 r_2 e^{-i\phi}}{1 - r_1 r_2 e^{-i\phi}}$$

$$\frac{E_{\text{t}}}{E_{\text{in}}} = \frac{t_1 t_2 e^{-i\phi/2}}{1 - r_1 r_2 e^{-i\phi}}$$



Very fast phase response

Fabry-Perot optical resonator

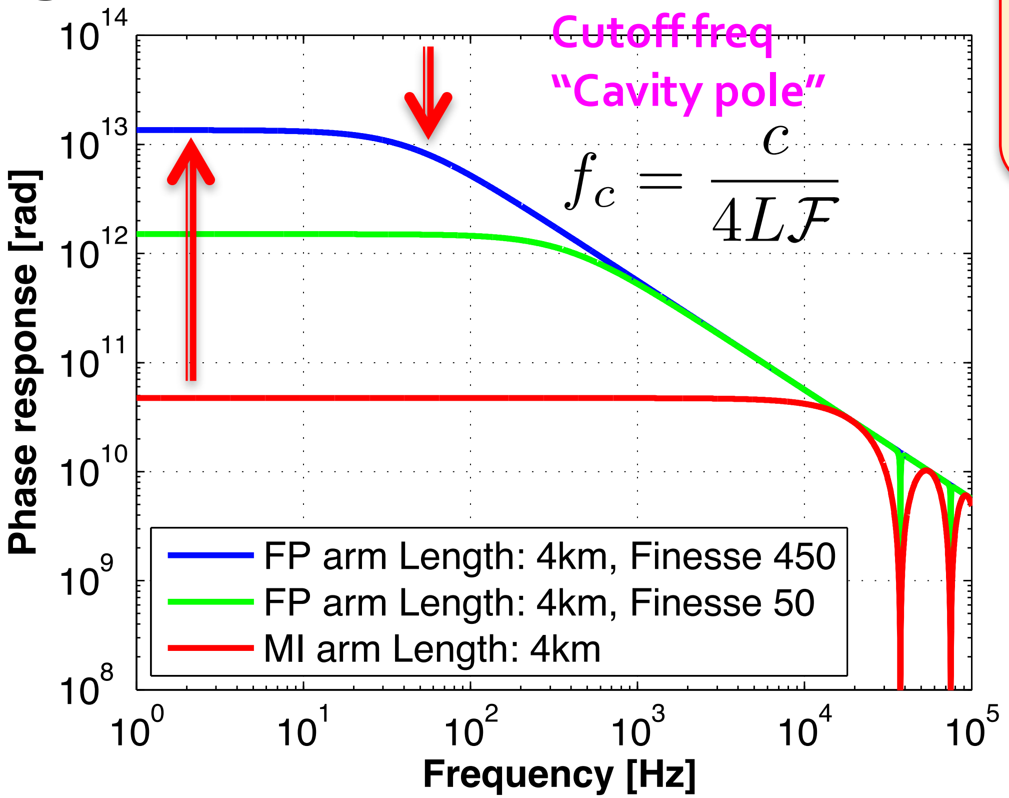
- **Storing light in an optical cavity**

Finesse

$$\mathcal{F} = \frac{\pi\sqrt{r_1 r_2}}{1 - r_1 r_2}$$

DC Response amplification

$$N = 2\mathcal{F}/\pi$$

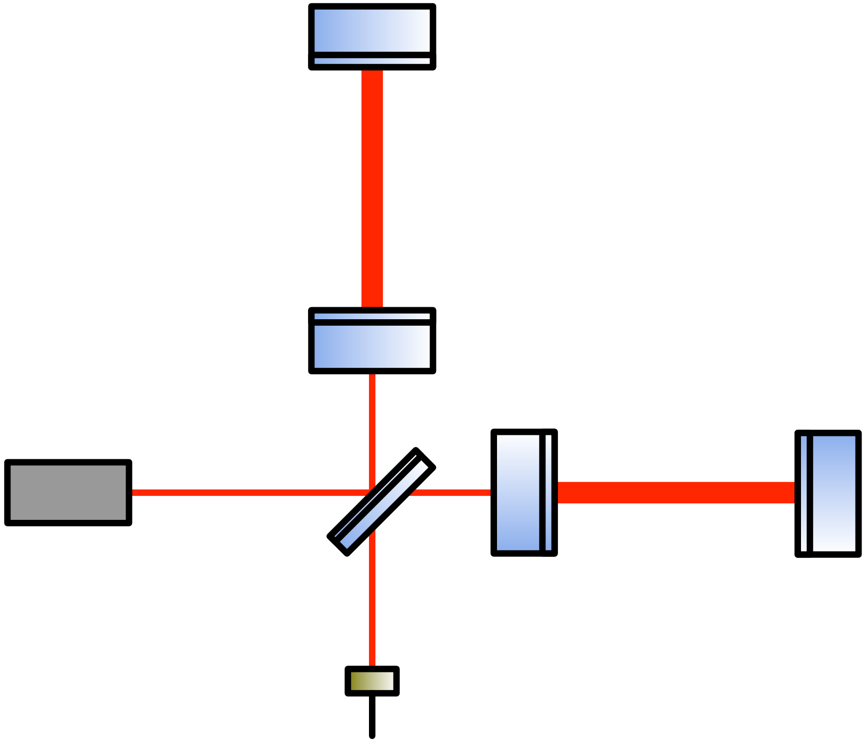


1. FP increases stored power in the arm
2. FP increases accumulation time of the signal

=> Above the roll-off, increasing F does not improve the response

Fabry-Perot Michelson Interferometer

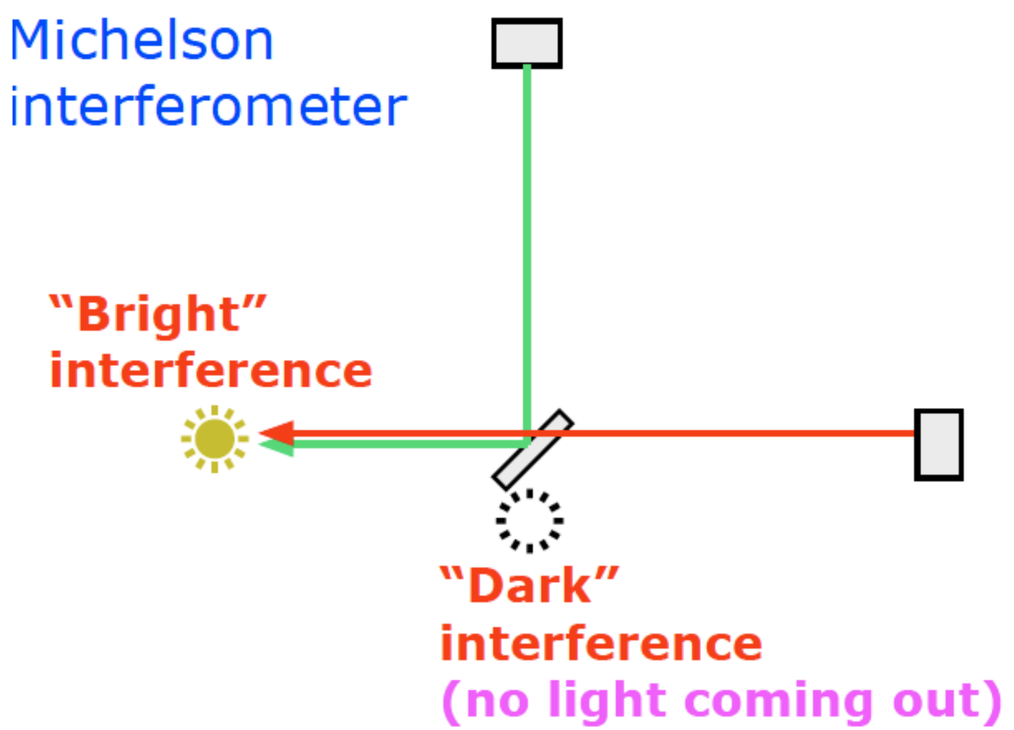
- Differential nature of the Michelson
  + Longer photon storage time of Fabry-Perot cavities
  = Fabry-Perot Michelson Interferometer



Basic form of the modern interferometer GW detector

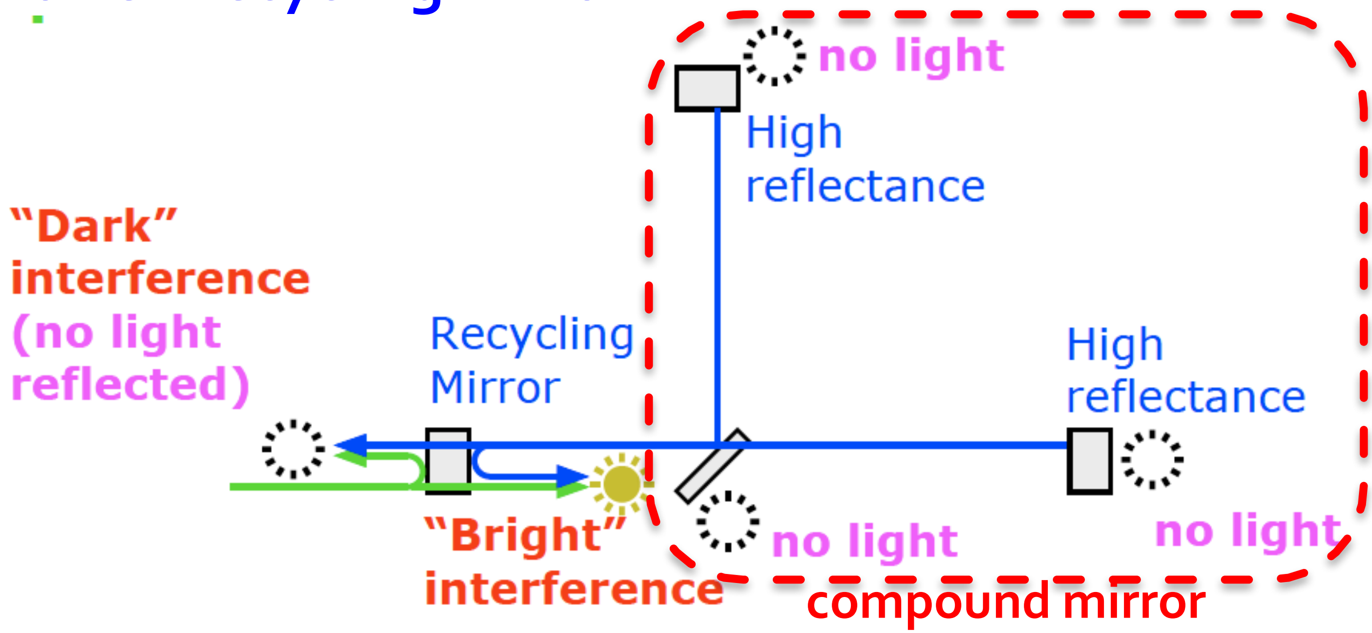
Optical Recycling Technique

- Power recycling
  - When the Michelson interferometer is operated at a “dark fringe”, most of the light goes back to the laser side



Optical Recycling Technique

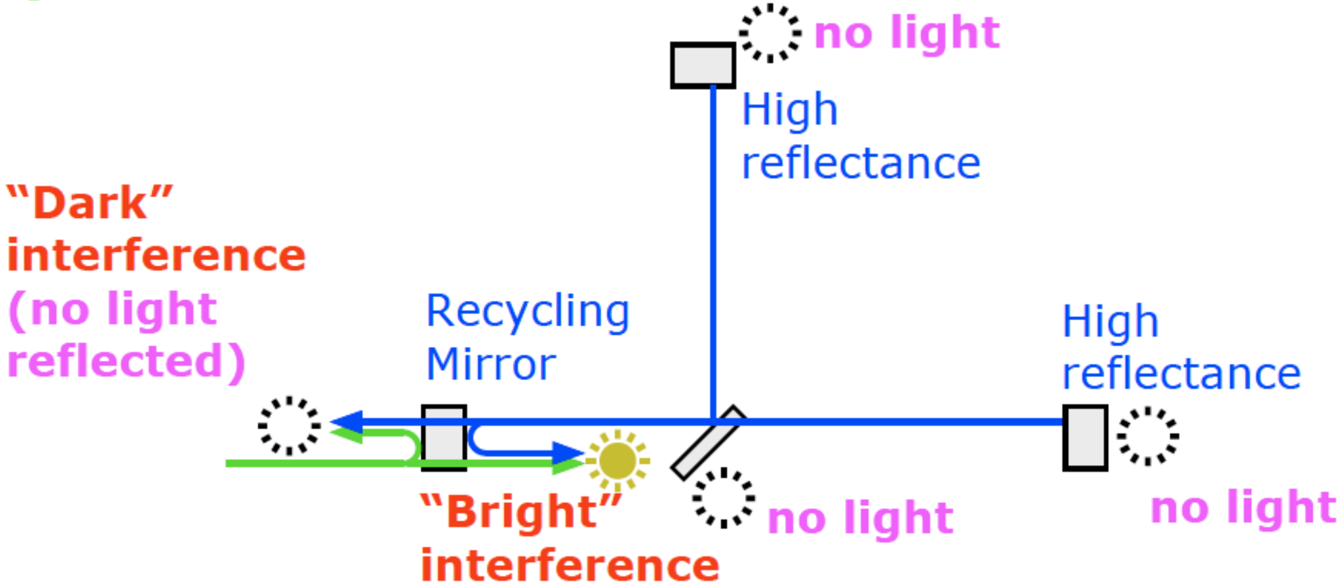
- Power recycling
  - Let's reuse the reflected light
  - Place a mirror in front of the interferometer
    to form a cavity with the Michelson (compound mirror)
    "Power Recycling Mirror"



  - The internal light power is increased
    = equivalent to the increase of the input laser power

Optical Recycling Technique

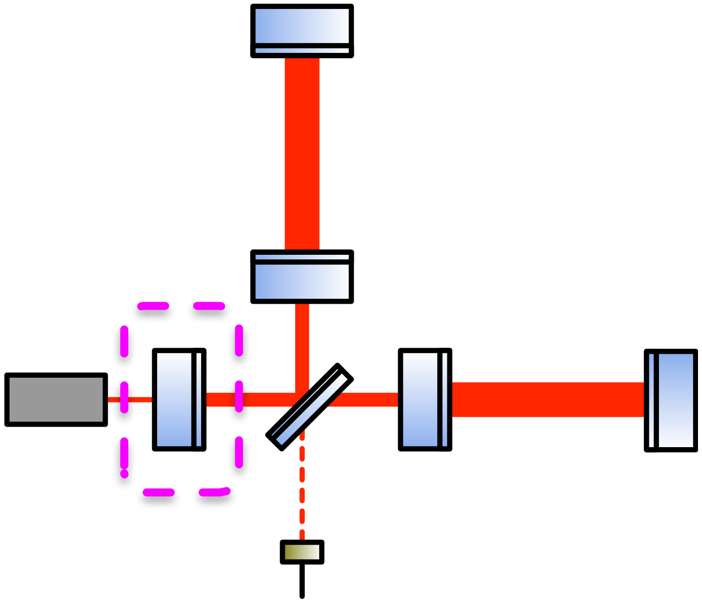
- Power recycling
  - BTW, all the output ports are made dark. Where does the light go?



  - In the ideal power recycling, all input power is internally consumed via optical losses (absorption & scattering)

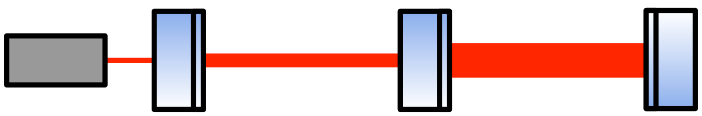
Optical Recycling Technique

- **Power-recycled Fabry-Perot Michelson Interferometer**
  - Internal light power in the arms is increased



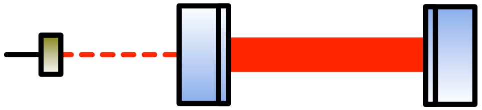
From the laser side / common arm length change

It looks like a three mirror caivty = high finesse cavity



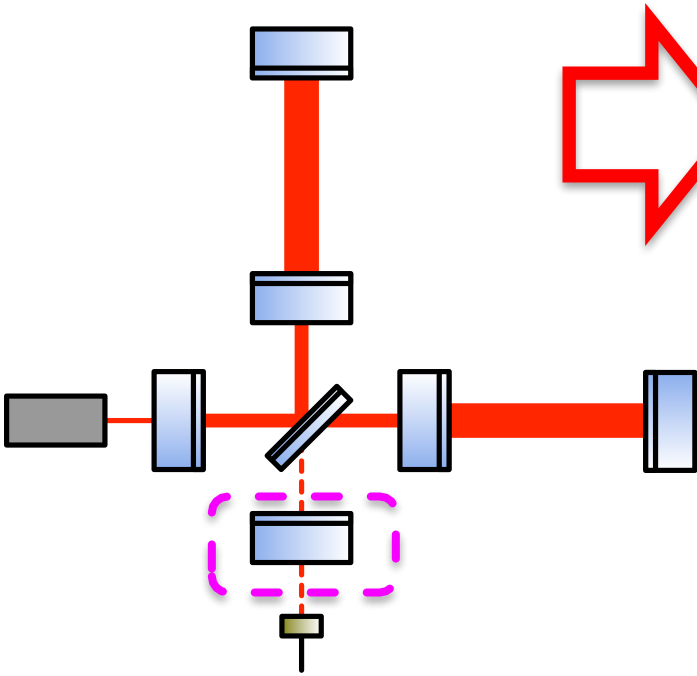
For the differential motion (=GW)

It looks like just an arm cavity



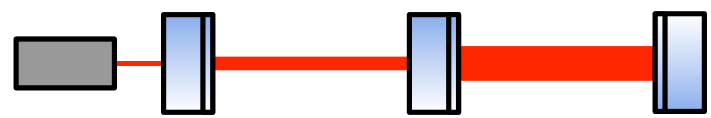
Optical Recycling Technique

- Dual-recycled Fabry-Perot Michelson Interferometer
  - Another mirror is added at the dark port
    "Signal Recycling Mirror"
  - Dual recycling allows us to set different storage times for common and differential modes



Common mode

= high finesse three mirror cavity

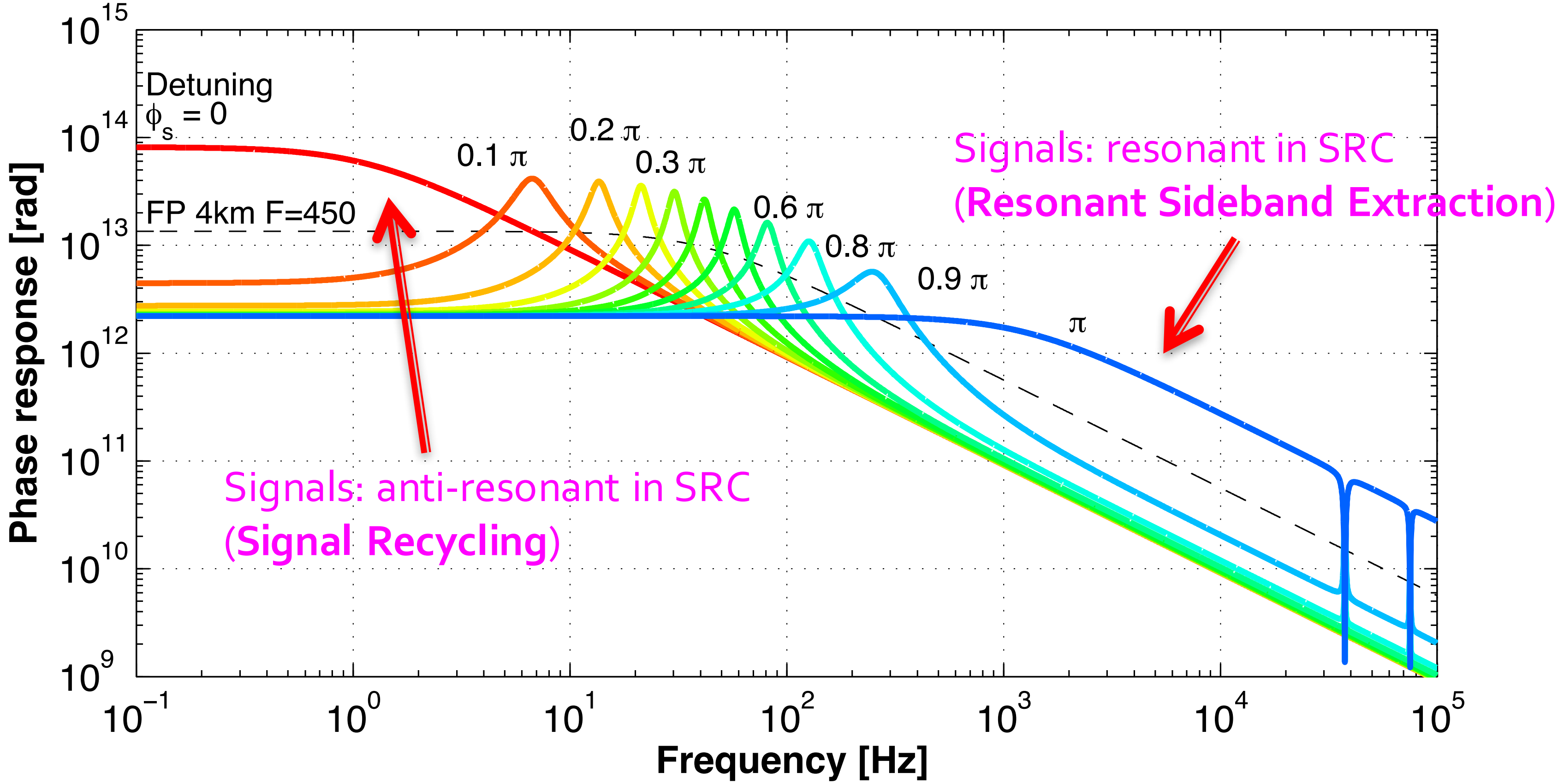


Differential mode (=GW)

= low (or high) finesse three mirror cavity

To tuned or not to tune

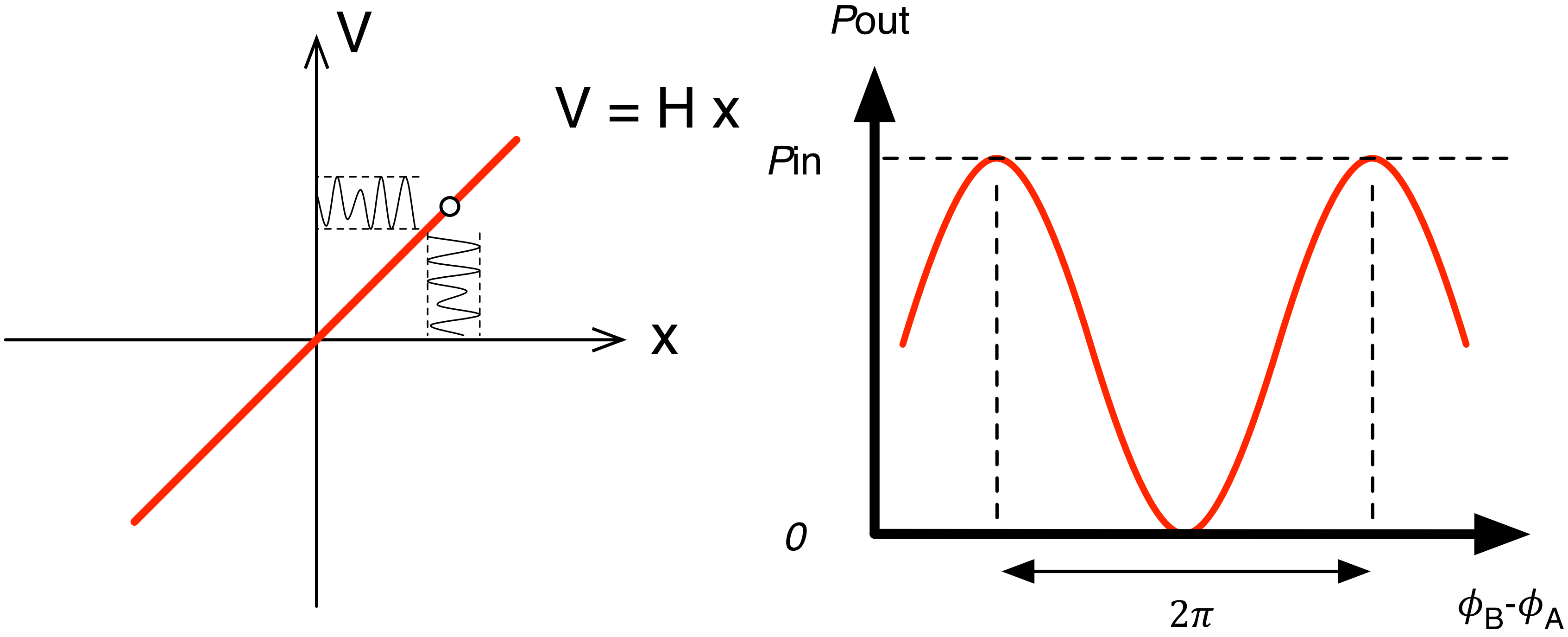
- **Bandwidth of the detector can be changed**
  - by changing the resonant phase of the signal recycling cavity (SRC)



- **Optimize the curve depending on the noise shape**
- **Dynamic signal tracking**

Signal Readout

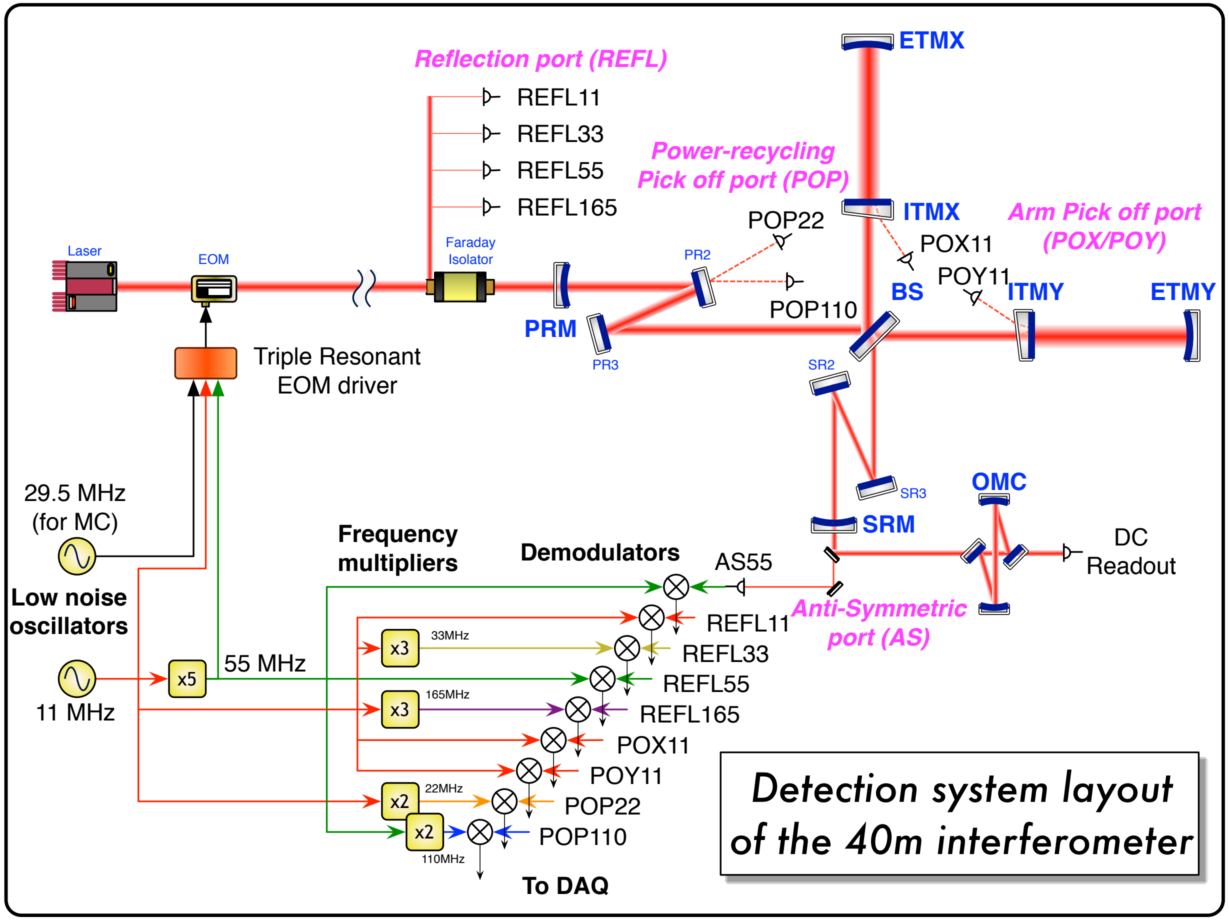
- **Interferometer response is nonlinear**



- **How do you read the GW signal (and other signals)?**
  => Signal readout scheme

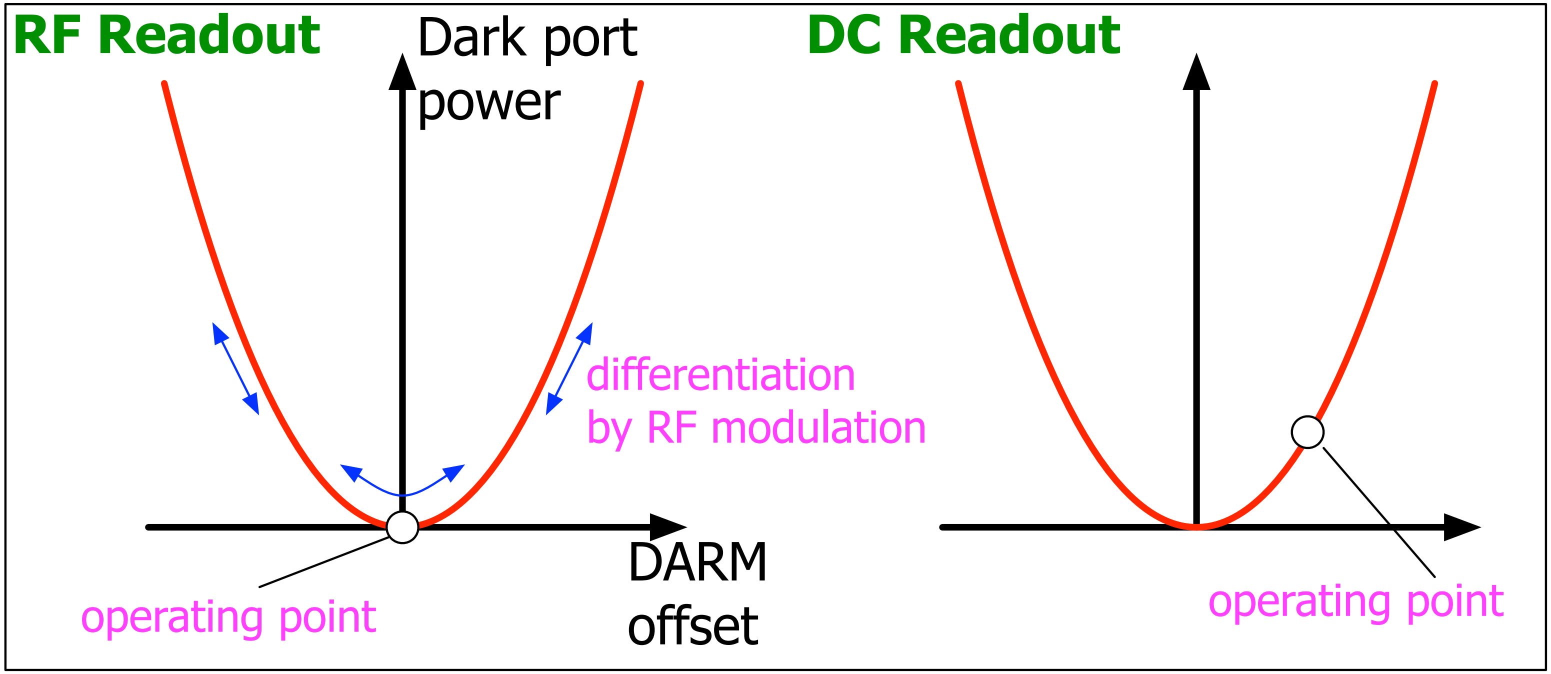
Signal Readout

- Signal readout scheme
- RF phase modulation / demodulation or DC Readout



Signal Readout

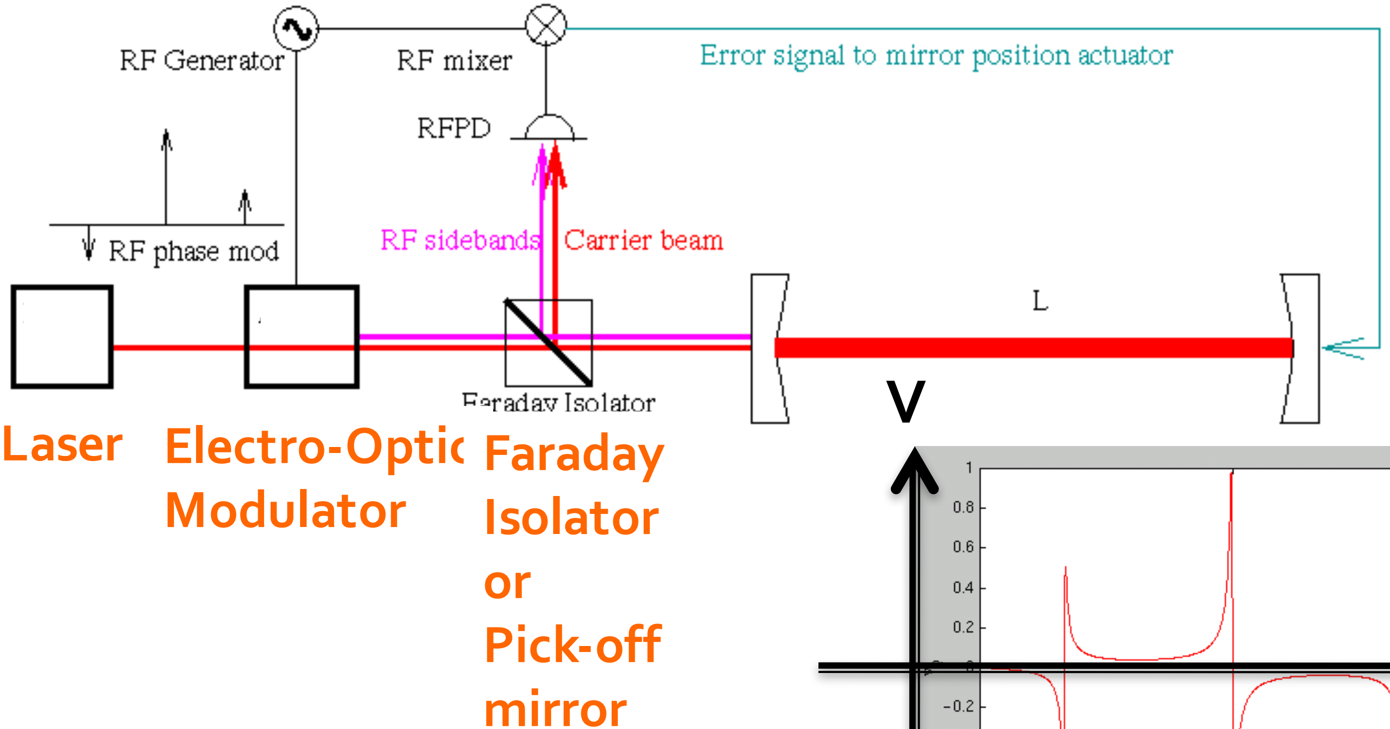
- **RF Readout and DC Readout**

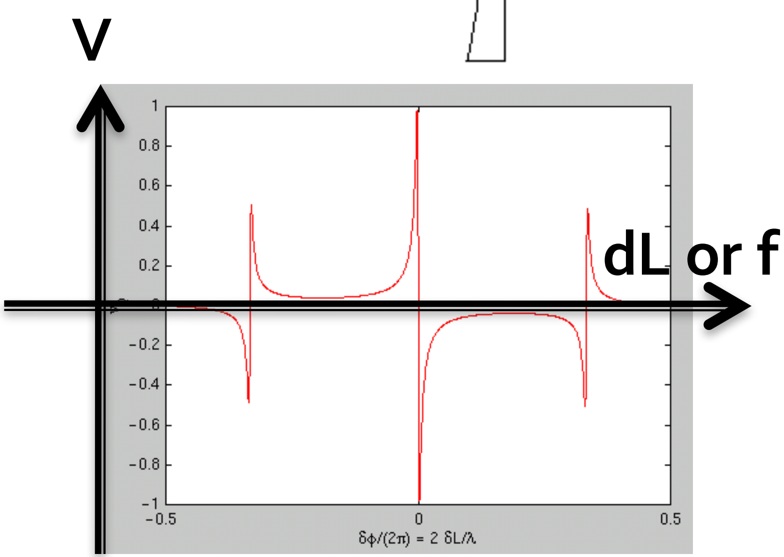


- **DC Readout is good for GW channel**
  - removes nonstationary shot noise
  - mitigates technical noises associated with the RF sidebands

Pound-Drever-Hall technique (PDH)

- **RF signal readout scheme for cavities**
  - Phase modulation -> RF optical sidebands
  - Reflected beam -> detected / demodulated





Summary

- Optical phase measurement => Interferometry

- Michelson interferometer: requires too long arm
- Fabry-Perot arm: longer light storage time
- Optical recycling technique:
  - more power in the arms
  - allows us to taylor the detector response to GW signals

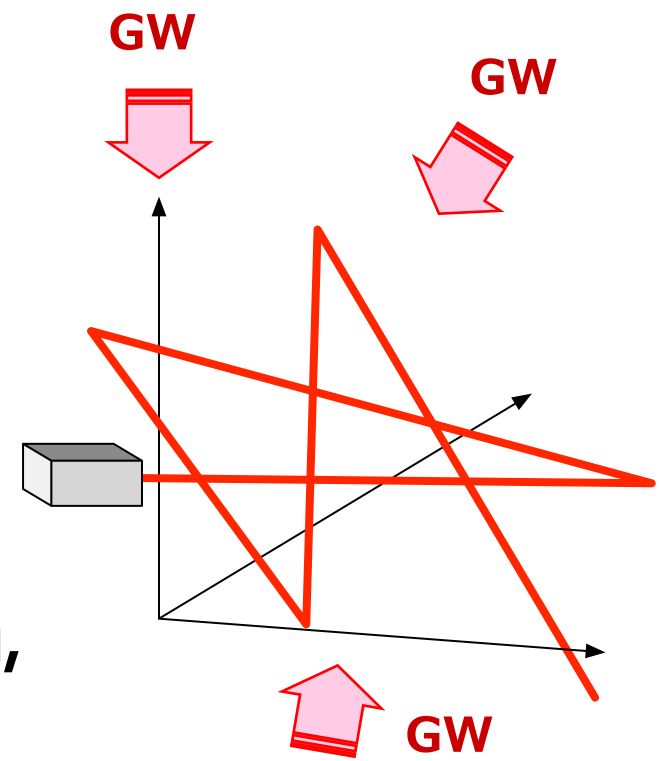
- Optical read-out schemes

Advanced topics

- Angular & frequency response of an interferometer
  - Up to this point GWs from the zenith was assumed.
  - What is the response to GWs with an arbitrary angle?
  - What is the frequency response of the detector for such GWs?

  - Draw an arbitrary optical path.
    What is the angular and frequency response of such a path?

  - Can we use numerical "optimization" for certain criteria?
    e.g.
    better sky coverage, directive beaming, for certain source frequency, etc…



Advanced topics

Angular & frequency response of an interferometer

- R. Schilling, Class. Quantum Grav. 14 (1997) 1513-1519

2.1. Single round trip

We will assume a gravitational wave propagating along the Z direction, with its polarization axes being parallel to the X/Y axes. In the simplest case the arm lies entirely in the X–Y plane, but in general there will be a tilt angle ϑ between the direction of the arm and the X–Y plane. With the single pass of a light beam travelling along the arm and measuring its length ℓ we find

$$\ell(t) = \ell_0 + \tfrac{1}{2} c \cos^2\vartheta \int_{t_0-\ell_0/c}^{t_0} h\left[t + t'(1-\sin\vartheta)\right] \mathrm{d}t'. \tag{1}$$

For a sinusoidal gravitational wave $h(t) = \hat{h}\exp(\mathrm{i}\omega t)$ and $t_0 = 0$ this becomes

$$\ell(t) = \ell_0 + \tfrac{1}{2}\hat{h}\ell_0 \cos^2\vartheta \,\mathrm{sinc}\left[\frac{\omega\ell_0}{2c}(1-\sin\vartheta)\right] \exp\left[\mathrm{i}\omega t - \mathrm{i}\frac{\omega\ell_0}{2c}(1-\sin\vartheta)\right], \tag{2}$$

where the sinc function is defined as $(\sin x)/x$. A complete round trip consists of the concatination of a forward and a return pass; for the latter we have to replace ϑ by $-\vartheta$, and we have to fulfil a continuation condition for the phase of the induced signal at the return point (mirror or transponder). For the time-varying part of ℓ this leads to

$$\delta\ell(t) = \tfrac{1}{2}\hat{h}\ell_0 \cos^2\vartheta \{\mathrm{sinc}[\pi\Omega(1-\sin\vartheta)] \exp[-\mathrm{i}\pi\Omega(3+\sin\vartheta)] + \mathrm{sinc}[\pi\Omega(1+\sin\vartheta)] \exp[-\mathrm{i}\pi\Omega(1+\sin\vartheta)]\} \exp(\mathrm{i}\omega t), \tag{3}$$

where we have introduced a normalized frequency Ω with $2\pi\Omega = \omega\ell_0/c$. The result of equation (3) can also be expressed in the form of a *normalized* antenna transfer function $\mathcal{T} = 2\delta\ell(t)/[\ell_0\hat{h}\exp(\mathrm{i}\omega t)]$ as

$$\mathcal{T} = \cos^2\vartheta \{\mathrm{sinc}[\pi\Omega(1-\sin\vartheta)] \exp[-\mathrm{i}\pi\Omega(3+\sin\vartheta)] + \mathrm{sinc}[\pi\Omega(1+\sin\vartheta)] \exp[-\mathrm{i}\pi\Omega(1+\sin\vartheta)]\}. \tag{4}$$

Figure 1(a) shows the magnitude of the normalized one-arm transfer function $\mathcal{T}_1$ for a single round trip and $\vartheta = 0°$, indicated separately for the forward pass, the return pass and the full round trip. In the case shown, the transfer functions for the forward and return pass are identical in magnitude, only differing in phase, which leads to the additional zeros in the full round-trip response at frequencies $\Omega = \frac{1}{2}(2k-1)$.

The response for a tilt of $\vartheta = 45°$ is shown in figure 1(b), revealing two interesting facts: the zeros of the round-trip response have moved up to much higher frequencies, from multiples of $\Omega = \frac{1}{2}$ to ones of $\Omega = 3.41$, and the transfer function can take values that are even above the envelope for $\vartheta = 0°$. It turns out that the well known response for the tilt $\vartheta = 0°$ is, in fact, the exception rather than the normal case, since most of the zeros (caused by cancellation) appear at normal incidence only.

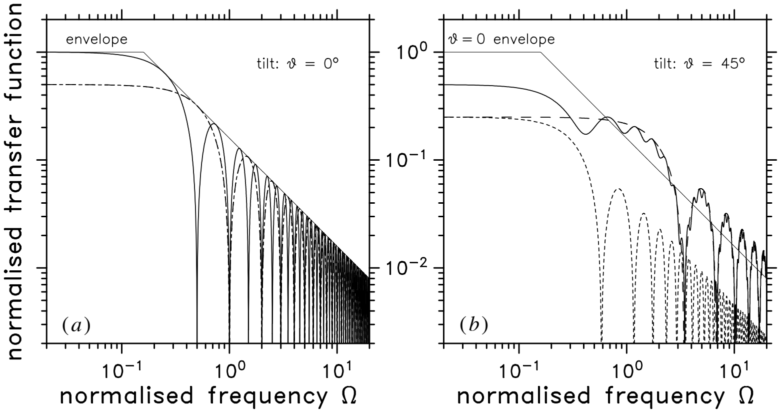


Figure 1. Magnitude of the normalized transfer function for a single round trip in a single arm and a tilt of (a) 0° and (b) 45°. Full curve, round trip; long broken curve, forward pass; short broken curve, return pass.

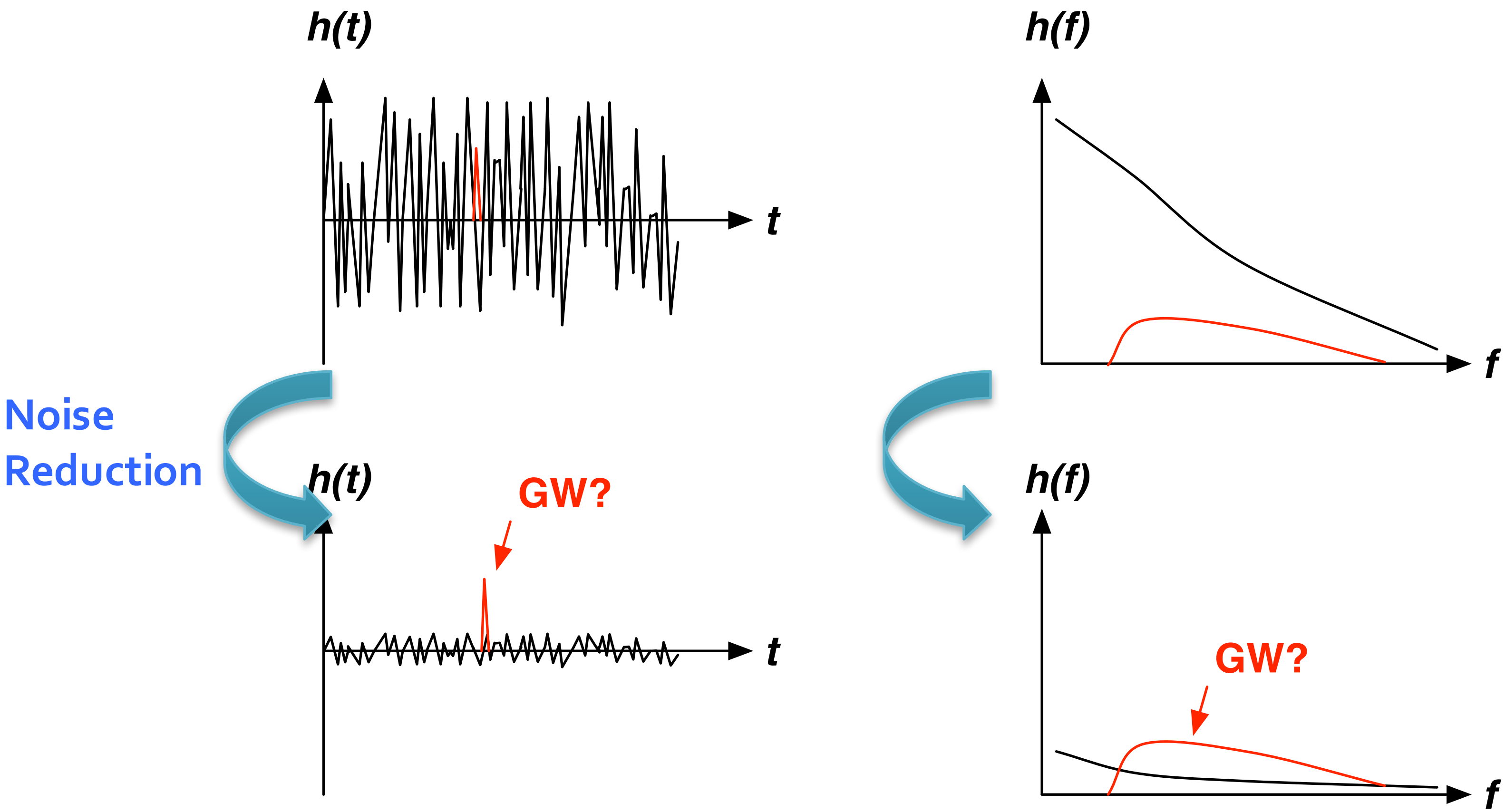
Noises
in Gravitational Wave Detectors

Introduction ~ Noise?

- **GW detection**
  Data stream of differential arm strain
- **Once recorded:**
  Signals and noises are indistinguishable
  What we can do is to catch "likely" features
- **Reduce any kind of noises!**

Introduction ~ Noise?

- Time domain vs frequency domain



- Time domain: transient noises
  Frequency domain: stationary noises

Introduction ~ Noise?

- **Power Spectral Density (PSD)**
  Double sided PSD (-Infinity < f < Infinity)

$$S_{\mathrm{DS}}(f) = \lim_{T\to\infty} \frac{1}{T} \left| \int_{-T/2}^{T/2} x(t) e^{-2\pi i f t} \mathrm{d}t \right|^2$$

- **Single sided PSD (0 <= f < Infinity)**

$$S_x(f) = 2S_{\mathrm{DS}}(f)$$ $[x_{unit}^2 / \mathrm{Hz}]$

- **Linearized PSD:**

$$G_x(f) = \sqrt{S_x(f)}$$ $[x_{unit} / \mathrm{sqrtHz}]$

Introduction ~ Noise?

- **Parseval's Theorem** for signal RMS and PSD

$$\overline{x^2(t)} = \int_0^\infty S_x(f)df$$
$$\equiv x^2_{\mathrm{RMS}}$$

Root Mean of *x*(*t*):

average signal power density (per sec)
(cf. variance, std deviation)

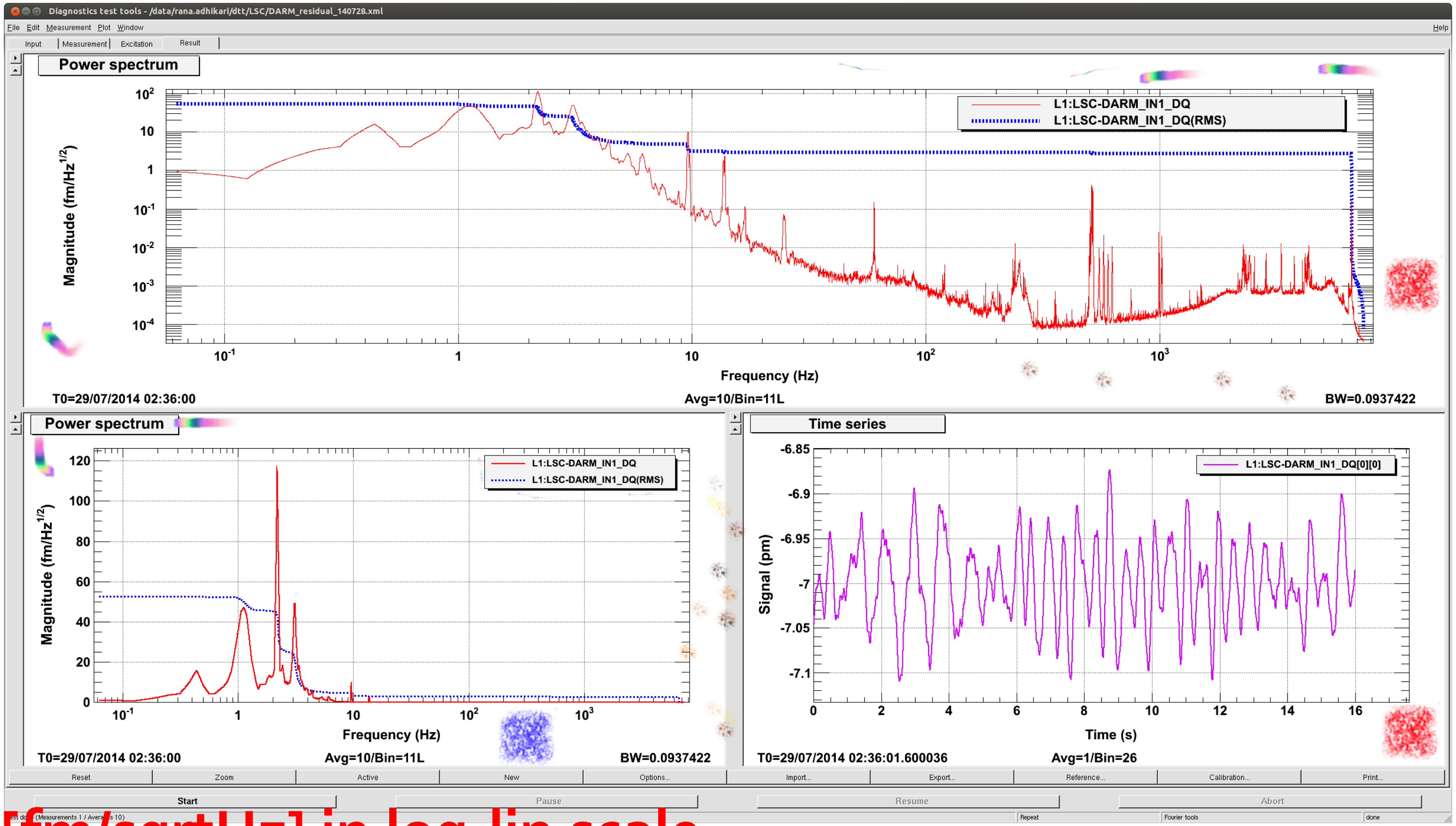
PSD *Sx*(*f*):

power density per frequency (per sec)

Introduction ~ Noise?

Example

PSD [fm/sqrtHz] in log-log scale, RMS [fm] ~ 50fm = 0.05pm



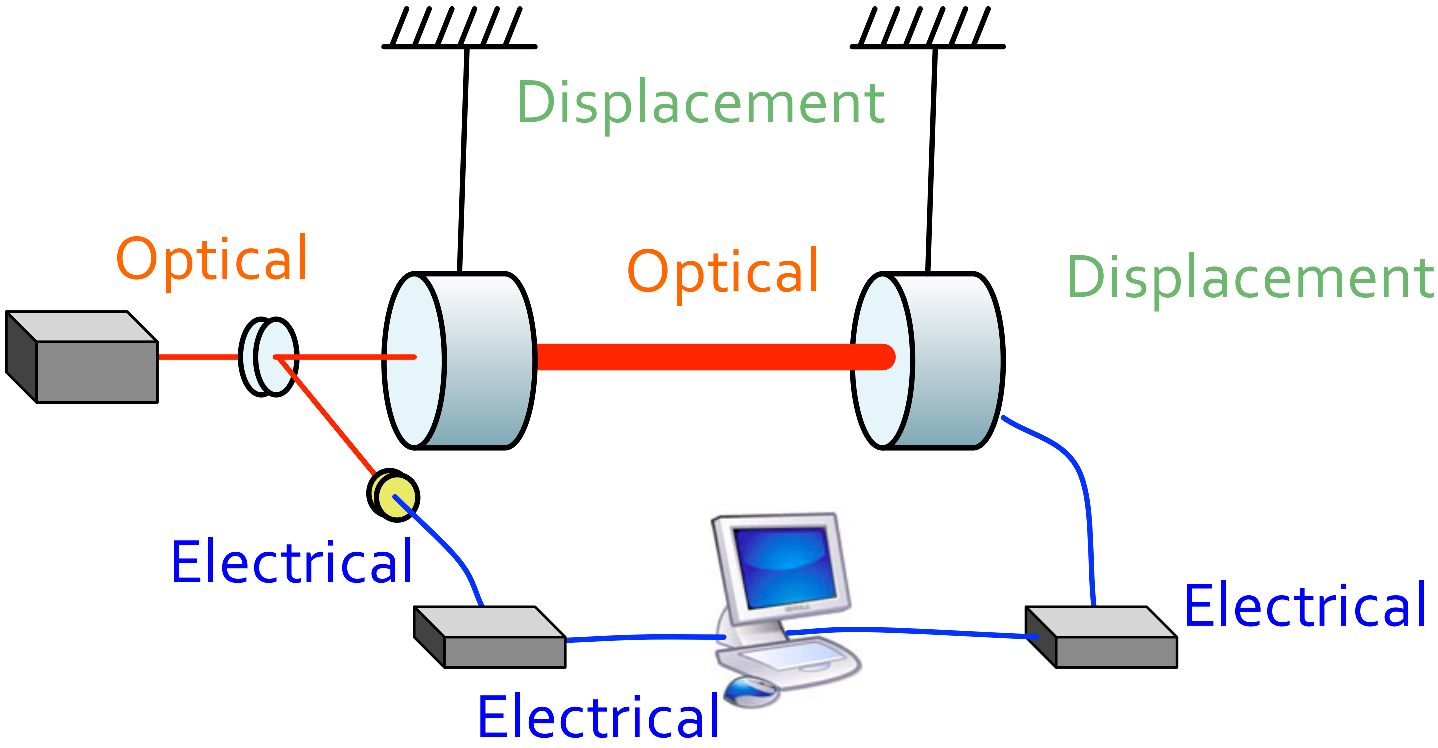
PSD [fm/sqrtHz] in log-lin scale

RMS [fm]

Time series [pm]

Noise categories

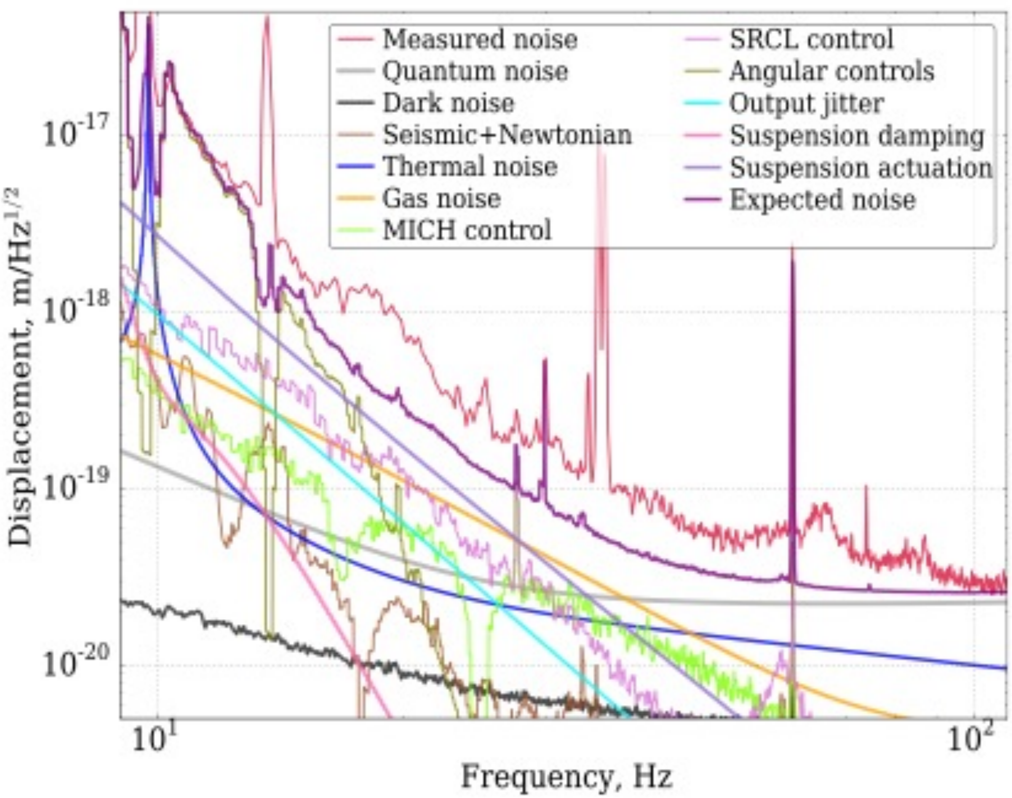
- 3 fundamentals of the GW detector
- Mechanics -> Displacement noises
- Optics -> Optical noises
- Electronics -> Electrical noises



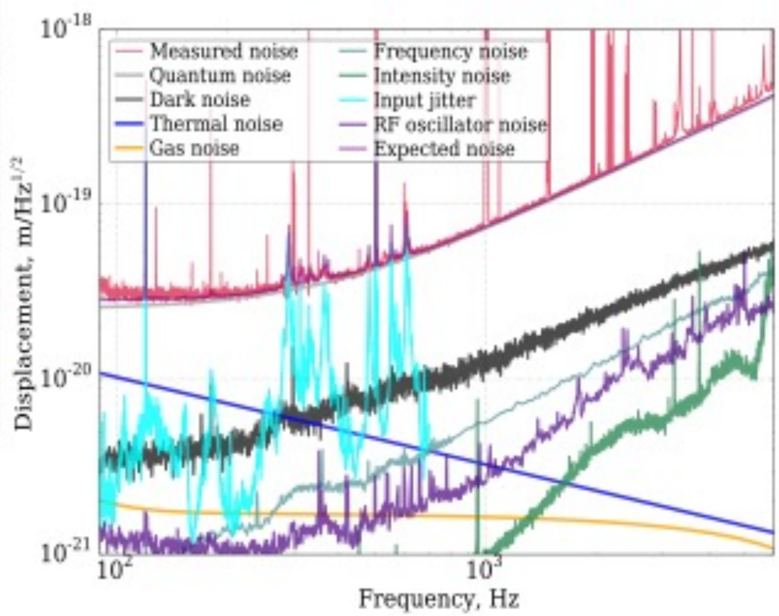
Sensitivity and noise

- Sensitivity (=noise level) of Advanced LIGO
- Noise budget

Laser shot noise
Laser radiation pressure noise
Thermal noise
Seismic noise
Laser intensity / frequency noise
Electronics noise
Digitization noise
Angular control noise
……



(a) LIGO Livingston Observatory

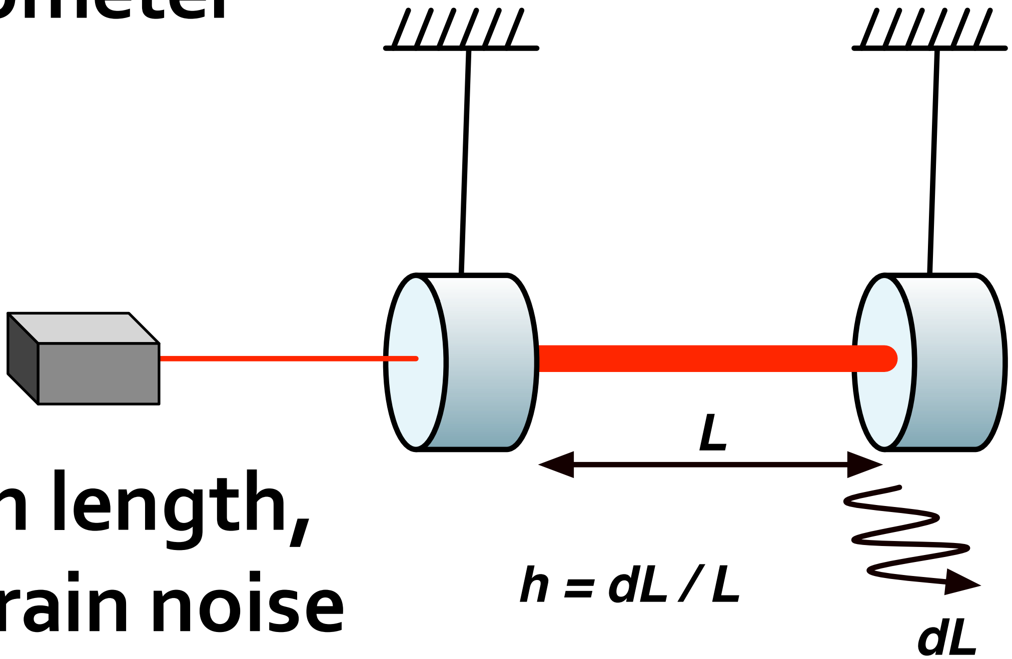


(b) LIGO Hanford Observatory

Displacement noises

Displacement noise

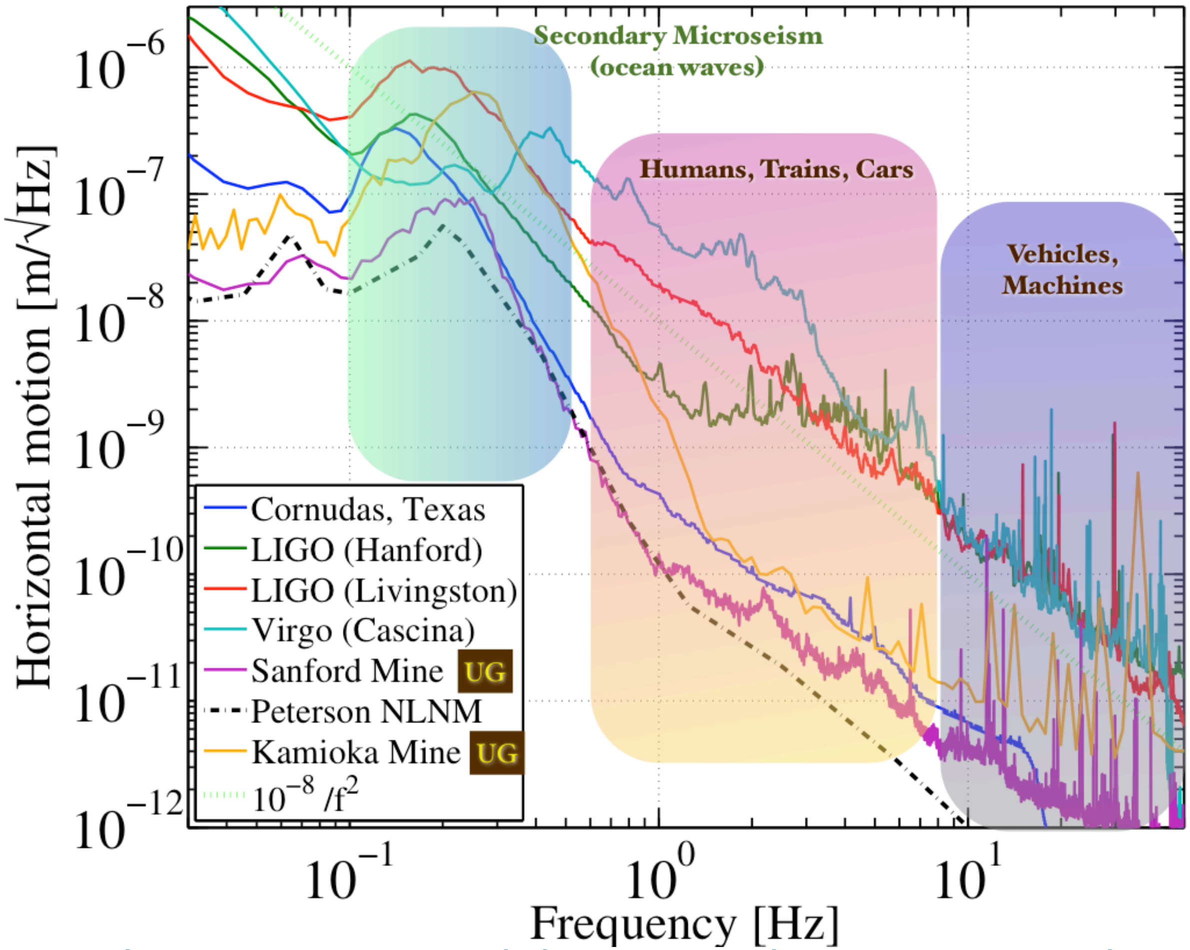
- **Mechanical displacement sensed by a laser interferometer**



- **The longer the arm length, the smaller the strain noise**
  - Seismic noise
  - Thermal noise
  - Newtonian Gravity noise

Displacement noise

- **Seismic noise**
  - Even when there is no noticeable earth quake…

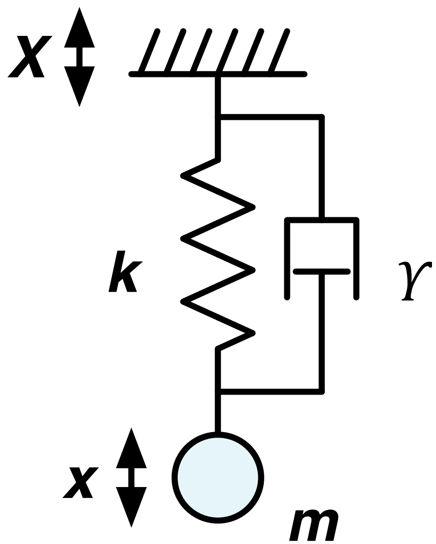


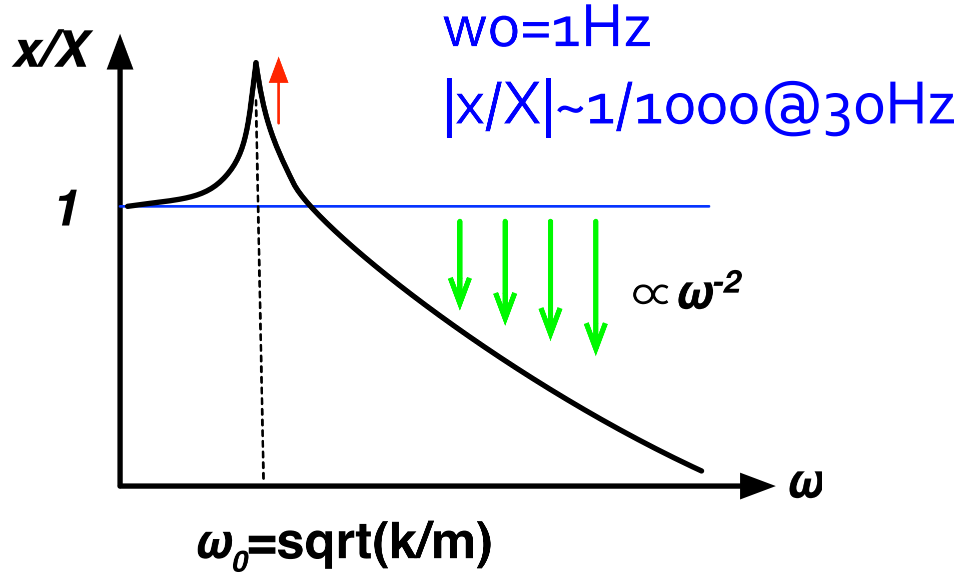
Target disp. noise 10^{-20}m/rtHz

http://link.aps.org/doi/10.1103/RevModPhys.86.121 (http://arxiv.org/abs/1305.5188)

Displacement noise

- Vibration isolation ~ utilize a harmonic oscillator
  - **A harmonic oscillator provides vibration isolation above its resonant frequency**





$$m\ddot{x} = -k(x - X) - \gamma(\dot{x} - \dot{X})$$

$$\left(\omega_0^2 + i\frac{\gamma}{m}\omega - \omega^2\right)\tilde{x} = \left(\omega_0^2 + i\frac{\gamma}{m}\omega\right)\tilde{X}$$

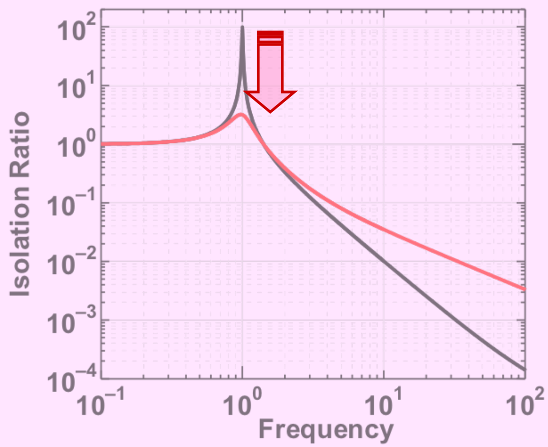
$$\frac{\tilde{x}}{\tilde{X}} = \frac{\omega_0^2 + i\frac{\gamma}{m}\omega}{\omega_0^2 + i\frac{\gamma}{m}\omega - \omega^2}$$

Displacement noise

- How to get more isolation?

Damping

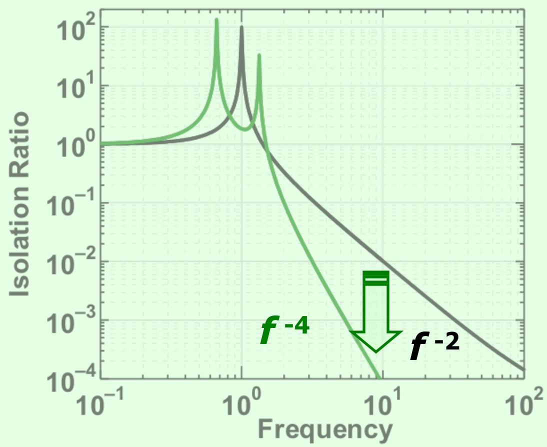
Lower the peak height



Worse isolation

Multi stage

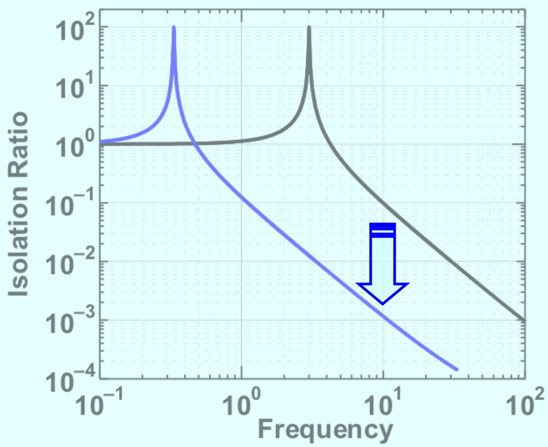
Steeper isolation curve



More peaks

Lower resonant freq

Better isolation

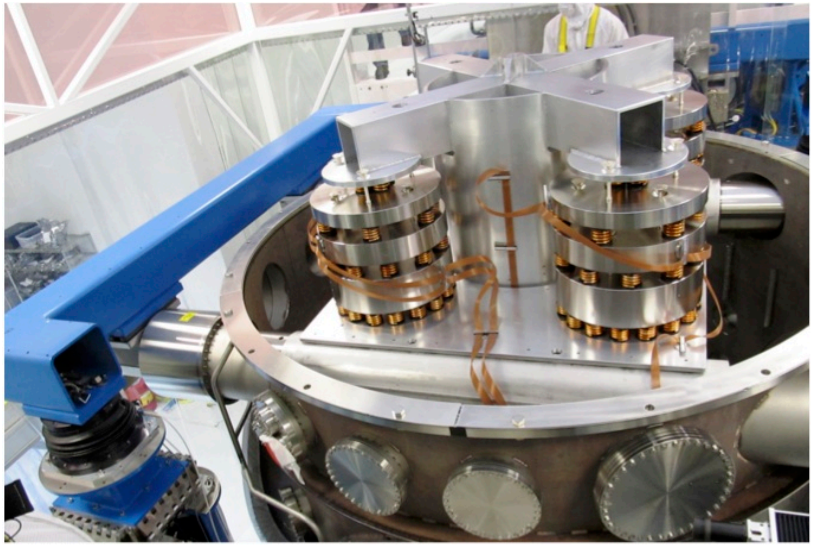


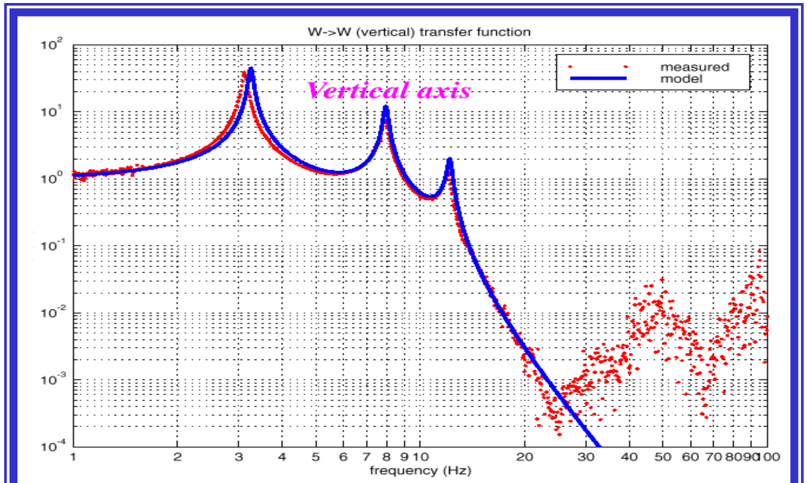
Complex to realize

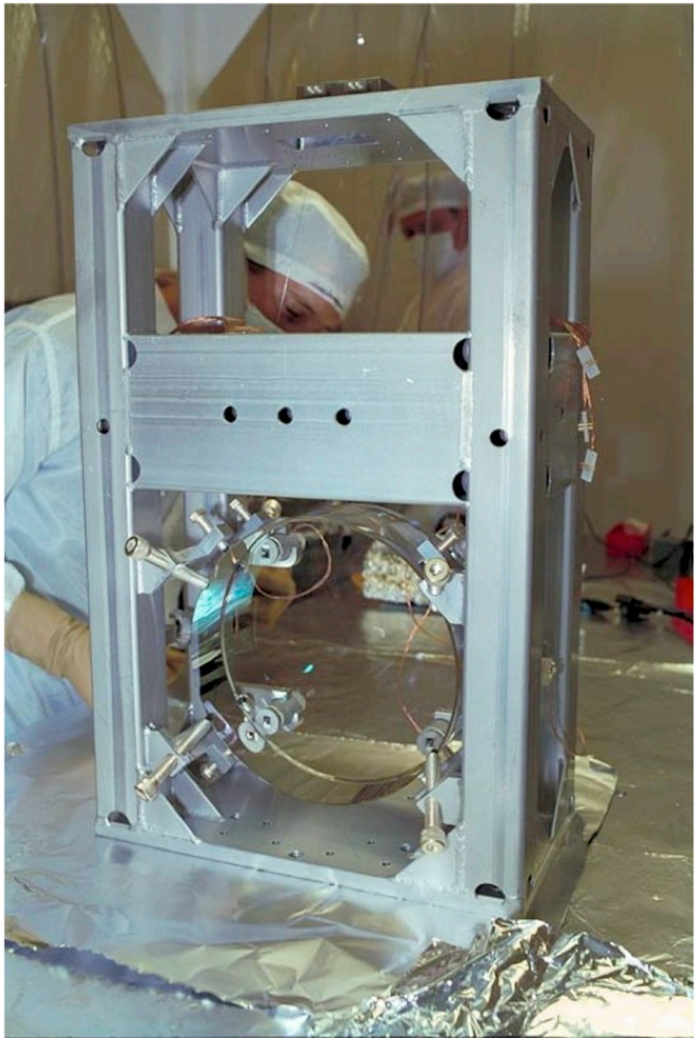
- In practice: employ combination of these measures

Displacement noise

- iLIGO vibration isolation
- Hydraulic active isolation / Isolation stack / Single Pendulum

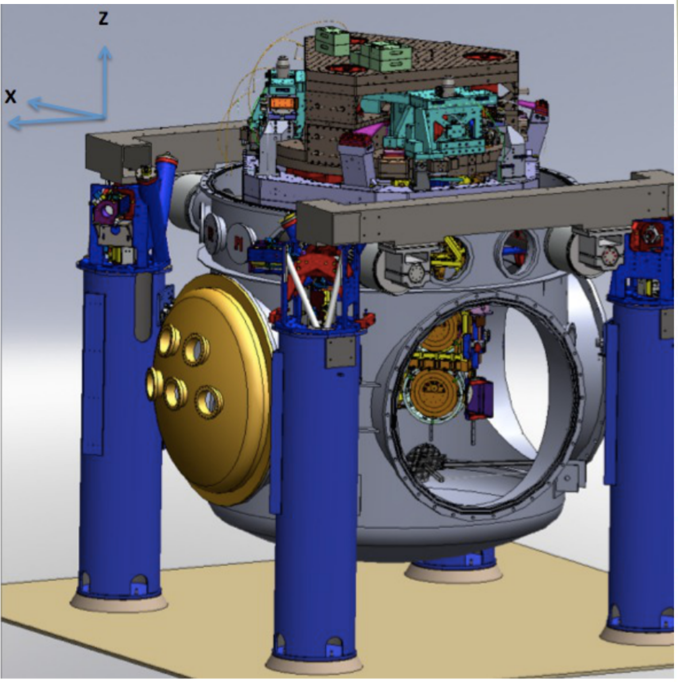


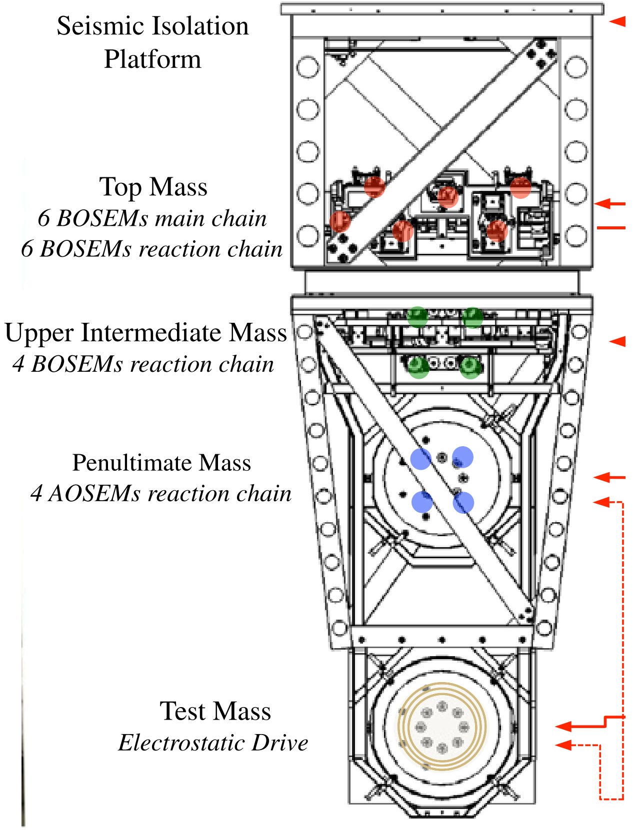




Displacement noise

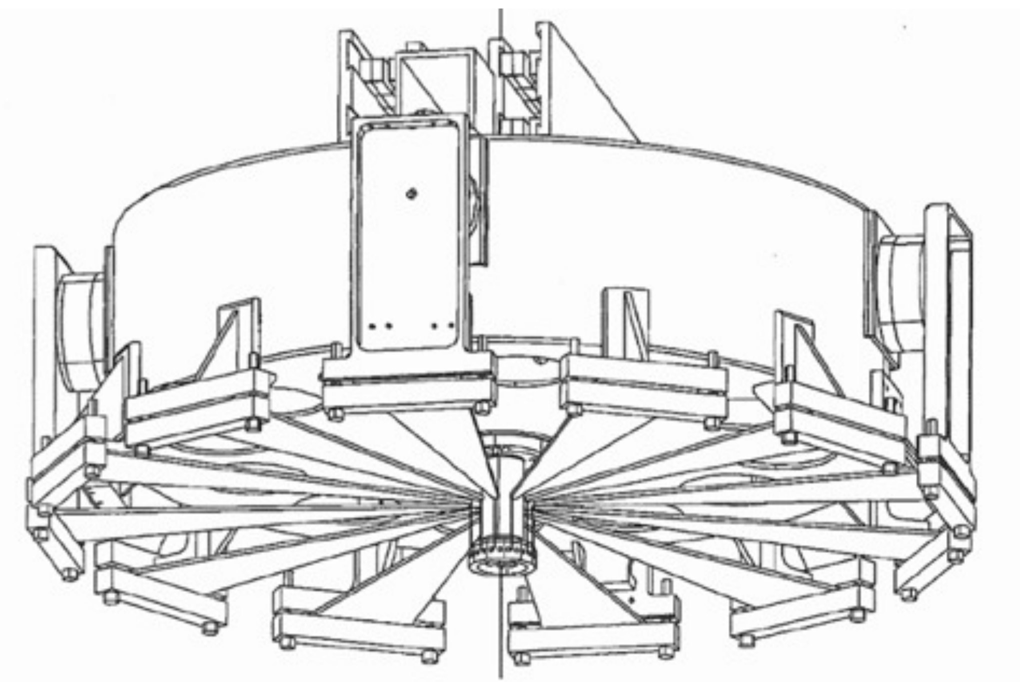
- aLIGO vibration isolation
- Hydraulic active isolation / Invacuum Active Isolation Platforms / Multiple Pendulum

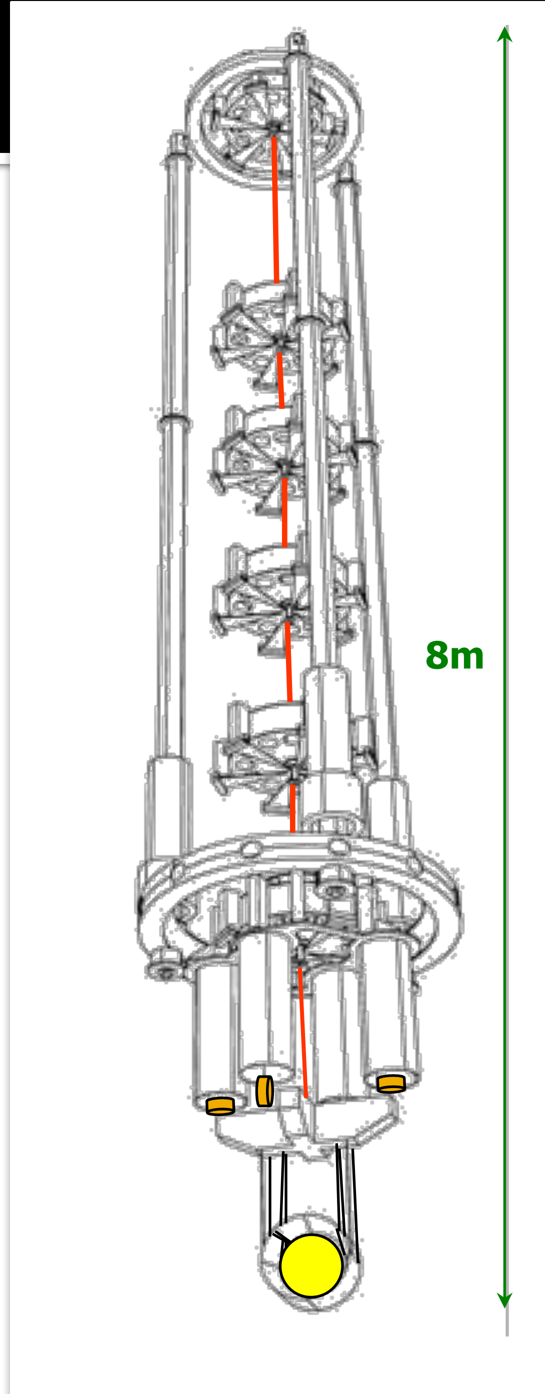




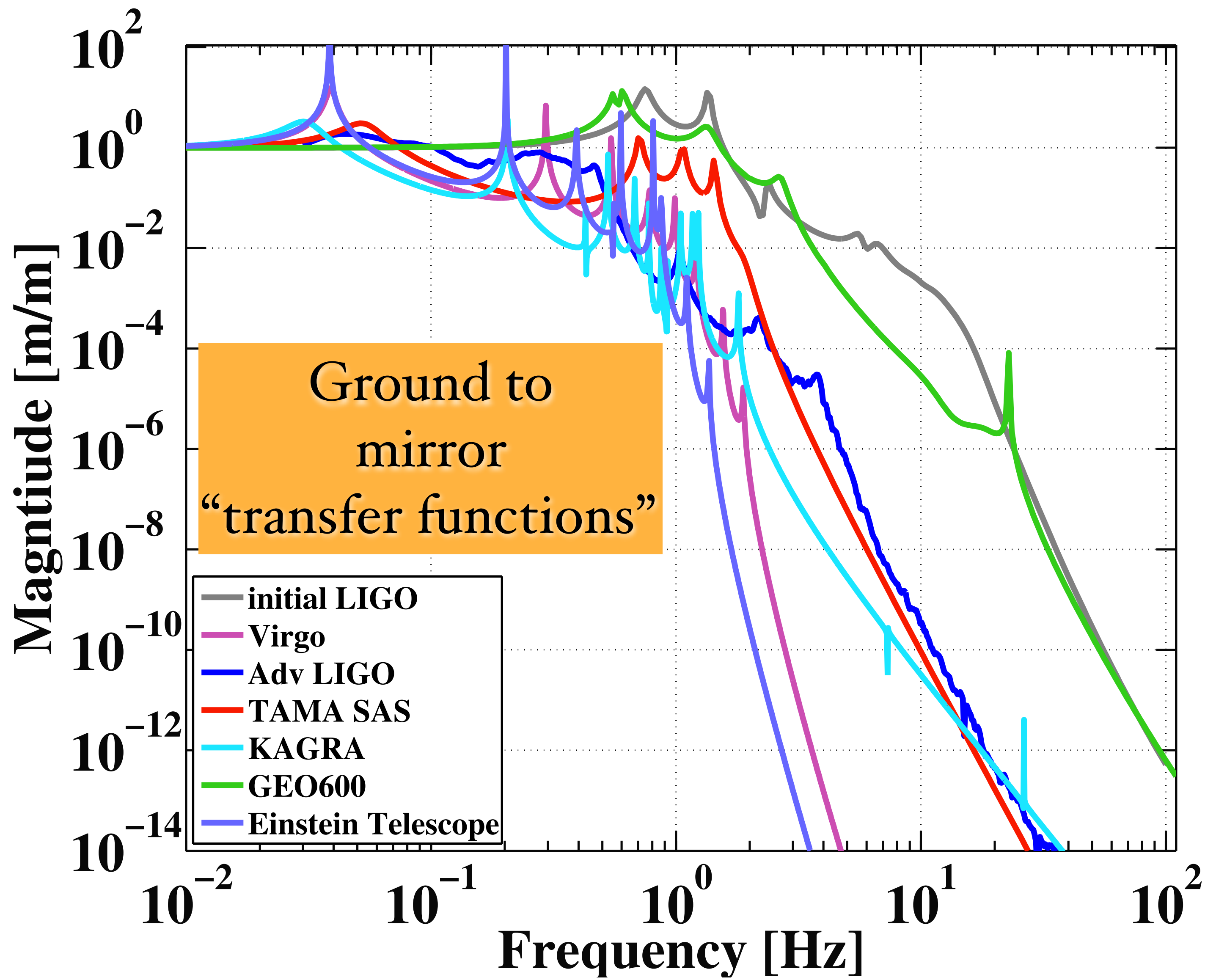
Displacement noise

- Virgo: super attenuator
  - 8m high
  - 9 stages in horizontal
  - 6 stages in vertical





Displacement noise

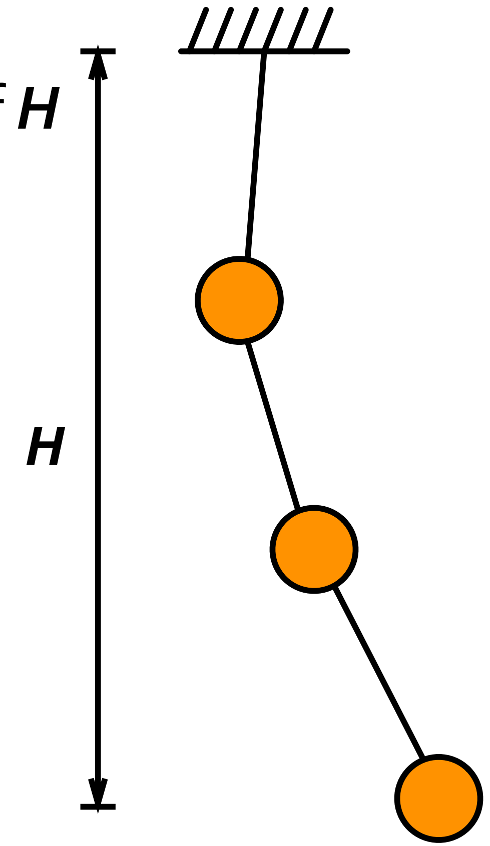


http://link.aps.org/doi/10.1103/RevModPhys.86.121 (http://arxiv.org/abs/1305.5188)

Displacement noise

Question:

- ***n***-stage multiple pendulum with fixed height of ***H***
- How many stages ***n*** do we need to realize the vibration isolation of ***A*** at frequency of ***f***?
- For a given ***A*** what is the minimum ***f***, we can realize by increasing ***n***?



(Mass distribution)

- For equal m for each stage
  or
- For arbitrary mass m_i and length h_i

Displacement noise

- Thermal noise:
- System in thermal equilibrium
  - the system can dissipate its energy to the heat bath
  - the system is thermally excited by thermal fluctuation

- Mechanical thermal noises
  - suspension thermal noise
  - mirror substrate thermal noise
  - mirror coating thermal noise

Displacement noise

- Fluctuation Dissipation Theorem
- Friction: interaction with “bath” = huge number of d.o.f.
- Fluctuation force: produced by huge number of d.o.f.

- Dissipation and fluctuation have certain relationship

  system description (Langevin equation)

$$m\ddot{q} + R\dot{q} = \mathcal{F} + F'(t)$$

q:	generalized coordinate	m: generalized mass
R:	friction (dissipation)	
F:	internal force (restoring force, etc)	
F’(t):	fluctuating force from heat bath	

Power spectrum density (PSD) of the fluctuation force

$$S_{F'}(\omega) = 4k_{\mathrm{B}}TR$$

Displacement noise

- **Transfer function approach**
- Equivalently, the fluctuation of the system can be obtained from the response of the system

$$S_q(\omega) = \frac{4k_B T \,\mathrm{Re}[1/Z(\omega)]}{\omega^2} = -\frac{4k_B T \,\mathrm{Im}[H(\omega)]}{\omega}$$

- where Z(w) and H(w) are the impedance and
- force-to-displacement transfer function of the system

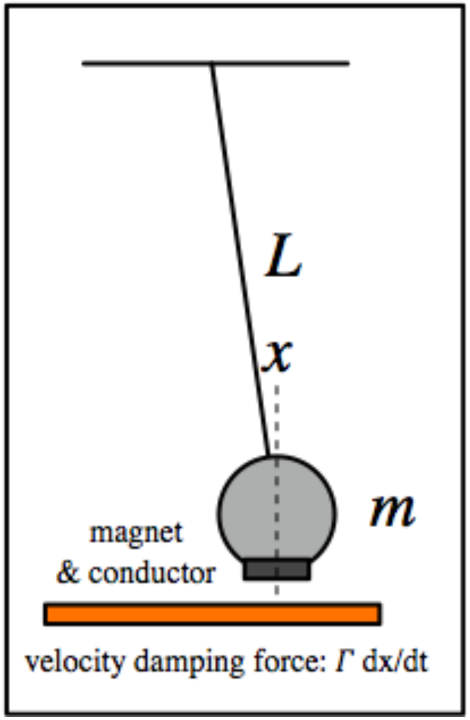
$$Z(\omega) = F(\omega)/\dot{q}(\omega), H(\omega) = q(\omega)/F(\omega)$$

Displacement noise

Question:

- Velocity damping of a pendulum

$$m\ddot{x} + \Gamma\dot{x} + m\omega_0^2 x = f$$



- Structure damping

loss angle: $0 < \phi \ll 1$

$$m\ddot{x} + m\omega_0^2(1 + \mathrm{i}\phi)x = f$$

- How anti-spring changes the thermal noise spectrum?

anti-spring parameter: $0<\alpha<1$

$$m\ddot{x} + m\omega_0^2(1 - \alpha + \mathrm{i}\phi)x = f$$

Displacement noise

- In some cases, calculating the system response is complicated (e.g. deformation of an elastic body)

- Systems response (impedance) at a certain freq:

$$Z(\omega) = F(\omega)/\dot{q}(\omega)$$

- Average rate of energy disspation

$$W_{\mathrm{diss}} = \langle \mathrm{Re}(F)\mathrm{Re}(\dot{q})\rangle$$

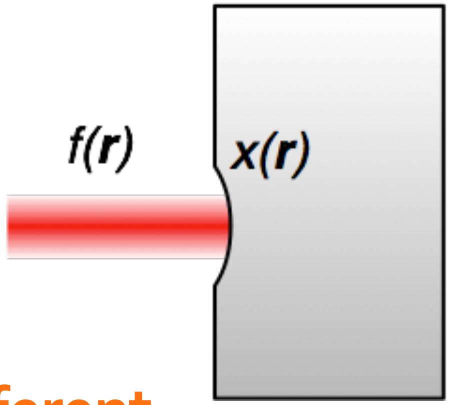
$$= \frac{1}{2}\mathrm{Re}[1/Z(\omega)]F_0^2 = \frac{1}{2}\frac{\mathrm{Re}[Z(\omega)]}{|Z(\omega)|^2}F_0^2$$

$$S_q(\omega) = \frac{4k_{\mathrm{B}}T}{\omega^2}\mathrm{Re}[1/Z(\omega)]$$

$$\boxed{S_q(\omega) = \frac{8k_{\mathrm{B}}TW_{\mathrm{diss}}}{F_0^2\omega^2}}$$

Displacement noise

- Sensing of the mirror surface deformation with a laser beam (with intensity profile of *f*(r))



- Apply periodic pressure with profile of *f*(r)

$$P(\mathbf{r}) = F_0 e^{i\omega t} f(\mathbf{r})$$

  This induces deformation of x(r) which is different from our sensing profile of *f*(r), but that's OK

- Calculate the rate of dissipation Wdiss

  analytically, using FEA, or etc

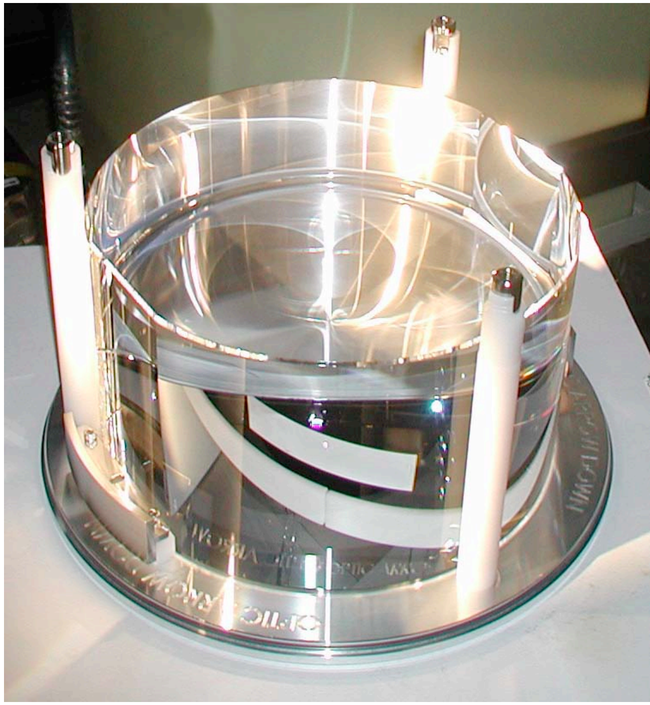
- Put this into the formula

$$S_x(\omega) = \frac{8 k_B T W_{\rm diss}}{F_0^2 \omega^2}$$

Y. Levin PRD **57**, 659-663 (1998)

Displacement noise

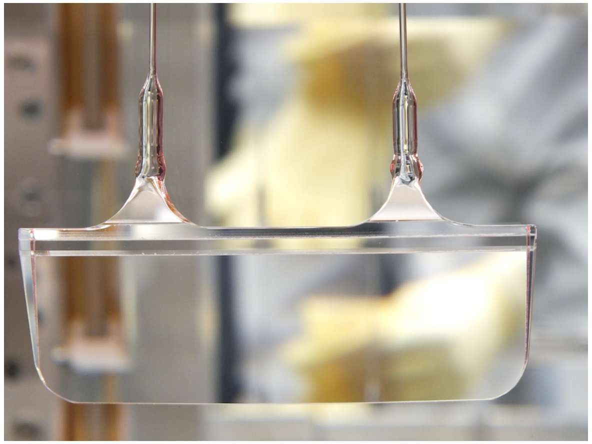
- **Mirror substrate thermal noise**
  - Brownian motion
    Mechanical loss associated
    with the internal friction
    ⇔Thermally excited body modes
    Optical coating (high mechanical loss)
    will be limiting noise source in aLIGO



  - Thermo elastic noise
    Elastic strain & thermal expansion coefficient
    => cause heat distribution & flow in the substrate
    ⇔Temperature fluctuation causes mirror displacement
  - Thermo-refractive noise
    ⇔Temp. fluctuation causes fluctuation of refractive index

Displacement noise

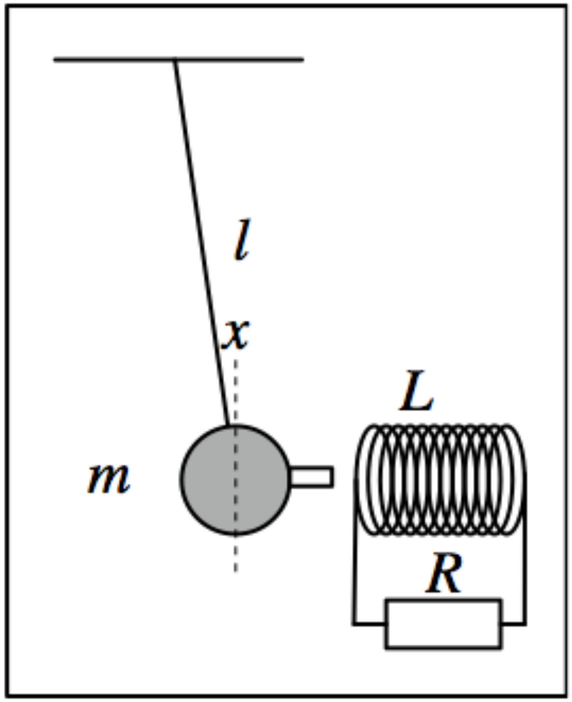
- **Suspension thermal noise**
  - Brownian motion
    Mechanical loss of the suspension fiber
    ⇔Thermally excited pendulum modes
  - Thermo elastic noise
    Elastic strain of the fiber & thermal expansion coefficient
    => cause heat distribution & flow in the fiber
    ⇔Temperature fluctuation causes mirror motion



<- Monolithic suspension for high pendulum Q

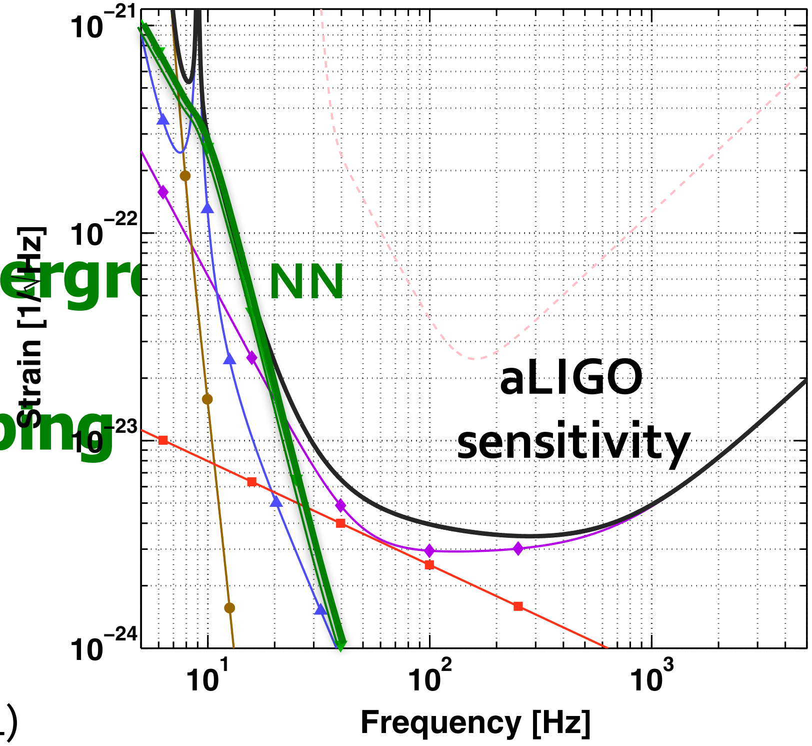
Displacement noise

- **Question**
  - **Induced current damping (electro-mechanical system)**
    1. How does the Q factor of the system depend on R?
    2. How much is the thermal noise displacement of the mass?
    3. How does the thermal noise of the resister shakes the mass?
    4. How are the above questions with a capacitive coupling instead of the coil?
  - **Cold damping**
    1. If the resister is cooled, how does the thermal noise motion change?
    2. Is the pendulum actually cooled? Down to what temperature?
    3. How fast the pendulum recovers the original temperature once R is returned to the room temp.?



Displacement noise

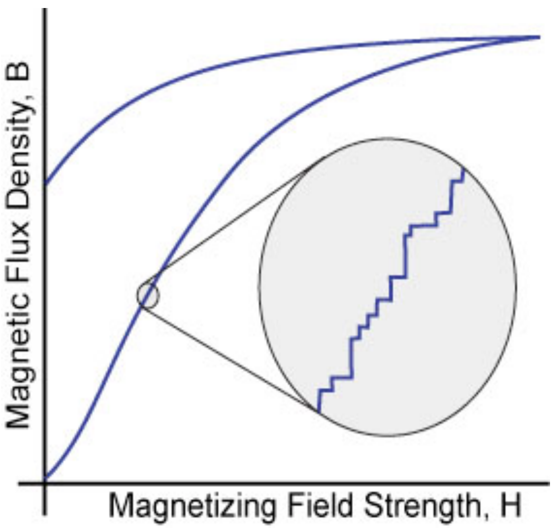
- **Newtonian Gravity noise**
  - **Mass density fluctuations around the test masses**
    => test mass motion via gravitational coupling
  - **Dominant source of Newtonian noise**
    = Seismic surface wave
  - **Mitigation**
    1) Going to quiet place (undergr
    2) Feedforward subtraction
    3) Passive reduction by shaping local topography

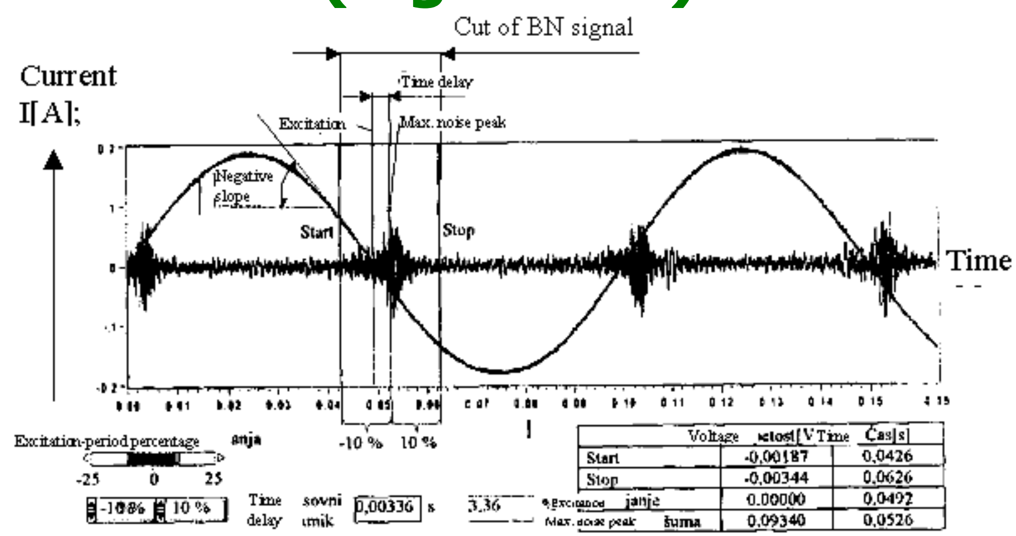


J Driggers, et al, PRD 86, 102001 (2012)
J Harms, et al, Class. Quantum Grav. 31 185011 (2014)

Displacement noise

- Mechanical upconversion noise
  - Large low frequency (f < 1Hz) motion
    => upconverted to 10~100Hz motion via nonliner processes
  - Barkhausen noise
    => low freq mirror actuation cause BH noise and upconversion
    Select better magnet materials (e.g. SmCo)



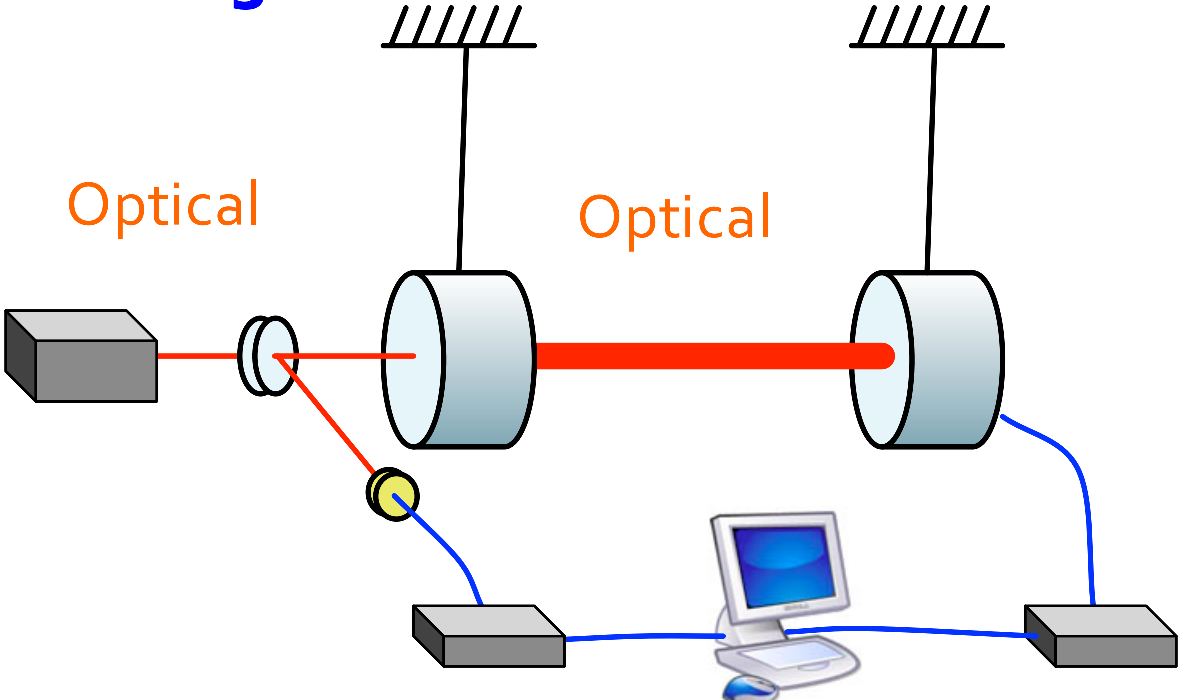


https://www.nde-ed.org/

Optical noises

Optical noises

- **Noises that contaminate the readout signal**
  - **Quantum noises (shot noise, radiation pressure noise)**
  - **Laser technical noises (frequency/intensity noise)**
  - **Modulation noises**
  - **Scattered light noise**



Optical noises

- **Quantum noises: Shot noise**
  - **Noise due to photon counting statistics**
  - **N detected photon => standard deviation $\sqrt{N}$**
  - **Increasing the incident power P_{in},**
    => The shot noise is increased by $\sqrt{P_{in}}$
    => The signal amplitude is increased by P_{in}
  - **In total, the signal-to-noise ratio is improved by**

$$\mathrm{SNR} \propto \sqrt{P_{\mathrm{in}}}$$

Optical noises

- **Quantum noises: Shot noise**
  - **Photon shot noise associated with photodetection**

$$i_{\rm shot} = \sqrt{2 e i_{\rm DC}} \;\; [{\rm A}/\sqrt{\rm Hz}]$$

  - **Michelson interferometer**

$$i_{\rm DC} = \frac{e \eta P_{\rm in}}{h\nu} \frac{1 - \cos\delta\phi}{2} \;\; [{\rm A}]$$

i_{DC} : DC Photocurrent
η : PD Quantum Efficiency
ν : Optical Frequency

at the limit of $d\phi$->0

$$i_{\rm shot} / \frac{d i_{\rm DC}}{d\phi} = \sqrt{\frac{2h\nu}{\eta P_{\rm in}}} \;\; [{\rm rad}/\sqrt{\rm Hz}]$$

Shot-noise limit of the Michelson phase sensitivity

  - **Michelson response (@DC)**

$$\frac{\delta\phi}{h_{\rm GW}} = \frac{4\pi L \nu}{c} \;\; [{\rm rad/strain}]$$

Michelson Strain Sensitivity

1.3x10^{-20} 1/sqrtHz @1W, 1064nm, 4km

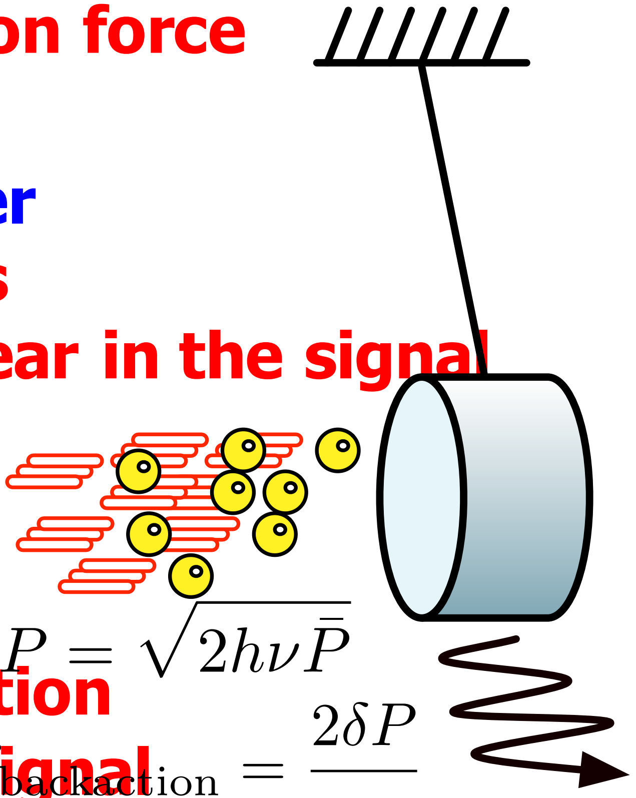
Optical noises

Supplemental slide ~ Shot noise derivation

- Take an average of Current $I(t)$ for a period of T, and sample it every T.
- Number of photons in this period T is $N = \bar{I}T/e$.
- Fluctuation of photon number in T is $\sigma_N = \sqrt{N}$. cf Poisson statistics
- Thus, the standard deviation (RMS) of $\bar{I}$ is $\sigma_I = e\sqrt{N}/T = \sqrt{e\bar{I}/T}$
- Think about the transfer function of this box car average filter. It is $H(f) = \mathrm{sinc}(\pi f T)$
- Parsevals theorem: $\sigma_I = \int_0^\infty H(f)^2 i_s^2 df$, where i_s is the linear power spectrum density of the current (white spectrum).
- According to the above integration, $i_s = \sigma_I \sqrt{2T}$.
- Therefore we obtain $i_s = \sqrt{2e\bar{I}}$.

Optical noises

- **Quantum noises ~ Radiation pressure noise**
  - **Photon number fluctuation in the arm cavity**
    => Fluctuation of the back action force
  - **Quantum noise of the input laser**
    => Common noise for two arms
    => cancelled and does not appear in the signal
  - **Vacuum fluctuation injected from the dark port**
    => Differentially power fluctuation
    => Cause the noise in the GW signal



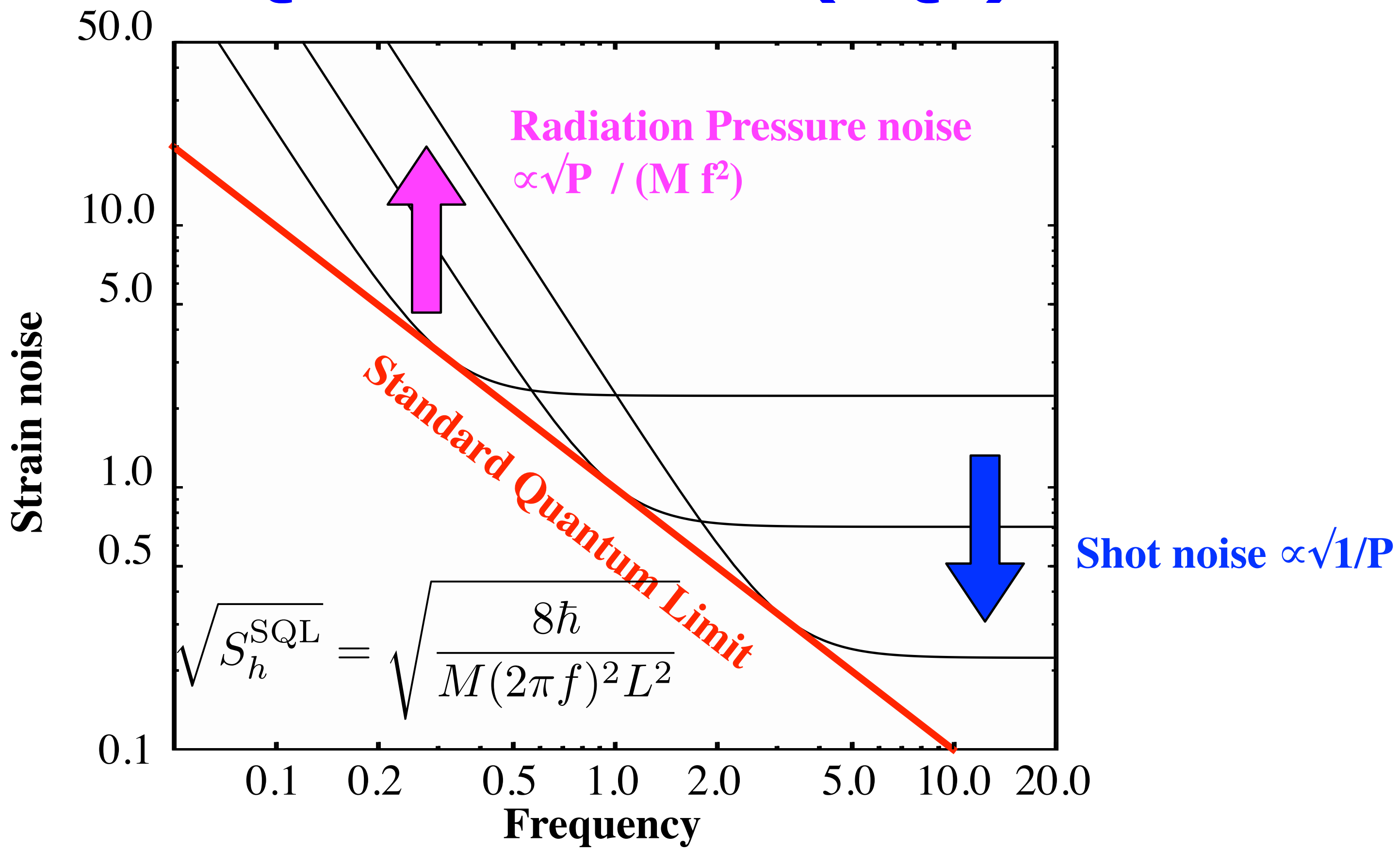
$$\delta P = \sqrt{2h\nu\bar{P}}$$

$$f_{\mathrm{backaction}} = \frac{2\delta P}{c}$$

$$\tilde{x} = \frac{f_{\mathrm{backaction}}}{M\omega^2}$$

Optical noises

- **Quantum noises**
  - **Standard Quantum Limit (SQL)**



- Trade-off Between Shot Noise and Radiation-Pressure Noise
- Uncertainty of the test mass position due to observation

Optical noises

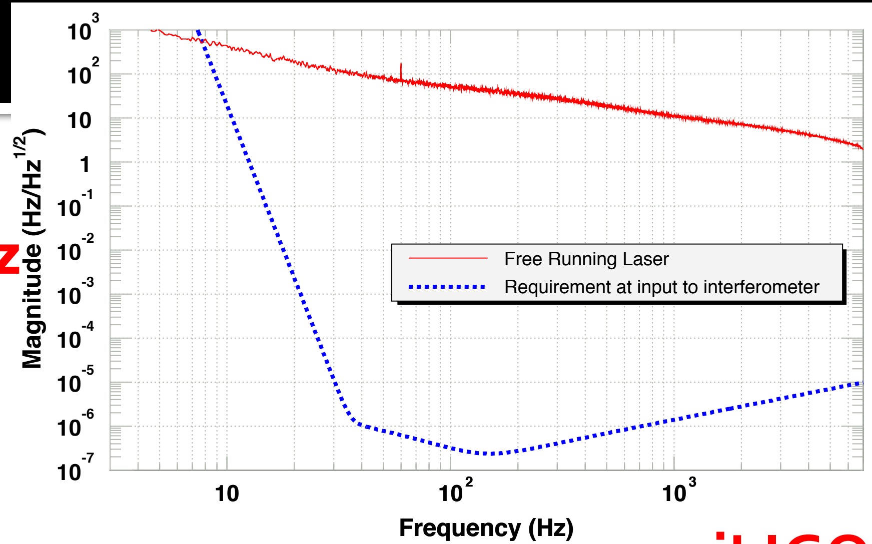
- **Laser frequency noise**
  - **Laser wavelength (λ = c / v)**
    = reference for the displacement measurement
  - **Optical phase ϕ = 2 pi v L / c**
    dϕ = 2 pi / c (L dv + v dL) <= indistinguishable

$$\frac{dL}{L} = \frac{d\nu}{\nu}$$

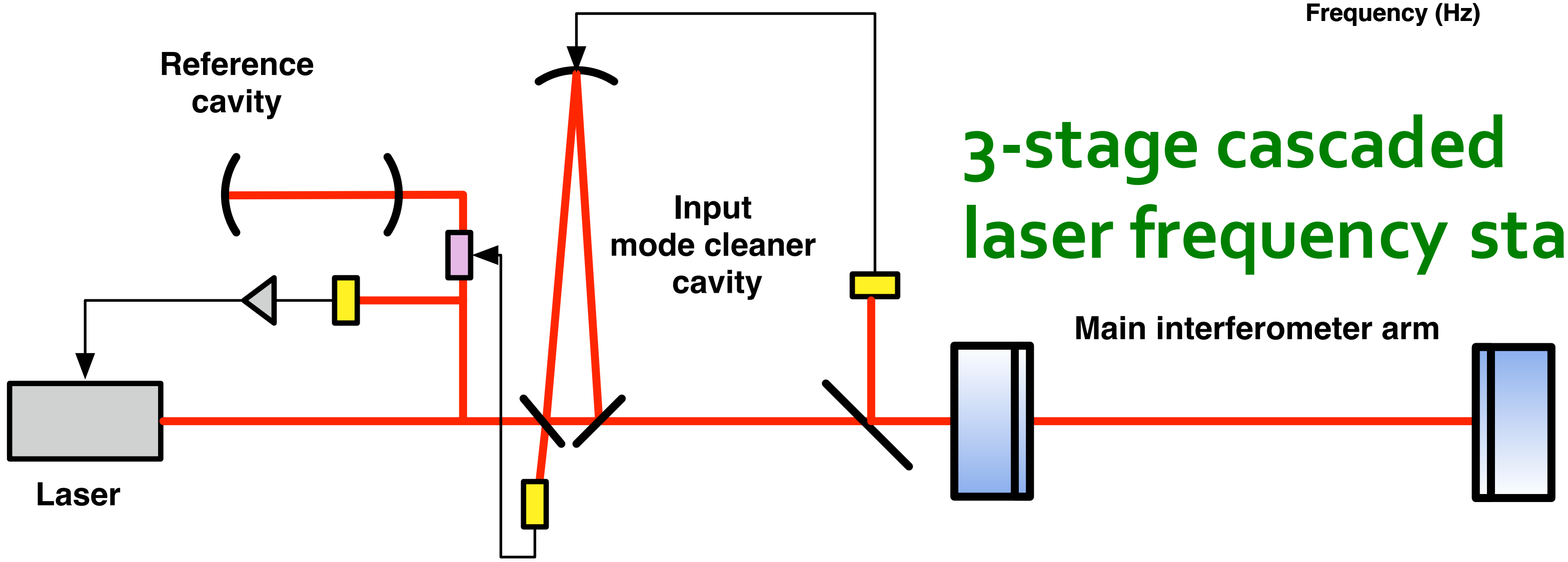
  - **dL/L target 10^{-24}**
    => dv = 10^{-24} x 300 THz **(1064nmYAG laser)**
    = 3 x 10^{-10} Hz/rtHz

Optical noises

- **Laser frequency noise**
  - **Target: dv_{eff} = 3 x 10^{-10} Hz/rtHz**
  - **Laser stability**
    dv = 10~100 Hz/rtHz @100Hz



iLIGO



3-stage cascaded
laser frequency stabilizatio

Michelson's differential sensitivity provides
Frequency noise cancellation of 1/100~1/1000
"Common Mode Rejection"

Optical noises

- **Laser intensity noise**
  - **Relative Intensity Noise (RIN): dP/P**
  - **Sensor output V = P x**
    => dV = P dx + x dP <= indistinguishable

$$\frac{dx}{x_{\mathrm{offset}}} = \frac{dP}{P}$$

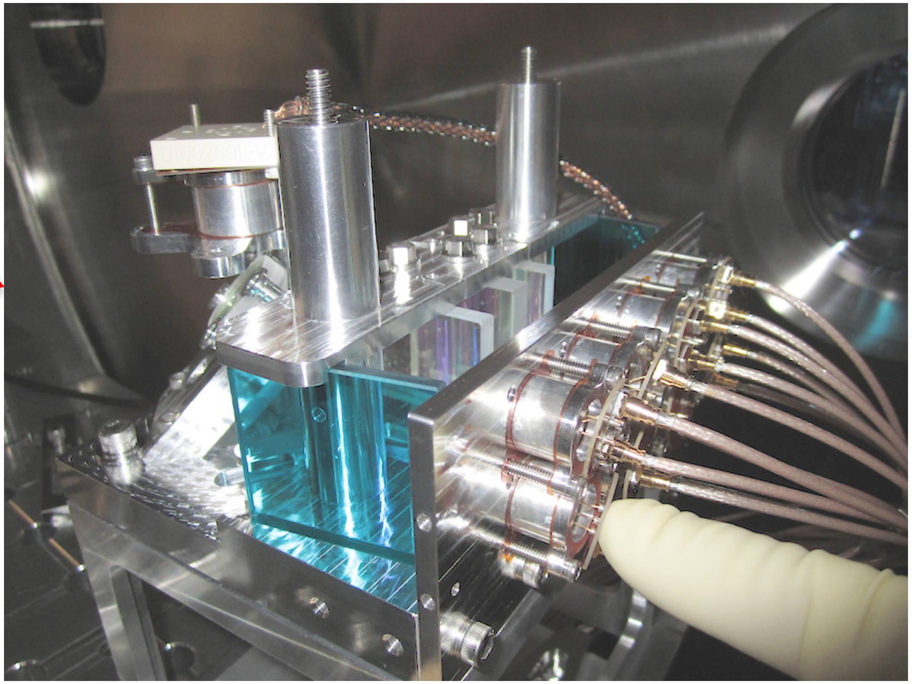
  - **Requirement: RIN = 10^{-9} 1/√Hz**
    x_{ofs}=10e-12 (DC Readout)
    => dx=1e-20 m/√Hz

Optical noises

- **Laser intensity noise ~ intensity stabiliaztion**
  - **Requirement: RIN = 10^{-9} $1/\sqrt{\text{Hz}}$**
  - **2-stage cascaded intensity stabilization control**
  - **Challenge: requires 300mA of photodetection**

Shot noise limited RIN $$\frac{i_\text{shot}}{i_\text{DC}} = \frac{\sqrt{2ei_\text{DC}}}{i_\text{DC}} = \sqrt{2e/i_\text{DC}}$$

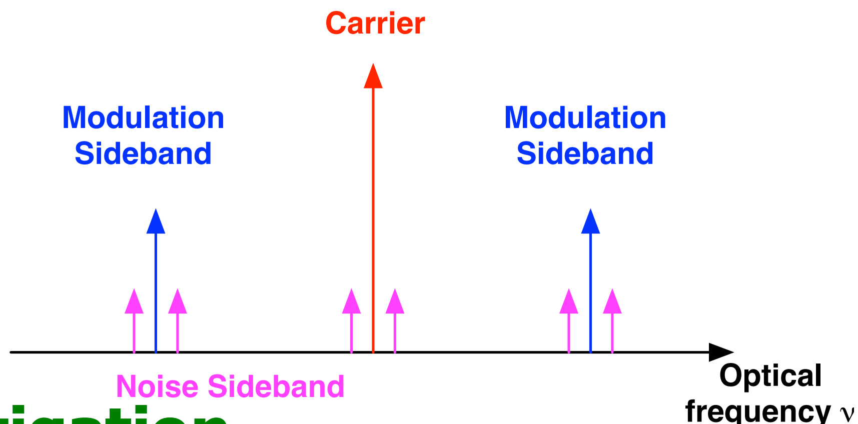
  - **In-vacuum 8-branch Photodiode array**

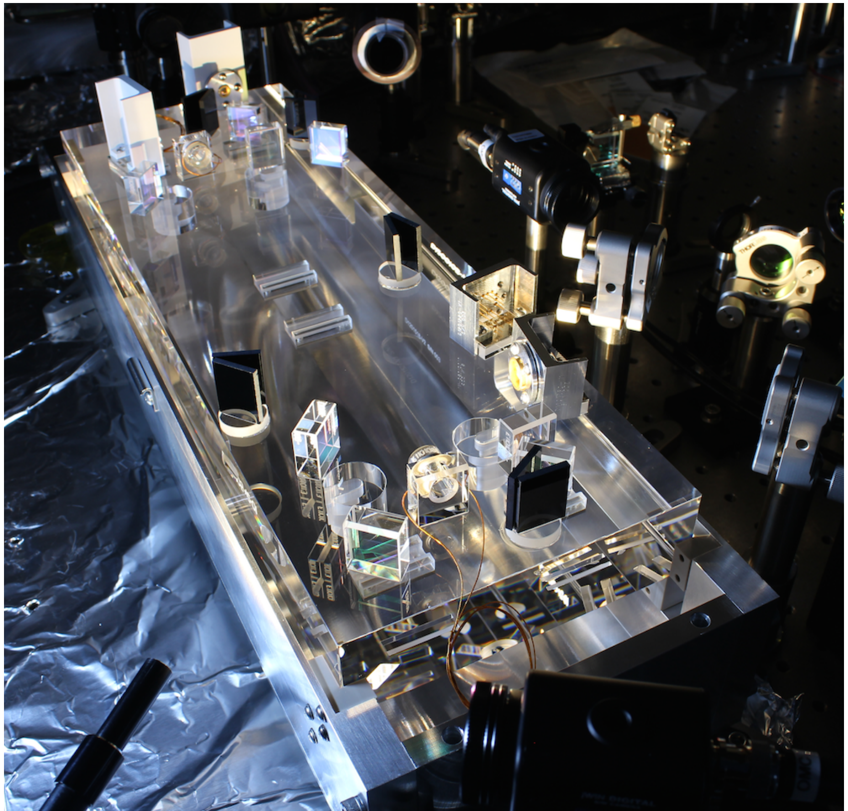


P. Kwee et al,
Optics Express **20** 10617-10634 (2012)

Optical noises

- **Modulation noises**
  - **RF Residual Amplitude Modulation**
  - **Modulation Oscillator Phase Noise**
  - **Modulation Oscillator Amplitude Noise**
- **Produce noise sidebands on the modulation sidebands**



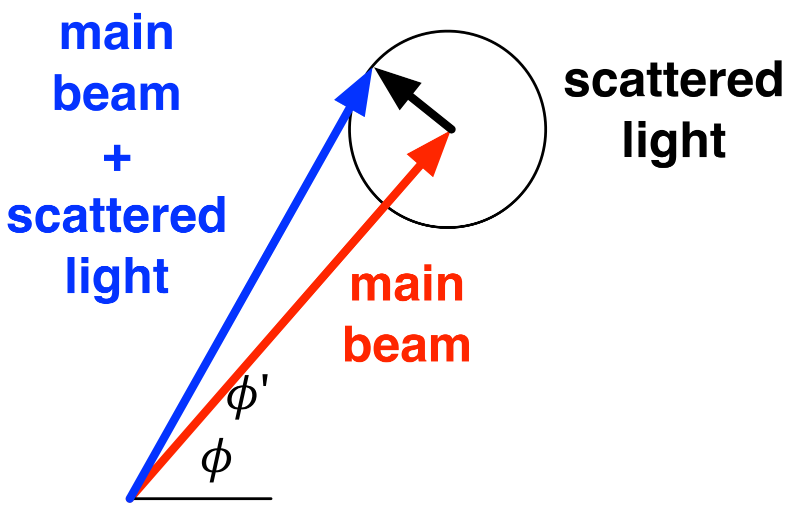


- **Mitigation**
  - For the GW signal:
    Use DC readout and eliminate them by an "output mode cleaner cavity"

J. Camp et al JOSA A **17** 120-128 (2000)

Optical noises

- **Scattered light noise**
  - **Scattered light recouples to the interferometer beam with an arbitrary phase**
    => causes amplitude and phase fluctuation
  - **Two effects:**
    1. Small motion regime: linear coupling of the phase fluctuation
    2. Large motion regime: low freq large motion of the scattering object => upconversion via fringe wrapping
  - **Mitigation**
    - **Reduce scattered light**
    - **Vibration isolation of the scattering object**



Electrical noises

Electrical noises

- **General rules for electrical noises**
- Electrical noise in photo detection
- Digitization noise (ADC/DAC) / Aliasing
- Control noise
- Actuator noise

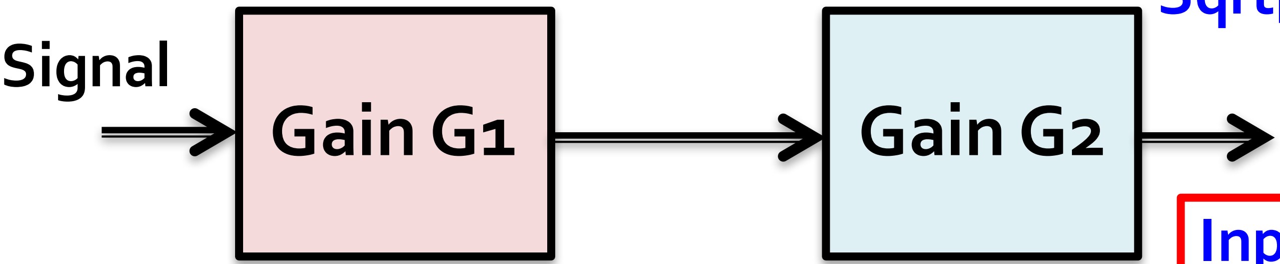
General rules for electrical noises

- **Low noise amplification at the beginning**
- **Give necessary gain as early as possible**
- **Don't attenuate (and amplify again)**

input noise: v_1

input noise: v_2

output noise Vout: $Sqrt[(v_1 G_1 G_2)^2+(V_2 G_2)^2]$



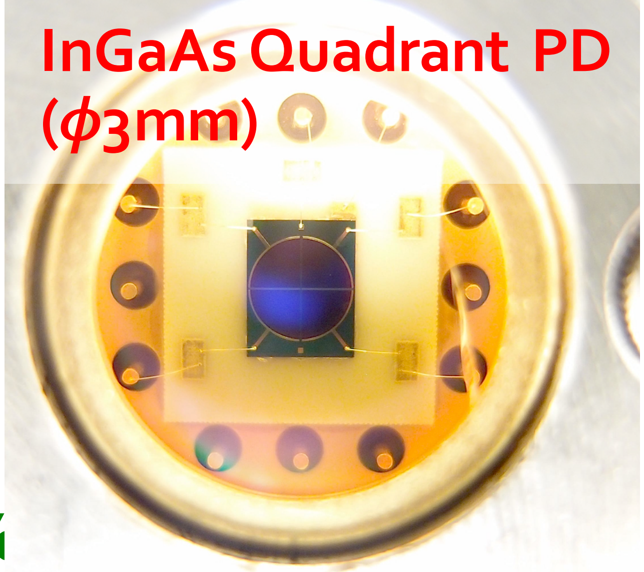
output noise: $v_1 G_1$

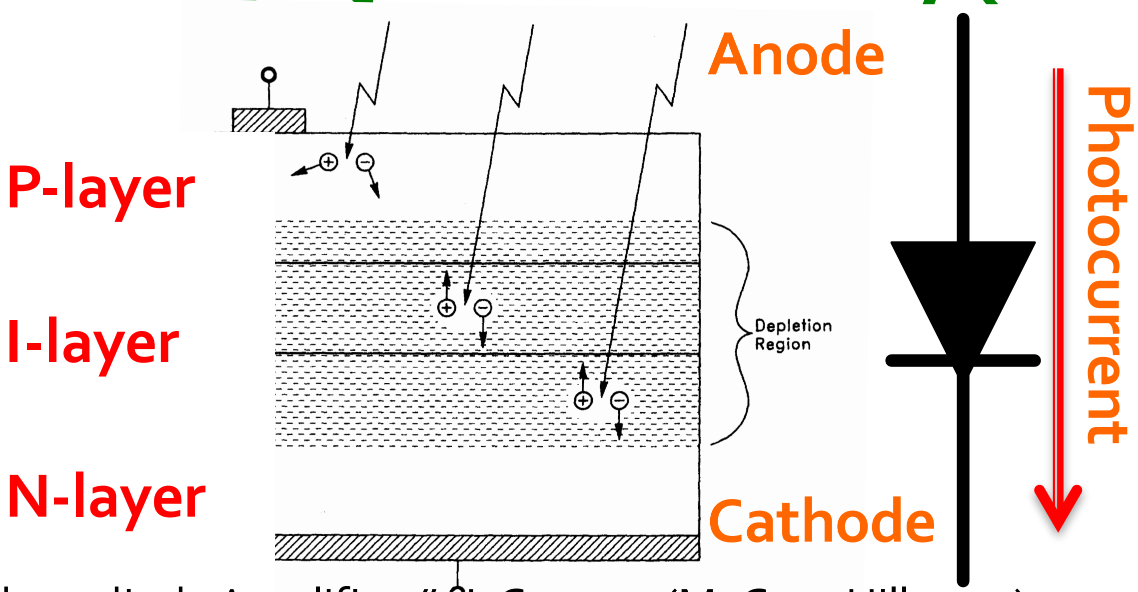
Input equivalent noise
Vout / (G_1 G_2)
$= Sqrt[v_1^2+(v_2/G_1)^2]$

- Lessons
  - The input referred noise is determined by v_1
  - It won't become better by the later stages
  - If G_1 is big enough, we can ignore the noise of later stages

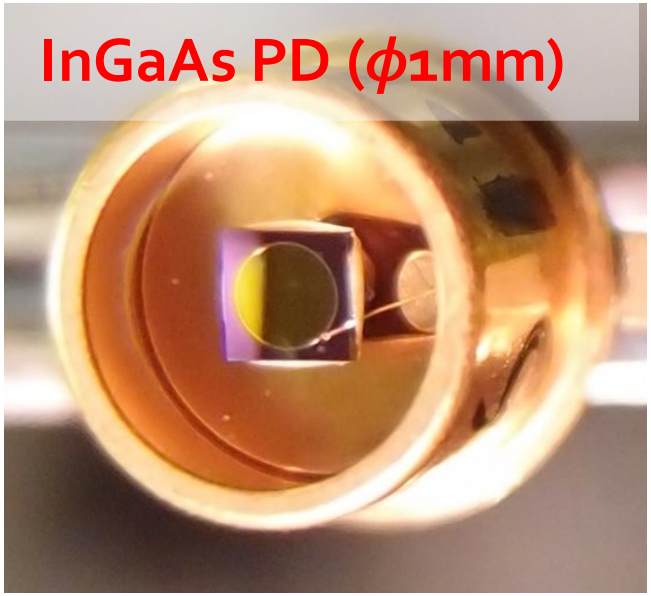
Noise in photodetectors

- Photodiodes
  - PIN photodiodes (InGaAs for near IR, Si for visible)
    - Good linearity
    - Low noise
    - High Quantum Efficiency (>90%





"Photodiode Amplifiers", J. Graeme (McGrawHill 1995)



Noise in photodetectors

- Photodetectors are the first electrical block of the control chains
  - **It is important to have low input-referred current noise**

- Photo detection
  - AF (Audio Frequency 0~100kHz)
    - **Prenty of light (photocurrent ~mA)**
      Not a big electrical issue
  - RF (Radio Frequency 10~200MHz)
    - **Large diode aperture -> high RF noise**
      Need careful consideration

Noise in photodetectors

- Noise in photodiodes
  - Photodiode equivalent circuit
    - Shunt Capacitance R_D (~100MΩ) **Usually not a problem**
    - Junction Capacitance C_D (1pF~1nF)
    - Series Resistance R_S (1Ω~100Ω)

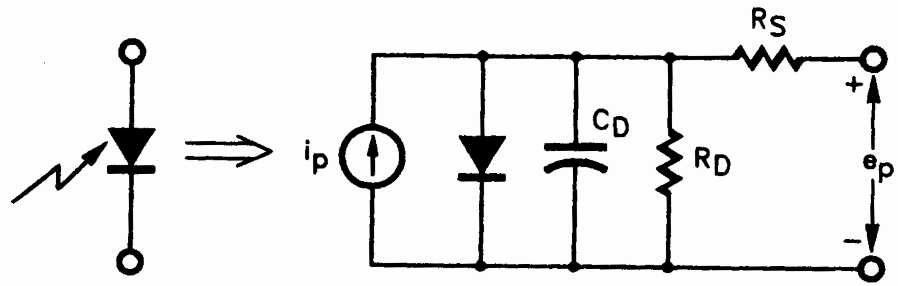


Figure 1.3 The circuit model of a photodiode consists of a signal current, an ideal diode, a junction capacitance, and parasitic series and shunt resistances.

input referred noise current

$$i_{Rs} \sim \omega C_d \sqrt{4 k_{\mathrm{B}} T R_s}$$

The diode aperture size needs to be ~mm => Cd tends to be big.

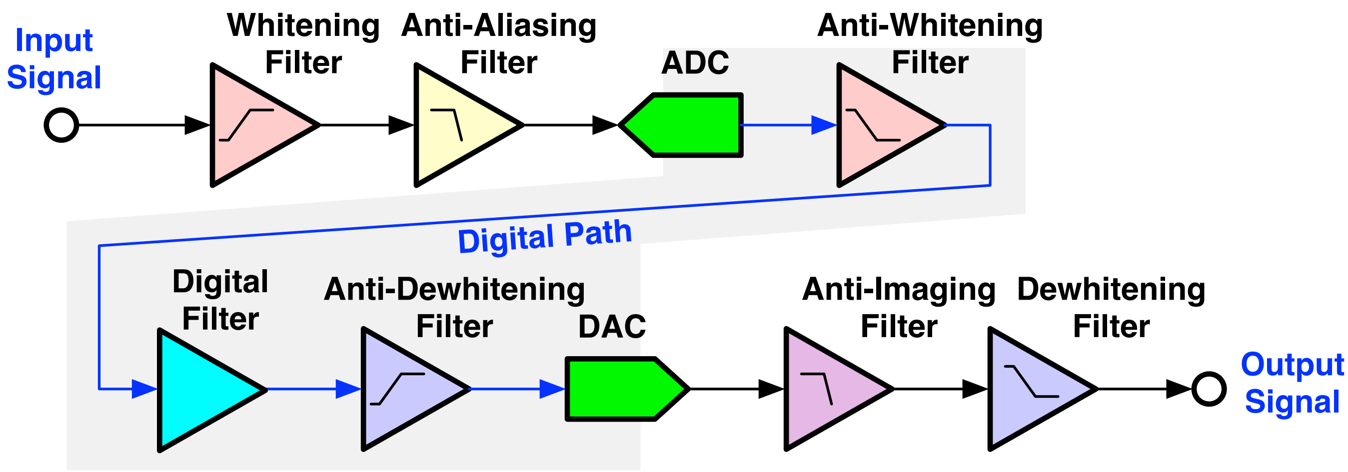
2mm InGaAs PD: Rs~10Ω, Cd~100pF

=> i_Rs = 20 pA/sqrtHz @100MHz

(equivalent to the shot noise of 1mA light ~ 1.3mW@1064nm)

Analog/Digital interface

- **Restriction of signal digitization**
  - **Voltage quantization: quantization noise**
    => limited dynamic range
    => Requires whitening/dewhitening filters
  - **Temporally discrete sampling: aliasing problem**
    => limited signal bandwidth
    => Requires anti-aliasing (AA) / anti-imaging (AI) filters
  - **Typical signal chain**



Digitization (Quantization) noise

- **Analog signals (~+/-10V) -> Digital signal**
  - **Digitized to a discrete N bit integer number**

quantization

quantization error

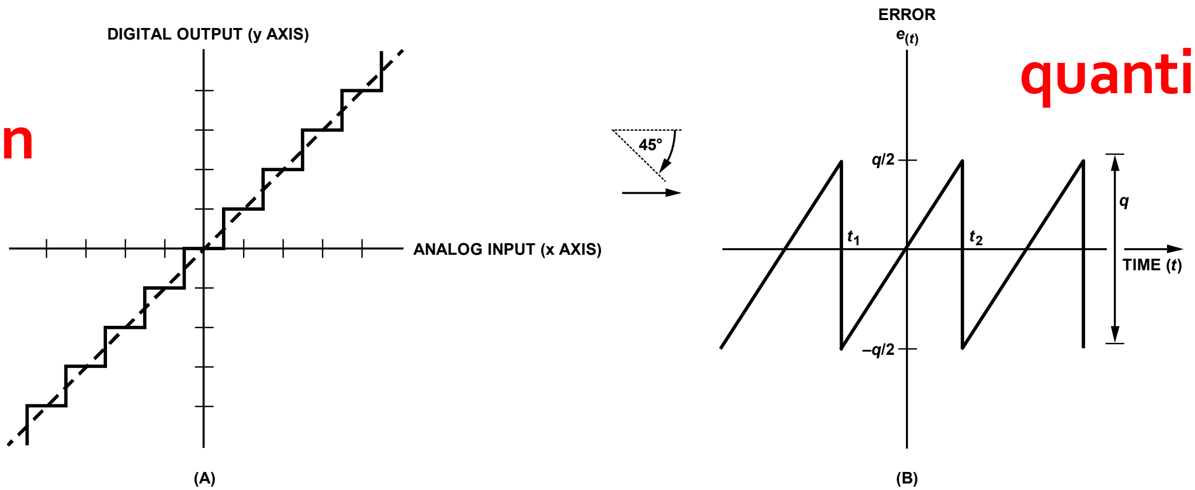


Figure 1. Ideal ADC Transfer Function (A) and Ideal N-Bit ADC Quantized Noise (B)

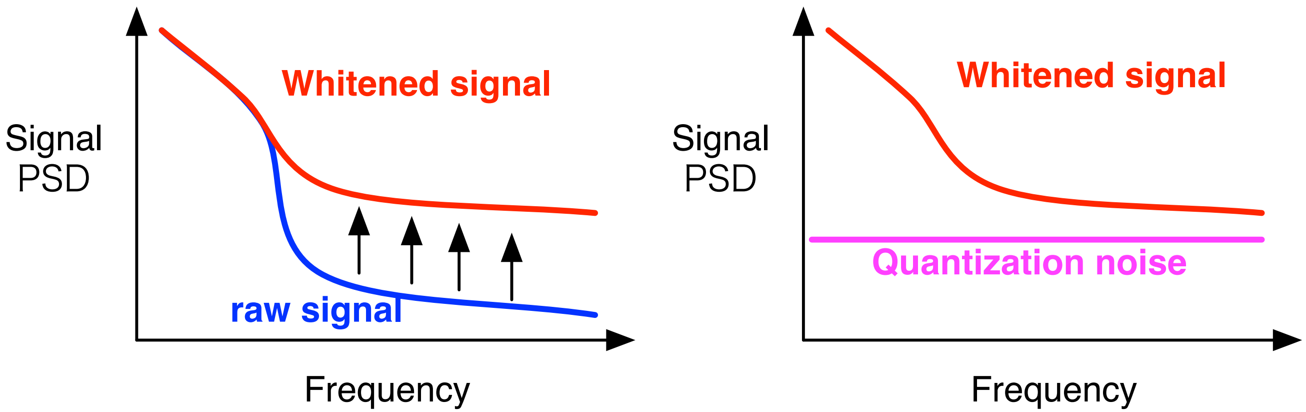
http://www.analog.com/static/imported-files/tutorials/MT-229.pdf

$$V_n = \frac{\Delta}{\sqrt{12}} \quad [\mathrm{V}/\sqrt{\mathrm{Hz}}]$$

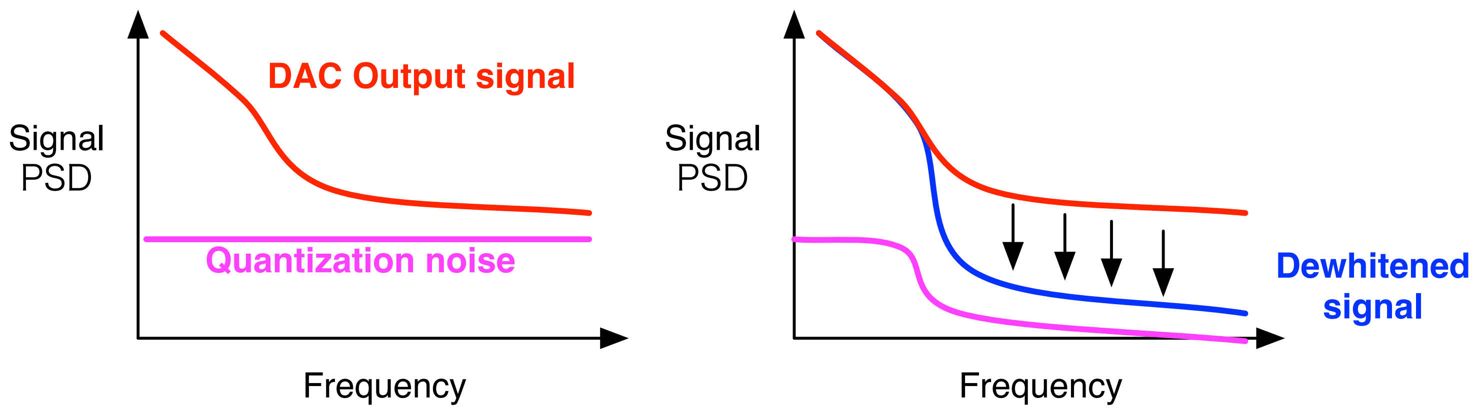
  - **Quantization causes a white noise**
    e.g. +/-10V 16bit => Δ = 0.3mV => Vn ~ 100 μV/sqrtHz
    cf. Input noise of a typical analog circuit 10nV/sqrtHz

Digitization (Quantization) noise

- **Whitening**
  - **Amplify a signal in the freq band where the signal is weak**



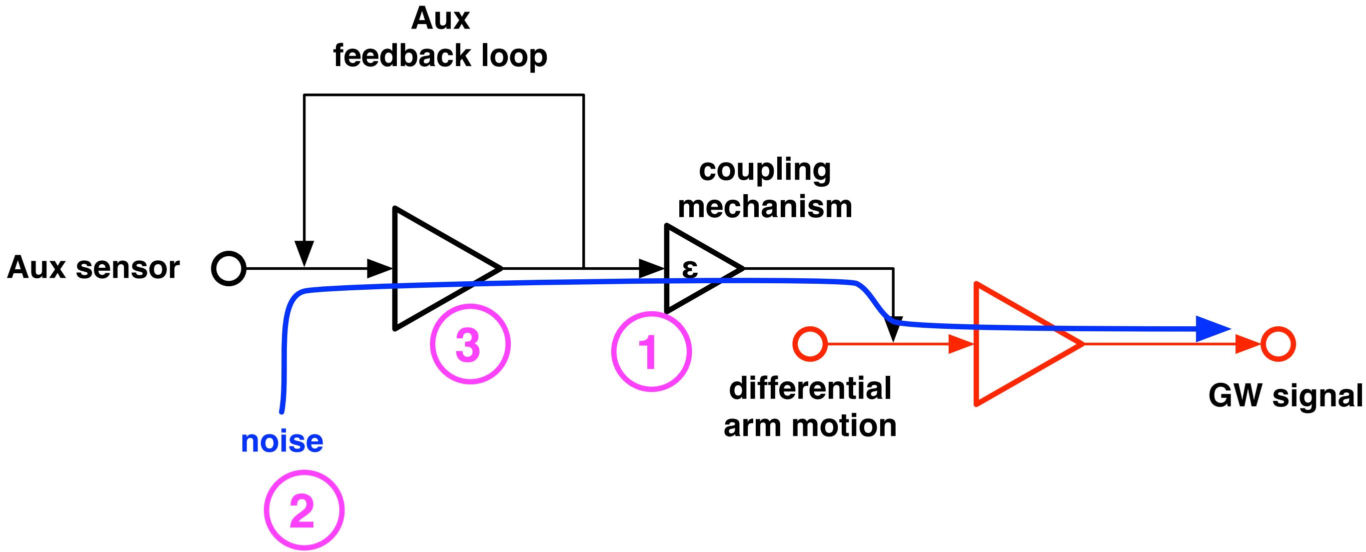
- **Dewhitening**
  - **Amplify a signal in the freq band where the signal is weak**



Control induced noise

- Noise couplings from auxiliary loops
  - e.g. Angle control feedback
    -> noise injection to the GW channel
  - Mitigation

    1) Make the coupling smaller
    2) Make the noise itself smaller
    3) Limit the control bandwidth of the aux loop



Actuator noise

- **Actuator noise appears in the GW signal as an external disturbance**
  - **Mitigation**
    1) Make the noise itself smaller
    2) Make the actuator response smaller
  - **We need to keep sufficient actuator strength for lock acquisition**
    => Transition to a low-noise mode after achieving lock

Summary

Summary

- **Summary**
  - There are such large number of noises
  - They are quite omnidisciplinary
  - Even only one noise can ruin our GW detection
  - GW detection will be achieved by
    - Careful design / knowledge / experience
    - Logical, but inspirational trouble shooting
  - Noise "hunting"