Determining the final spin of a binary black hole system including in-plane spins: Method and checks of accuracy

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(Dated: August 6, 2016)

We describe a simple extension to aligned-spin fits for the final spin of a binary black hole system that includes the contribution from the in-plane spins. We show that this addition gives good agreement with the final spin from a suite of 752 numerical relativity simulations when applied to the aligned-spin fit from Healy, Lousto, and Zlochower (HLZ) [1]. This agreement is increased if one evolves the spins using post-Newtonian expressions. We also show that the unmodified HLZ final mass fit gives good agreement with the numerical relativity data.

I. THE FINAL SPIN EXPRESSION

There exist quite accurate fits for the final mass and spin of binary black hole systems with aligned spins, e.g., the ones by Healy, Lousto, and Zlochower (HLZ) [1]. Moreover, these aligned-spin final mass fits still perform well even for precessing systems if one uses the aligned components of the spins. However, the final spin fits do not perform so well for precessing cases, since the in-plane spins make a direct contribution to the final spin. Nevertheless, we can augment aligned-spin fits for the final spin to include the contribution from the in-plane spins in a simple way (first introduced in [2]). The basic idea is the same as that used in the precessing IMRPhenom waveform model (introduced in an earlier form in [3]) to extend the IMRPhenomD aligned-spin fit [4] to the precessing case (see [5]):[*,†]

$$\chi_f^{\text{full}} = \sqrt{\left(\chi_f^{\text{aligned}}\right)^2 + \left(S^{\text{in-plane}}/M^2\right)^2}. \tag{1}$$

Here χ_f^{aligned} is the final (dimensionless) spin obtained from the fit using the components of the spins along the orbital angular momentum, $S^{\text{in-plane}}$ is the magnitude of the sum of the in-plane components of the dimensionful spins, and M is the binary's initial (total) mass. Using the initial mass gives better agreement with the numerical relativity (NR) data than does using the final mass. Additionally, if one uses the initial mass, one obtains a χ_f^{full} that is always less than the Kerr bound of 1 when using either the HLZ or IMRPhenomD aligned-spin fits, even for extremal initial spins. This is not the case if one uses the final mass.

One can obtain even better agreement with NR results if one uses post-Newtonian expressions to evolve the initial spins up to orbital velocity of the Schwarzschild innermost stable circular orbit (ISCO), i.e., $v = 6^{-1/2} \simeq 0.41$, before applying Eq. (1). Here we use the expressions from [8]. When evolving parameter estimation samples, we initialize the evolution using f_{ref} (the $2, 2$ mode gravitational wave frequency at which the spins are defined in the waveform; 20 Hz for O1 analyses) to set the binary's initial orbital velocity by $v_0 = (\pi M_z f_{\text{ref}})^{1/3}$. Here we use the binary's redshifted mass M_z, since f_{ref} is defined in the detector frame.

When comparing with NR simulations, we either use $M\omega_0$ (obtained from the initial frequency of the waveform) instead of $\pi M_z f_{\text{ref}}$ or, if this is not available, the magnitude of the initial orbital angular momentum, L_0, using the first post-Newtonian (1PN) relation $v_0 = M\eta/L_0 + (3/2 + \eta/6)(M\eta/L_0)^3$, where M is the total mass and η is the symmetric mass ratio. Note that this evolution only affects the final spin in double spin cases: The post-Newtonian evolution equations we use preserve the component of the spin along the orbital angular momentum in single spin cases. We also use the direction of the initial orbital angular momentum from the numerical simulation (obtained from the initial ADM angular momentum $\boldsymbol{J}_{\text{ADM}}$ and the coordinate components of the binary's dimensionful spins $\boldsymbol{S}_{1,2}$ by $\boldsymbol{L}_0 = \boldsymbol{J}_{\text{ADM}} - \boldsymbol{S}_1 - \boldsymbol{S}_2$ if not given explicitly) to initialize the spin evolution when they are available; when the initial orbital angular momentum is not available, we take it to be in the z-direction.

II. COMPARISON WITH NUMERICAL RELATIVITY

We compare with 752 numerical relativity simulations from four different collaborations, including 473 precessing simulations with mass ratios up to 8 and dimensionless spins up to 0.8 in most cases; a few have spins of up to 0.99 on one hole. We use

[*] Note, however, that the IMRPhenomPv2 final spin expression computes the in-plane spin from χ_p, while we compute it from the spin magnitudes, tilt angles, and ϕ_{12}; these are defined in [6].

[†] After the review of these results was mostly complete, a new final spin fit for generic quasicircular binaries appeared [7]. This fit includes the in-plane spins in a way similar to what we do here, though it does not evolve the spins, and performs some further adjustments to improve the agreement in precessing cases.

the 144 quasicircular simulations (eccentricity $< 10^{-3}$) from the Simulating eXtreme Spacetimes catalogue (using the SPEC code) [9, 10], considering catalogue numbers up to 201, and the 341 simulations from the Georgia Institute of Technology catalogue (using the MAYA code) [11, 12] that give final mass and spin data (leaving off GT0392, which is the shortest waveform in the catalogue, and whose final spin has uncertain accuracy). Additionally, we use 244 simulations from the Rochester Institute of Technology group (using the LAZEV code [13])—the 140 simulations from [14] plus 104 further simulations [15]—and 23 precessing simulations by the Cardiff University and Universitat de les Illes Balaers groups (using the BAM code [16, 17]) with parameters close to those inferred for GW150914. Though the full waveforms of some of these simulations are still unpublished, they are all used in [18], and their initial and final state quantities will be available as supplementary material with that paper.

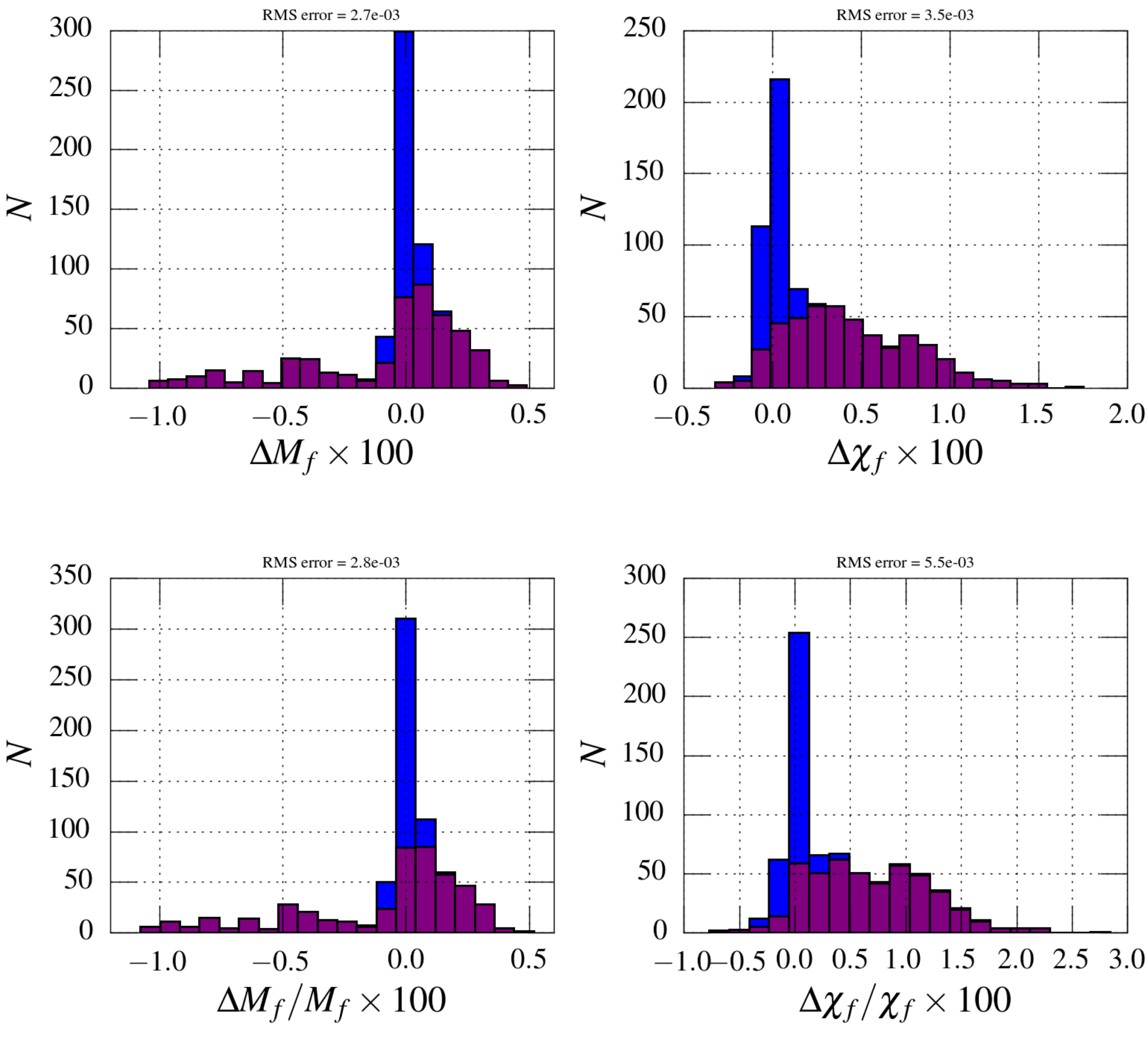

FIG. 1. Histograms of absolute *(top)* and fractional *(bottom)* errors in the final mass and spin comparing to all the simulations we consider using HLZ final mass fit and the augmented HLZ final spin fit with spin evolution. We show just precessing simulations in purple and all simulations in blue.

In Fig. 1 we show a histogram of the errors (absolute and fractional) in the final mass and spin when comparing with the HLZ fit, with the final spin augmented with the in-plane spins using spin evolution, as described above, and the final mass fit evaluated using the components of the spins along the orbital angular momentum. When computing the histogram of fractional errors on the final spin, we omit a few cases with final spin magnitudes < 0.05, for which there is a large fractional error, even though the magnitude of the absolute error is $\lesssim 0.002$. We compute the errors as fit $-$ data and denote the final mass (scaled by the total mass) and final (dimensionless) spin by M_f and χ_f, respectively.

The intervals containing 90% of the errors on χ_f are $[-1.2, 9.2] \times 10^{-3}$ (absolute) and $[-2.3, 12] \times 10^{-3}$ (fractional). For M_f they are $[-8.1, 3.4] \times 10^{-3}$ (absolute) and $[-8.4, 3.6] \times 10^{-3}$ (fractional). If we just restrict to all the precessing systems, all these intervals remain unchanged, except for the 90% interval for the fractional error on χ_f, which becomes $[-0.5, 14] \times 10^{-3}$.‡ Note that the small remaining bias in M_f and χ_f for precessing systems is negligible for LIGO O1 accuracy needs, but can be addressed in the future using new fitting formulae.

For comparison, if we just consider the plain HLZ fit, without including the in-plane spins, we obtain 90% intervals on the final spin error of $[-98, 7.8] \times 10^{-3}$ (absolute) and $[-130, 17] \times 10^{-3}$ (fractional), so these intervals are factors of ~ 50 to 100 larger on the lower side, though they are comparable on the higher side. This is to be expected, since we expect the plain HLZ fit to underestimate the final spin for precessing cases, due to the neglect of in-plane spins. The 90% final spin error intervals for just the precessing systems are $[-140, 7.8] \times 10^{-3}$ (absolute) and $[-340, 12] \times 10^{-3}$ (fractional), more than 100 times larger on the lower side. The RMS errors on the final spin (absolute and fractional) are ~ 20 to 30 times larger than with the augmented fit.

ACKNOWLEDGMENTS

We acknowledge the Simulating eXtreme Spacetimes project and Georgia Institute of Technology for their public NR waveform catalogues, and the following people for providing us with results from yet-to-be-published simulations: Edward Fauchon-Jones, Mark Hannam, Chinmay Kalaghatgi, Sebastian Khan, Lionel London, Francesco Pannarale, and Alex Vañó-Viñuales (Cardiff University); Manuela Campanelli, James Healy, Carlos Lousto, and Yosef Zlochower (Rochester Institute of Technology).

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‡ The small differences in these intervals and Fig. 1 compared to v5 of this document are due to small corrections to the data set provided with [18].