Teaching Gravitational Waves: A Lesson in Heuristic (Mis)understanding of How an Interferometer Detects Gravitational Waves

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Outline

1. Early confusion about the physical reality of gravitational waves, resolved by Pirani in 1957
2. Rai Weiss followed Pirani's prescription to work out how an interferometer can sense gravitational waves.
3. A puzzle: If light waves are stretched by gravitational waves, how can we use light as a ruler to detect gravitational waves?

Einstein's prediction of grav waves didn't settle the matter…

… even for Einstein. He doubted their physical reality until the end of his life.

Einstein proposed many experiments, including really hard ones, but never the search for gravitational waves.

The resolution of this question came in 1957.

The Chapel Hill Conference

In January 1957, the U.S. Air Force (under program officer Josh Goldberg) sponsored the *Conference on the Role of Gravitation in Physics*, a.k.a. the Chapel Hill Conference, a.k.a. GR1. The organizers were Bryce and Cecile DeWitt. 44 of the world's leading relativists attended.

Much of the future of gravitational physics was launched then. (Numerical relativity was prefigured in a remark by Charles Misner.)

The "gravitational wave problem" was solved there, and the quest to detect gravitational waves was born.

The “gravitational wave problem”

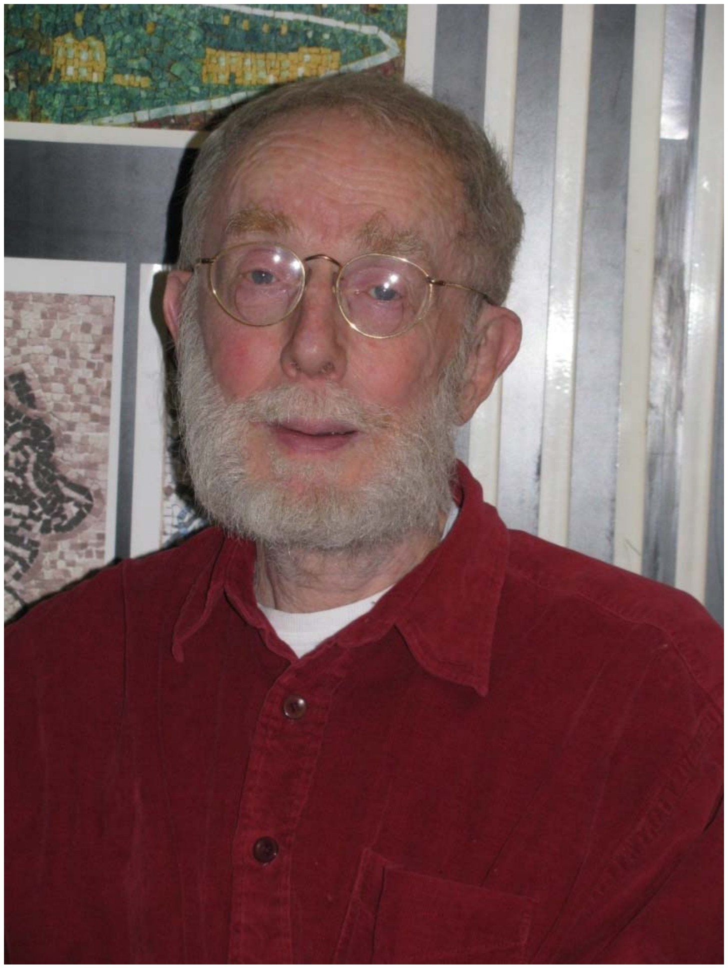
Were gravitational waves real, or were they “pure gauge”?

Before Chapel Hill, debate raged. Einstein wavered. Eddington suggested that gravitational waves “traveled at the speed of thought.”

One main approach was to solve the equations of motion of a binary star, and show that they generated waves that couldn’t be transformed away.

It was hard. People were still hard at work on it when Hulse and Taylor found the binary pulsar in 1974 …

Felix Pirani solved the problem of the reality of gravitational waves



Felix Pirani was a student of Alfred Schild's and then of Hermann Bondi's. In 1957 he was a junior colleague of Bondi at King's College, London.

At Chapel Hill, he gave the solution of the gravity wave problem, although Bondi (or Feynman) usually get the credit.

Photo by Josh Goldberg

Pirani's 1957 papers

Pirani's breakthrough was to analyze the reception of gravitational waves, not their generation.

He showed that, in the presence of a gravitational wave, a set of freely-falling particles would experience genuine motions with respect to one another. Thus, gravitational waves must be real.

He made this case in two papers submitted before the Chapel Hill conference, and presented there.

Pirani's talk

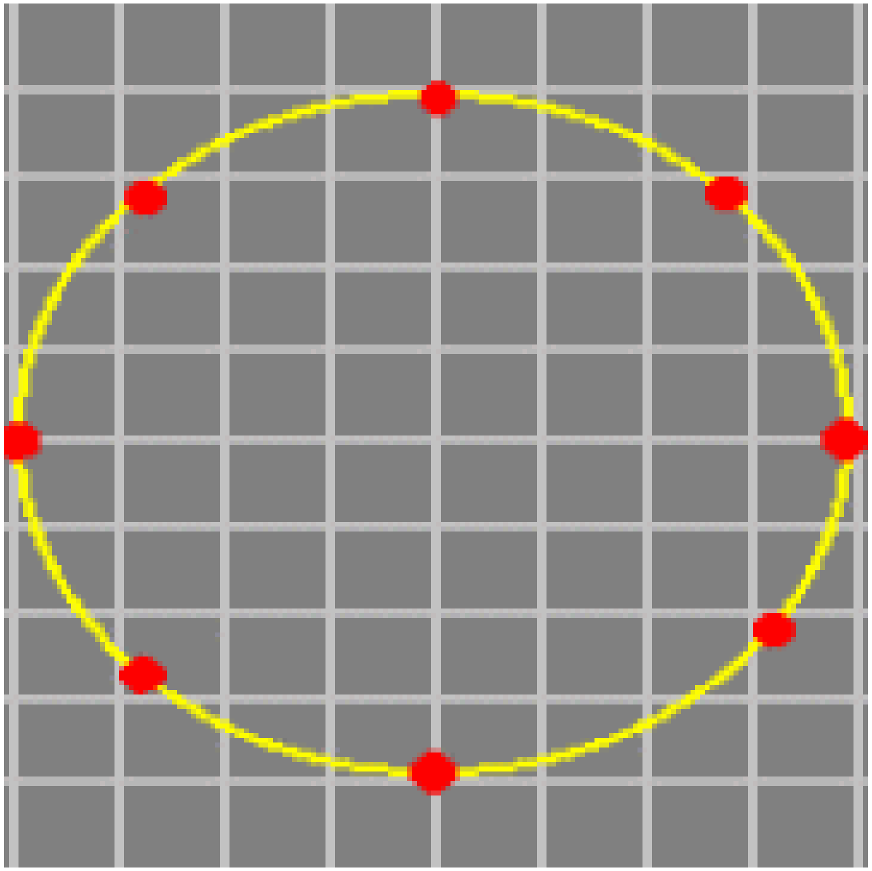
If now one introduces an orthonormal frame on ζ, v^μ being the timelike vector of the frame, and assumes that the frame is parallelly propagated along ζ (which insures that an observer using this frame will see things in as Newtonian a way as possible) then the equation of geodesic deviation (1) becomes

$$\frac{d^2\eta^a}{d\tau^2} + R^a{}_{obo}\,\eta^b = 0 \qquad (a,b = 1,2,3) \qquad (2)$$

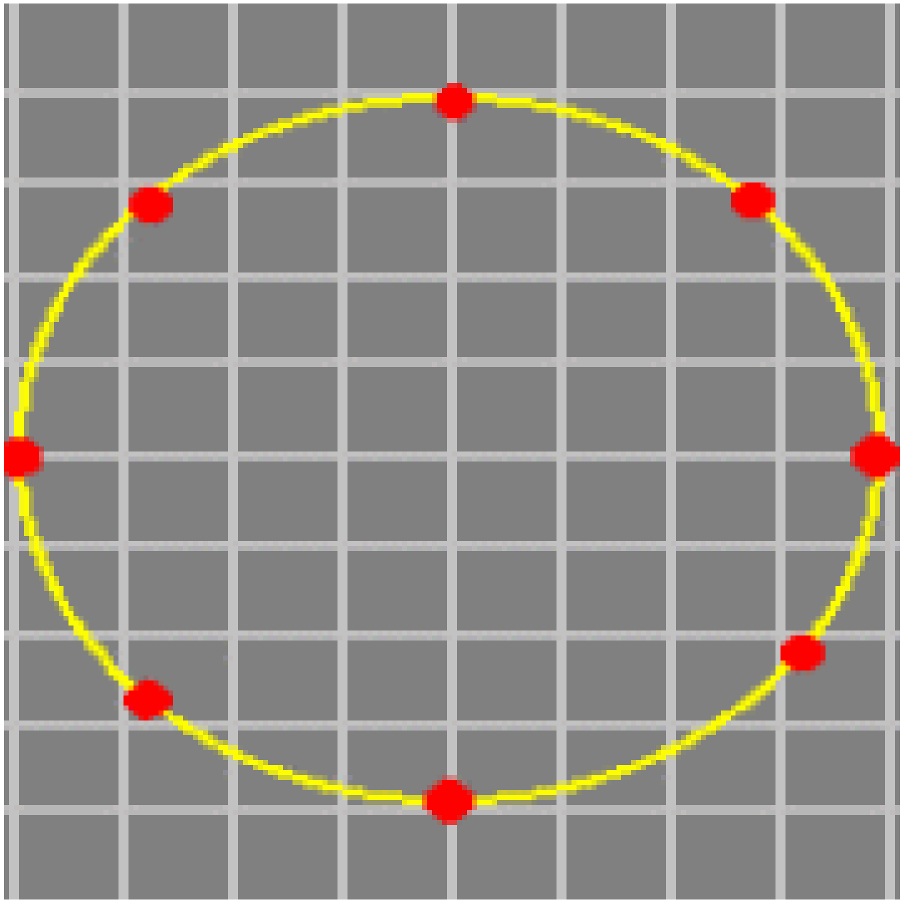
Here η^a are the physical components of the infinitesimal displacement and $R^a{}_{obo}$ some of the physical components of the Riemann tensor, referred to the orthonormal frame.

By measurements of the relative accelerations of several different pairs of particles, one may obtain full details about the Riemann tensor. One can thus very easily imagine an experiment for measuring the physical components of the Riemann tensor.

Pirani's set of neighboring freely-falling test masses



They respond in a measurable way to a gravitational wave



Bondi clarifies Pirani's point

Pirani's mentor Bondi arrived at Chapel Hill unsure about gravitational waves.

Listening to Pirani's talk, he asked whether you could connect two nearby masses with a dashpot, thus absorbing energy from the wave.

Energy absorption is the ultimate test of physical reality.

Pirani replied: "I have not put in an absorption term, but I have put in a 'spring'. You could invent a system with such a term quite easily."

Bondi is credited with the "sticky bead argument."

Proof by dialog that gravitational waves are real

BONDI: Can one construct in this way an absorber for gravitational energy by inserting a $\frac{d\eta}{d\tau}$ term, to learn what part of the Riemann tensor would be the energy-producing one, because it is that part that we want to isolate to study gravitational waves?

PIRANI: I have not put in an absorption term, but I have put in a "spring." You can invent a system with such a term quite easily.

Pirani got there first

Peter Bergmann’s summary talk emphasizes that Pirani’s contribution was considered to be one of the most important outcomes of the meeting.

(We can see in the proceedings that Bondi learned the argument from Pirani, and Feynman’s talk on the same subject came later in the meeting.)

Joe Weber at Chapel Hill

Joe Weber, co-inventor of the maser, was working with John Wheeler at Princeton on gravitational waves.

The two of them were at Chapel Hill, and listened well to Pirani's talk.

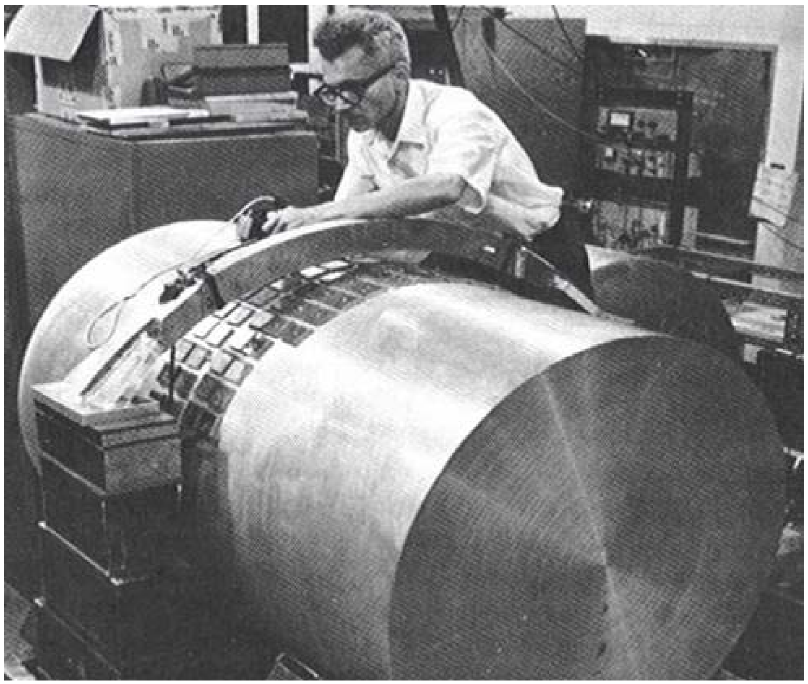
Joe Weber starts GW detection

Weber and Wheeler recapped Pirani's argument in a paper written within weeks of the Chapel Hill conference.

He expanded on the experimental ideas in two Gravity Research Foundation essays (3rd prize 1958, 1st prize 1959), leading to his 1960 Phys. Rev. paper laying out the bar program.

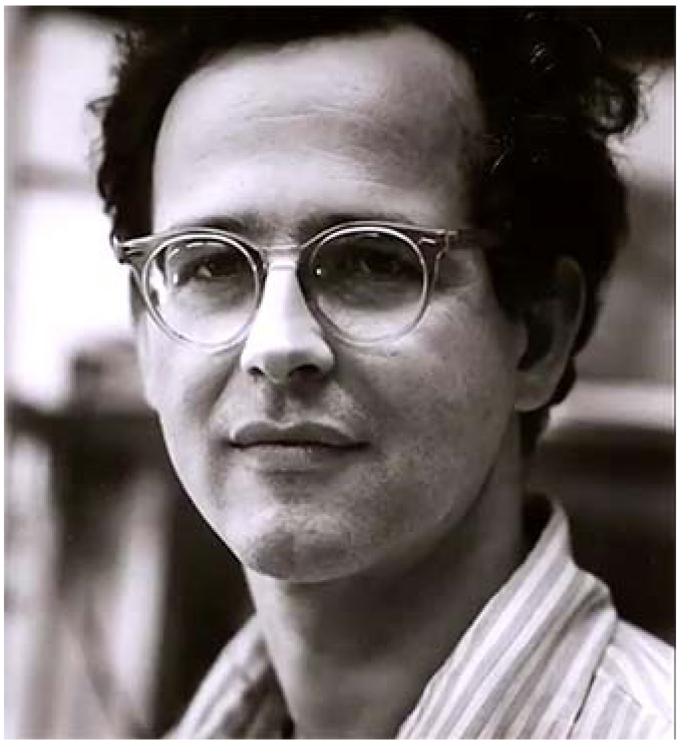
In other words: <u>The search for gravitational waves started in January 1957 during Pirani's talk at Chapel Hill.</u>

Weber’s bar



Weber’s gravitational wave detector was a cylinder of aluminum. Each end is like a test mass, while the center is like a spring. PZT’s around the midline are Bondi’s dashpots, absorbing energy to send to an electrical amplifier.

Rainer Weiss, not at Chapel Hill



In 1957, Rai Weiss was a grad student of Jerrold Zacharias at MIT, working on atomic beams.

In the early '60's, he spent two years working with Bob Dicke at Princeton on gravity experiments.

Rainer Weiss and Joe Weber

In 1964, Rai was back at MIT as a professor. He was assigned to teach general relativity. He didn't know it, so he had to learn it one day ahead of the students.

He asked, What's really measurable in general relativity? He found the answer in Pirani's papers presented at Chapel Hill in 1957.

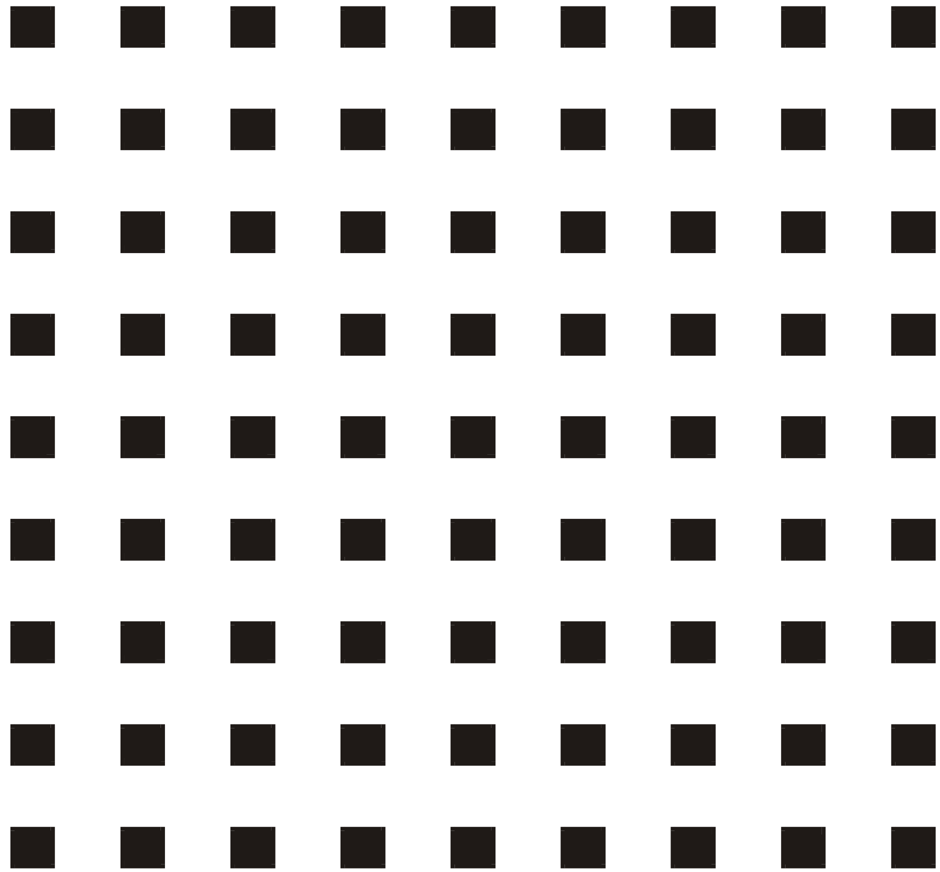
What Pirani actually proposed

In Pirani's papers, he didn't "put in" either a spring or a dashpot between the test masses. Instead, he said:

"It is assumed that an observer, by the use of light signals or otherwise, determine the coordinates of a neighboring particle in his local Cartesian coordinate system."

Zach's lab at MIT was in the thick of the new field of lasers. Rai read Pirani, and knew that lasers could do the job.

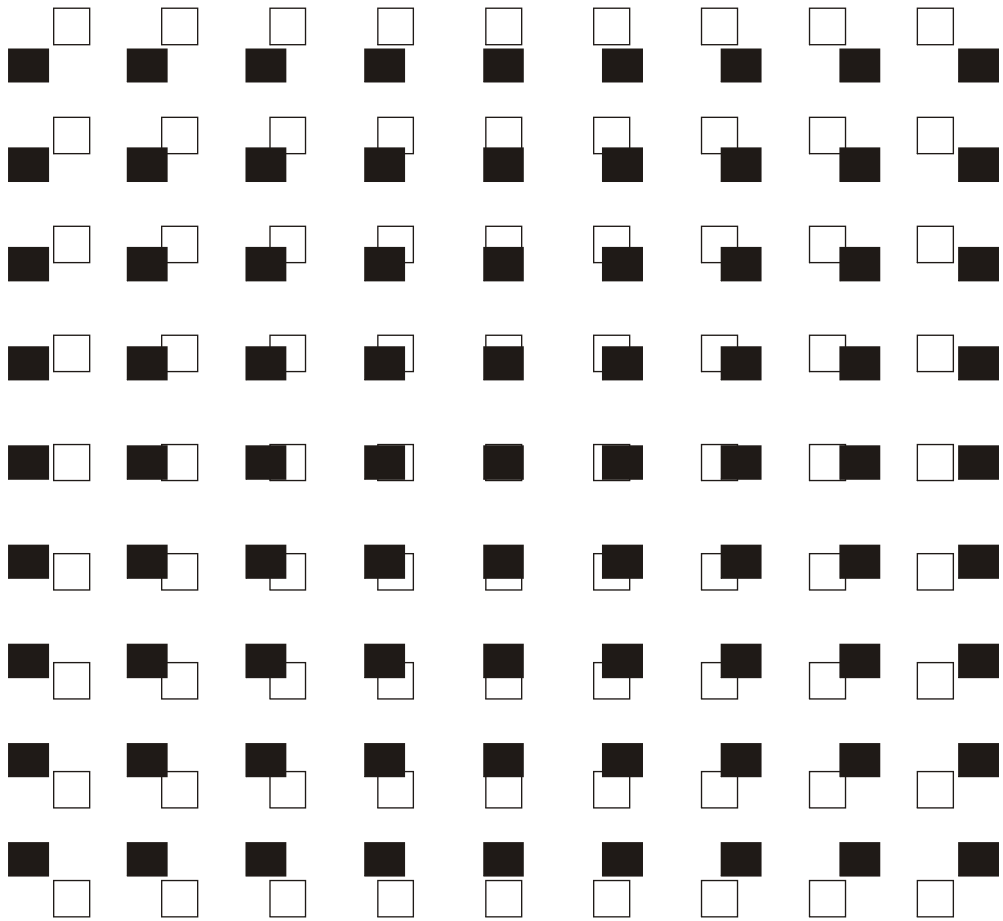
A set of freely-falling test particles



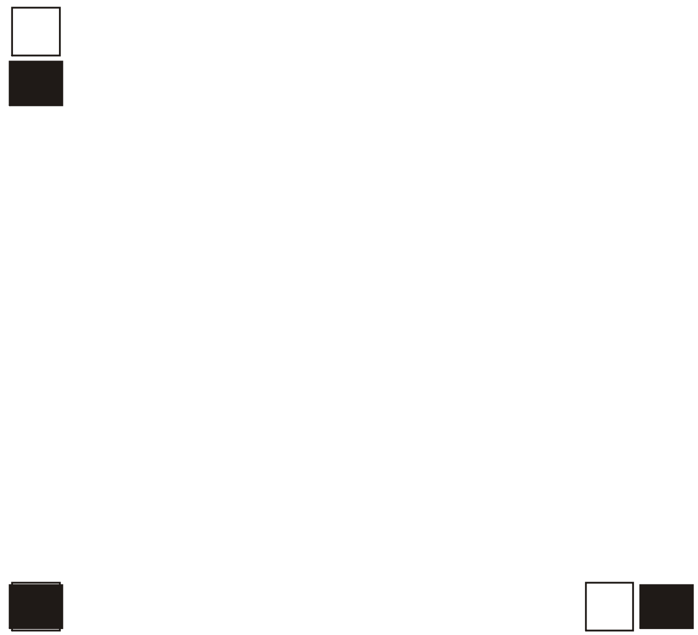
Gravitational wave:
a transverse quadrupolar strain

strain amplitude:

$h = 2\Delta L/L$



Three test masses



Gravitational waves

Gravitational waves propagating through flat space are described by

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

A wave propagating in the *z*-direction can be described by

$$h_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & a & b & 0 \\ 0 & b & -a & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Two free parameters implies two polarizations

Here is Rai Weiss's calculation, as he learned to do it from Pirani

Rai knew that an interferometer compares the light travel time through one arm with the light travel time through the other arm.

For light moving along the x axis, we are interested in the interval between points with non-zero *dx* and *dt*, but with *dy* = *dz* = 0:

$$ds^2 = 0 = -c^2 dt^2 + (1 + h_{11}) dx^2$$

Solving for variation in light travel time: start with *x* arm

$$ds^2 = -c^2 dt^2 + \left(1 + h_{11}\right) dx^2 = 0$$

h(t) can have any time dependence, but for now assume that *h(t)* is constant during light's travel through ifo.

Rearrange, take square root, and replace square root with 1st two terms of binomial expansion

$$\int dt = \frac{1}{c} \int \left(1 + \frac{1}{2} h_{11}\right) dx$$

then integrate from $x = 0$ to $x = L$:

$$\Delta t = h_{11} L / 2c$$

Solving for variation in light travel time (II)

In doing this calculation, we choose coordinates that are marked by free masses.

"Transverse-traceless (TT) gauge"

Thus, end mirror is always at $x = L$.

Round trip back to beam-splitter:

$$\Delta t = h_{11} L / c$$

y-arm ($h_{22} = - h_{11} = -h$):

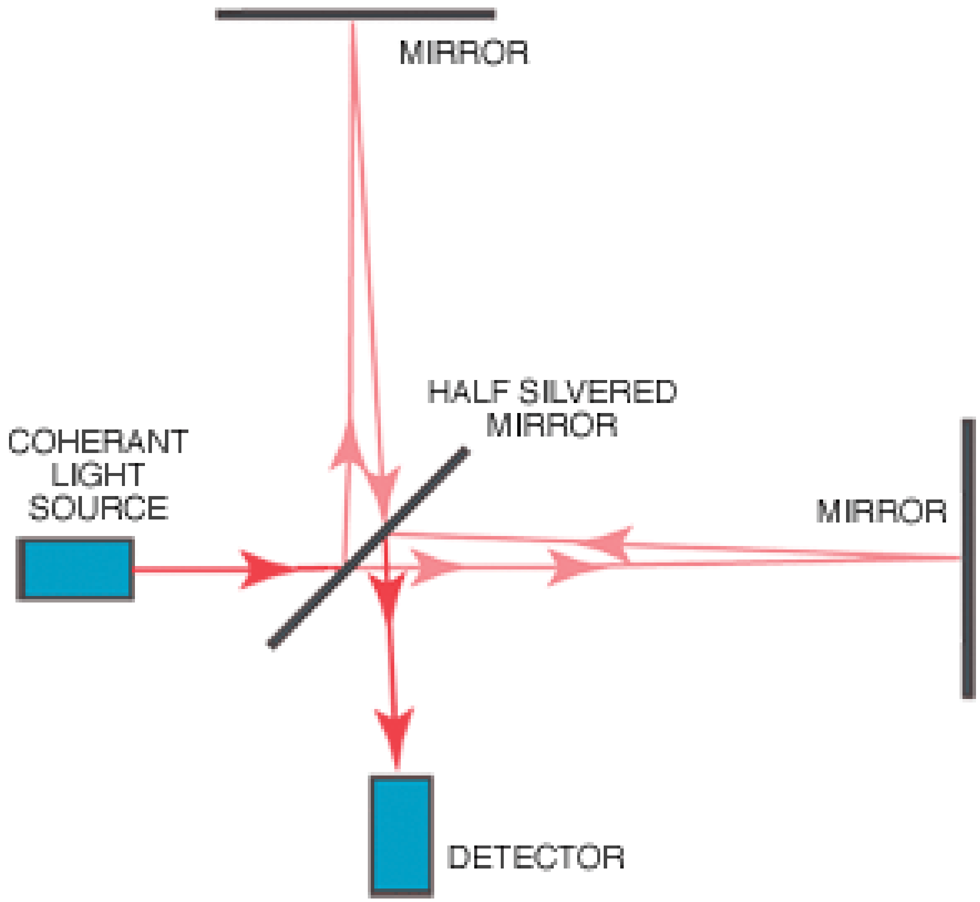
$$\Delta t_y = -hL / c$$

Difference between x and y round-trip times:

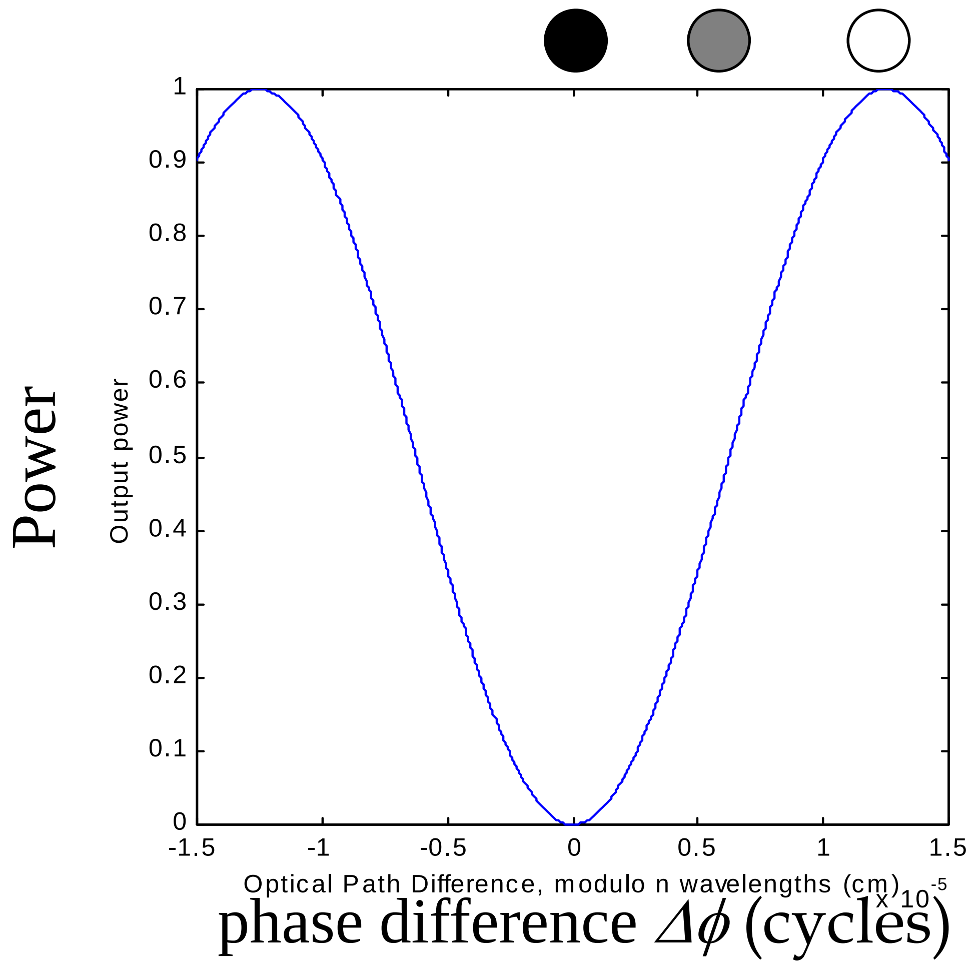
$$\Delta \tau = 2hL / c$$

Sensing relative motions of distant free masses

Michelson interferometer



Interferometer output vs. arm length difference



Interpretation

A gravitational wave's effect on one-way travel time:

$$\Delta t = \frac{h}{2}\frac{L}{c}$$

Just as if the arm length is changed by a fraction

$$\frac{\Delta L}{L} = \frac{h}{2}$$

In the TT gauge, we say that the masses didn't move (they mark coordinates), but that the separation between them changed.

Metric of the space between them changed.

Comparison with rigid ruler, force picture

- We can also interpret the same physics in a different picture, using different coordinates. Here, define coordinates with rigid rods, not free masses.
- With respect to a rigid rod, masses <u>do</u> move apart. In this picture, it is as if the gravitational wave exerts equal and opposite forces on the two masses.
- This is the best way to understand Weber's bars.

At this moment, the LIGO discovery is being challenged by the "rubber ruler puzzle"

If a gravitational wave stretches space, doesn't it also stretch the light traveling in that space?

If so, the "ruler" is being stretched by the same amount as the system being measured.

And if so, how can a gravitational wave be observed using light?

How can interferometers possibly work?

A related case: the expanding universe

In cosmology, one typically uses *co-moving coordinates,* marked by freely-falling test masses (i.e., galaxies).

As the universe expands

- galaxies get farther apart
- light traveling through the universe is stretched (cosmological redshift)

Do galaxies move? Depends who you ask …

Light in an interferometer arm

Imagine many freely-falling masses along arms of interferometer.

Test case: imagine that a *step function* gravitational wave, with amplitude h and + polarization, encounters interferometer.

Along x arm, test masses suddenly farther apart by $(1+h/2)$.

Wavefronts near each test mass stay near the mass. (No preferred frames in GR!)

If the arms are stretched, then the light is stretched.

To the extent that we're willing to use language that says that the arms of an interferometer are lengthened by a gravitational wave, then the wavelength of the light in an interferometer is also lengthened by a gravitational wave, by the same factor.

OK, so how can interferometers work?

The argument given above proves that there is no *instantaneous* response to a gravitational wave.

But, we don't just care about the instantaneous response. We watch the entire history of the interferometer output.

The time-dependent response

- The x arm was lengthened by the gravitational wave.
- Light travels at *c*. So, light will start to arrive late, as it has to traverse longer distance than it did before the wave arrived.
- Delay builds up until all light present at wave's arrival is flushed out. Then delay stays constant at $\Delta\tau = h(2NL/c)$.

Consider the DC response …

New light produced by the laser (after gravitational wave has passed by) isn't affected by the gravitational wave.

(Its wavelength is determined by the length of a rigid resonant cavity.)

So if we wait to measure using all "new light", it must reveal the changed arm lengths.

We never (or never should have) said that we were using light as a ruler.

Pirani taught us to use the travel time of light signals between free masses to sense the passage of a gravitational wave.

That is what Rai Weiss did from the beginning.

In the end, there is no puzzle: Interferometers *can* work.