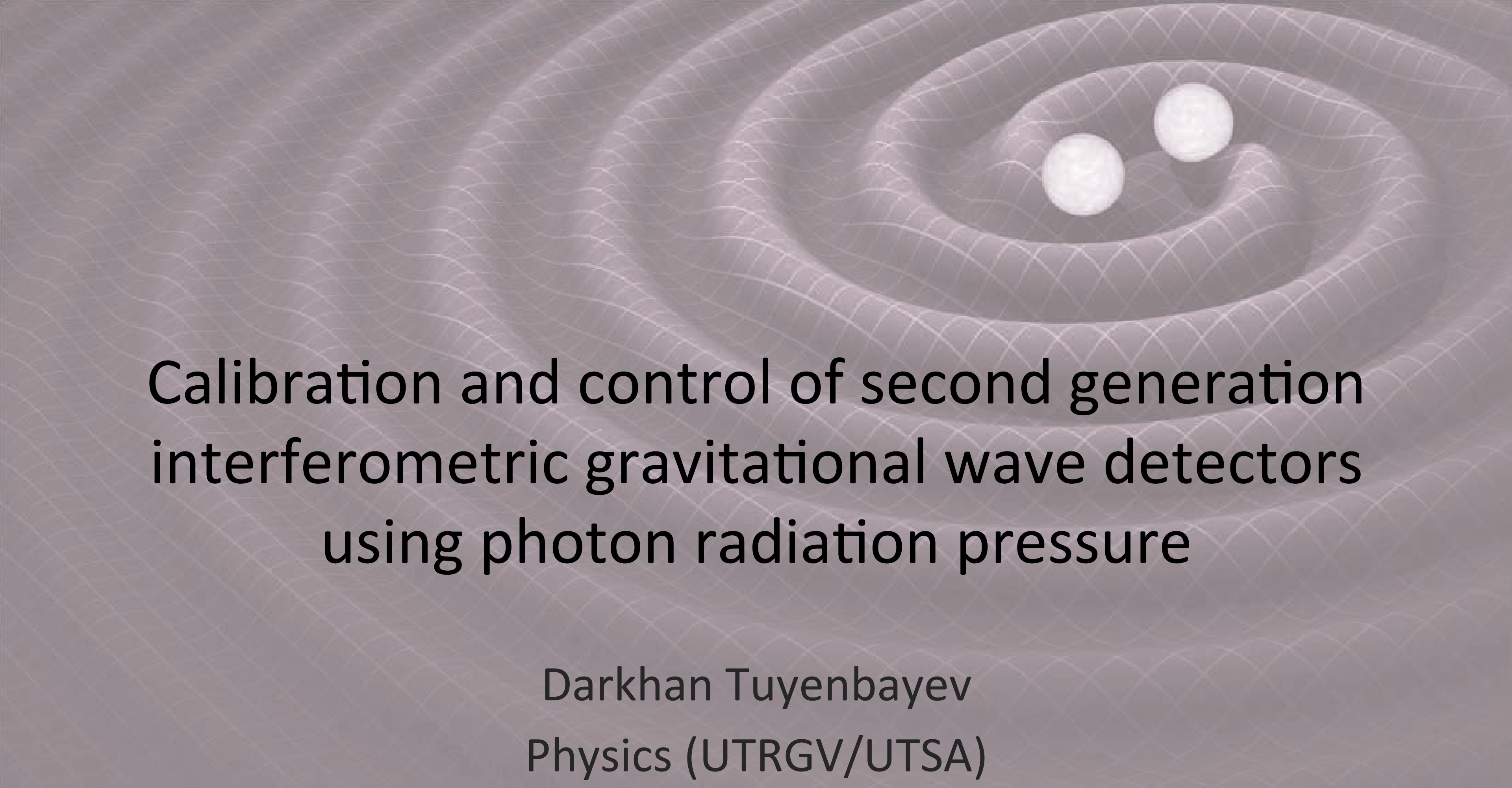


Calibration and control of second generation interferometric gravitational wave detectors using photon radiation pressure

Darkhan Tuyenbayev
Physics (UTRGV/UTSA)

Overview

- Gravitational waves
- LIGO detectors
- Interferometer calibration
  - Photon calibrator system
- Differential arm length (DARM) signal / GW signal
  - DARM control loop model
  - Tracking temporal variations
- Future work
  - Calibration upgrades
  - DARM actuation with photon calibrators
- Summary

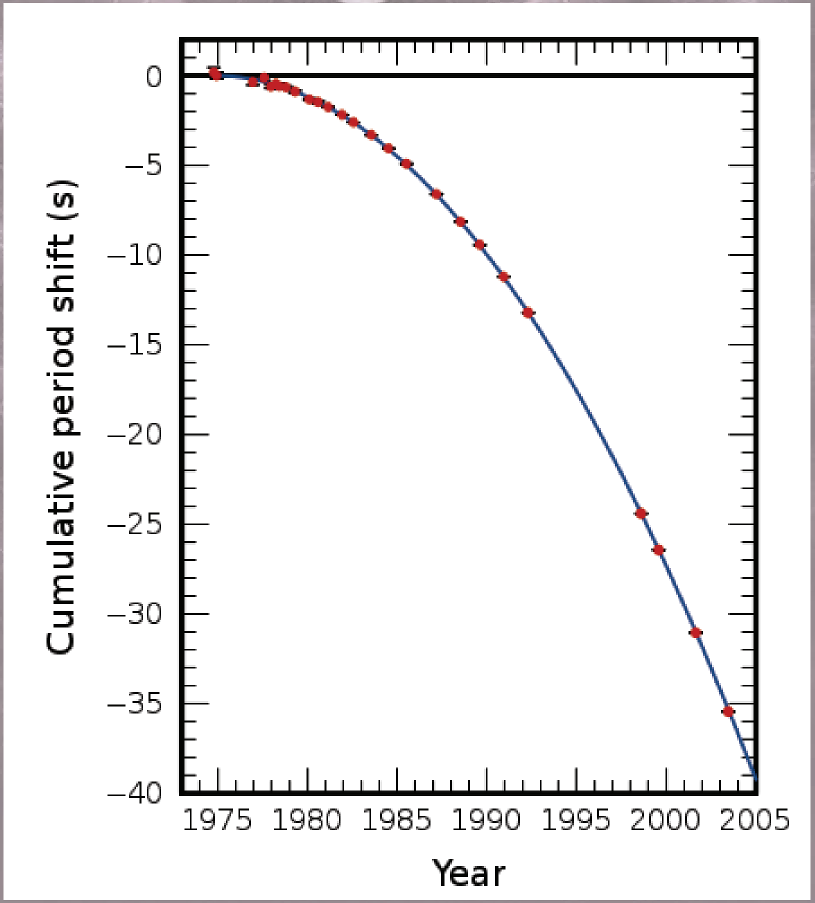
Gravitational Waves

- 1915: A. Einstein - general relativity (GR);
- 1916: A. Einstein – gravitational waves (GW);
- GR -> existence of gravitational waves (GW), from assymetrically accelerating masses;
- Observable GW: inspiral and coalescence of binary systems such as e.g. neutron stars, black holes or supernova collapses.

Neutron star merger animation
Credit: NASA/Goddard Space Flight Center

Indirect observation of GWs

- 1974: PSR 1913+16 was discovered by R.A. Hulse and J.H. Taylor.
- Observations have shown that the pulsar's orbit is gradually contracting; rate of contraction agrees very well with general relativity (emitting GWs).
- 1993 earned Noble Prize in Physics.

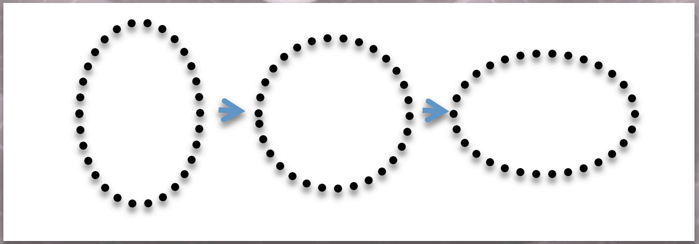


Orbital decay of PSR B1913+16.[1]

1. Weisberg, Joel M., and Joseph H. Taylor. Binary Radio Pulsars. Vol. 328. 2005.

Michelson interferometer – ideal for detecing GWs

- Mass quadrupole moment;
- The interference of the two beams produces a “dark fringe” at the photodetector;
- Changes in the measured photodetector beam power can be an indication of passing GW;
- GW sources at distances over 100 million light years away have an extremely small apparent length changes _~10-19 m_ in a 4-km long interferometer (1/10,1000 of the diameter of a proton);



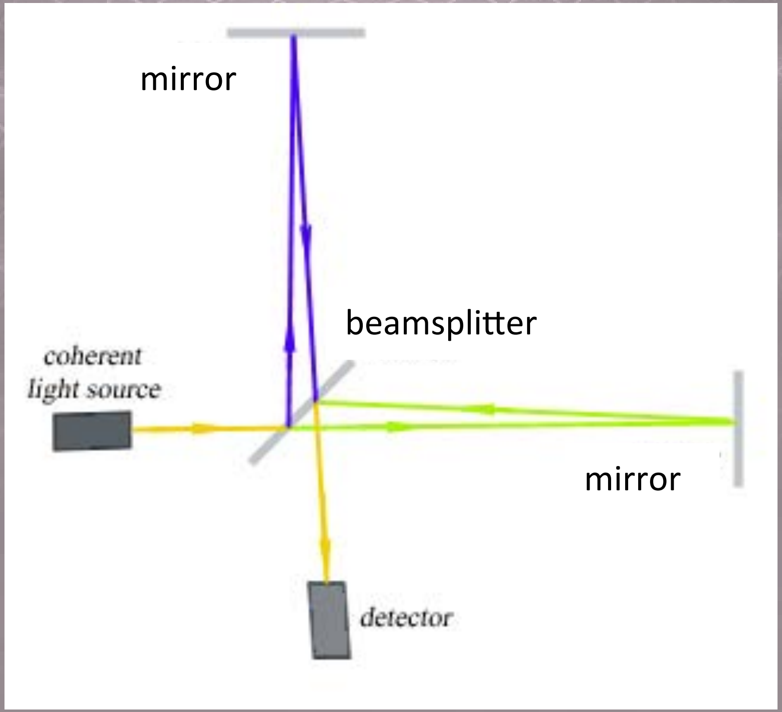
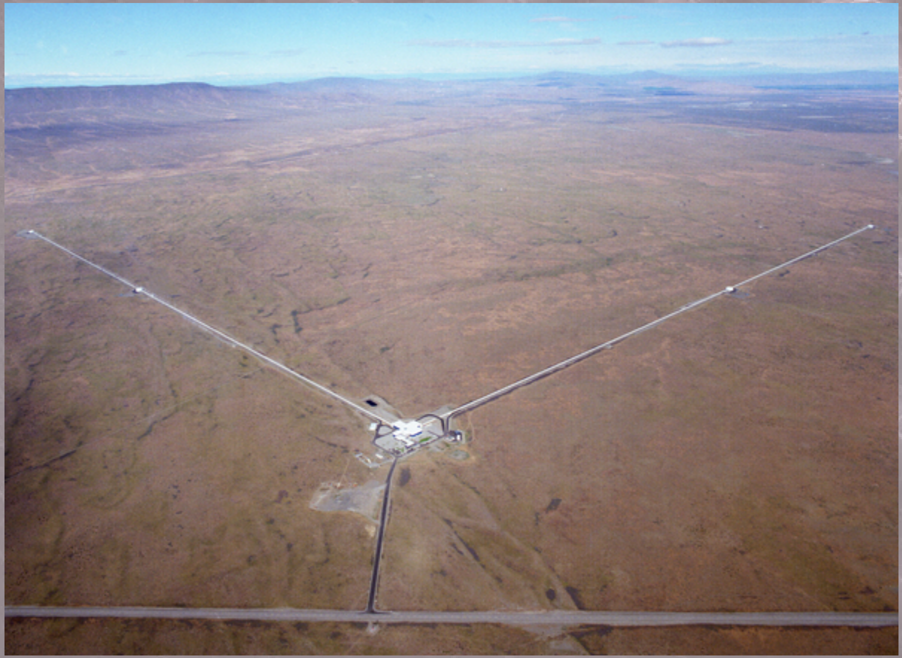
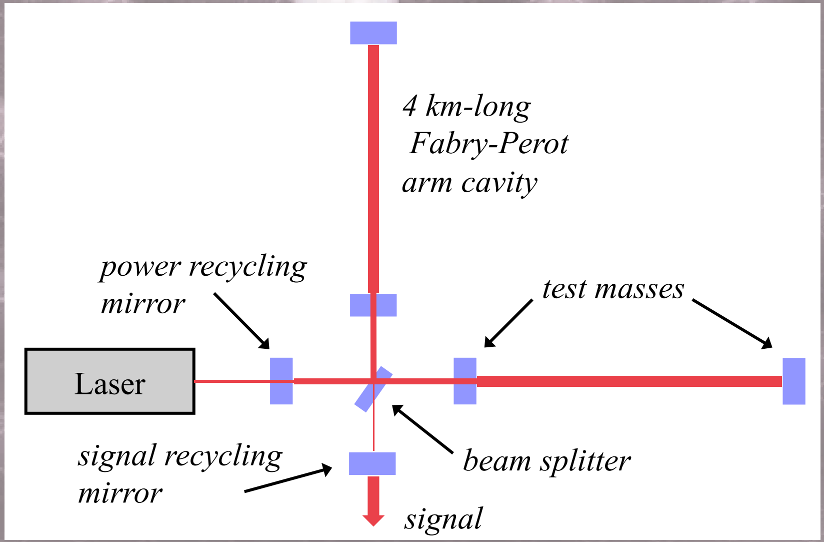


Image credit: Wikipedia

What is LIGO?



LIGO Hanford



4 km-long
Fabry-Perot
arm cavity

power recycling
mirror

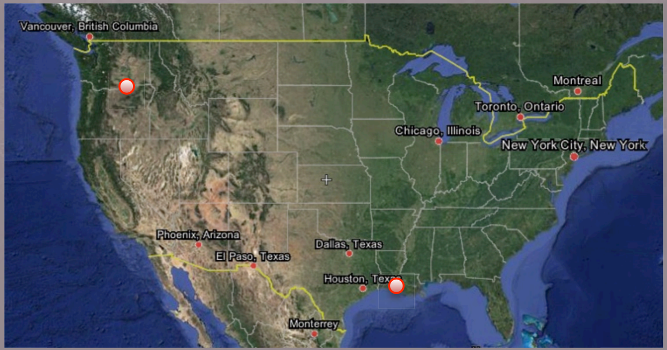
test masses

Laser

beam splitter

signal recycling
mirror

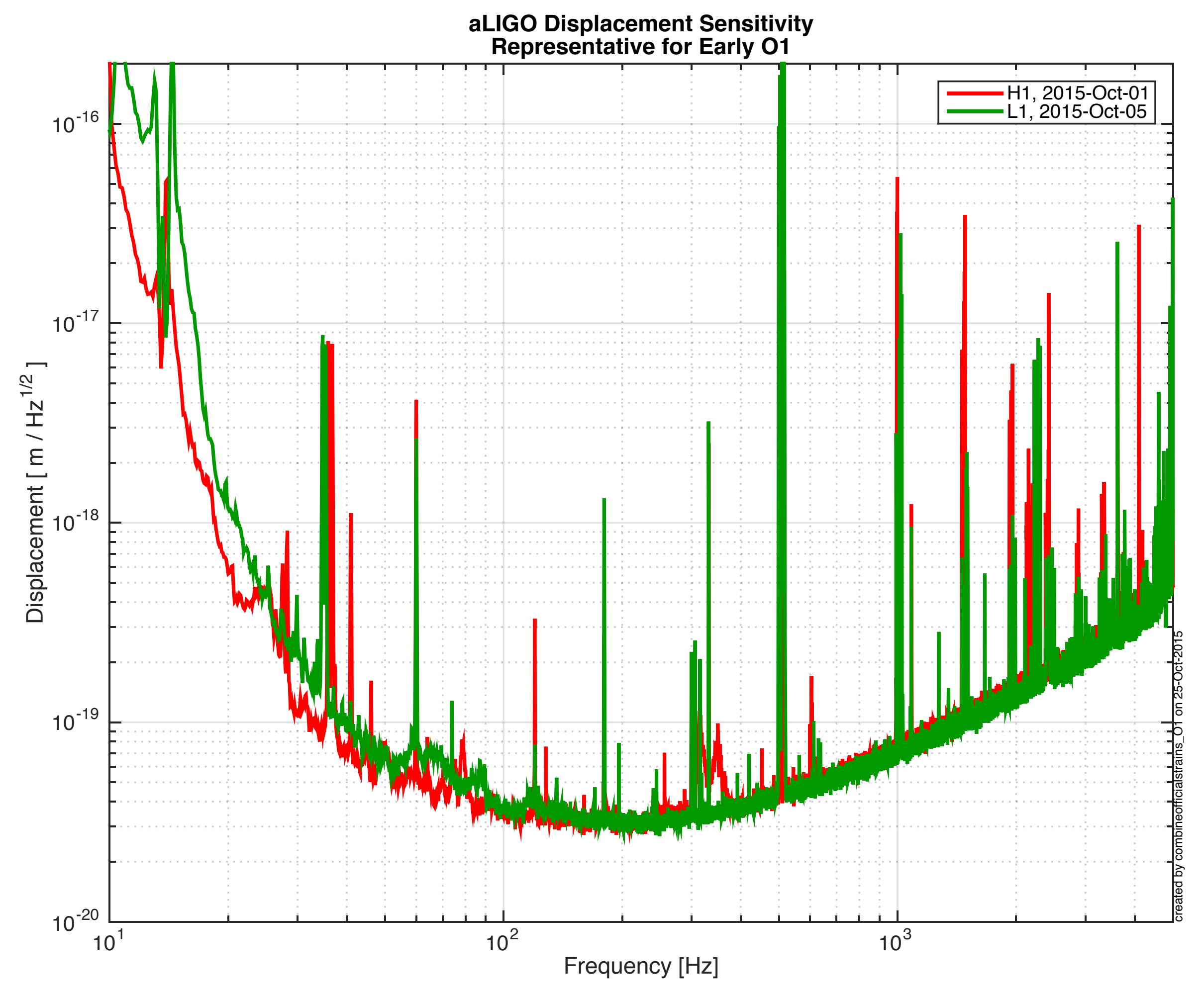
signal



LIGO Livingston

ΔL_{ext} – calibrated interferometer signal

- Goal: accurate reconstruction of the ΔL_{ext}



Required calibration accuracy

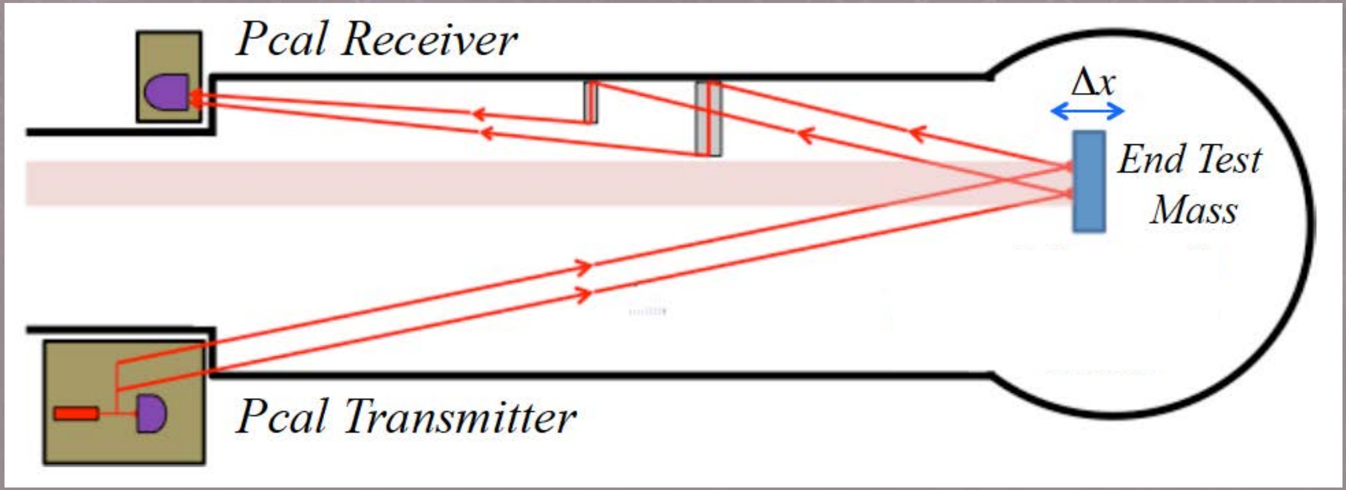
- For ***Detecting*** of the GW signal from a binary black-hole with a signal to noise ratio of 8 the required 1σ errors must be *under 3.5%* in magnitude [1];
- For *Measuring* the parameters of the GW source - *0.35%*;
- The calibration goal for operations in 2015 was set to 9% in amplitude and 5° in phase [2].
- Photon Calibrator is used as a calibration tool in the Advanced LIGO.

1. Lindblom, Lee “Optimal calibration accuracy for gravitational-wave detectors” Physical Review D 80.4 (2009): 042005
2. LIGO-T1300950, Calibration Group, “Calibration uncertainty budget requirements for early aLIGO” (2013)

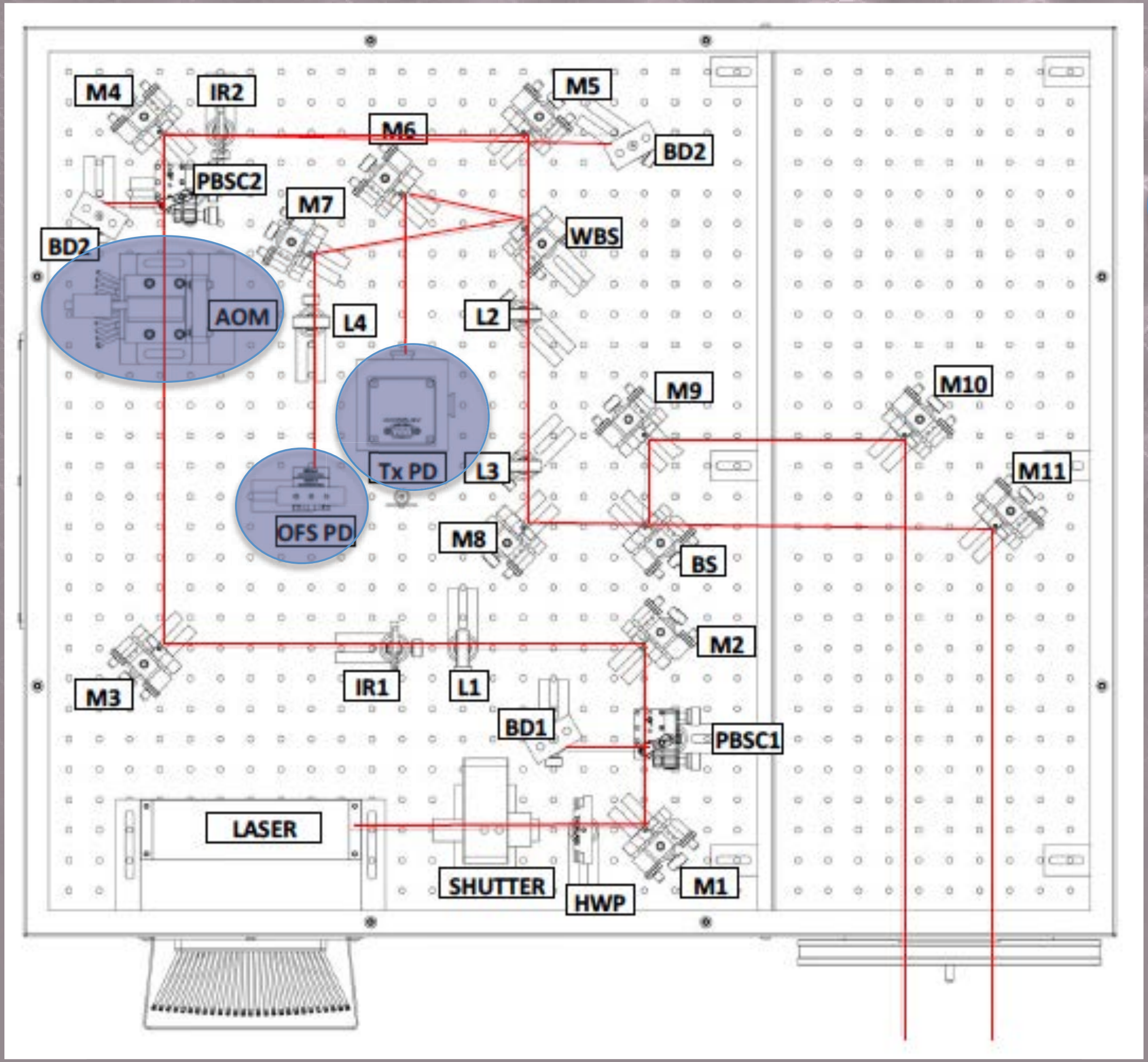
Photon radiation pressure

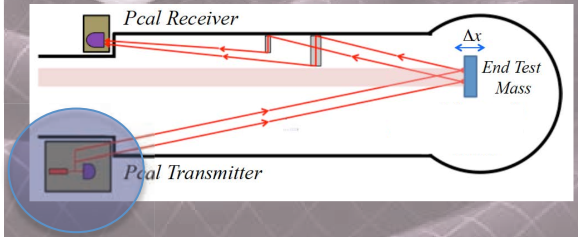
- A force exerted on a surface due to photon radiation pressure can be expressed as: $F(t) = \frac{2cos\theta}{c} \cdot P(t)$

- A displacement of a free swinging end test mass (mirror) due to radiation pressure is $\Delta x_{pcal}(\omega) = -\frac{|F(\omega)|}{M\omega^2} = -\frac{2|P(\omega)|cos\theta}{M\omega^2 c}$



Photon Calibrator transmitter module layout



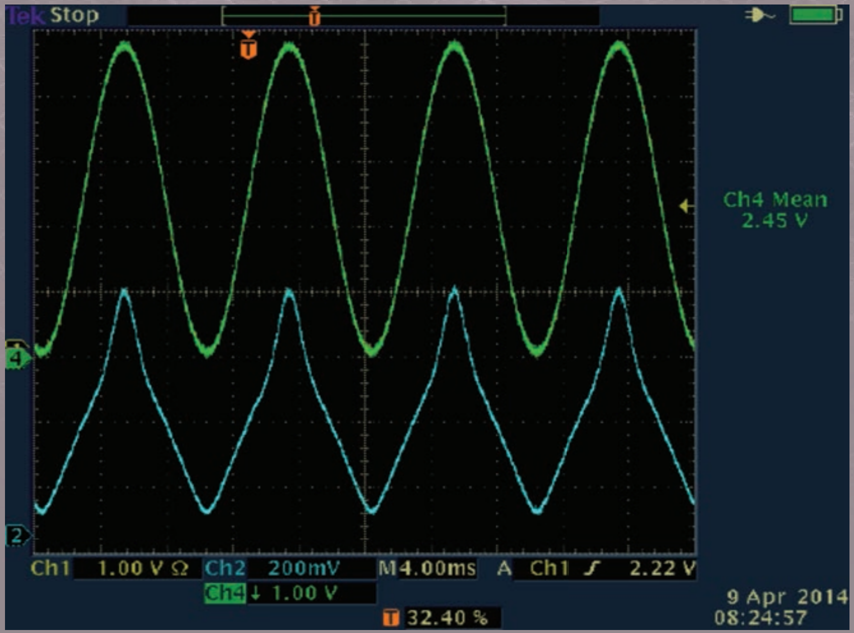


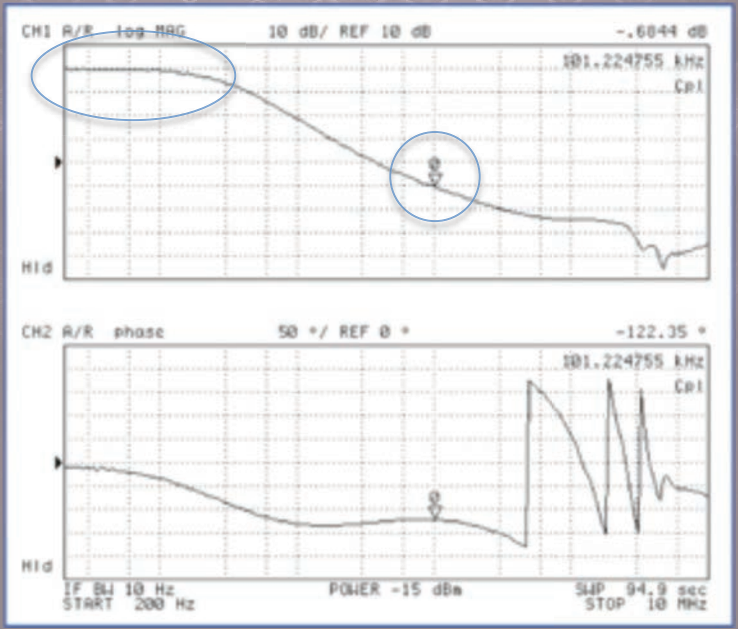
Photon calibrator optical follower servo

Photon calibrator system uses an optical follower servo (OFS) that ensures a linear response to the requested actuation:

- The OFS servo controls amount acousto-optic modulator (AOM) drive level;
- Unity gain frequency of the OFS is 100 kHz and the phase margin is 58 degrees;
- The OFS control loop has a 50 dB gain at low frequencies.

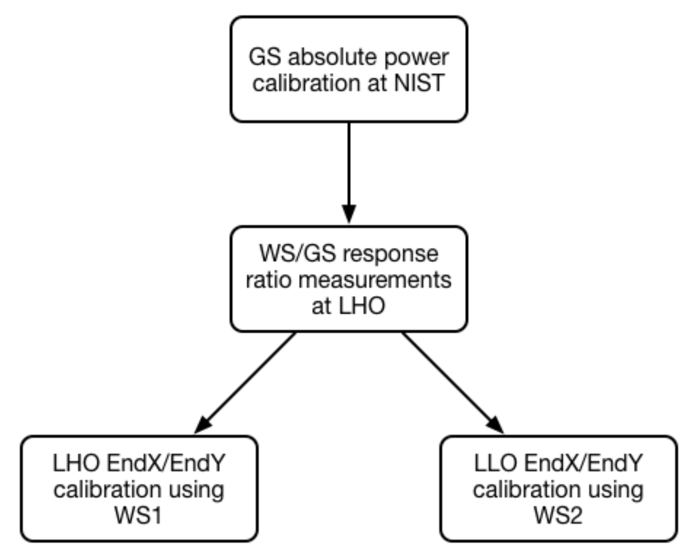
Legend: requested waveform (yellow), output waveform (green), AOM drive level.





Calibration Standards, GS calibration at NIST

- Calibration standards are an integrating sphere based photodetector assemblies;
- The Gold Standard (GS), Working Standards (WS*n*);
- The GS is calibrated at NIST once a year.

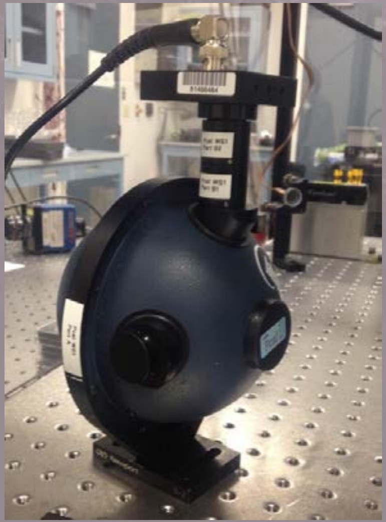


GS absolute power calibration at NIST

WS/GS response ratio measurements at LHO

LHO EndX/EndY calibration using WS1

LLO EndX/EndY calibration using WS2

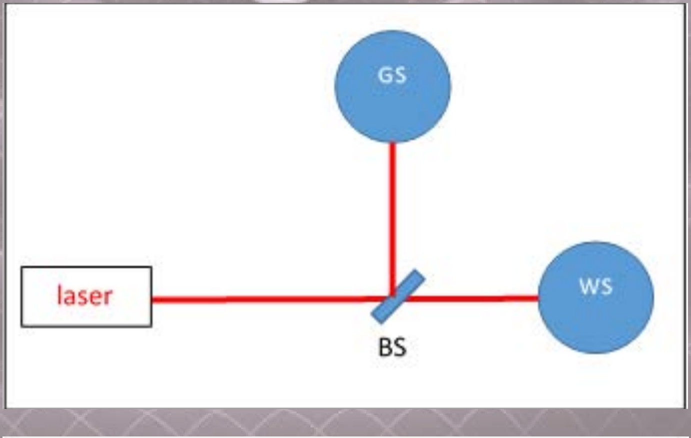


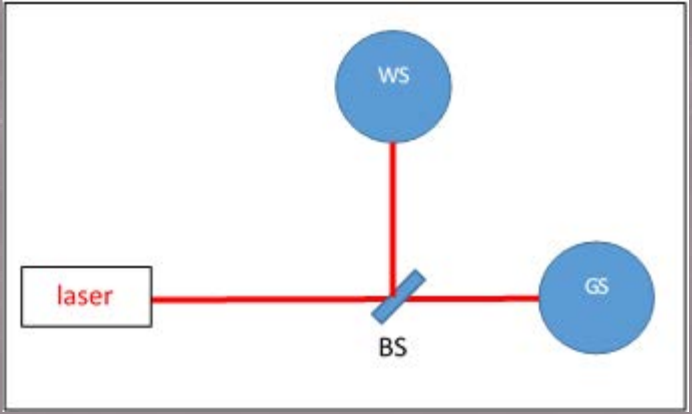
WS / GS response ratio

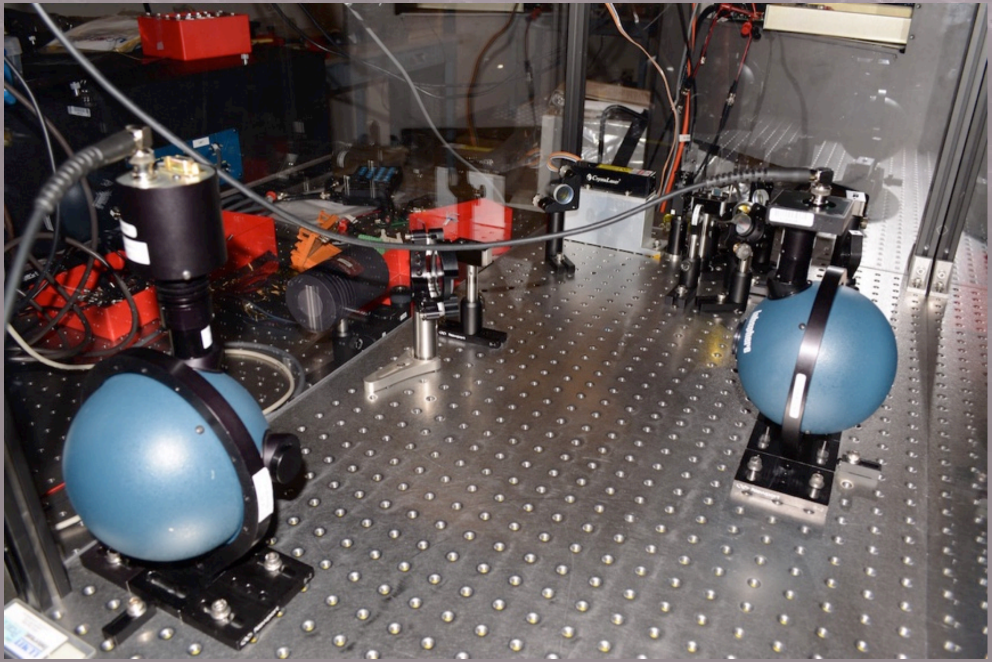
This procedure is done by taking measurements in two configuations:

1) WS at transmitted, GS at reflected beam;

2) WS at reflected, GS at transmitted beam.



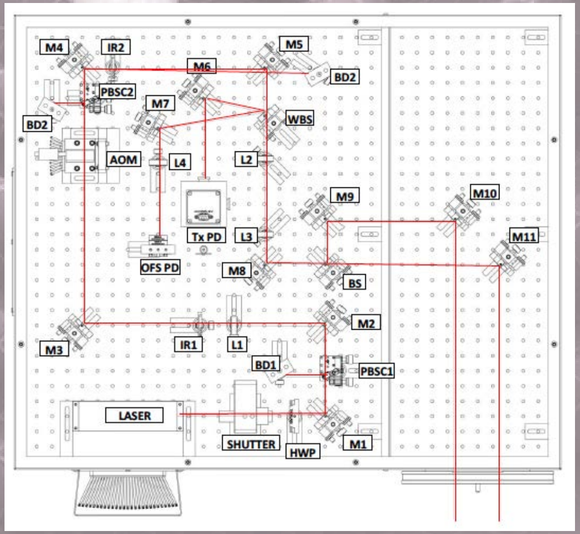


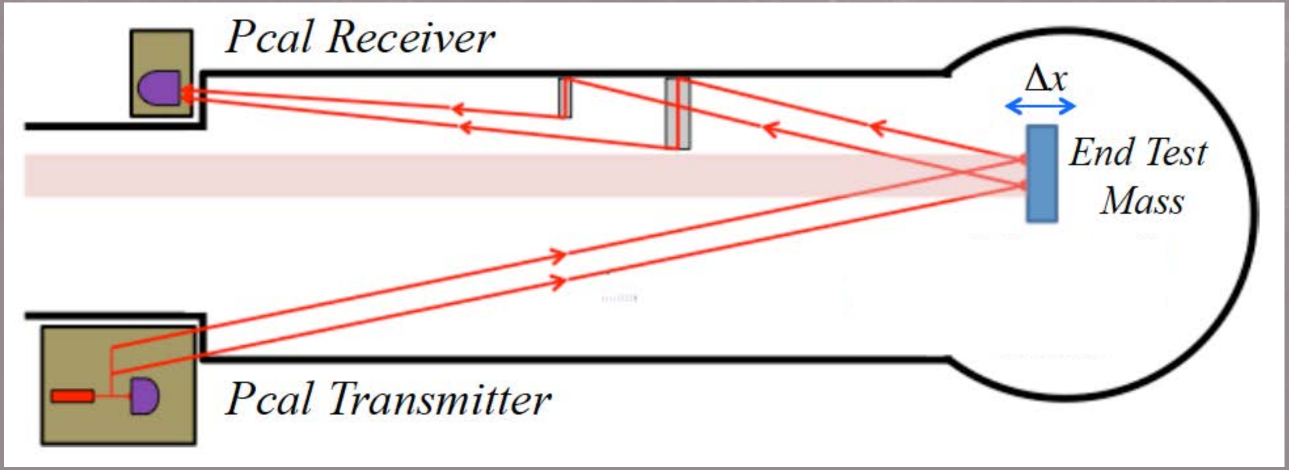


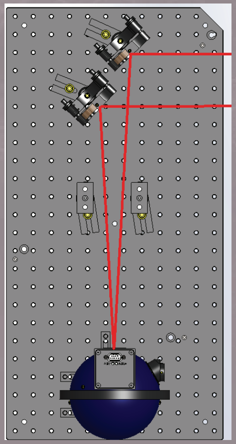
System calibration, N/V

The measurement procedure at the end-station includes taking a set of measurements with a WS from which we obtain:

- Calibrate transmitter and receiver module photodetectors;
- Measure overall optical efficiency of the Photon calibrator (power losses between transmitter and receiver modules).







Beam localization: centering of the beams

Calibration errors could be introduced due to not well centered Pcal beams [1]:

$$x_{\text{tot}}(\omega) \simeq -\frac{2P_m \cos\theta}{Mc\omega^2}\left(1 + \frac{M}{I}\vec{a}\cdot\vec{b}\right)$$

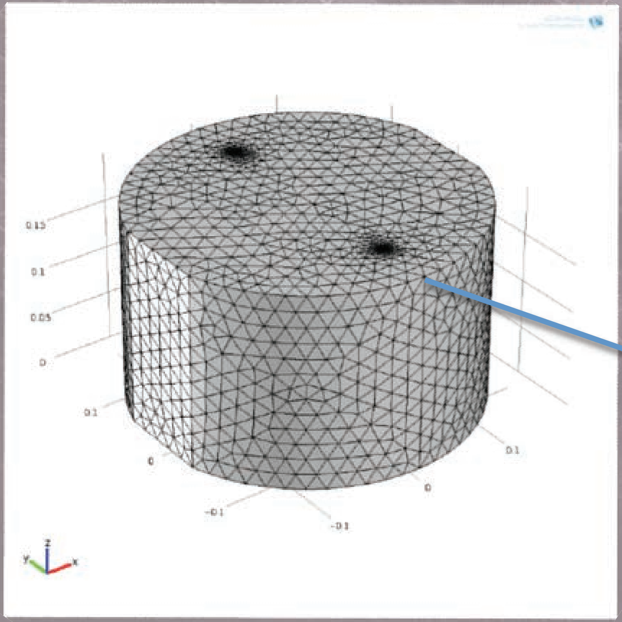
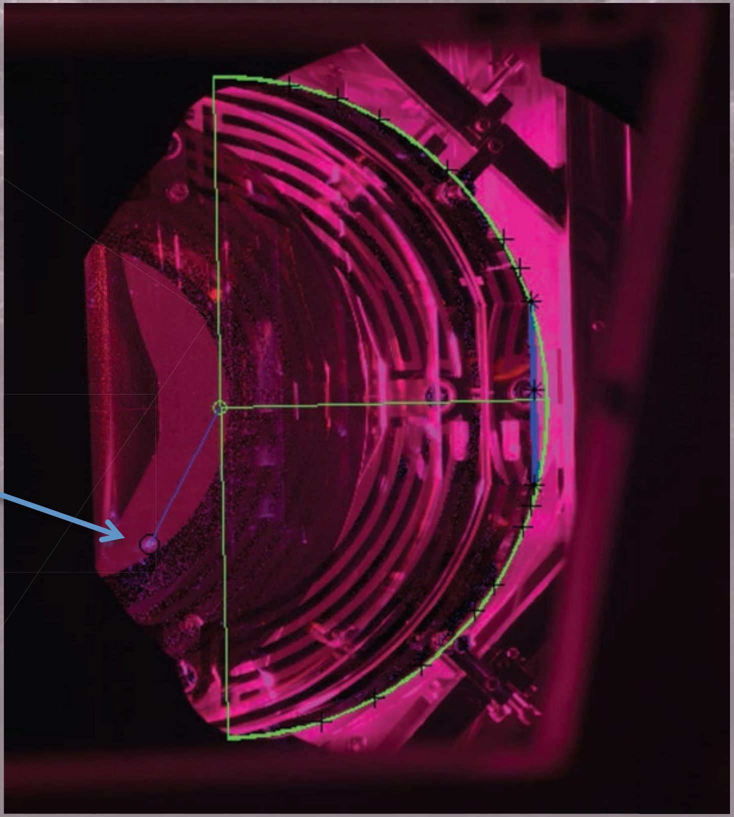


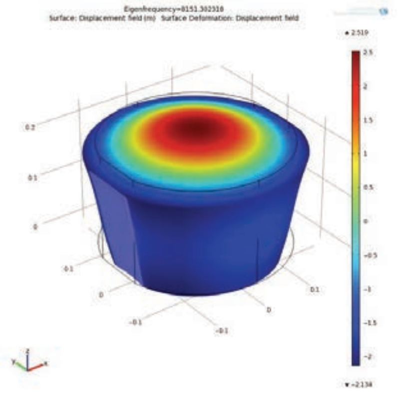
Image credit: ETM deformation simulations [2]



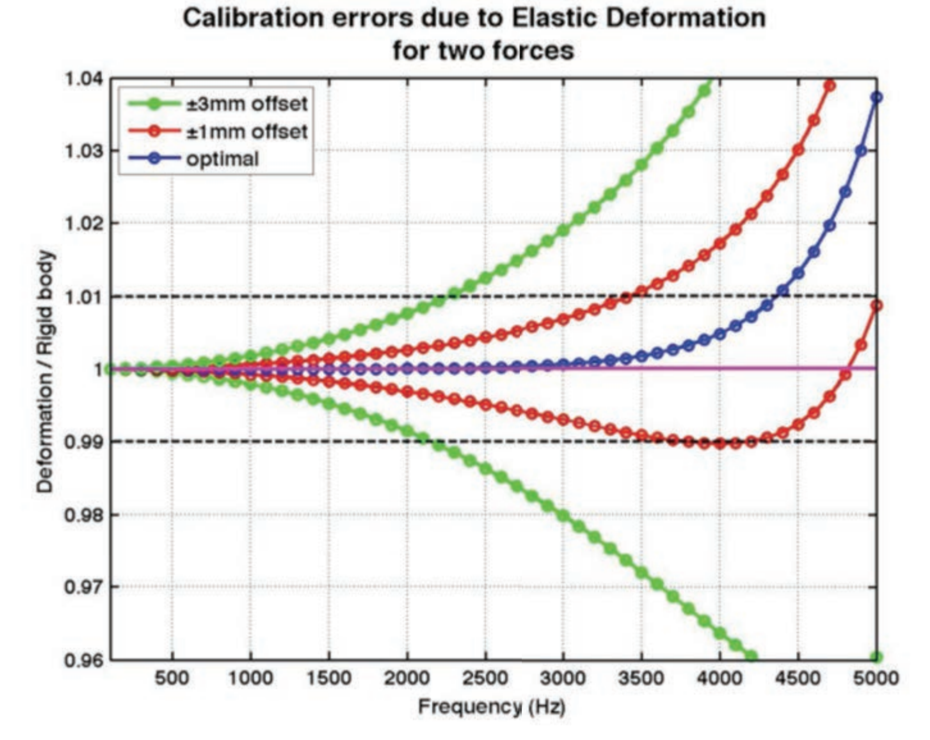
1. Goetz, E., et al. *Classical and Quantum Gravity* 26.24 (2009): 245011.
2. Daveloza, H. Pablo, et al. Journal of Physics: Conference Series. Vol. 363. No. 1. IOP Publishing, 2012.

Beam localization: bulk elastic deformations

- Mispositioned Pcal beams -> bulk elastic deformations -> calibration errors



Drumhead mode of the ETM

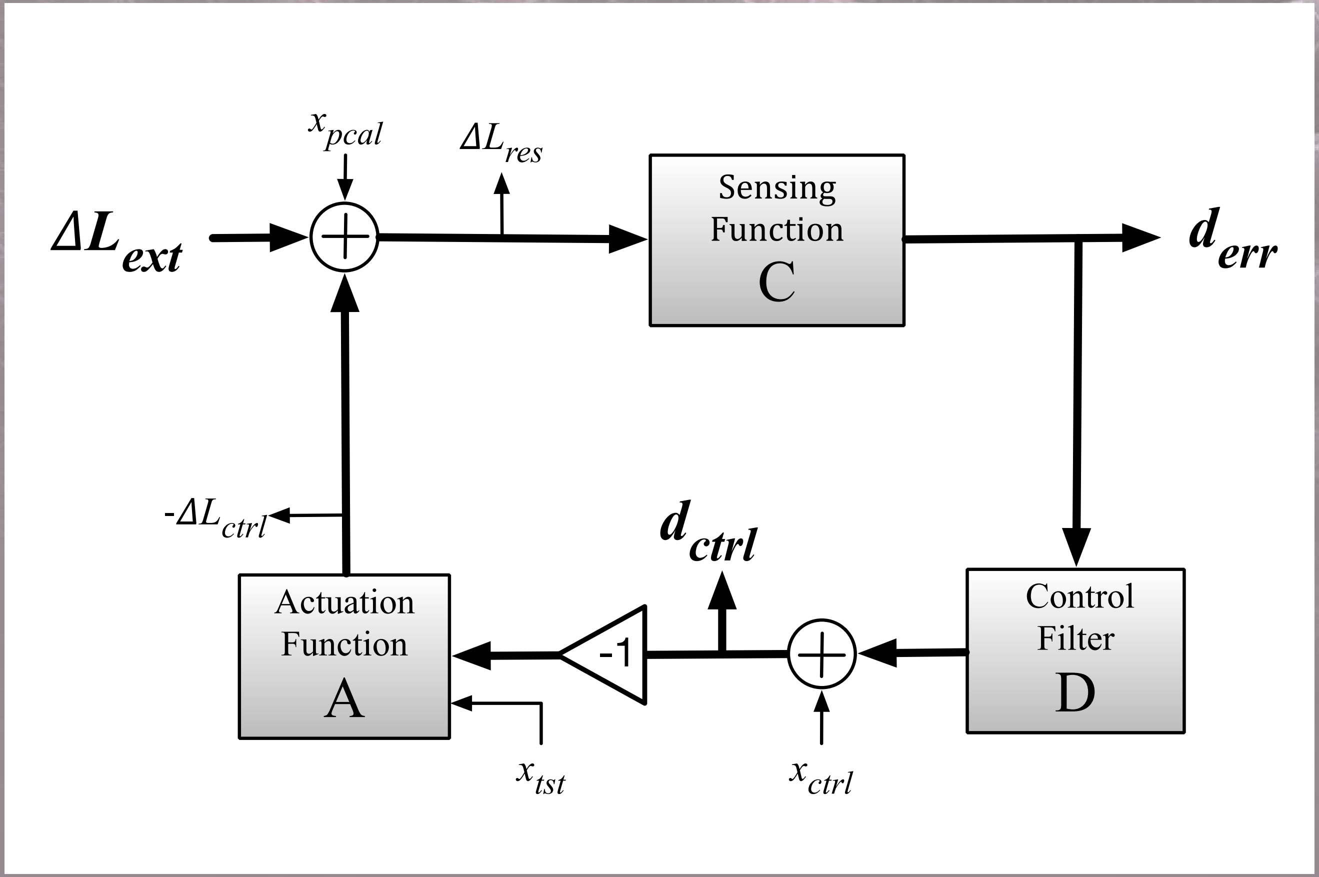


1. Daveloza, H. Pablo, et al. Journal of Physics: Conference Series. Vol. 363. No. 1. IOP Publishing, 2012.

Modeling differential arm length (DARM) control loop

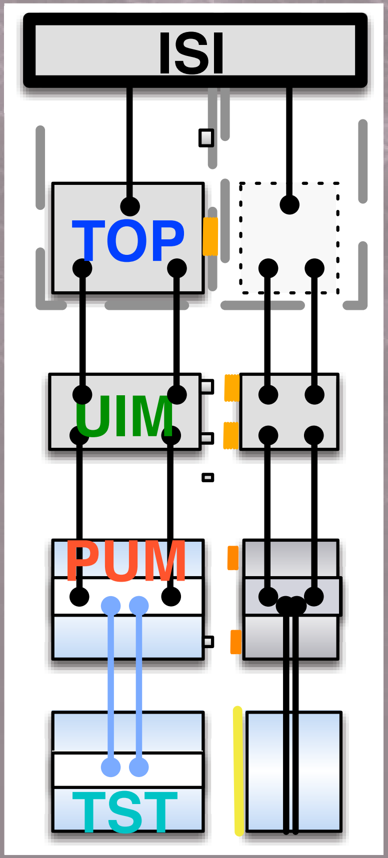
$$h(t) = \frac{\Delta L_{ext}(t)}{L}$$

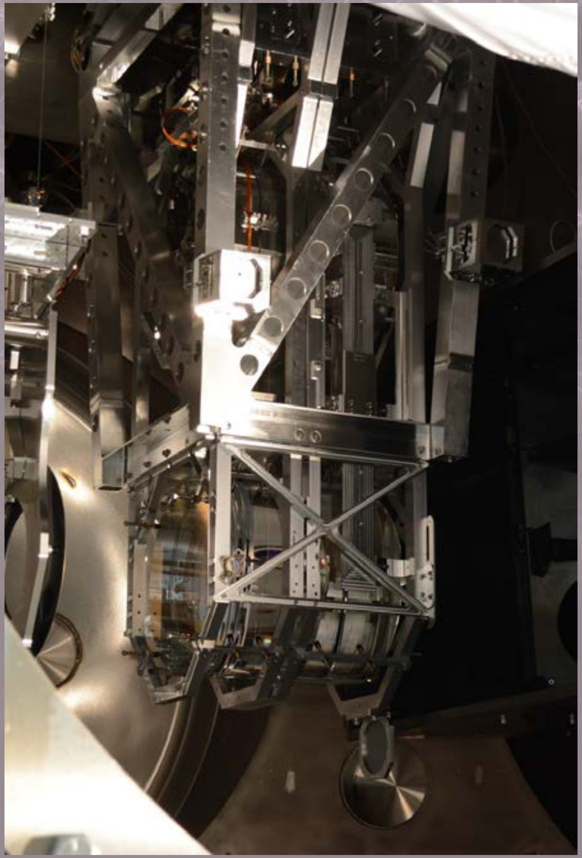
$$\Delta L_{ext}(t) = \Delta L_{res} + \Delta L_{ctrl} = \frac{d_{err}(t)}{\boxed{C(t)}} + d_{ctrl}(t)\boxed{A(t)}$$



DARM actuation function details

DARM actuation is done through the actuating the stages of the quadruple pendulum.



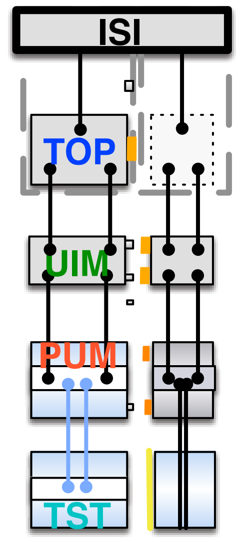


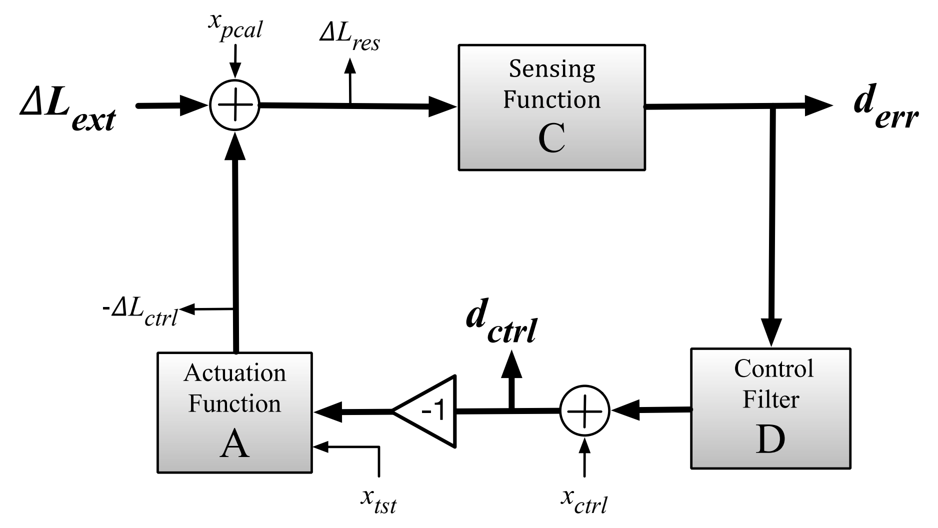
DARM model

Actuation and sensing function models are created in Matlab. This model takes into account:

- Interferometer cavities' frequency response;
- Light travel time in a 4km long arm;
- Electronics transfer functions (actuation and sensing);
- Anti-aliasing, anti-imaging electronics transfer functions;
- Signal up/downsampling filters;
- Digital time delays (signal processing computer cycles);

Temporal variations in the DARM





$$A(t) = \kappa_{tst}(t) A_0^{tst} + \kappa_{pu}(t) A_0^{pu}$$

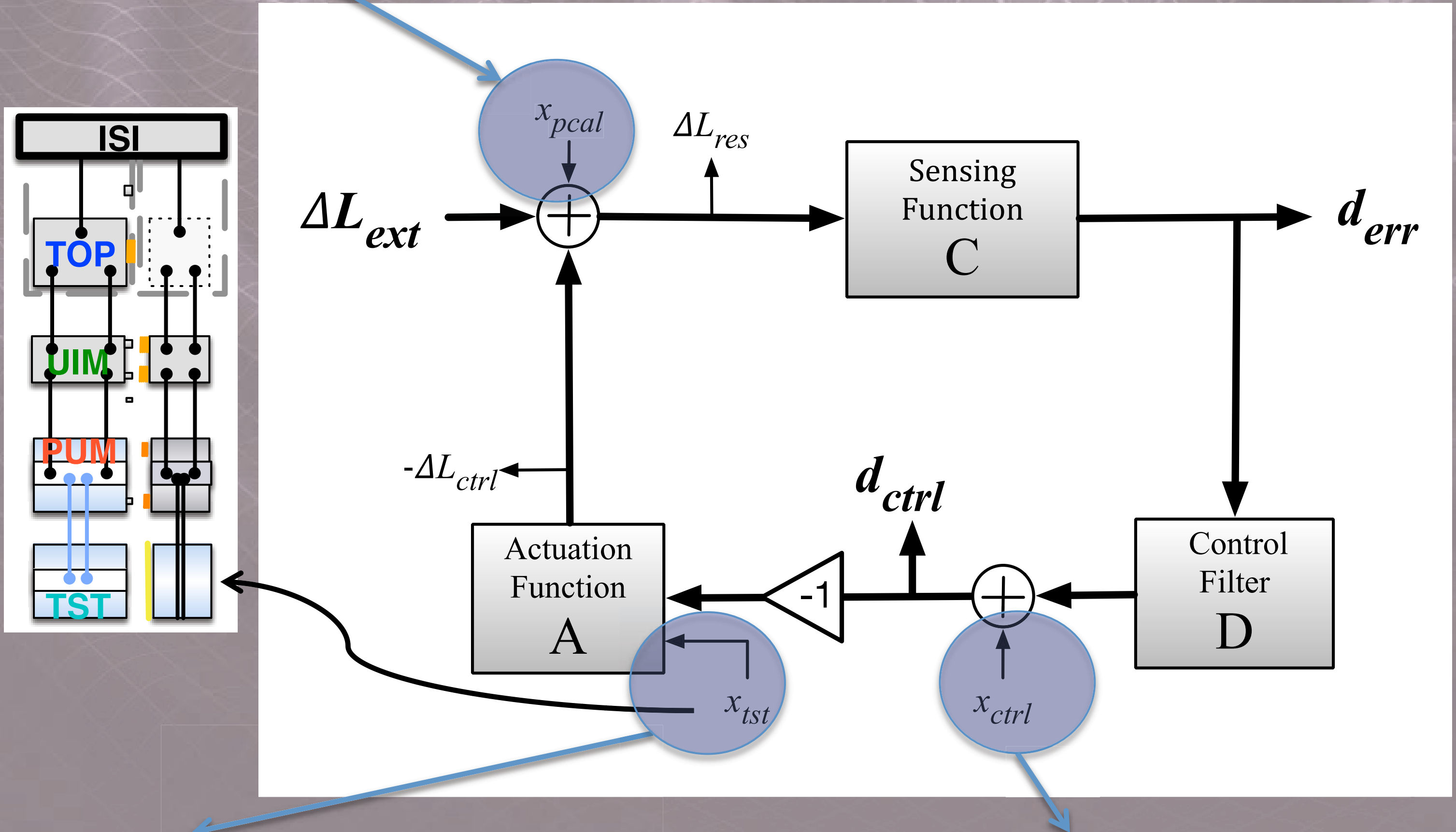
$$C(t) = \kappa_C(t) \underbrace{\frac{C_{res}}{1 + if/f_c(t)}}_{C_0 \text{ when } f_c = f_c(t_0)}$$

$$C_{res} = C_0 \left(\frac{1}{1 + if/f_c(t_0)} \right)^{-1}$$

$$G(t) = C(t)\ D_0\ A(t) = \frac{\kappa_C(t)}{1 + if/f_c(t)}\ C_{res}\ D_0 \left[\kappa_{tst}(t) A_0^{tst} + \kappa_{pu}(t) A_0^{pu} \right]$$

Tracking temporal variations: cal. line injection points

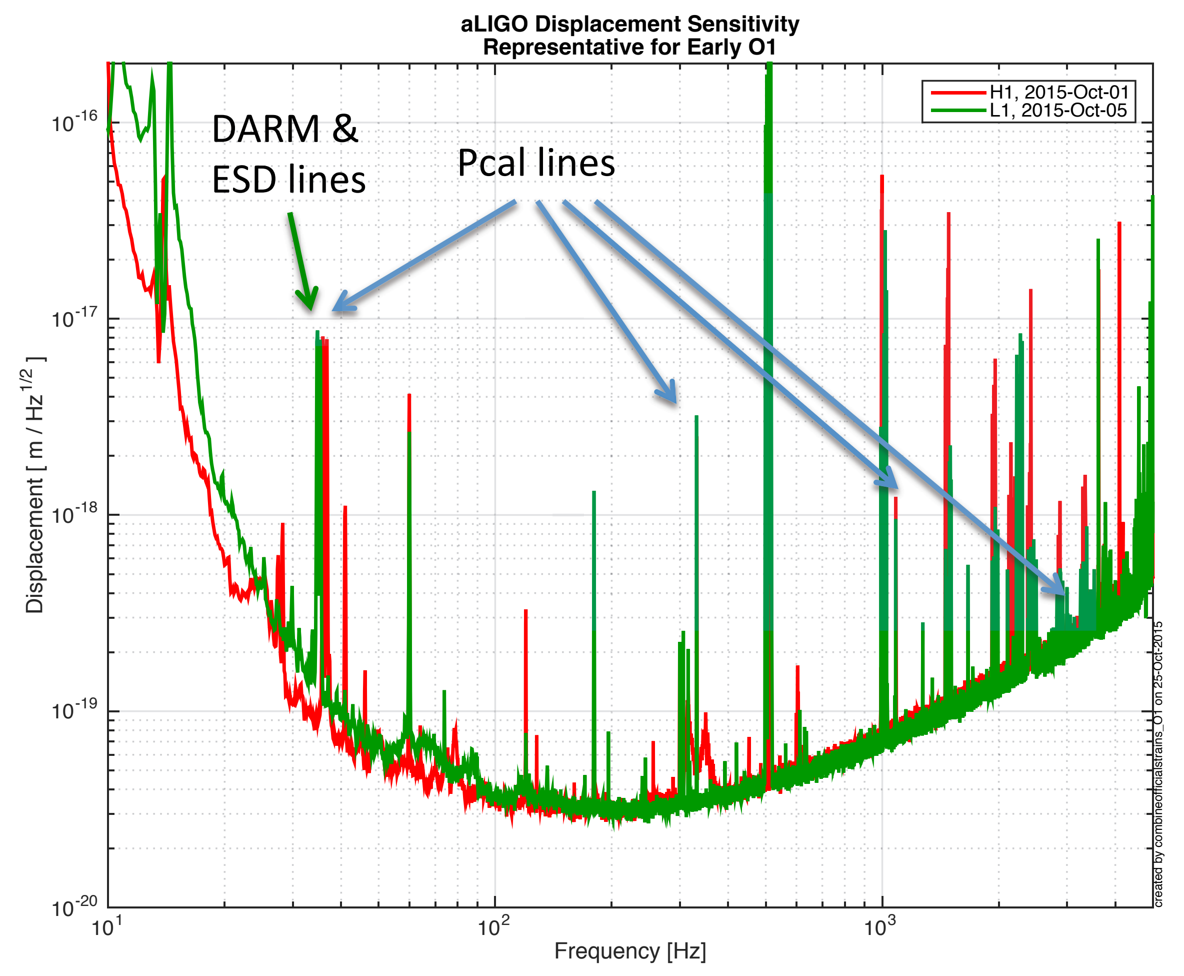
$$\tilde{d}_{err}(f_{pcal}) = \frac{C(f_{pcal})}{1+G(f_{pcal})} \cdot \tilde{x}_{pcal}(f_{pcal})$$



$$\tilde{d}_{err}(f_{tst}) = \frac{C(f_{tst})}{1+G(f_{tst})} \cdot \kappa_{tst}\, A_0^{tst}(f_{tst})\, \tilde{x}_{tst}(f_{tst})$$

$$\tilde{d}_{err}(f_{ctrl}) = -\frac{C(f_{ctrl})}{1+G(f_{ctrl})} \cdot A(f_{ctrl})\, \tilde{x}_{ctrl}(f_{ctrl})$$

Calibration lines in the DARM spectrum



DARM & ESD lines

Pcal lines

Tracking and application of the parameters

$$\kappa_{tst} = \frac{1}{A_0^{tst}(f_{tst})} \frac{\widetilde{d}_{err}(f_{tst})}{\widetilde{x}_{tst}(f_{tst})} \left(\frac{\widetilde{d}_{err}(f_{pcal})}{\widetilde{x}_{pcal}(f_{pcal})} \right)^{-1} \frac{C_0(f_{pcal})}{1+G_0(f_{pcal})} \left(\frac{C_0(f_{tst})}{1+G_0(f_{tst})} \right)^{-1}$$

$$\kappa_{pu} = \frac{1}{A_0^{pu}(f_{ctrl})} \left[A(f_{ctrl}) - \kappa_{tst} A_0^{tst}(f_{ctrl}) \right]$$

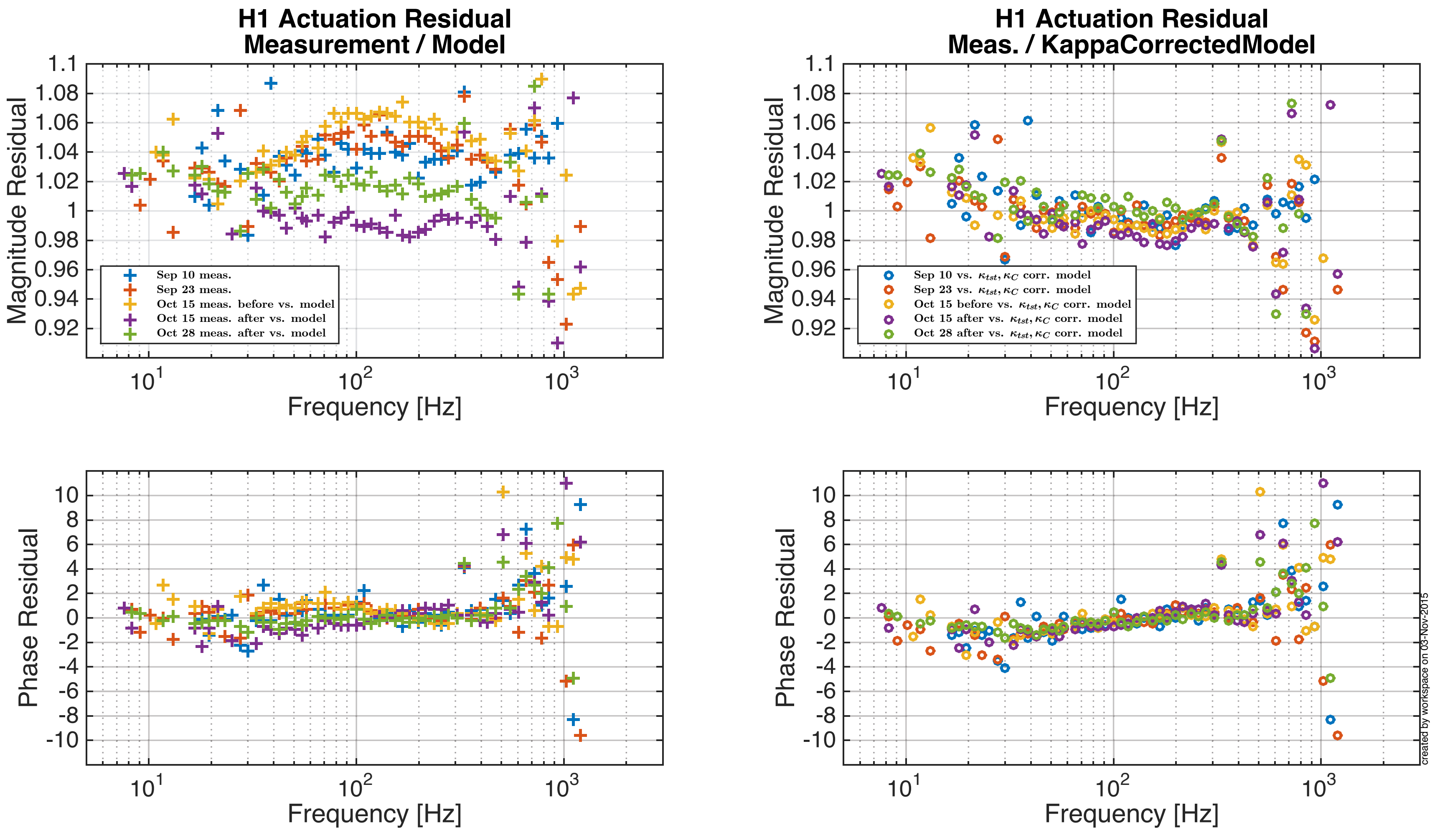
$$\frac{C}{1+i f_{pcal2}/f_c} = \frac{1}{C_{res}(f_{pcal2})} \left(\frac{\widetilde{x}_{pcal}(f_{pcal2})}{\widetilde{d}_{err}(f_{pcal2})} - D_0(f_{pcal2}) \left[\kappa_{tst} A_0^{tst}(f_{pcal2}) + \kappa_{pu} A_0^{pu}(f_{pcal2}) \right] \right)^{-1} \equiv S$$

$$\kappa_C = \frac{|S|^2}{\Re(S)} \qquad f_c = -\frac{\Re(S)}{\Im(S)} f_{pcal2}$$

$$\begin{aligned} \Delta L_{ext}(t) &= \Delta L_{res} + \Delta L_{ctrl} = \frac{d_{err}(t)}{C(t)} + d_{ctrl}(t) A(t) \\ &= \frac{1+if/f_c(t)}{\kappa_C(t) C_{res}} d_{err}(t) + \left(\kappa_{tst}(t) A_0^{tst} + \kappa_{pu}(t) A_0^{pu} \right) d_{ctrl}(t) \end{aligned}$$

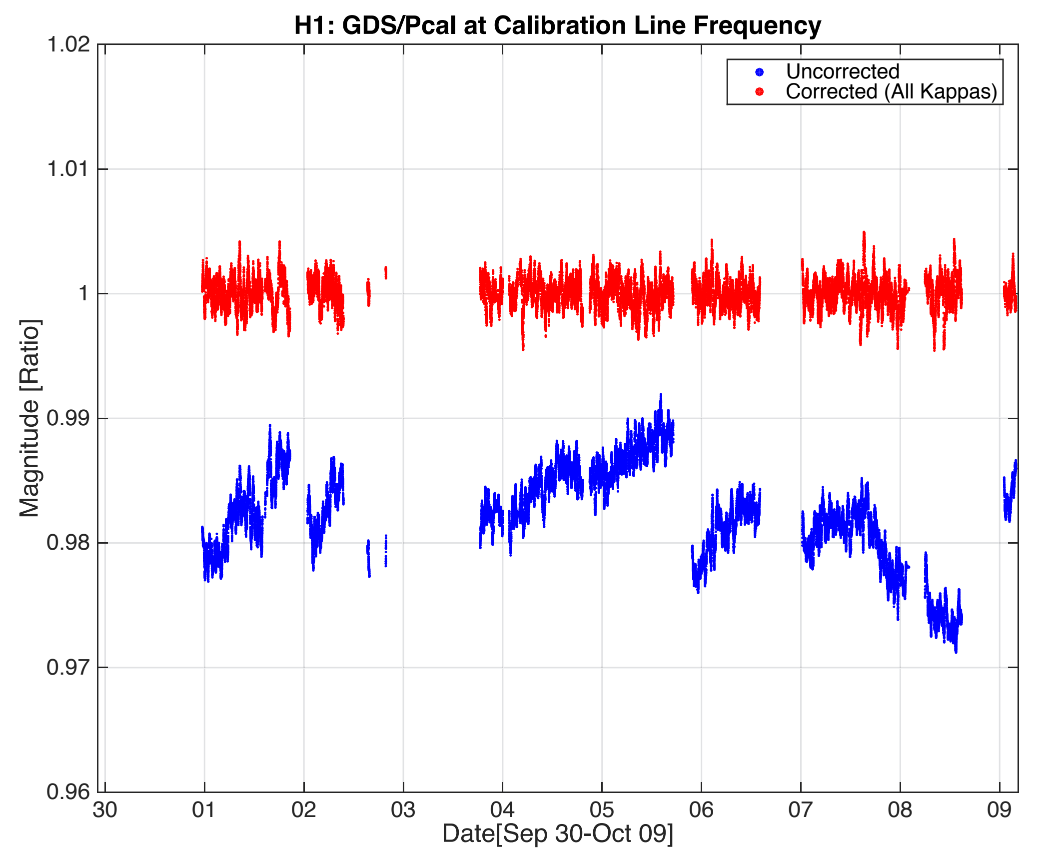
LIGO-T1500377, D. Tuyenbayev, et.al. "Tracking temporal variations in the DARM calibration parameters"

Comparison. Measurements over the model



Recent DARM correction factor trends (all corrections)

Single frequency (line at 332 Hz) check of the calibrated ΔL_{ext} time-series show that applying all parameter corrections highly improve our calibration.

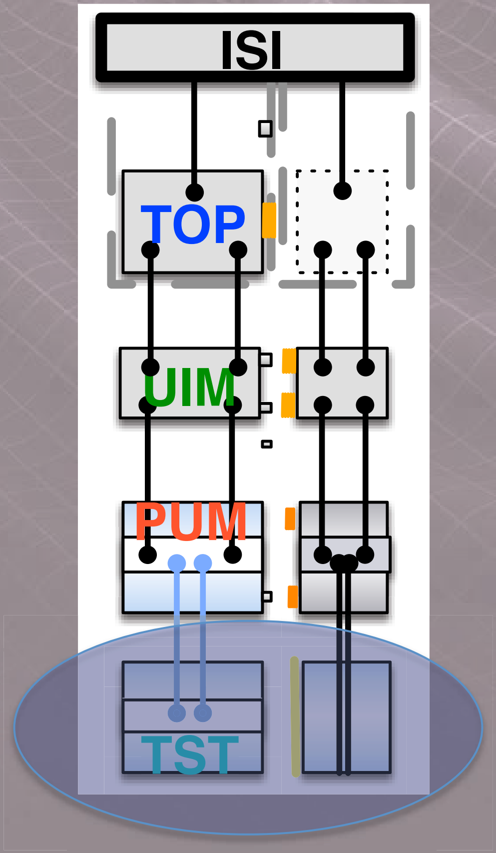


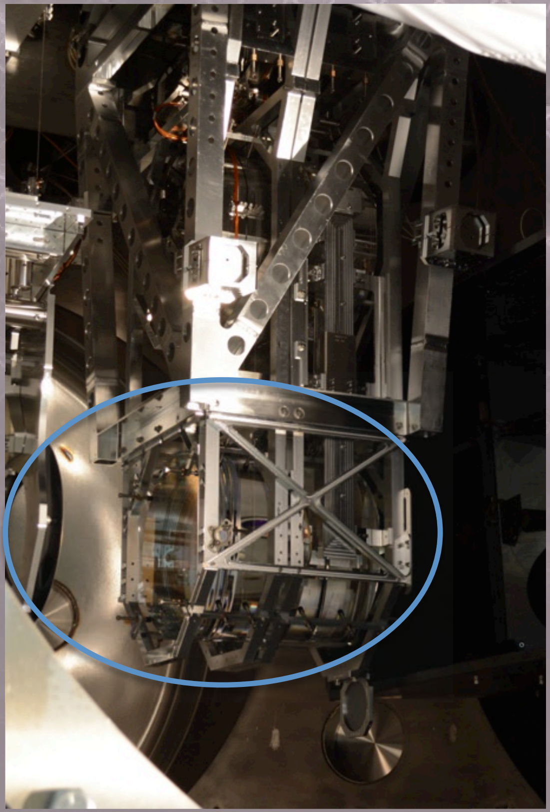
Future research

- Calibration upgrades
  - Applying varying coupled-cavity pole frequency correction
  - apply full length response function
  - Optimizing calibration line frequencies
- DARM actuation using photon calibrators
  - much simpler actuator, no reaction mass
  - may be less noisy actuator
  - possible driver for 3rd generation GW detectors

DARM actuation using photon radiation pressure

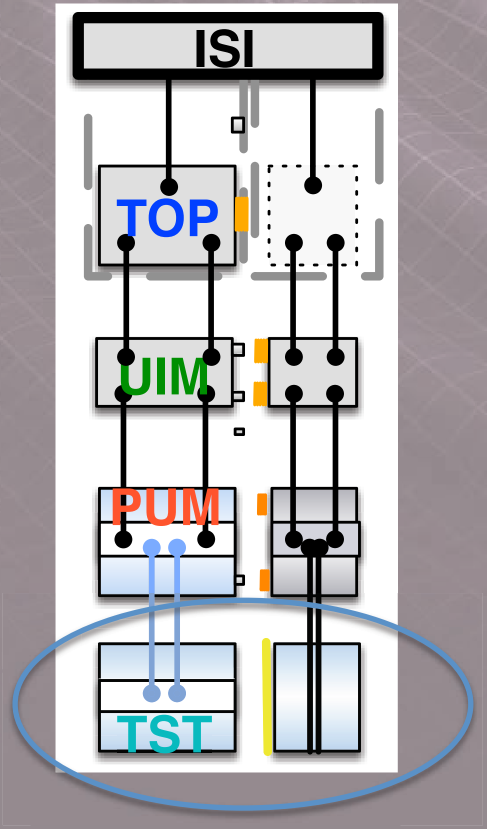
- Currently the ETM is actuated with the electrostatic drive (ESD).
- We propose actuating the ETM using photon radiation pressure.

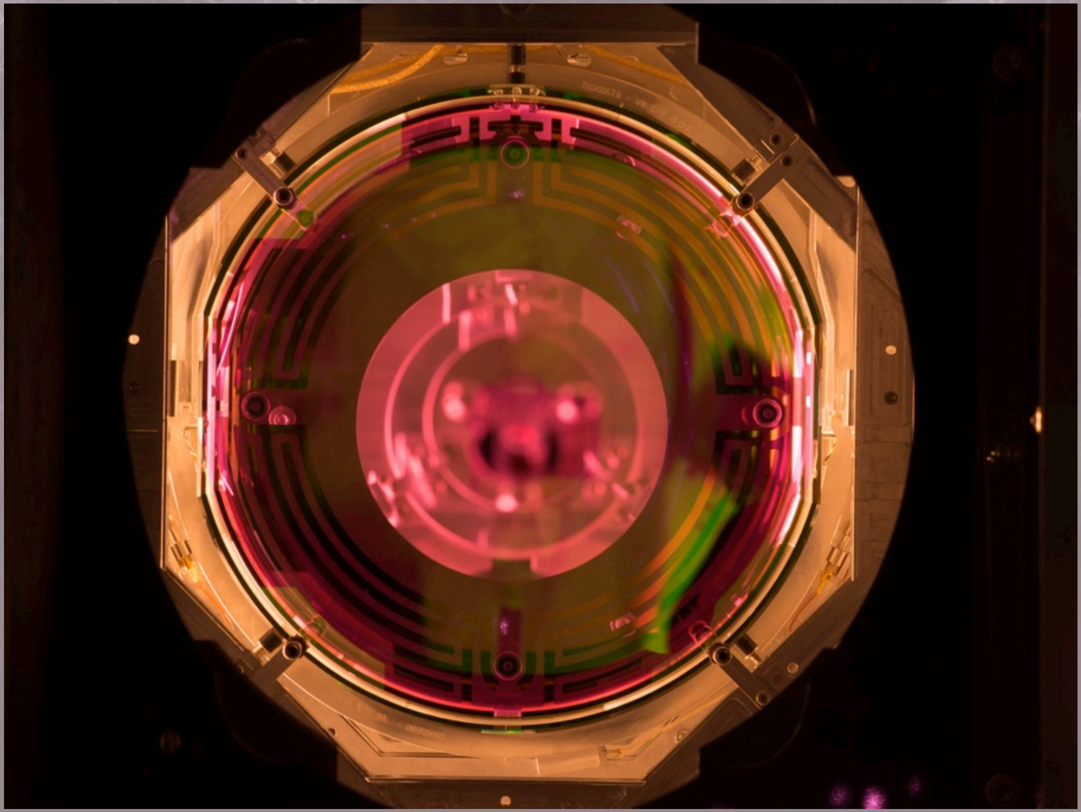




Actuation using photon radiation pressure: simplicity of Photon Calibrator

- The residual gas (in the vacuum) in this gap -> noise
- The ESD driver actuation strength has a slow drift due to charge accumulation on the ETM, this requires flipping the sign of the ESD drive voltage.

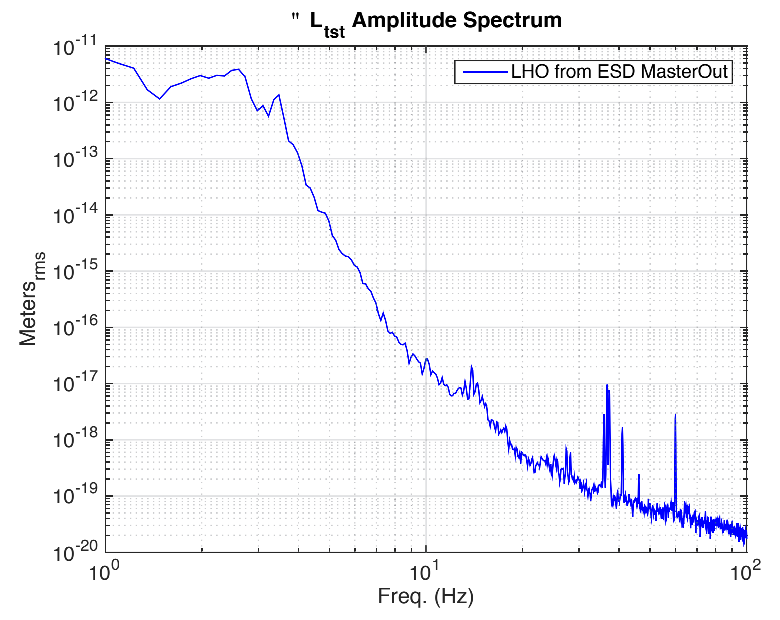


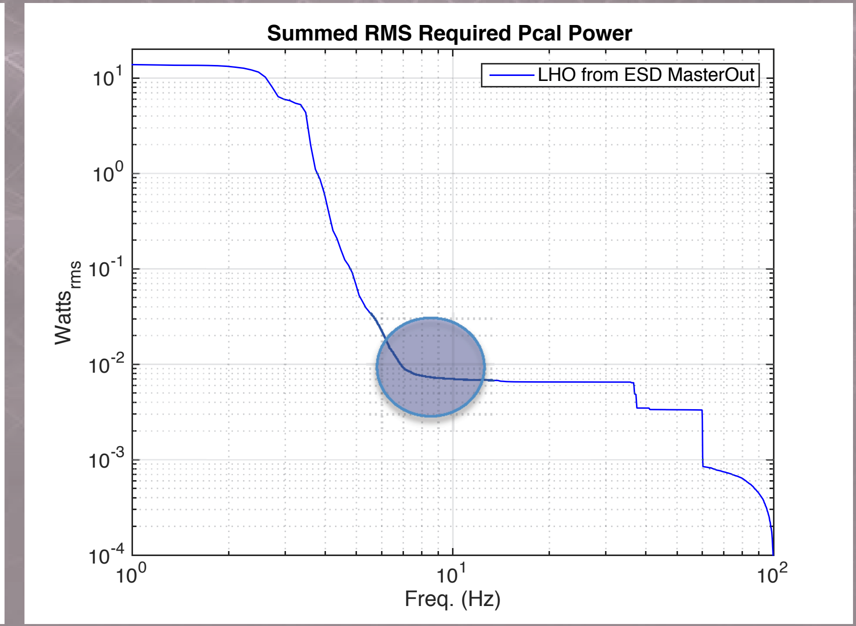


Actuation using photon radiation pressure: required drive level

Displacements of the ETMs induced by the ESD during nominal operation state.

- Photon calibrator laser: 0.5 W modulation power.

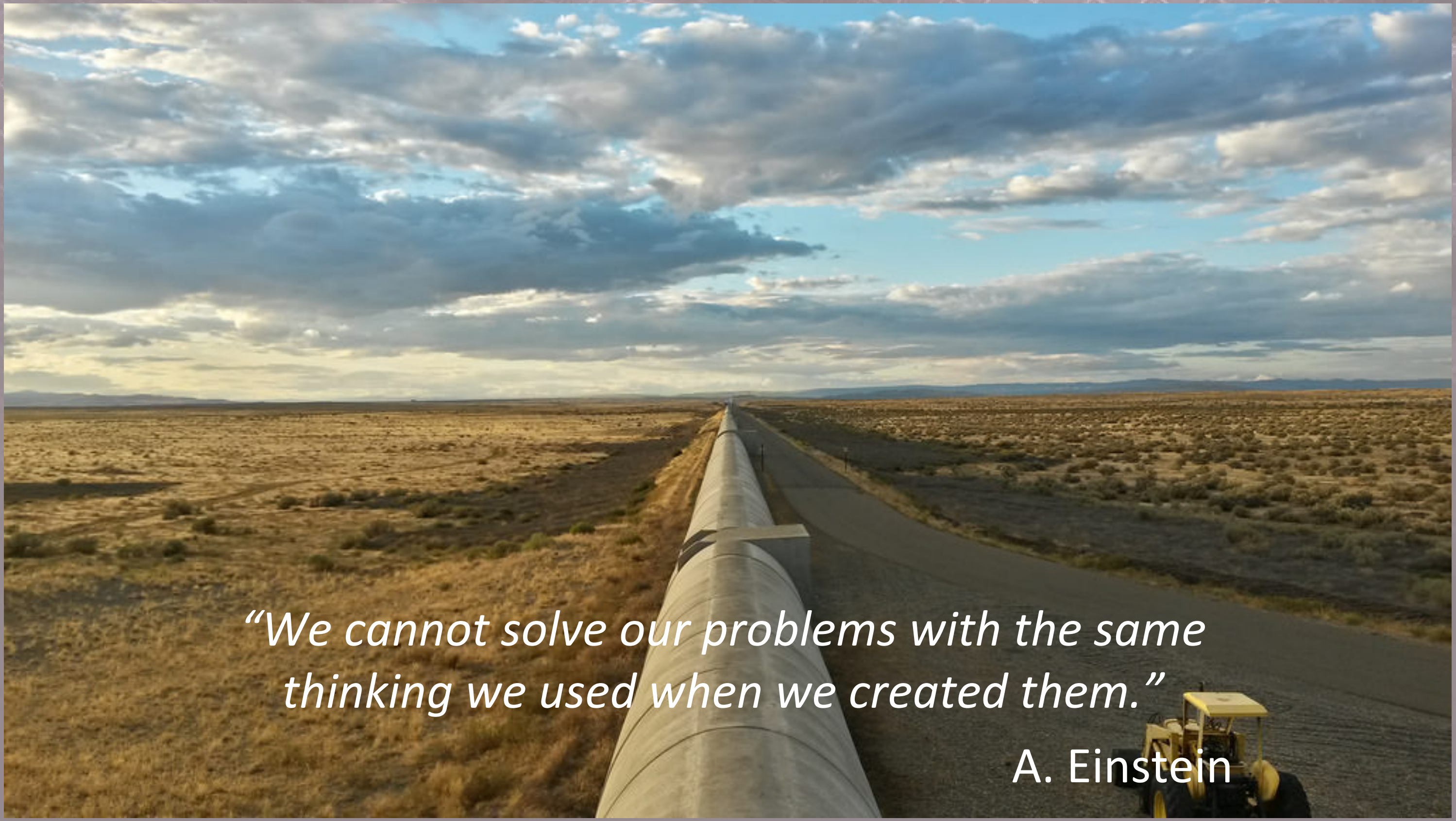




Summary

- Gravitational waves are generated by for example the inspiral and coalescence of binary systems consisting of neutron stars or black holes;
- There has been an indirect confirmation for existence of GWs and LIGO is hoping to directly observe them;
- A DARM feedback-control loop model with time-varying parameters was shown;
- The interferometer data confirm that the ΔL_{ext} reconstructed by using corrected DARM model improves calibration uncertainties;
- Future work: study topics for improving calibration of the LIGO interferometers were discussed;
- Future work: using photon radiation pressure for controlling an interferometer will be considered for the future scientific runs of LIGO interferometers.

Thanks for your attention!



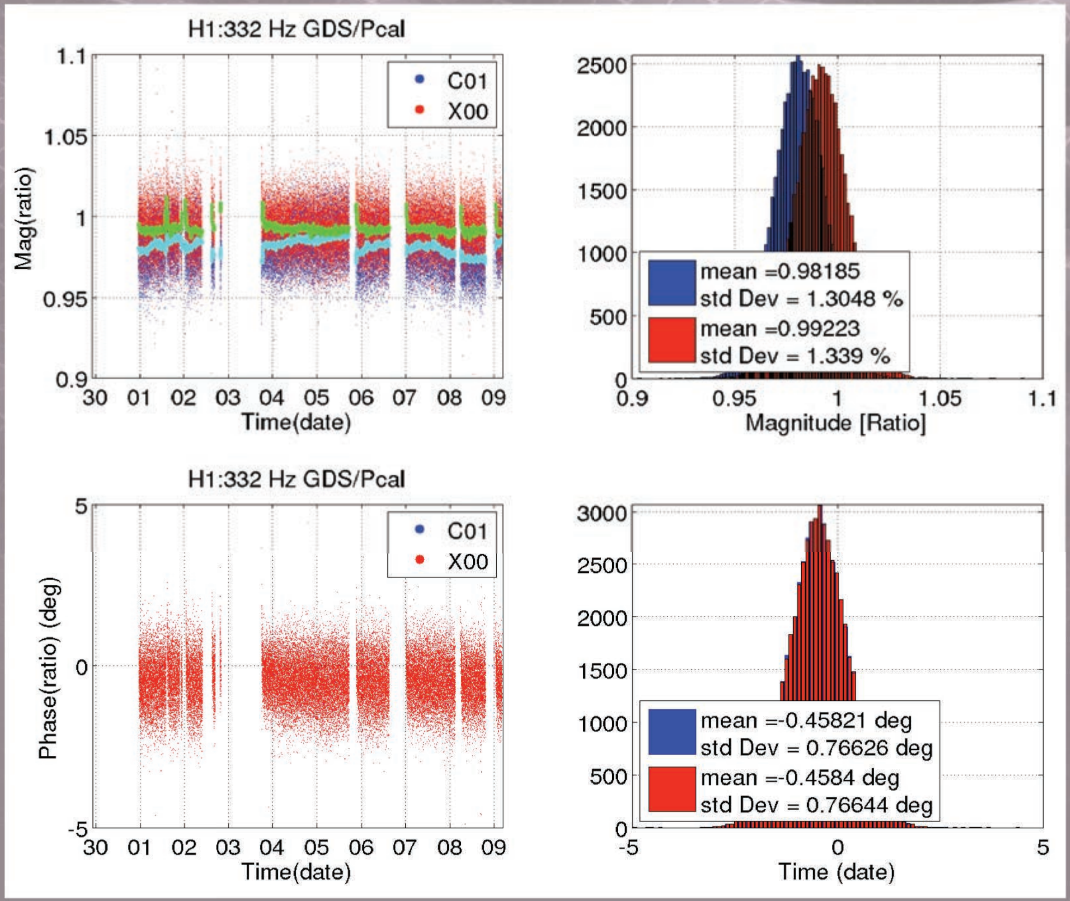
“We cannot solve our problems with the same thinking we used when we created them.”

A. Einstein

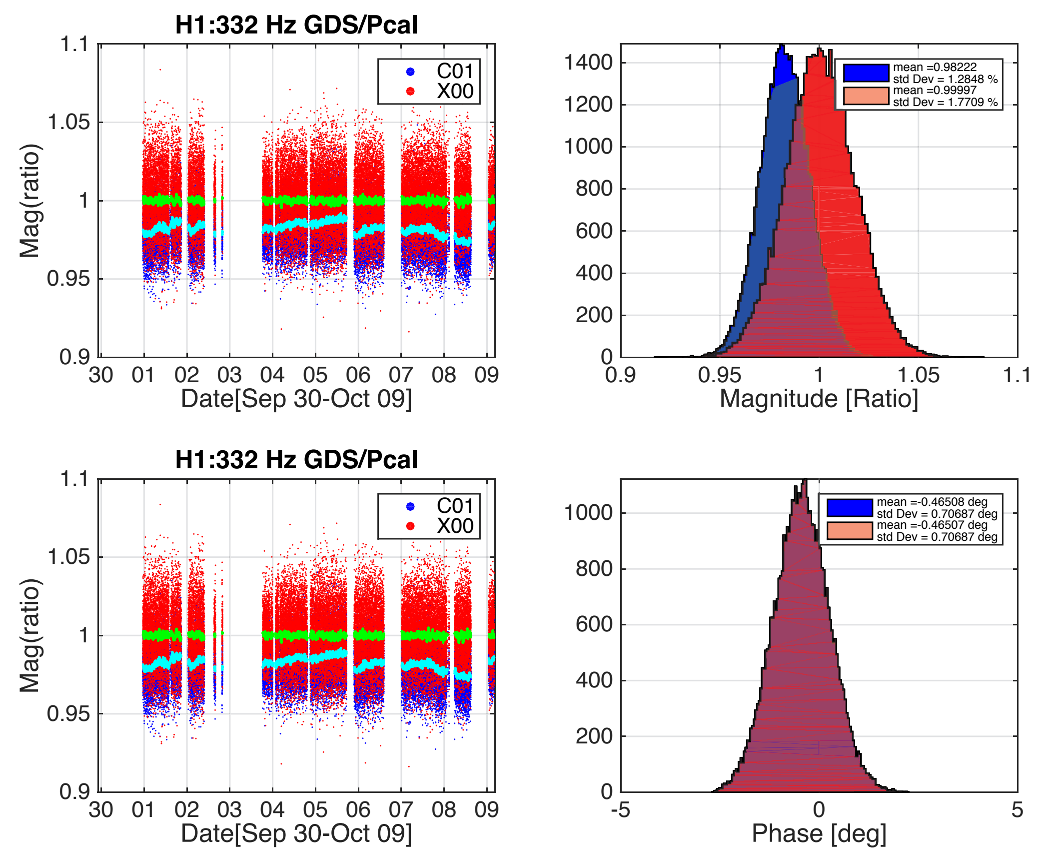
Future work. Applying varying coupled-cavity pole correction

- Our studies show that applying time-varying coupled cavity pole response function improves uncertainties of our calibration;
- Applying κ_{pu}, κ_{tst} and κ_C is essentially applying a gain factor to a particular output signal;
- In the low-latency pipeline a frequency dependent function is applied with the use of a finite-impulse response (FIR) filter;
- At the point it is not so clear how a new set of coefficients for FIR filter should be calculated for a new interferometer response due to different CC pole frequency;
- How to smoothly replace old FIR coefficients with the new ones.

Correction factors (2x2 plot)



Correction factors (2x2 plot)



$$V_{GSR}(t) = P(t)R_{BS}\rho_{GS}$$
$$V_{WST}(t) = P(t)T_{BS}\rho_{WS}$$

$$r_1 = \frac{V_{WST}}{V_{GSR}} = \frac{T_{BS}\rho_{WS}}{R_{BS}\rho_{GS}}$$

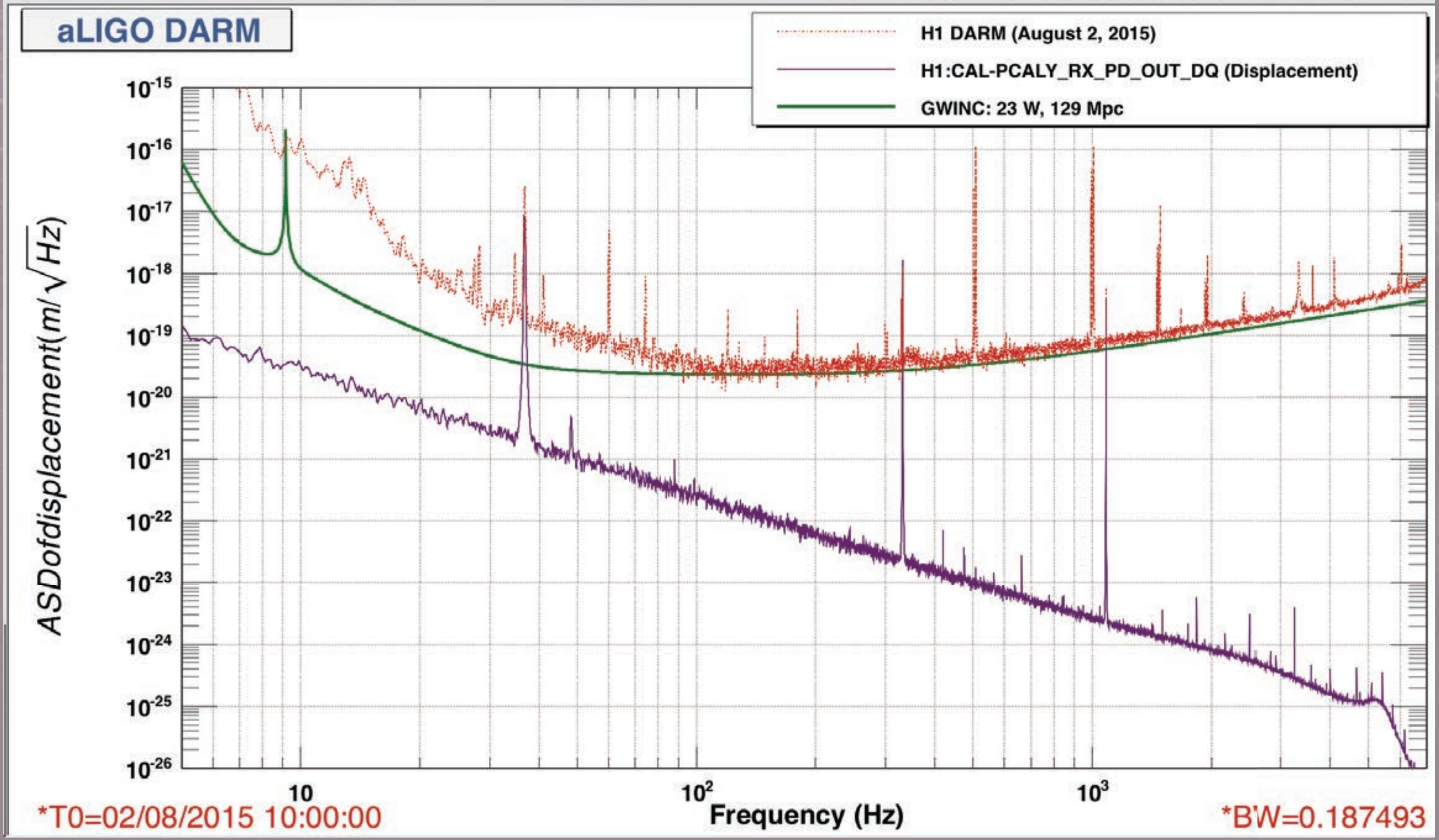
$$V_{GSR}(t) = P(t)R_{BS}\rho_{GS}$$
$$V_{WST}(t) = P(t)T_{BS}\rho_{WS}$$

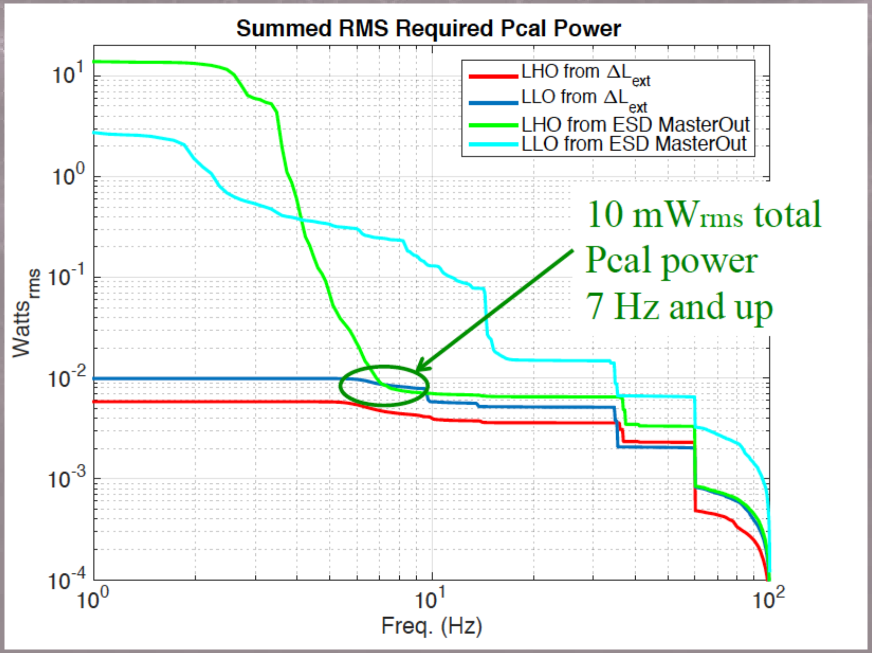
$$r_2 = \frac{V_{WSR}}{V_{GST}} = \frac{R_{BS}\rho_{WS}}{T_{BS}\rho_{GS}}$$

$$\rho_{WS} = \sqrt{r_1 r_2}\rho_{GS}$$

Future work. Actuation of ETM using photon radiation pressure: Pcal background noise level

Pcal background noise level is over one order of magnitude lower than the noise seen in ASD of ΔL_{ext}.

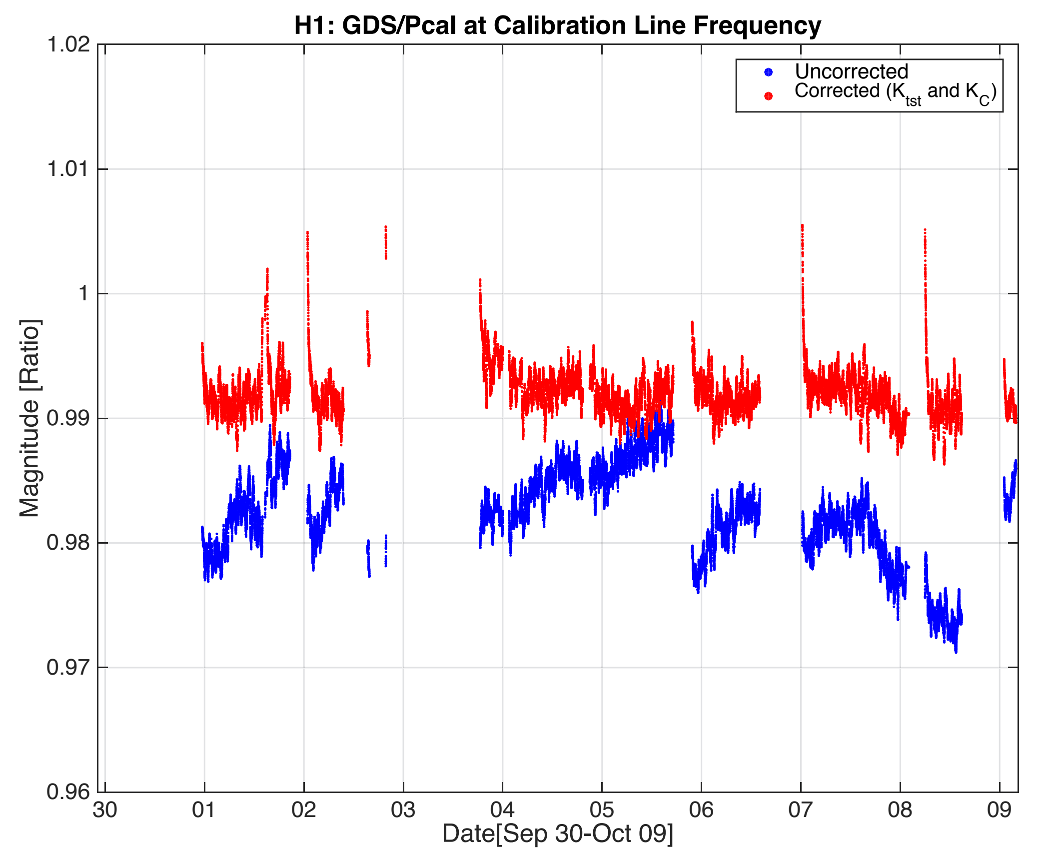




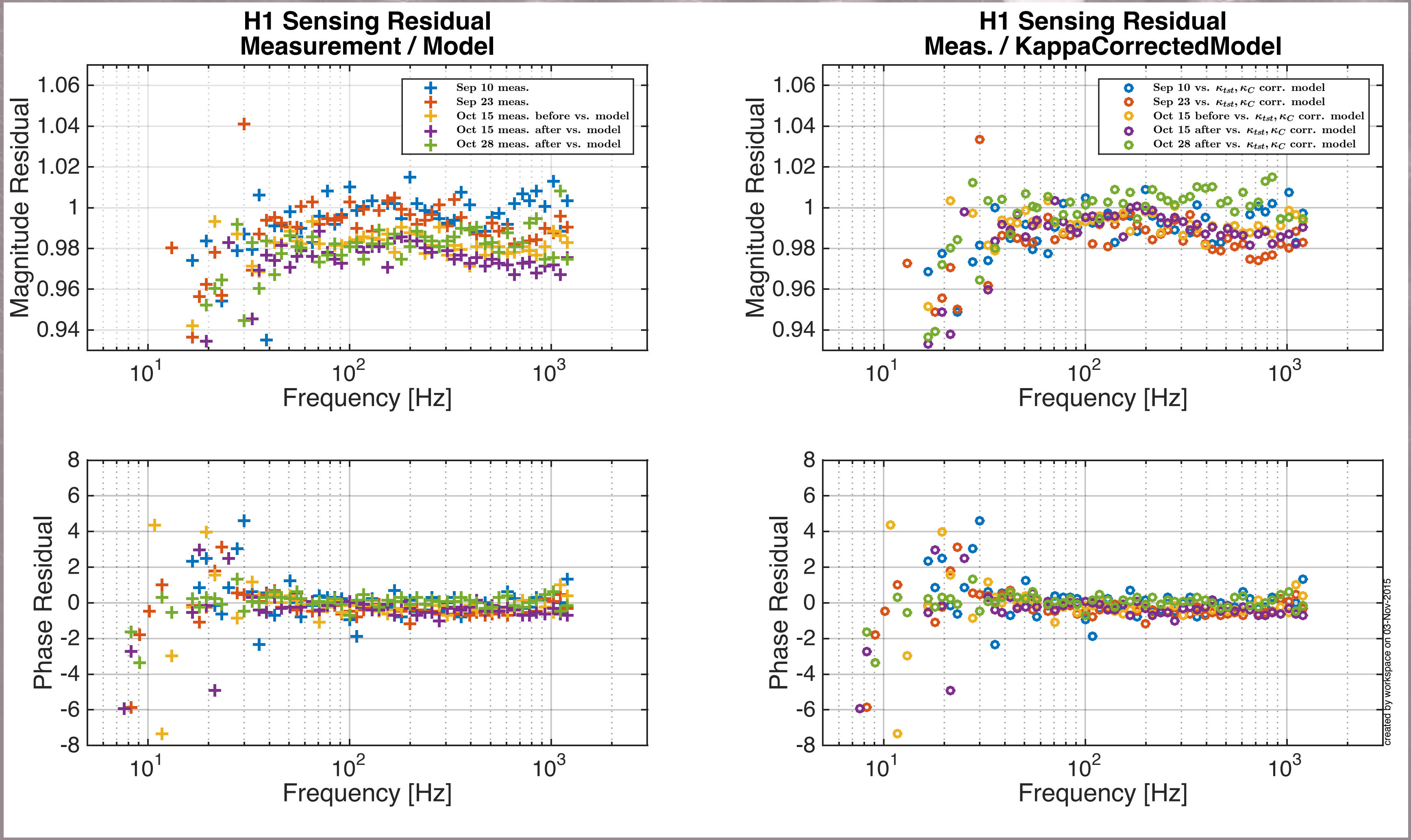
10 mW_{rms} total
Pcal power
7 Hz and up

Recent DARM correction factor trends

Single frequency check (line at 332 Hz) of the calibrated ΔL_{ext} time-series show that applying correction factors improve our calibration.

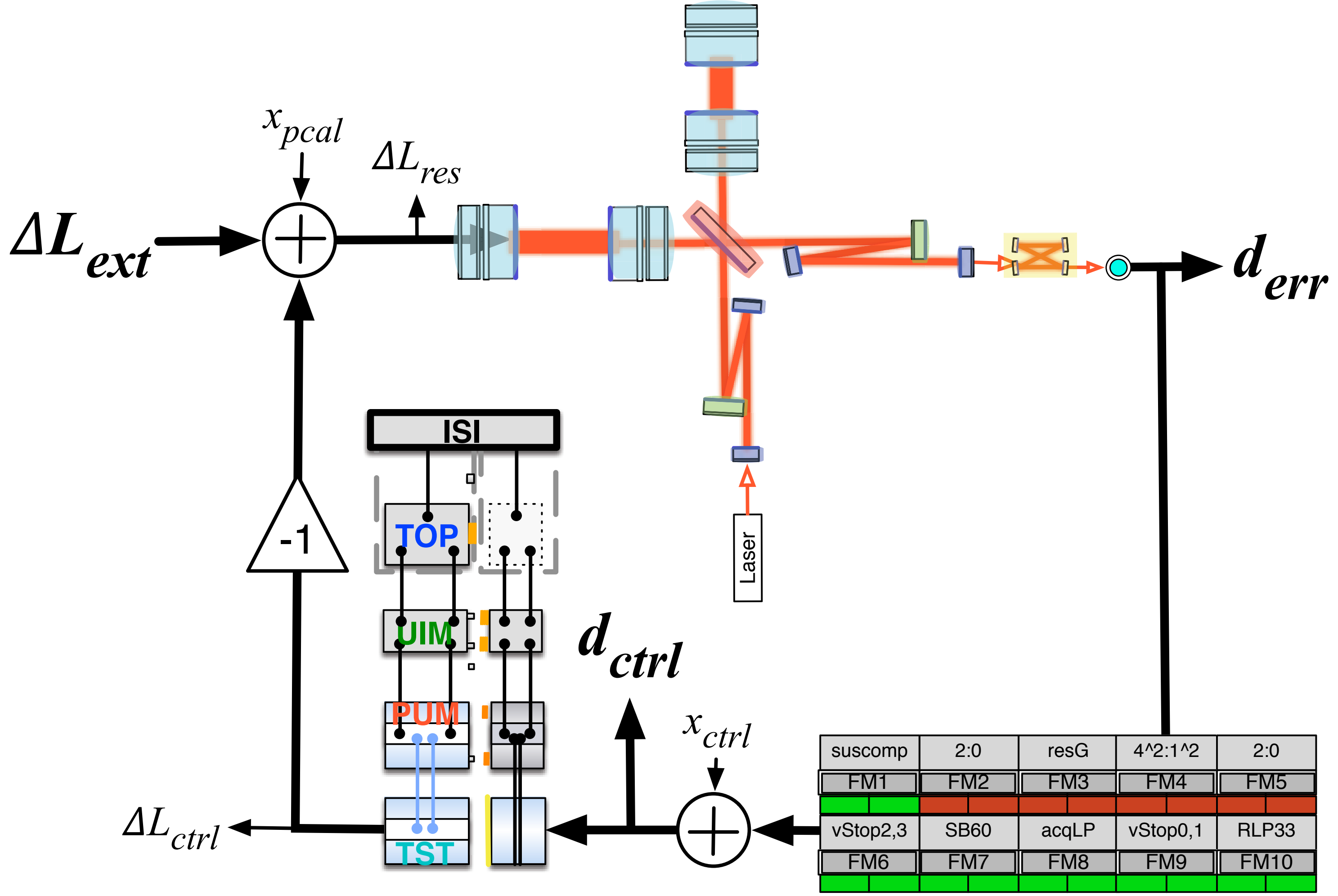


Comparison. DARM TF measurements



H1 Sensing Residual
Measurement / Model

H1 Sensing Residual
Meas. / KappaCorrectedModel



suscomp	2:0	resG	4^2:1^2	2:0
FM1	FM2	FM3	FM4	FM5
🟩	🟥	🟥	🟥	🟥
vStop2,3	SB60	acqLP	vStop0,1	RLP33
FM6	FM7	FM8	FM9	FM10
🟩	🟩	🟩	🟩	🟩

Taken from LIGO-G1500221

Differential arm closed-loop signal

Strain of the interferometer is obtained by dividing the variations in the difference in the arm lengths (DARM) caused by external sources, ΔL_{ext}, by the unperturbed arm length, L.

$$h(t) = \frac{\Delta L_{ext}(t)}{L}$$

Since the LIGO interferometer is kept at the state of destructive interference through the DARM feedback control loop, the value of ΔL_{ext} has to be reconstructed from the d_{err} and d_{ctrl} signals.

- DARM control loop -> mirrors do not move by ΔL_{ext}
- Accurate reconstruction of the ΔL_{ext} is the main goal of the calibration effort.