Actuation force of the ESD with charges and cage effects

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Overview

In this document we discuss the forces acting on the test mass from the ESD. It includes careful consideration of action of charges and ground surroundings. We write all the equations one-dimensional, investigating only in longitudinal movement of the test mass. Generalizing to 3D seems to be not necessary for this consideration of possible effects of charging and ground surroundings on the ESD strength.

1 Force acting on the test mass

Total electrostatic force on the test mass from ESD include several components.

$$F = A(V_b - V_s)^2 + B(V_b - V_s) + C\left(\frac{V_b + V_s}{2} - V_{ref}\right)^2 + D\left(\frac{V_b + V_s}{2} - V_{ref}\right) + E \quad (1)$$

where V_b is the electric potential of the bias electrode, V_s - of the signal electrode, V_{ref} - potential of the cage and other surroundings.

First term of this equation is the force acting on the TM due to the dipole attraction of the dielectric test mass to ESD (interaction of dielectric test mass and electrical field between two ESD electrodes). This force is proportional to squared field of the ESD, so it's proportional to the squared differential voltage.

The next force component due to electrical charge located on the test mass in the field of the ESD. It's proportional to the strength of the electrical field and depends on the amount of charge and it's distribution. Charges close to ESD will act the most.

Force component with C characterize the dipole attraction of the test mass to electrical field from ESD to grounded surroundings. This field is proportional to the difference between mean voltage of ESD electrodes and ground. Usually, the cage voltage V_{ref} should be written as zero.

Electrical charge interaction with this field (from ESD to cage) results in the forth term. If B mostly include acting of the charges close to the ESD, D include all the charges on the test mass due to this part of electric field is more homogeneous. One of this terms may dominate: B if we have a huge charge near by ESD or D if it is sitting on the other side of the test mass.

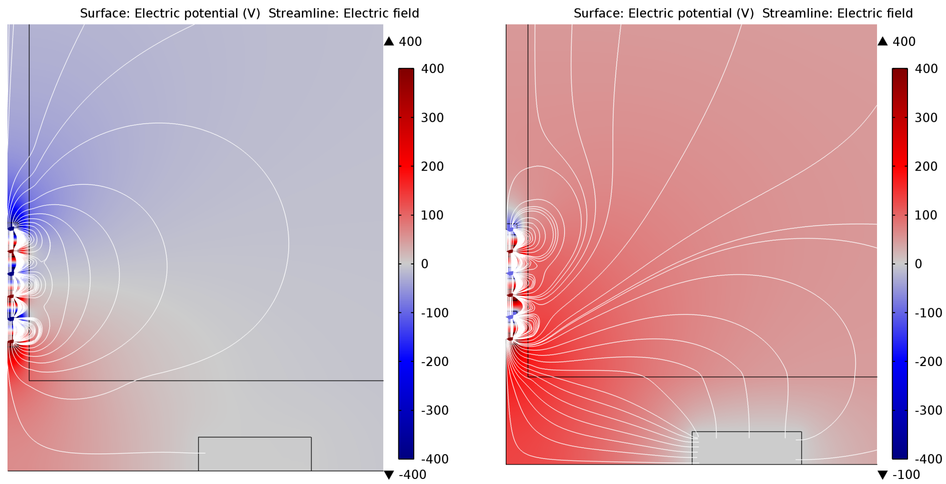

Figure 1: Result of raw modelling of voltage (color) and electrical field (lines) distribution if ESD voltage is: *(a)* $V_s = -V_b = -400V$; *(b)* $V_s = -100V, V_b = +400V$. ESD electrodes are on the left and grounded ring heater is at the bottom

The last term in equation (1) shows the interaction of charged test mass with surroundings due to the image charge. This term does not depend on bias or signal voltages but vary with electrical charge distribution and changes of geometry.

Figure 1 illustrate how the electric field change if we use $V_b + V_s = 0$, where we can ignore C and D, versus $V_b + V_s \neq 0$. In the last case we have a lot of lines of electric field goes through the test mass to the grounded surroundings so we have to use all the terms of equation.

Equation (2) may be written in more common way if we use $V_{ref} = 0$:

$$F = \alpha(V_b - V_s)^2 + \beta(V_b + V_s) + \beta_2(V_b - V_s) + \gamma(V_b + V_s)^2 + \delta \tag{2}$$

α characterize the dipole attraction of the TM to ESD. It depends mostly on the distance between them.

β characterize the charge amount and distribution at all the test mass

β_2 characterize the charge amount and distribution on the test mass near the ESD electrodes. β and β_2 depends on amount of charge and it's distribution on the test mass so they may vary significantly.

γ characterize the dipole interaction between uncharged dielectric TM and nonuniform electric field from ESD to grounded surroundings. γ changes with movement of grounded surroundings (cage) vs the test mass.

Equation (2) could be transformed to:

$$F = \alpha(V_b - V_s + \beta_2/2\alpha)^2 + \gamma(V_b + V_s + \beta/2\gamma)^2 + f_o,$$

where f_o is combination of $\alpha, \beta, \beta_2, \gamma$ and δ: $f_o = \delta - \frac{\beta_2^2}{4\alpha} - \frac{\beta^2}{4\gamma}$

We can use voltage terms:

$$V_{ch2} = \beta_2/2\alpha, V_{ch1} = \beta/2\alpha, \text{ so}$$

$$F = \alpha(V_b - V_s + V_{ch2})^2 + \gamma(V_b + V_s + V_{ch1})^2 + f_o, \tag{3}$$

Note, that there are two different charge voltages here. V_{ch2} in term with α characterize the charges near the ESD, and V_{ch1} in term with γ characterize the charge at the test mass in general.

Force components associated with action of electric fields on electric charge of the test mass could be written using β_s – charge coupling to signal voltage and β_b – charge coupling to the bias voltage. We can find $\beta_s = \beta - \beta_2$, $\beta_b = \beta + \beta_2$. Equation (2) then should be written as:

$$F = \alpha(V_b - V_s)^2 + \beta_b V_b + \beta_s V_s + \gamma(V_b + V_s)^2 + \delta, \tag{4}$$

Linear term of the force applied to test mass using signal voltage

$$V_s = V_{so} + V_{s1} sin(\omega t)$$

is:

$$F_\omega = [2\alpha(-V_b + V_{so}) + \beta_s + 2\gamma(V_b + V_{so})]V_{s1} sin(\omega t), \text{ or} \tag{5}$$

$$F_\omega = [-2V_b(\alpha - \gamma) + \beta_s + 2V_{so}(\alpha + \gamma)]V_{s1} sin(\omega t), \text{ or} \tag{6}$$

$$F_\omega = \left[-2\alpha V_b \left(1 - \frac{\gamma}{\alpha} - \frac{\beta_s}{2\alpha V_b}\right) + 2\alpha V_{so}\left(1 + \frac{\gamma}{\alpha}\right)\right] V_{s1} sin(\omega t) \tag{7}$$

Now LIGO use ESD with $V_{so} = 0$, so the linear term of force is

$$F_\omega = [2V_b(\gamma - \alpha) + \beta_s]V_{s1} sin(\omega t) \tag{8}$$

We see that in this case force acting on TM depend both on β and γ.

We think that α is a constant with good precision, and we had a lot of discussions and care about charges on test masses, nevertheless the effect of γ needs a careful consideration as well as charge review involving β and β_2.

Charge measurements

On charge measurements in LIGO observatories we apply the set of vias voltages while electrode voltage is sine: $V_s = V_o \cdot sin(\omega t)$. At different bias voltage the linear term of the test mass response is different, proportional to the linear force term (8). We use linear fit of response dependence on the bias voltage to find the bias voltage with zero response. This voltage named the effective charge bias voltage V_{EFF}. We can find the relation between V_{EFF} and others:

$$F_\omega = [2V_{EFF}(\gamma - \alpha) + \beta_s]V_{s1} sin(\omega t) = 0$$

$$2V_{EFF}(\gamma - \alpha) + \beta_s = 0$$

$$V_{EFF} = \frac{\beta_s}{2(\alpha - \gamma)}, \text{ or} \tag{9}$$

$$V_{EFF} = \frac{\beta_s}{2\alpha} \cdot \frac{1}{1 - \gamma/\alpha} = (V_{ch1} - V_{ch2}) \cdot \frac{1}{1 - \gamma/\alpha} \tag{10}$$

We see that V_{EFF} characterize the charges coupled to signal voltage. It does not include β_b and we measure the combination of β and β_2. It might be good due to it include both nearby charges and charges located far from ESD. But there are some part of charge near bias electrode which is ignored.

Using the effective charge bias voltage V_{EFF} we can write equation for linear term of the force acting to ESD (7):

$$F_\omega = \left[-2\alpha\left(1-\frac{\gamma}{\alpha}\right)(V_b - V_{EFF}) + 2\alpha V_{so}\left(1+\frac{\gamma}{\alpha}\right)\right] V_{s1} sin(\omega t) \tag{11}$$

While we use the signal voltage without offset, this equation should be simplified:

$$F_\omega = -2\alpha\left(1-\frac{\gamma}{\alpha}\right)(V_b - V_{EFF}) V_{s1} sin(\omega t) \tag{12}$$

2 Linearization

Previous studies of linearization [4, 5] use simplified version of equation (2) so that it does not include γ and use simplified model of charge interaction. Using equation (4) we can write:

$$\alpha(V_b^2 - 2V_bV_s + V_s^2) + \beta_b V_b + \beta_s V_s + \gamma(V_b^2 + 2V_bV_s + V_s^2) + \delta - F = 0 \text{ , so}$$

$$V_s^2 + V_s\left[-2V_b\frac{\alpha-\gamma}{\alpha+\gamma} + \frac{\beta_s}{\alpha+\gamma}\right] + V_b^2 + \frac{\beta_b V_b}{\alpha+\gamma} + \frac{\delta - F}{\alpha+\gamma} = 0$$

Solve this quadratic equation:

$$V_s = V_b\frac{\alpha-\gamma}{\alpha+\gamma} - \frac{\beta_s}{2(\alpha+\gamma)} \pm \frac{1}{2}\sqrt{\left(\frac{\beta_s - 2V_b(\alpha-\gamma)}{\alpha+\gamma}\right)^2 - 4\left(V_b^2 + \frac{\beta_b V_b}{\alpha+\gamma} + \frac{\delta-F}{\alpha+\gamma}\right)}$$

Denote $\xi = \frac{\gamma-\alpha}{\gamma+\alpha}$:

$$V_s = (V_b - V_{EFF})\xi \pm \sqrt{(V_b - V_{EFF})^2\xi^2 - V_b^2 + \frac{F - \beta_b V_b - \delta}{\alpha+\gamma}}$$

Usually we are interested in changing of force and don't care about additional constant force. While we don't change the bias voltage, we can write this equation using $F_o = \beta_b V_b - \delta$:

$$V_s = (V_b - V_{EFF})\xi \pm \sqrt{(V_b - V_{EFF})^2\xi^2 - V_b^2 + \frac{F - F_o}{\alpha+\gamma}} \tag{13}$$

This equation consist with Linearization equation from [5] if we use $\gamma \ll \alpha$. Unfortunately, results of LLO measurements [1] shows that $\frac{\gamma}{\alpha}$ is about 0.3. Preliminary results obtained in LHO are of the same order. So estimation of ξ is about 0.5.

3 Conclusion and Plans

We should worry about γ changes the actuation force. Probably we should measure γ with charge measurements and/or measure the actuation strength of ESD with CAL team.

We are planning to continue the charge measurements using optical levers and to check the correlation between charge and ESD actuation strength using the ESD calibration line.

It was founded in LLO measurements in Jan, 2015 that the charge force is proportional to $(V_b + V_s)$. It cause the using of simplified formula without β_2 in [2, 3]. We think that result of this measurements is: "β was greater than β_2", what means that the charge existed on the test mass at that time was located far from the ESD. In other measurements with different charge location and distribution we should use both β and β_2.

In further investigation we should take into consideration that charges on different sides of the test mass act in different way, according the terms 2 and 3 in equation (2)

Probably, we will find that γ changes could be a trouble. In this case one of the possible decisions is changing the bias voltage with the signal voltage: $V_b = -V_s$. We can do it while the ESD is using only for longitudinal actuation. Case of angular correction is not a subject of our investigation, but the possible solution is probably using the bias voltage equal to mean voltage of four quadrants $V_b = -0.25(V_{UL} + V_{UR} + V_{LL} + V_{LR}$. Using this changing bias we minimize the terms 2 and 4 in equation (2). The linear term of actuation force so does not depend on γ:

$$F_\omega = [-2\alpha V_s + \beta_2] 2\Delta V_s \tag{14}$$

Charge measurement procedure should be optimized for this kind of operation to measure the β_2.

References

[1] D. Martynov, PhD Thesys

[2] R.Weiss, LIGO document **T1400647**

[3] S. Aston et al, LIGO document **G1500264**

[4] Jeff Kissel, LIGO document **T1400321**

[5] Joseph Betzwiesser, LIGO document **T1400490**

Addition (Gamma measurements)

The measurements of relationship between γ and α was described in [1] and [2] using the simplified formula for force acting on the test mass. Show that using the equation (2) gives us the similar results. The same DC voltage was applied to bias and to electrode: $V_{so} = V_b = V_{DC}$. According the equation (6), linear term of force is:

$$F_\omega = [4\gamma V_{DC} + \beta_s] V_{s1} sin(\omega t) \tag{15}$$

Changing of the F_ω with changing of the DC voltage gives us γ. For simplest case using of $V_{DC} = V_o$ and $V_{DC} = -V_o$ we have:

$$\left.\frac{F_\omega}{V_{s1} sin(\omega t)}\right|_{\substack{V_b=V_o,\\ V_{so}=V_o}} - \left.\frac{F_{\omega 2}}{V_{s1} sin(\omega t)}\right|_{\substack{V_b=-V_o,\\ V_{so}=-V_o}} = 8\gamma V_{DC} \tag{16}$$

Using $V_{so} = -V_b$, $V_b = \pm V_{DC}$, we can find the same equation for α:

$$\left.\frac{F_\omega}{V_{s1} sin(\omega t)}\right|_{\substack{V_b=V_o,\\ V_{so}=-V_o}} - \left.\frac{F_{\omega 2}}{V_{s1} sin(\omega t)}\right|_{\substack{V_b=-V_o,\\ V_{so}=V_o}} = 8\alpha V_{DC} \tag{17}$$

So we can definitely find relation between γ and α:

$$\frac{\gamma}{\alpha} = \frac{\left.\frac{F_\omega}{V_{s1} sin(\omega t)}\right|_{\substack{V_b=V_o,\\ V_{so}=V_o}} - \left.\frac{F_{\omega 2}}{V_{s1} sin(\omega t)}\right|_{\substack{V_b=-V_o,\\ V_{so}=-V_o}}}{\left.\frac{F_\omega}{V_{s1} sin(\omega t)}\right|_{\substack{V_b=V_o,\\ V_{so}=-V_o}} - \left.\frac{F_{\omega 2}}{V_{s1} sin(\omega t)}\right|_{\substack{V_b=-V_o,\\ V_{so}=V_o}}} \tag{18}$$

If we want to make the measurement of γ with the conventional charge measurements, we can use couple of points measured on charge measurements with $V_{so} = 0, V_b = \pm V_o$. At this measurements we have linear term of force:

$$F_\omega = [\mp 2V_o(\alpha - \gamma) + \beta_s] V_{s1} sin(\omega t) \tag{19}$$

For γ measurements we will apply the signal voltage $V_{so} = V_o, V_b = 0$. So the linear term of acting force will be:

$$F_\omega = [2V_o(\alpha + \gamma) + \beta_s] V_{s1} sin(\omega t) \tag{20}$$

The relation between γ and α can be found as:

$$\frac{F_\omega|_{\substack{V_b=0,\\ V_{so}=V_o}} - F_\omega|_{\substack{V_b=-V_o,\\ V_{so}=0}}}{F_\omega|_{\substack{V_b=0,\\ V_{so}=V_o}} - F_\omega|_{\substack{V_b=V_o,\\ V_{so}=0}}} = \frac{(2V_o(\alpha+\gamma)+\beta) - (2V_o(\alpha-\gamma)+\beta)}{(2V_o(\alpha+\gamma)+\beta) - (-2V_o(\alpha-\gamma)+\beta)} = \frac{\gamma}{\alpha} \tag{21}$$