LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY
- LIGO -
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Tracking temporal variations in the DARM calibration parameters

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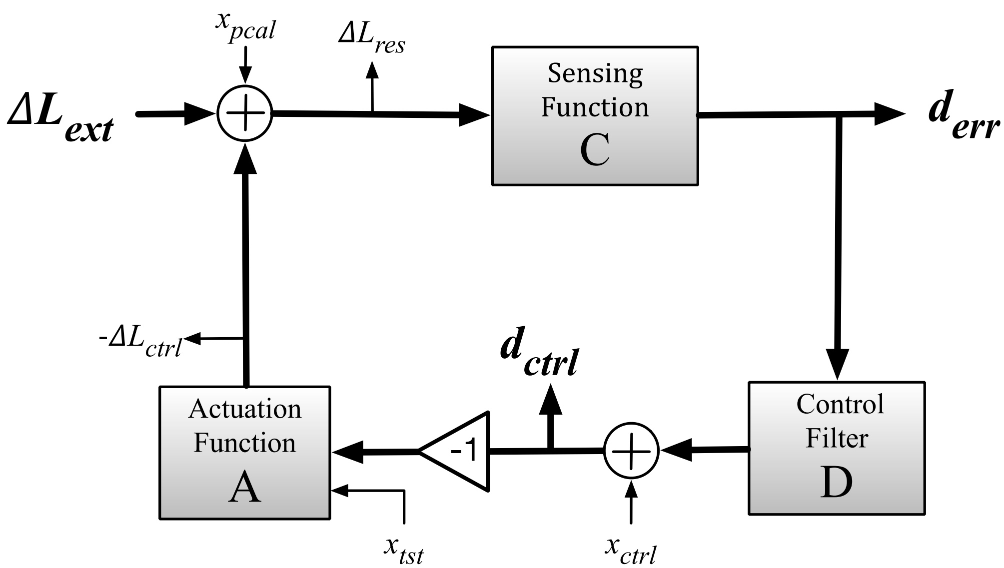

Figure 1: Simplified schematic of the DARM control loop.

1 Introduction

Data from recent engineering runs showed that the DARM open loop gain, G, is changes over time due to changes in the loop actuation and sensing functions. In this document we describe the implementation of calibration lines for the LIGO interferometers and their use to calculate time-varying actuation and sensing function parameters. In Fig. 1, we include a simplified schematic of the DARM control loop for reference. Note that the "-1" has been located to reflect where it actually occurs in the loops implemented at LHO and LLO. Following earlier conventions, ΔL is defined as $A \times d_{ctrl}$; due to the embedded minus sign it appears as $-\Delta L_{ctrl}$ in the diagram.

2 Calibration lines

Six calibration lines are injected for calculation and tracking of calibration-related parameters. Their frequencies, target signal-to-noise ratios (SNRs), injection points, and purpose are detailed in Table 1.

[1] using 10-sec.-long FFTs

Ifo.	**#**	**Freq. (Hz)**	**Sym.**	**Type**	**Target SNR**[1]	**Purpose**
H1	1	35.9	f_{tst}	L3(ESD)	100	L2/L3 actuation
H1	2	36.7	f_{pcal}	Pcal	100	DARM control actuation
H1	3	37.3	f_{ctrl}	x_{ctrl}	100	DARM control actuation
H1	4	331.9	f_{pcal2}	Pcal	100	DARM sensing
H1	5	1083.7	f_{pcal3}	Pcal	13	Calibration check
H1	6	3001.3	f_{pcal4}	Pcal	1	Calibration check
L1	1	35.3	f_{tst}	L3(ESD)	100	L2/L3 actuation
L1	2	34.7	f_{pcal}	Pcal	100	DARM control actuation
L1	3	33.7	f_{ctrl}	x_{ctrl}	100	DARM control actuation
L1	4	331.3	f_{pcal2}	Pcal	100	DARM sensing
L1	5	1083.1	f_{pcal3}	Pcal	13	Calibration check
L1	6	3001.1	f_{pcal4}	Pcal	1	Calibration check

Table 1: Calibration lines for the H1 and L1 interferometers. Note that the frequency ordering of the first and third lines at LLO are swapped to reflect the error that was made during implementation.

3 Calculating time-varying parameters

We assume that there is a DARM loop model that was generated for a reference time, t_0, that includes the following parameters, valid at $t = t_0$:

- A_0^{tst}, the frequency-dependent complex actuation function for the *L3* (TST) ETM actuation stage
- A_0^{pu}, the combined frequency-dependent complex actuation function for the *L1* (UIM) and *L2* (PUM) ETM actuation stages
- C_0, the frequency-dependent complex sensing function
- D_0, the frequency-dependent complex digital filter transfer function

The DARM open loop transfer function at $t = t_0$ can thus be written as

$$G_0 = C_0 D_0 (A_0^{tst} + A_0^{pu}) \tag{1}$$

For $t > t_0$, we can write the actuation and sensing functions as

$$A(t) = \kappa_{tst}(t) A_0^{tst} + \kappa_{pu}(t) A_0^{pu} \tag{2a}$$

$$C(t) = \kappa_C(t) \underbrace{\frac{C_{res}}{1 + if/f_c(t)}}_{C_0 \text{ when } f_c = f_c(t_0)} \tag{2b}$$

where $\kappa_{tst}(t)$ and $\kappa_{pu}(t)$ are time-dependent correction factors for the *L3* and *L1+L2* actuation functions, $\kappa_C(t)$ is a sensing function correction factor, $f_c(t)$ is the time-dependent coupled-cavity pole frequency, C_{res} is the residual of the sensing function at $t = t_0$, C_0, when the frequency-dependent cc pole expression (including the cc pole frequency at the reference time) is explicitly removed.

$$C_{res} = C_0 \left(\frac{1}{1 + if/f_c(t)} \right)^{-1} \tag{3}$$

The time-dependent DARM open loop transfer function, $G(t)$, can thus be written as

$$G(t) = C(t)\ D_0\ A(t) = \frac{\kappa_C(t)}{1 + if/f_c(t)}\ C_{res}\ D_0 \left[\kappa_{tst}(t) A_0^{tst} + \kappa_{pu}(t) A_0^{pu}\right] \tag{4}$$

While we will describe how to calculate $\kappa_{pu}(t)$ in Sec. 3.2, we expect it not to change appreciably, i.e. $\kappa_{pu}(t) \simeq 1$.

3.1 Calculating the TST actuation correction factor

To calculate κ_{tst}, we use the x_{tst} excitation (line 1 in Table 1) and the Pcal excitation (line 2 in Table 1) by monitoring the line amplitudes in the `ifo:CAL-DARM_ERR_WHITEN_OUT_DQ` channel. We also use the line amplitude in the calibrated (in meters) Pcal photodetector signal, `ifo:CAL-PCALX_RX_PD_OUT_DQ` or `ifo:CAL-PCALY_RX_PD_OUT_DQ`[2] and the line amplitude in the x_{tst} excitation readback channel, `ifo:SUS-ETMY_L3_CAL_LINE_OUT_DQ`. In addition, we apply a small correction from the response function at the reference time, $t = t_0$. Thus,

$$\widetilde{d}_{err}(f_{tst}) = \frac{C(f_{tst})}{1 + G(f_{tst})} \cdot \kappa_{tst}\ A_0^{tst}(f_{tst})\ \widetilde{x}_{tst}(f_{tst}) \tag{5}$$

$$\widetilde{d}_{err}(f_{pcal}) = \frac{C(f_{pcal})}{1 + G(f_{pcal})} \cdot \widetilde{x}_{pcal}(f_{pcal}) \tag{6}$$

where the tildes denote Fourier transforms. Dividing Eq. 5 by Eq. 6 and rearranging terms, we have

$$\kappa_{tst} = \frac{1}{A_0^{tst}(f_{tst})}\ \frac{\widetilde{d}_{err}(f_{tst})}{\widetilde{x}_{tst}(f_{tst})} \left(\frac{\widetilde{d}_{err}(f_{pcal})}{\widetilde{x}_{pcal}(f_{pcal})}\right)^{-1} \frac{C(f_{pcal})}{1 + G(f_{pcal})} \left(\frac{C(f_{tst})}{1 + G(f_{tst})}\right)^{-1} \tag{7}$$

Now, we assume that the slope of $C/(1 + G)$ over the $\sim$1 Hz frequency span between the first and second calibration lines (see Table 1) doesn't change appreciably over time[3], that is,

$$\left.\frac{C(f_{pcal})}{1 + G(f_{pcal})} \left(\frac{C(f_{tst})}{1 + G(f_{tst})}\right)^{-1}\right|_{t>t_0} \approx \underbrace{\frac{C_0(f_{pcal})}{1 + G_0(f_{pcal})} \left(\frac{C_0(f_{tst})}{1 + G_0(f_{tst})}\right)^{-1}}_{at\ \ t=t_0} \tag{8}$$

Substituting the right side of Eq. 8 into Eq. 7 gives

$$\kappa_{tst} = \frac{1}{A_0^{tst}(f_{tst})}\ \frac{\widetilde{d}_{err}(f_{tst})}{\widetilde{x}_{tst}(f_{tst})} \left(\frac{\widetilde{d}_{err}(f_{pcal})}{\widetilde{x}_{pcal}(f_{pcal})}\right)^{-1} \frac{C_0(f_{pcal})}{1 + G_0(f_{pcal})} \left(\frac{C_0(f_{tst})}{1 + G_0(f_{tst})}\right)^{-1} \tag{9}$$

Note that κ_{tst} is, in general, a complex quantity. However, if the slope of $C/(1+G)$ over the $\sim$1 Hz frequency span between the first and second calibration lines (see Table 1) doesn't change appreciably over time it will be real. The imaginary component of κ_{tst} differing from zero is an indication of deviations from constant slope.

3.2 Calculating the combined PUM and UIM actuation correction factor

To calculate κ_{pu} we use the third calibration line, at frequency f_{ctrl}, and the second calibration line, at frequency f_{pcal}. The complex amplitudes of the DARM loop signals at f_{ctrl} are related by

$$\widetilde{d}_{err}(f_{ctrl}) = -\frac{C(f_{ctrl})}{1 + G(f_{ctrl})} \cdot A(f_{ctrl})\ \widetilde{x}_{ctrl}(f_{ctrl}) \tag{10}$$

[2] Note that the calibrated Pcal `TX_PD` channels could be used instead of the `RX_PD` channels.

[3] Modeling indicates that variations in the response function as large as 20% induce fractional changes in the ratio of the response function at the two calibration line frequencies on the order of 1×10^{-4} to 1×10^{-5}.

Using Eq. 6 and Eq. 10 and assuming that the slope of $C/(1+G)$ doesn't change with time as we did in Sec. 3.1 we can solve for $A(f_{ctrl})$ as

$$A(f_{ctrl}) = -\frac{\widetilde{d}_{err}(f_{ctrl})}{\widetilde{x}_{ctrl}(f_{ctrl})}\left(\frac{\widetilde{d}_{err}(f_{pcal})}{\widetilde{x}_{pcal}(f_{pcal})}\right)^{-1}\frac{C_0(f_{pcal})}{1+G_0(f_{pcal})}\left(\frac{C_0(f_{ctrl})}{1+G_0(f_{ctrl})}\right)^{-1} \tag{11}$$

Solving Eq. 2a for κ_{pu} yields

$$\kappa_{pu} = \frac{1}{A_0^{pu}(f_{ctrl})}\left[A(f_{ctrl}) - \kappa_{tst}A_0^{tst}(f_{ctrl})\right] \tag{12}$$

where $A_(f_{ctrl})$ is given by Eq. 11.

3.3 Calculating an overall actuation scaling factor

For comparison with earlier methods, we can calculate an overall actuation scaling factor, κ_A, at f_{ctrl} by defining

$$A(f_{ctrl}) = \kappa_A A_0(f_{ctrl}) \tag{13a}$$

$$A_0(f_{ctrl}) = A_0^{tst}(f_{ctrl}) + A_0^{pu}(f_{ctrl}) \tag{13b}$$

then solving for κ_A as

$$\kappa_A = \frac{A(f_{ctrl})}{A_0^{tst}(f_{ctrl}) + A_0^{pu}(f_{ctrl})} \tag{14}$$

Note that κ_A is only strictly valid at $f = f_{ctrl}$.

3.4 Calculating the cc pole frequency and the sensing correction factor

Once we have calculated κ_{tst} as described in Sec. 3.1, above, f_c and κ_C can be calculated using the Pcal line at f_{pcal2}, near the cc pole frequency, line 4 in Table 1.[4]

Starting with Eq. 6 evaluated at $f = f_{pcal2}$, substituting Eq. 4 for G, then rearranging terms, we have

$$\frac{C}{1+if_{pcal2}/f_c} = \frac{1}{C_{res}(f_{pcal2})}\left(\frac{\widetilde{x}_{pcal}(f_{pcal2})}{\widetilde{d}_{err}(f_{pcal2})} - D_0(f_{pcal2})\left[\kappa_{tst}A_0^{tst}(f_{pcal2}) + \kappa_{pu}A_0^{pu}(f_{pcal2})\right]\right)^{-1} \equiv S \tag{15}$$

Equating the real and imaginary parts of both sides of Eq. 15 we obtain

$$\kappa_C = \frac{|S|^2}{\Re(S)} \tag{16}$$

$$f_c = -\frac{\Re(S)}{\Im(S)}f_{pcal2} \tag{17}$$

[4]A notch may be implemented in D_0 for this line so that the DARM loop doesn't consume ESD drive range acting on this line. If implemented, G vanishes in the denominator of the first terms in Eq. 6 when evaluated at $f = f_{pcal2}$, which may improve accuracy or simplify calculations.

4 Implementation for the ER8 and O1 runs

Here we will describe the utilization of EPICS records that, together with interferometer data, are used to calculate the time-varying parameters derived in Sec. **??**. The relevant equations from Sec. **??** are translated to include the EPICS records that are pre-calculated from the model.

In the following sections, we will recast the equations for κ_{tst}, κ_{pu}, κ_A, κ_C, and f_c by replacing factors that are calculated at the reference time, $t = t_0$, with EPICS record abbreviated constants. To simplify writing the equations in terms of the EPICS records, in this section we define simply-named constants for the EPICS records as shown in Table 2. Note that all of the EPICS records are either the real or the imaginary components of complex factors. When used in this document, EPICS constants that appear without `_R` or `_I` are the complex constant formed by adding the real and imaginary components, i.e. $\texttt{EP} = \texttt{EP\_R} + i\,\texttt{EP\_I}$.

4.1 Equations for time-varying factors in terms of EPICS records

For κ_{tst} we can rewrite Eq. 9 as

$$\kappa_{tst} = \frac{\widetilde{d}_{err}(f_{tst})}{\widetilde{x}_{tst}(f_{tst})} \left(\frac{\widetilde{d}_{err}(f_{pcal})}{\widetilde{x}_{pcal}(f_{pcal})} \right)^{-1} \times \quad \texttt{EP1} \tag{18}$$

where

$$\texttt{EP1} = \frac{1}{A_0^{tst}(f_{tst})} \cdot \frac{C_0(f_{pcal})}{1 + G_0(f_{pcal})} \left(\frac{C_0(f_{tst})}{1 + G_0(f_{tst})} \right)^{-1} \tag{19}$$

For κ_{pu}, we first write Eq. 11 as

$$A(f_{ctrl}) = -\frac{\widetilde{d}_{err}(f_{ctrl})}{\widetilde{x}_{ctrl}(f_{ctrl})} \left(\frac{\widetilde{d}_{err}(f_{pcal})}{\widetilde{x}_{pcal}(f_{pcal})} \right)^{-1} \times \quad \texttt{EP2} \tag{20}$$

where

$$\texttt{EP2} = \frac{C_0(f_{pcal})}{1 + G_0(f_{pcal})} \left(\frac{C_0(f_{ctrl})}{1 + G_0(f_{ctrl})} \right)^{-1} \tag{21}$$

Then, we write Eq. 12 as

$$\kappa_{pu} = \texttt{EP3} \times [A(f_{ctrl}) - \kappa_{tst} \cdot \texttt{EP4}] \tag{22}$$

where

$$\texttt{EP3} = \frac{1}{A_0^{pu}(f_{ctrl})} \tag{23}$$

and

$$\texttt{EP4} = A_0^{tst}(f_{ctrl}) \tag{24}$$

For κ_A we rewrite Eq. 14 as

$$\kappa_A = \frac{A(f_{ctrl})}{\texttt{EP4} + \texttt{EP5}} \tag{25}$$

Const.	**EPICS record name**
EP1_R	ifo:CAL-TDEP_REF_INVA_CLGRATIO_TST_REAL
EP1_I	ifo:CAL-TDEP_REF_INVA_CLGRATIO_TST_IMAG
EP2_R	ifo:CAL-CS_TDEP_REF_CLGRATIO_CTRL_REAL
EP2_I	ifo:CAL-CS_TDEP_REF_CLGRATIO_CTRL_IMAG
EP3_R	ifo:CAL-CS_TDEP_DARM_LINE1_REF_A_USUM_INV_REAL
EP3_I	ifo:CAL-CS_TDEP_DARM_LINE1_REF_A_USUM_INV_IMAG
EP4_R	ifo:CAL-CS_TDEP_DARM_LINE1_REF_A_TST_REAL
EP4_I	ifo:CAL-CS_TDEP_DARM_LINE1_REF_A_TST_IMAG
EP5_R	ifo:CAL-CS_TDEP_DARM_LINE1_REF_A_USUM_REAL
EP5_I	ifo:CAL-CS_TDEP_DARM_LINE1_REF_A_USUM_IMAG
EP6_R	ifo:CAL-CS_TDEP_PCALY_LINE2_REF_C_NOCAVPOLE_REAL
EP6_I	ifo:CAL-CS_TDEP_PCALY_LINE2_REF_C_NOCAVPOLE_IMAG
EP7_R	ifo:CAL-CS_TDEP_PCALY_LINE2_REF_D_REAL
EP7_I	ifo:CAL-CS_TDEP_PCALY_LINE2_REF_D_IMAG
EP8_R	ifo:CAL-CS_TDEP_PCALY_LINE2_REF_A_TST_REAL
EP8_I	ifo:CAL-CS_TDEP_PCALY_LINE2_REF_A_TST_IMAG
EP9_R	ifo:CAL-CS_TDEP_PCALY_LINE2_REF_A_USUM_REAL
EP9_I	ifo:CAL-CS_TDEP_PCALY_LINE2_REF_A_USUM_IMAG

Table 2: Definition of constants for EPICS records used in the equations in Sec.4.1. See [4] for a list of the DARM parameter EPICS records.

where $A(f_{ctrl})$ and `EP4` are given by Eqs. 20 and 24, respectively, and `EP5` is given by

$$\texttt{EP5} = A_0^{pu}(f_{ctrl}) \tag{26}$$

For κ_C and f_c, we rewrite Eq. 15 as

$$S = \frac{1}{\texttt{EP6}} \left(\frac{\widetilde{x}_{pcal}(f_{pcal2})}{\widetilde{d}_{err}(f_{pcal2})} - \texttt{EP7}\left[\kappa_{tst} \cdot \texttt{EP8} + \kappa_{pu} \cdot \texttt{EP9}\right] \right)^{-1} \tag{27}$$

where

$$\texttt{EP6} = C_{res}(f_{pcal2}) \tag{28}$$

$$\texttt{EP7} = D_0(f_{pcal2}) \tag{29}$$

$$\texttt{EP8} = A_0^{tst}(f_{pcal2}) \tag{30}$$

$$\texttt{EP9} = A_0^{pu}(f_{pcal2}) \tag{31}$$

and κ_C and f_c are given by Eqs. 16 and 17 and f_{pcal2} is given in Table 1. Note that when deriving $\widetilde{x}_{pcal}$ from the Pcal Rx PD signals (e.g. `ifo:CAL-PCALX_RX_PD_OUT_DQ`), one must compensate for whitening and both digital and analog anti-aliasing filters as detailed in LIGO-G1501014.

5 Calculation of $h(t)$

In LIGO, detector strain, $h(t)$, is the fractional variation in the LIGO differential arm length caused by external sources, defined as

$$h(t) \equiv \frac{\Delta L_{ext}(t)}{L} \tag{32}$$

where $\Delta L_{ext}(t)$ is the change in the differential arm length, $L_X - L_Y$, and L is the mean length of the interferometer arms, $(L_X + L_Y)/2$. The calibrated time series $\Delta L_{ext}(t)$ can be calculated from the d_{err} and d_{ctrl} signals (refer to Fig. 1) as

$$\Delta L_{ext}(t) = \Delta L_{res} + \Delta L_{ctrl} = \frac{d_{err}(t)}{C(t)} + d_{ctrl}(t)A(t) \tag{33}$$

$$= \frac{1 + if/f_c(t)}{\kappa_C(t)C_{res}} d_{err}(t) + \left(\kappa_{tst}(t)A_0^{tst} + \kappa_{pu}(t)A_0^{pu}\right) d_{ctrl}(t) \tag{34}$$

where κ_{tst}, κ_{pu}, κ_C and f_c, are the time-varying parameters described in Sec. 3.

References

[1] LIGO-T1400256: Time Domain Calibration in Advanced LIGO.

[2] LIGO-T1500121: aLIGO Front-end Optical Gain Compensation, or $\gamma(t)$.

[3] LIGO-T1500422: Compensating for time variations in DARM actuation.

[4] LHO aLog entry #20361 by J. Kissel, Aug. 9, 2015.

[5] LIGO-T1501014: Pcal signal chain topology.